Performance of Distillation Protocols in the presence of Noise

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Abstract

In this report, the performance of three entanglement distillation protocols (BBPSSW, DEJMPS, and EPL) is analyzed in the presence of environmental interactions (depolarization) and gate noise. The protocols are applied to the Werner state to determine the increase in the pair fidelity with respect to the input pair and gate fidelities. Numerical simulations were performed using the NetSquid simulator, and the results showed that all protocols displayed an increase in pair fidelity for a gate fidelity above 0.95, with a trade-off between success fidelity and success probability. The report helps in providing a simple recipe for selecting the most helpful protocol based on specific requirements and hardware performance. The results emphasize practical limitations of entanglement distillation and despite the simplicity of the error model we employ, we are able to obtain a good insight of the behaviour of all three protocol under real circumstances.

I. INTRODUCTION

Quantum information theory has opened up many paths for us to securely send and receive messages, to perform vast computations with merely a single call and to correct errors. And, almost all of these advanced techniques require 'entanglement'. Thus entanglement is crucial and therefore the need arises to study how it can be generated over large distances and preserved. Entanglement distillation is the process in which a 'bipartite' system attempts to convert some large number of copies of a known pure state into as many copies of a Bell state as possible using local operations and classical communication[1]. The main aim of this paper is to study the performance of different distillation protocols in the presence of noise using a quantum network simulator. Through a depolarizing channel, which yields the initial state with probability p and the maximally mixed state with probability $1-p$, it is possible to represent the pair initialization noise and imperfect gates. Then, the noisy EPR state shared between two parties is in the form of a Werner state:

$$\rho_{AB}(p) = p|\phi_{00}\rangle\langle\phi_{00}| + \frac{1-p}{4}\mathbb{1} \tag{1}$$

where, $|\phi_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and p is the parameter of the depolarizing channel. The fidelity of such state with respect to $|\phi_{00}\rangle$ is given by $F = \frac{(1+3p)}{4}$.

Entanglement Distillation enables us to obtain high-fidelity EPR pairs. Ideally, the entanglement distillation schemes work to obtain M pairs with higher entanglement and higher degree of purity from N noisy EPR-pairs ($M < N$). We are going to focus on three 2-to-1 entanglement distillation protocols: (1)Extreme Photon Loss (EPL) Protocol, (2) BBPSSW Protocol and the (3) DEJMPS Protocol. These protocols will be explained in detail in the further subsections.

A. EPL

The Extreme Photon Loss Protocol (EPL), as its name implies, was created [2] to support entanglement generation systems that utilize a single photon detection setup. The circuit executing this protocol is depicted in Figure 1. The distillation procedure is successful if the measurement result is 11. Applying this protocol to two copies of an initial state which is of the form of 1, gives the success fidelity $F_{succ} = 1$ and the success probability $p_{succ} = p^2/2$.

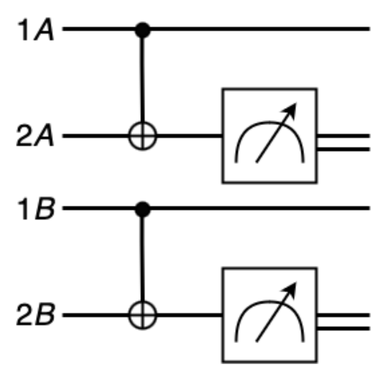

Figure 1. Implementation of EPL Protocol. The A indices refer to qubits of Alice and the B indices refer to qubits of Bob.

B. BBPSSW

The BBPSSW protocol was introduced by Bennet *et. al.* [2] in 1996. The BBPSSW protocol allows for the production of maximally entangled states with a desired level of accuracy from several copies of Werner states, as long as the initial fidelity F is greater than 1/2. The Figure 2 demonstrates the protocol and the success outcome is achieved when the measurements results are identical.

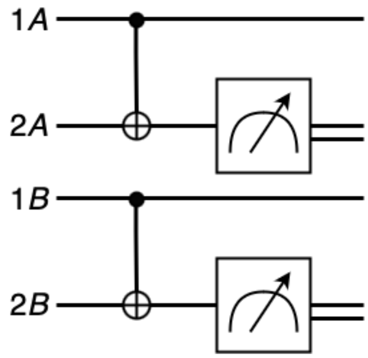

Figure 2. Implementation of BBPSSW Protocol. The A indices refer to qubits of Alice and the B indices refer to qubits of Bob.

