

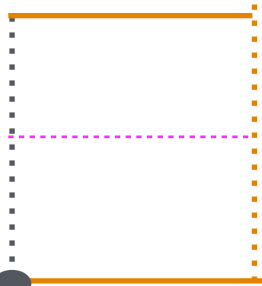
Dimension Reduction

Dimension

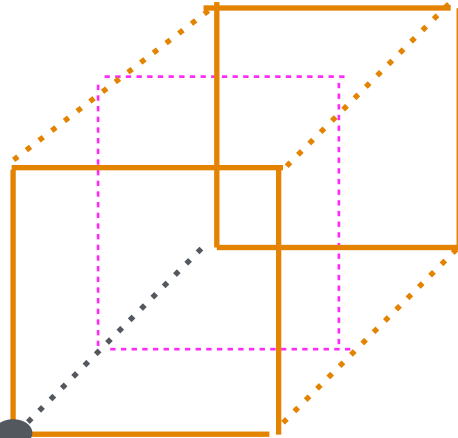
1D moves within 1 line
(0)



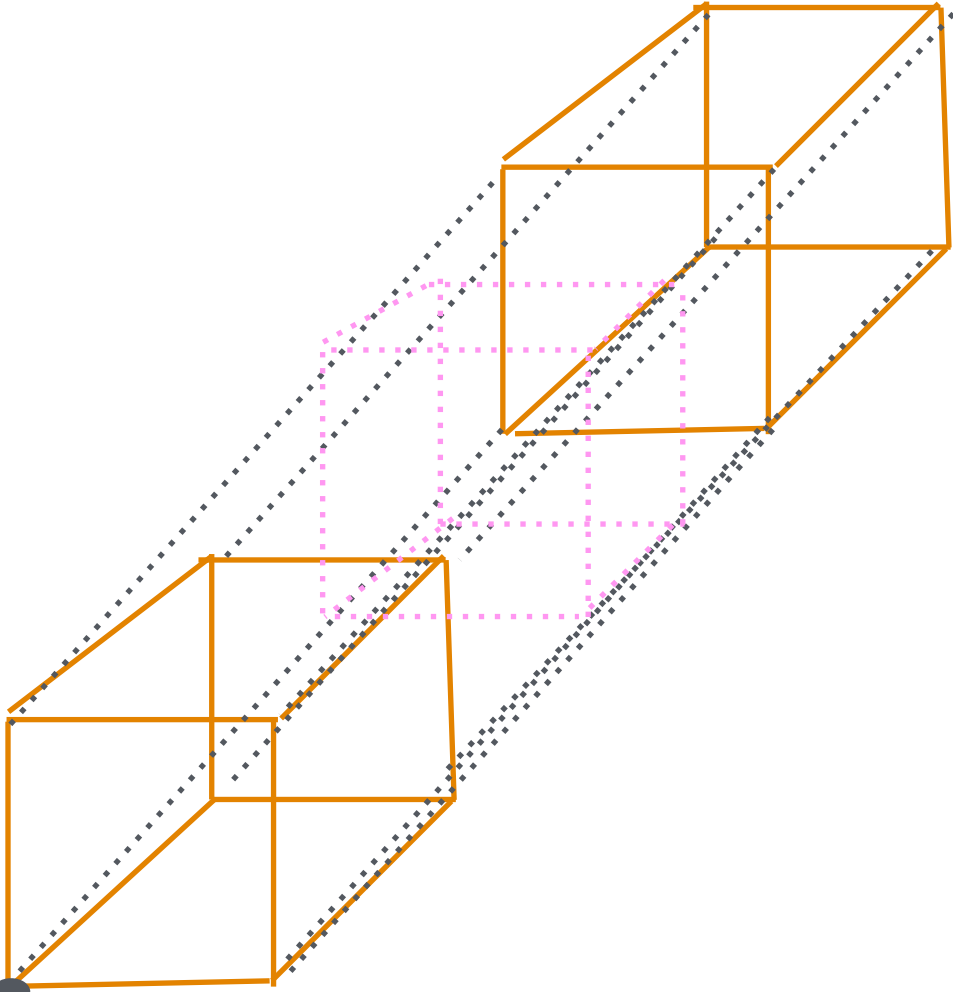
2D , 1D plane moves within 2 lines
(0,0)



3D, 2D plane moves within 4 lines
(0,0,0)



4D, 3D plane moves within 8 lines
(0,0,0,0)



Prior
dimension
capable of
moving its
plane in next
dimension

(0,0,0,0)



