IEC 61850 Response Time Analysis

$$R_i = \max_{q=0\ldots Q_i-1} R_i(q) \tag{1}$$

where

$$R_i(q) = J_i + w_i(q) - qT_i + C_i \tag{1a}$$

$$Q_i = \left\lceil \frac{t_i + J_i}{T_i} \right\rceil \tag{1b}$$

$$t_i^{n+1} = B_i + \sum_{\forall j \in hp(i) \cup ep(i)} \left\lceil \frac{t_i^n + J_j}{T_j} \right\rceil C_j \tag{1c}$$

and

$$w_i^{n+1}(q) = B_i + qC_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{w_i^n + J_j + \tau_{bit}}{T_j} \right\rceil C_j \tag{1d}$$

Message stream $\mathcal{T}_7$ (SV messages sent by SB1)

Calculation of Q_7: starting with C_7 as the initial value of t_7 and assuming $B_7 = 13.76\mu s$, whose value corresponds to the computation time of $\mathcal{T}_4$, we have:

$$t_7^1 = B_7 + \left\lceil \frac{t_7^0 + J_7}{T_7} \right\rceil C_7 \therefore t_7^1 = 13.76 + \left\lceil \frac{12.16 + 1}{208.33} \right\rceil 12.16 = 25.92$$

Iterating again:

$$t_7^2 = 13.76 + \left\lceil \frac{25.92 + 1}{208.33} \right\rceil 12.16 = 25.92$$

Therefore, 7-busy-period value converged in $25.92\mu s$. By applying Eq. 1b:

$$Q_7 = \left\lceil \frac{t_7 + J_7}{T_7} \right\rceil \therefore Q_7 = \left\lceil \frac{25.92 + 1}{208.33} \right\rceil = 1$$

The value of $Q_7 = 1$ indicates that the q value in Eq. 1 is equal to 0. Therefore, in order to obtain the worst-case response time for $\mathcal{T}_7$, it is enough to examine its first instance within the 7-busy period.

Calculation of R_7: Since $\mathcal{T}_7$ is the highest priority message stream, in Eq. 1d there is no element in $hp(i)$. Because $q = 0$, this equation can be simplified to $w_7^{n+1}(0) = B_7$. Therefore, $w_7(0)$ converges to $13.76\mu s$. Through Eq. 1a, $R_7(0) = 1 + 13.76 + 12.16 = 26.92\mu s$. We conclude that this worst-case response time is smaller than the deadline D_7 ($208.33\mu s$).

Message stream $\mathcal{T}_6$ (GOOSE messages sent by BP2)

Calculation of Q_6: starting with C_6 as the initial value of t_6:

$$t_6^1 = B_6 + \left\lceil \frac{t_6^0 + J_7}{T_7} \right\rceil C_7 + \left\lceil \frac{t_6^0 + J_6}{T_6} \right\rceil C_6 \quad \therefore$$

$$t_6^1 = 13.76 + \left\lceil \frac{13.76 + 1}{208.33} \right\rceil 12.16 + \left\lceil \frac{13.76 + 1}{31000} \right\rceil 13.76 = 39.68\mu s$$

Iterating again:

$$t_6^2 = 13.76 + \left\lceil \frac{39.68 + 1}{208.33} \right\rceil 12.16 + \left\lceil \frac{39.68 + 1}{31000} \right\rceil 13.76 = 39.68\mu s$$

This iteration converges with the value of 6-busy-period equal to $39.68\mu s$, then:

$$Q_6 = \left\lceil \frac{t_6 + J_6}{T_6} \right\rceil \therefore Q_6 = \left\lceil \frac{39.68 + 1}{31000} \right\rceil = 1$$

As with $\mathcal{T}_7$, this value of Q indicates that the q value in Eq. 1 is equal to 0 and it is enough to examine the first instance of $\mathcal{T}_6$ within the 6-busy period.

Calculation of R_6: starting with B_6 as the initial value of $w_6^0(0)$, this results in $w_6^0(0) = 13.76\mu s$. By applying Eq. 1d:

$$w_6^1(0) = 13.76 + \left\lceil \frac{13.76 + 1 + 0.01}{208.33} \right\rceil 12.16 = 25.92\mu s$$

$$w_6^2(0) = 13.76 + \left\lceil \frac{25.92 + 1 + 0.01}{208.33} \right\rceil 12.16 = 25.92\mu s$$

As $w_6(0)$ converges in $25.92\mu s$:

$$R_6(0) = 1 + 25.92 + 13.76 = 40.68\mu s;$$

This time value is smaller than D_6 ($3000\mu s$).