The success Fidelity for the BBBPSSW protocol is given by:

$$F_{out} = \frac{F^2 + \left(\frac{1-F}{3}\right)^2}{p_{succ}} \tag{2}$$

and,

$$p_{succ} = F^2 + 2F\frac{1-F}{3} + 5\left(\frac{1-F}{3}\right)^2 \tag{3}$$

Where we can note that $F_{out} > F$ for $F > 1/2$.

C. DEJMPS

The DEJMPS protocol was designed by Deutsch *et al* [3] in 1996. Its implementation is described in Figure 1 and the success outcome is when the measurement results are the same. The key distinction between the BBPSSW and the DEJMPS is that the latter is capable of working on Bell basis diagonal states and exhibits greater efficiency

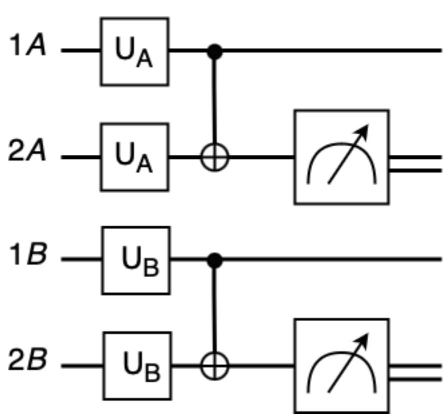

Figure 3. DEJMPS Protocol where $U_A = R_X(\pi/2)$ and $U_B = R_X(-\pi/2)$.

II. RESEARCH QUESTIONS

Real-world devices always have imperfections and non-ideal behaviours that cause various types of errors, leading to entangled state noise and hence a decrease in the fidelity of shared EPR pairs. The distillation protocols like EPL, BBPSW and DEJMPS aim to increase the fidelity. However, there are also experimental errors related to the performance of the gates used in the protocols that are not accounted for in determining the theoretical F_{succ} and p_{succ}. This gate noise must also be considered in order to study and better understand how these protocols work in reality. In our simulations we fixed the gate fidelity to be the same for all gates. However, we are aware that this is a big simplification, since 1-qubit and 2-qubit gates usually have very different errors.

In particular, it is estimated that single qubit gates have error rates smaller than 0.1 % , Error rates for two-qubit CNOT gates average 1.8% (they range from 0.5-6.5%) and readout errors on single qubits are approximately 4.8% [4]. For this reason, we will consider gate fidelities between 1 (ideal case) and 0.95 (worst possible scenario).

Dealing with depolarizing errors can become extremely difficult, especially when we consider large distances. Nevertheless, this effect can be mitigated by the use of quantum repeaters. In this project, we will not worry about the decoherence time nor the success probability of entangling two nodes through a network or quantum repeaters but we will just assume that the depolarizing error is not so large, in order to successfully perform entanglement distillation. For all three protocols we will consider three cases, with noise fidelity F=1 (ideal case), 0.99 and 0.9 (worst scenario but still reasonable).

In this project, we try to answer the following questions:

- For achieving the probability success fidelity of fault tolerant limit 0.95, what should be the corresponding values of gate fidelity and depolarising parameter (F) for all three protocols?
- Which protocol among DEJMPS and BBPSSW works the best for the chosen regime of gate fidelity and depolarising parameter?
- Assuming that the initial entangled fidelity is 0.9, what is the minimum gate fidelity that enables entanglement distillation using the EPL protocol?

Answering the above research questions would allow us to analyze the variation in the final entangled state fidelity under the influence of different values of entangled state noise and gate noise. This analysis would provide us with a general understanding of the impact of noise on the distillation protocols and enable us to choose the best protocol based on the desired output pair fidelity, initial pair fidelity, and hardware performance.

III. RESULTS

In this section, an analysis of the EPL protocol and a comparison between DEJMPS and BBPSSW protocols will be performed with regards to the research question. The analysis will focus on the gate fidelity and depolarizing error fidelity within the ranges of Fg from 0.95 to 1 and Fepr from 0.9 to 1. The number of simulations, Nsim, chosen for each point is 1000.

A. Analysing EPL

The EPL protocol operates differently from the DEJMPS and BBPSSW protocols. The starting shared state in the EPL protocol has a phase difference, which is $\left(|01\rangle_{1A,1B} + e^{i\phi}|10\rangle_{1A,1B}\right)$. The objective of the EPL protocol is to eliminate the phase difference and create a high-fidelity EPR pair.