2D

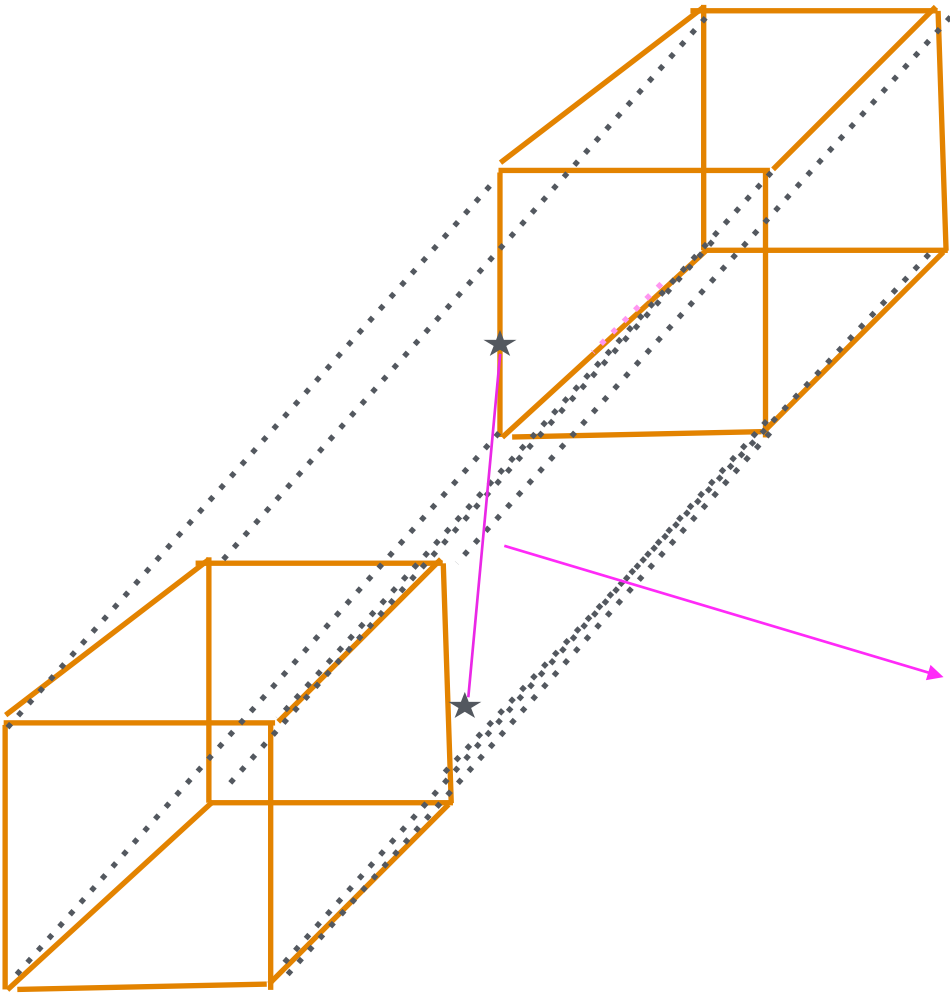
Border is this 2D plane



Lower probability of a random point touching the border

4D

Border is this entire 3D block



Higher probability of a random point touching the border

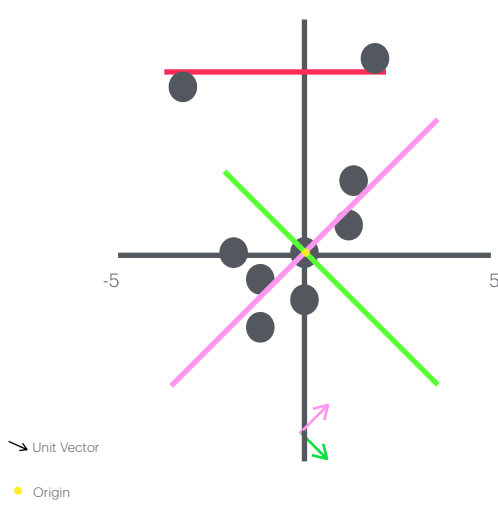
Because random samples are likely to be near borders. The distance between two randomly picked points in higher dimensions will be large

In higher dimensions, training instances are likely to be far away from another

Predictions aren't so reliable because higher dimensions have so much 'ether' between instances. Patterns are such high dimensions are almost impossible to find

In order for a model to fit a high dimensional dataset it must overfit because patterns in higher dimensions are so hard to find (think of a model fitting random noise, not able to generalize(fit) a noise pattern)

PCA



Axis with small variance

Axis with highest variance (c_1)

Axis perpendicular to the highest variance (largest amount of remaining variance) (c_2)

Find Principal

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$$

\mathbf{X} = Training Data



SVD – SingleValueDecomposition

$$\mathbf{V} = \begin{pmatrix} | & | & & | \\ c_1 & c_2 & \dots & c_n \\ | & | & & | \end{pmatrix}$$

c_n = unit Vectors where principal components lives (**line on the top left figure** $c_{1,1} \rightarrow y = c_1 \cdot x_1$)

