2.2

2.2a)

In[ ]:=
```
A = {{σ + 1, 3}, {-2, σ - 1}};
Eigenvalues[A]
```

Out[ ]=

$\{-i\sqrt{5}+\sigma, i\sqrt{5}+\sigma\}$

In[ ]:=
```
ClearAll["Global`*"];
solution =
   DSolve[{ X'[t] == (sigma + 1) X[t] + 3 Y[t], Y'[t] == -2 X[t] + (sigma - 1) Y[t],
     X[0] == u, Y[0] == v}, {X[t], Y[t]}, t]; // Simplify

(*To be able to copy paste to Open TA*)
equationString = ToString[solution, InputForm];
equationModifiedString =
 ToLowerCase[StringReplace[equationString, {"[" → "(", "]" → ")"}]]
Export["test.txt", {equationModifiedString}]
```

Out[ ]=

```
{{x(t) -> (e^(sigma*t)*(5*u*cos(sqrt(5)*t) +
   sqrt(5)*u*sin(sqrt(5)*t) + 3*sqrt(5)*v*sin(sqrt(5)*t)))/5,
   y(t) -> -1/5*(e^(sigma*t)*(-5*v*cos(sqrt(5)*t) +
   2*sqrt(5)*u*sin(sqrt(5)*t) + sqrt(5)*v*sin(sqrt(5)*t)))}}
```

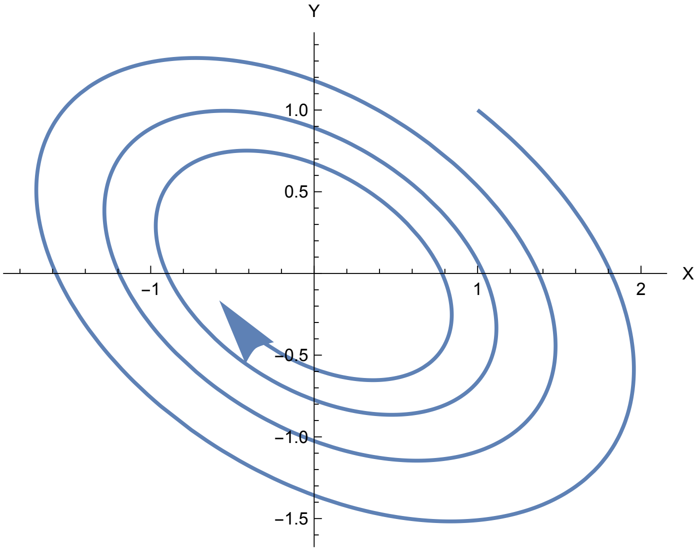
2.2c)

In[1]:=
```
ClearAll["Global`*"];
sigma=-1/10;
solution =DSolve[{ X'[t]==(sigma+1)X[t]+3Y[t],Y'[t]== -2 X[t]+(sigma-1) Y[t],X[0]==u,Y[0

solX=X[t]/. solution[[1]];
solY=Y[t]/. solution[[1]];

streamPlot = StreamPlot[
  Evaluate[{eqns} /. {solX → vx, solY → u}],
  {X[t], -2, 2}, {Y[t], -2, 2}
];
trajPlot=ParametricPlot[{solX,solY}/. {u→1,v→1},{t,0,10},AxesLabel→{"X","Y"}];
trajPlot /. Line[x_] :> {Arrowheads[{.1}], Arrow[x]}
```