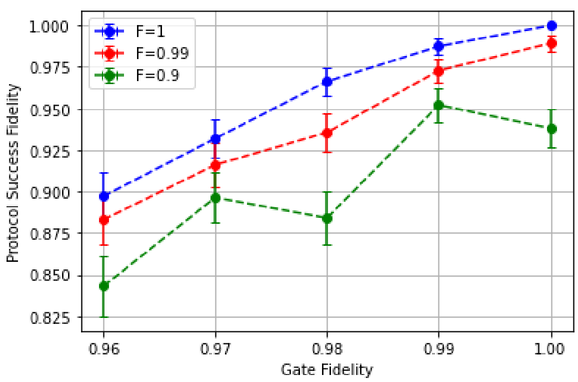

Figure 4. Overall fidelity of the EPL protocol against the gate fidelity for depolarizing fidelity F=0.9, 0.99 and 1.

The protocol success fidelity of a protocol is considered to be fault tolerant at 0.95 in real-world applications. When the depolarizing parameter (F) is equal to 1, the desired results can be obtained with a gate fidelity of more than 0.97. However, if F is equal to 0.99, the gate fidelity needs to be higher, more than 0.98. It's important to note that when F is equal to 0.9, it never satisfies the required gate fidelity, highlighting the importance of always maintaining F above 0.9.

As hardware performance (gate fidelity) improves, the protocol success fidelity also improves as expected. For very low depolarizing errors, the trend becomes more difficult to predict. In this case, for a depolarizing parameter of 0.9, even with good hardware performance, instability in results occurs.

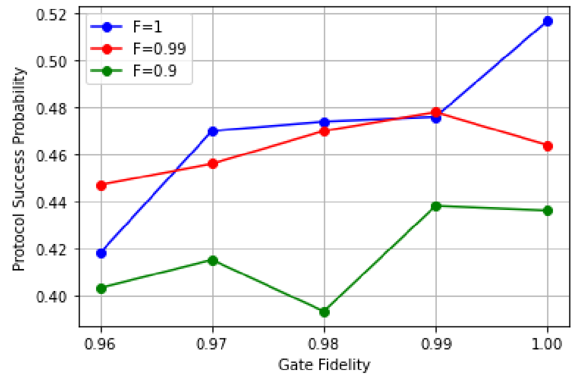

Figure 5. Overall success probability of the EPL protocol against gate fidelity for depolarizing fidelity F=0.9, 0.99 and 1.

The analysis of the plot reveals that the success probability is less than 0.5. The protocol is considered successful only if both the receiver's and sender's circuit measurements result in 1. This results in a success probability that is half as compared to DEJMPS and BBPSSW, where the protocol is considered successful when the measurement results from both circuits match.

Similarly, as explained in Figure 4, an increase in depolarizing error leads to unpredictable results. The improvement in gate fidelity does not consistently increase the success probability, emphasizing the significance of maintaining depolarizing error above 0.9.

B. DEJMPS and BBPSSW

Here we present the results of our simulations for both the DEJMPS and BBPSSW protocols.

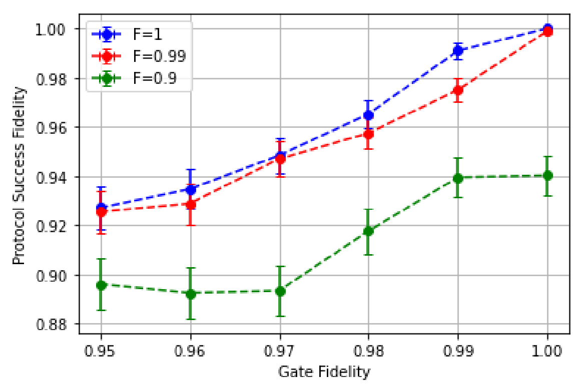

Figure 6. Overall fidelity of the BBPSSW protocol against gate fidelity for depolarizing fidelity F=0.9, 0.99 and 1.

Figure III B shows the average output fidelity after the BBPSSW suceeds. As expected, lower gate fidelity and depolarizing fidelity reduce the output fidelity. Interestingly, for the case F=0.9, gate fidelity does not seem to play an important role under the value 0.97, indicating that the error in this regime is essentially due to depolarization.