Message stream $\mathcal{T}_5$ (GOOSE messages sent by BP1)

Calculation of Q_5: starting with C_5 as the t_5 value, we have: $t_5^0 = 13.76\mu s$. Therefore:

$$t_5^1 = B_5 + \left\lceil \frac{t_5^0 + J_7}{T_7} \right\rceil C_7 + \left\lceil \frac{t_5^0 + J_6}{T_6} \right\rceil C_6 + \left\lceil \frac{t_5^0 + J_5}{T_5} \right\rceil C_5$$

As $J_6 = J_5$, $T_6 = T_5$ and $C_6 = C_5$, iterating:

$$t_5^1 = 13.76 + \left\lceil \frac{13.76 + 1}{208.33} \right\rceil 12.16 + 2 \times \left\lceil \frac{13.76 + 1}{31000} \right\rceil 13.76 = 53.44\mu s$$

$$t_5^2 = 13.76 + \left\lceil \frac{53.44 + 1}{208.33} \right\rceil 12.16 + 2 \times \left\lceil \frac{53,44 + 1}{31000} \right\rceil 13.76 = 53.44\mu s$$

The 5-busy-period converged with the value $53.44\mu s$ and with this result we have:

$$Q_5 = \left\lceil \frac{t_5 + J_5}{T_5} \right\rceil \therefore Q_5 = \left\lceil \frac{53.44 + 1}{31000} \right\rceil = 1$$

Consequently, it is enough to analyze the first instance of $\mathcal{T}_5$.

Calculation of R_5: Starting with $w_5^0(0) = 13.76\mu s$:

$$w_5^1(0) = 13.76 + \left\lceil \frac{13.76 + 1 + 0.01}{208.33} \right\rceil 12.16 + \left\lceil \frac{13.76 + 1 + 0.01}{31000} \right\rceil 13.76$$

As $w_5^1(0) = 39.68\mu$, a second iteration is required:

$$w_5^2(0) = 13.76 + \left\lceil \frac{39.68 + 1 + 0.01}{208.33} \right\rceil 12.16 + \left\lceil \frac{39.68 + 1 + 0.01}{31000} \right\rceil 13.76$$

As $w_5^2(0) = 39.68\mu$, $w_5(0)$ converged to this value, therefore $R_5(0) = 1 + 39.68 + 13.76 = 54.44\mu s$, which is lower than the D_5.

Message stream $\mathcal{T}_4$ (GOOSE messages sent by SB2)

Calculation of Q_4: Starting with $t_4^0 = 13.76\mu s$ and noting that the $\mathcal{T}_4$ is not blocked because it is the message stream of lower priority:

$$t_4^1 = \left\lceil \frac{t_4^0 + J_7}{T_7} \right\rceil C_7 + \left\lceil \frac{t_4^0 + J_6}{T_6} \right\rceil C_6 + \left\lceil \frac{t_4^0 + J_5}{T_5} \right\rceil C_5 + \left\lceil \frac{t_4^0 + J_4}{T_4} \right\rceil C_4$$

Since $J_6 = J_5 = J_4$, $T_6 = T_5 = T_4$ and $C_6 = C_5 = C_4$:

$$t_4^1 = \left\lceil \frac{13.76 + 1}{208.33} \right\rceil 12.16 + 3 \times \left\lceil \frac{13.76 + 1}{31000} \right\rceil 13.76 = 53.44\mu s$$

$$t_4^2 = \left\lceil \frac{53.44 + 1}{208.33} \right\rceil 12.16 + 3 \times \left\lceil \frac{53.44 + 1}{31000} \right\rceil 13.76 = 53.44\mu s$$

The 4-busy-period converged with $53.44\mu s$:

$$Q_4 = \left\lceil \frac{t_4 + J_4}{T_4} \right\rceil \therefore Q_4 = \left\lceil \frac{53.44 + 1}{31000} \right\rceil = 1$$

That is, it is enough to analyze just the first instance of $\mathcal{T}_4$.

Calculation of R_4: Assuming $B_4 = 0$ and $w_4^0(0) = 13.76\mu s$:

$$w_4^1(0) = \left\lceil \frac{13.76 + 1 + 0.01}{208.33} \right\rceil 12.16 + 2 \times \left\lceil \frac{13.76 + 1 + 0.01}{31000} \right\rceil 13.76$$

As $w_4^1(0) = 39.68\mu$, a second iteration is required:

$$w_4^2(0) = \left\lceil \frac{39.68 + 1 + 0.01}{208.33} \right\rceil 12.16 + 2 \times \left\lceil \frac{39.68 + 1 + 0.01}{31000} \right\rceil 13.76$$

As $w_4^2(0) = 39.68\mu$, $w_4(0)$ converge to this value, therefore $R_4(0) = 1 + 39.68 + 13.76 = 54.44\mu s$, which is less than D_4.