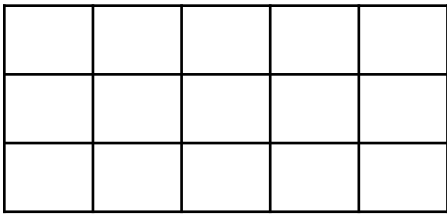
n = features

Dataset must be centered
around the origin when
using PCA

Reduced Projection

$$X_{d-proj} = XW_d$$

$$V^T = \begin{bmatrix} - & c_1 & - \\ - & c_2 & - \\ - & c_3 & - \\ - & \vdots & - \\ - & c_n & - \end{bmatrix}$$



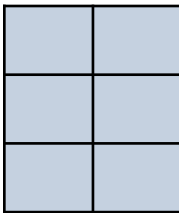
3x5



d = 2



$$W_d = V^T_{truncated}$$

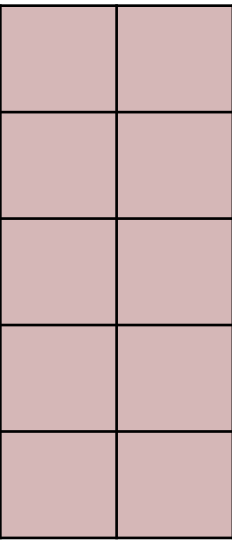


3x2

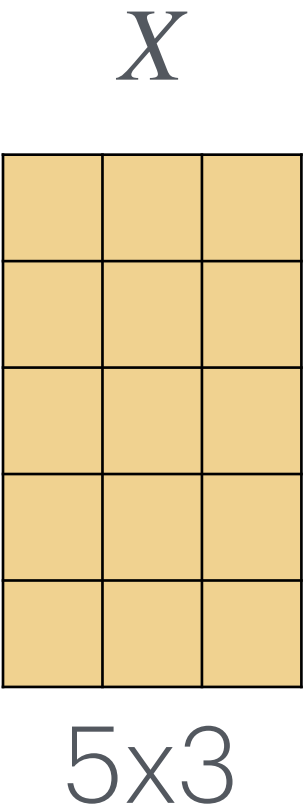
.