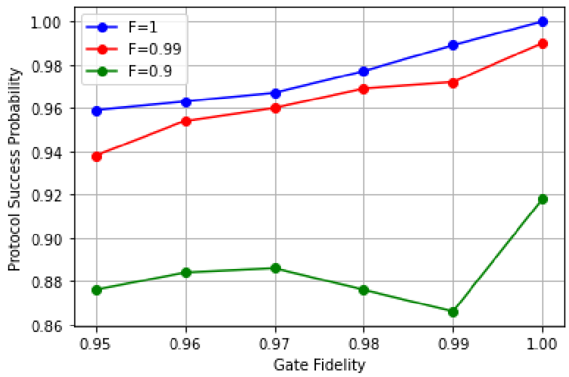

Figure 7. Overall success probability of the BBPSSW protocol against gate fidelity for depolarizing fidelity F=0.9, 0.99 and 1.

The success probability also behaves as expected for high values of the depolarizing fidelity. However, again for F=0.9 the results seem quite erratic and the dependence on gate fidelity vanishes.

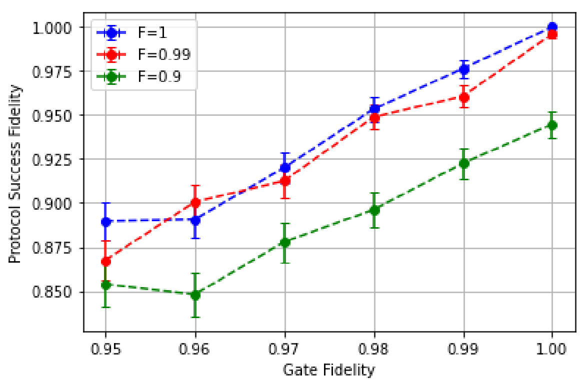

Figure 8. Overall fidelity of the DEJMPS protocol against gate fidelity for depolarizing fidelity F=0.9, 0.99 and 1.

Observing the plots, first we focus on the the overall success fidelity of the DEJMPS protocol as a function of the gate fidelities for multiple depolarization fidelities. This is portrayed in Figure III B. We observe rising trends of the overall protocol fidelity with increasing gate fidelity which is expected. There is an initial small decrease in the overall protocol fidelity when the gate fidelity increases from 0.95 to 0.96. This marginal decrease is however only observed for an imperfect depolarization case. This is an interesting observation.

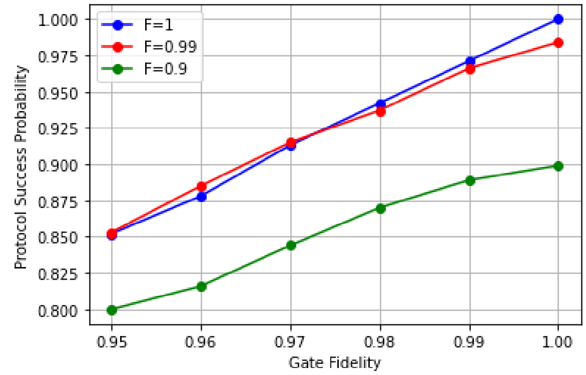

Figure 9. Overall success probability of the DEJMPS protocol against gate fidelity for depolarizing fidelity F=0.9, 0.99 and 1.

Next, we go to the plots of the overall protocol success probability vs. the gate fidelity for various depolarization fidelities. Figure III B depicts the following plot. Contrarily to the BBSPPW case, the decrease is very smooth and clearly linear with the gate fidelity. We also observe that for all three values of F, the BBPSSW yields both higher success fidelity and success probability, and thus it will be the preferred choice in this scenario.

IV. CONCLUSIONS

The trend observed in all the above three protocols shows that as gate fidelity improves, the probability success fidelity also increases. However, when the depolarizing parameter (F) is less than 0.9, the success probability becomes unreliable for any value of gate fidelity, highlighting the importance of keeping F above 0.9.

In order to reach the real-world fault-tolerant limit of 0.95 for success probability, all three protocols show that the gate fidelities need to be greater than 0.97 when F is equal to 1 and greater than 0.98 when F is equal to 0.99.

For the chosen regime of gate fidelities and depolarizing error, DEJMPS never outperforms BBPSSW. Hence, BBPSSW is recommended to be used in real-life scenarios for better performance. Nevertheless, more specific studies with a more realistic error model with different gate fidelities must be conducted in order to arrive at a definitive conclusion.

V. CITATIONS AND REFERENCES

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