=

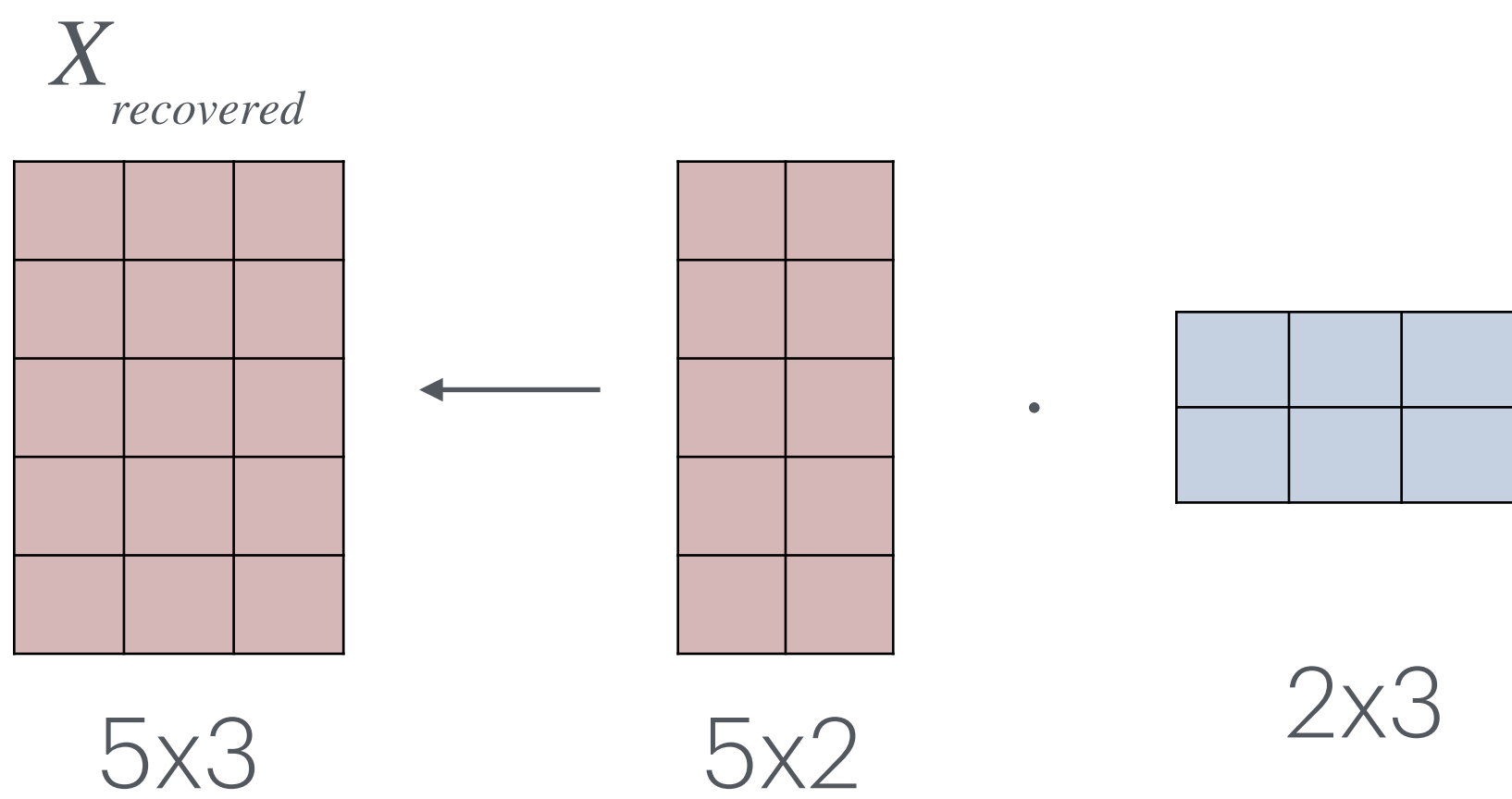
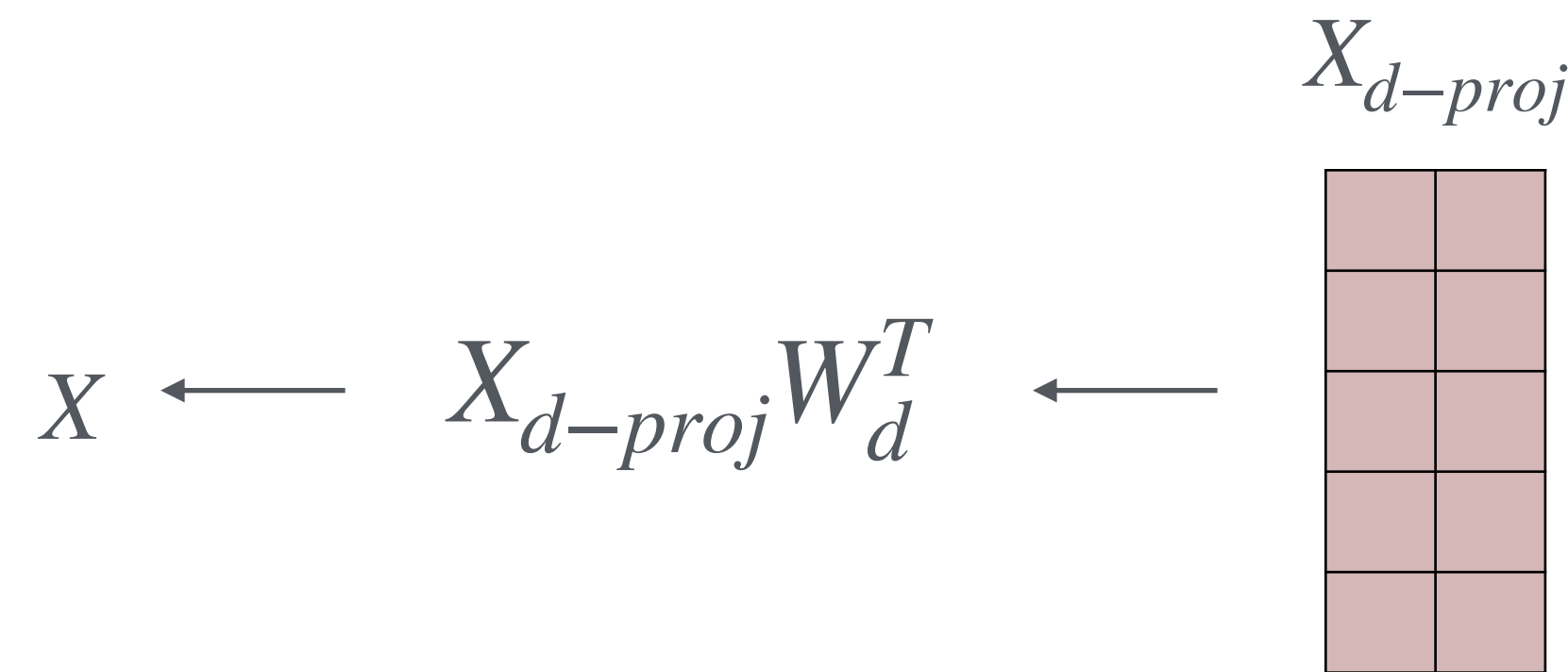
$$X_{d-proj}$$



5x2



Recover Reduced Projection



TSNE MNIST