Out[8]=

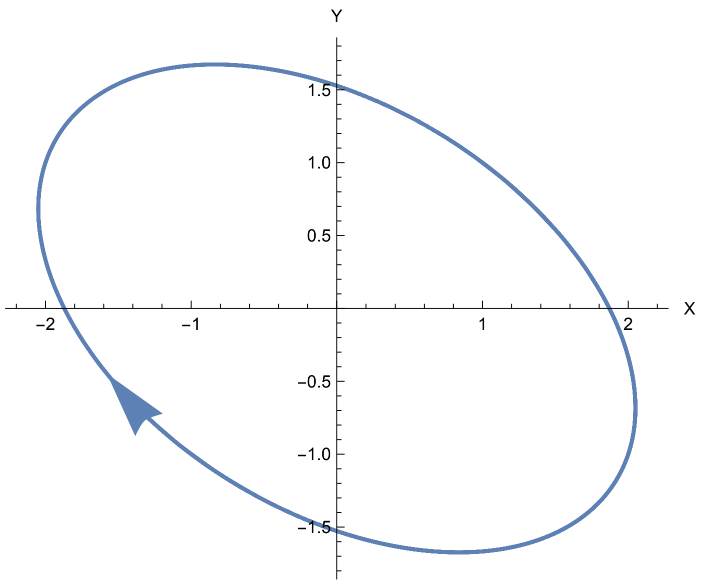


In[ ]:=
```
ClearAll["Global`*"];
sigma = 0;
solution = DSolve[{ X'[t] == (sigma + 1) X[t] + 3 Y[t],
    Y'[t] == -2 X[t] + (sigma - 1) Y[t], X[0] == u, Y[0] == v}, {X[t], Y[t]}, t];

solX = X[t] /. solution[[1]];
solY = Y[t] /. solution[[1]];

trajPlot = ParametricPlot[
   {solX, solY} /. {u → 1, v → 1}, {t, 0, 10}, AxesLabel → {"X", "Y"}];
trajPlot /. Line[x_] :→ {Arrowheads[{.1}], Arrow[x]}
```

Out[ ]=

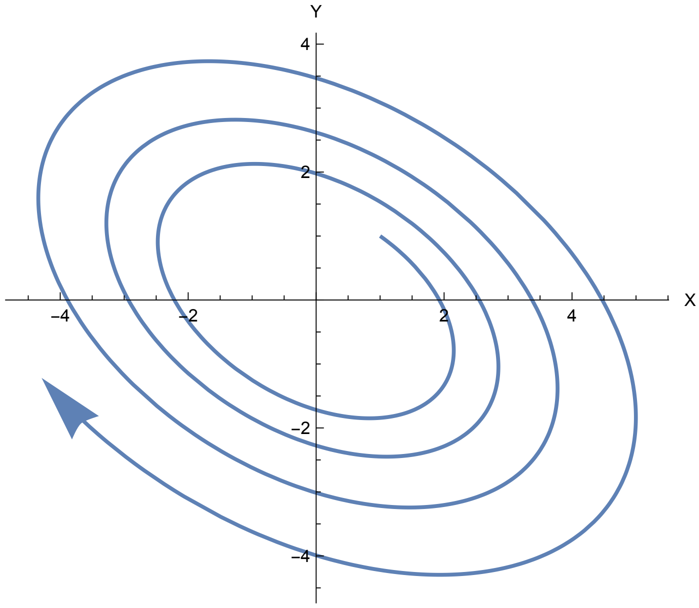


In[ ]:=
```
ClearAll["Global`*"];
sigma = 1 / 10;
solution = DSolve[{ X'[t] == (sigma + 1) X[t] + 3 Y[t],
     Y'[t] == -2 X[t] + (sigma - 1) Y[t], X[0] == u, Y[0] == v}, {X[t], Y[t]}, t];

solX = X[t] /. solution[[1]];
solY = Y[t] /. solution[[1]];

trajPlot = ParametricPlot[
   {solX, solY} /. {u → 1, v → 1}, {t, 0, 10}, AxesLabel → {"X", "Y"}];
trajPlot /. Line[x_] :→ {Arrowheads[{.1}], Arrow[x]}
```

Out[ ]=



2.2d)

In[ ]:=
```
solution = DSolve[{ X'[t] == (sigma + 1) X[t] + 3 Y[t],
    Y'[t] == -2 X[t] + (sigma - 1) Y[t], X[0] == u, Y[0] == v}, {X[t], Y[t]}, t]
```

Out[ ]=

$$\left\{\left\{X[t] \to \frac{1}{5} e^{-t/10} \left(5\, u \, \text{Cos}\left[\sqrt{5}\, t\right] + \sqrt{5}\, u \, \text{Sin}\left[\sqrt{5}\, t\right] + 3 \sqrt{5}\, v \, \text{Sin}\left[\sqrt{5}\, t\right]\right),\right.\right.$$
$$\left.\left. Y[t] \to -\frac{1}{5} e^{-t/10} \left(-5\, v \, \text{Cos}\left[\sqrt{5}\, t\right] + 2 \sqrt{5}\, u \, \text{Sin}\left[\sqrt{5}\, t\right] + \sqrt{5}\, v \, \text{Sin}\left[\sqrt{5}\, t\right]\right)\right\}\right\}$$

All cos and sin has the term sqrt(5) in front of the variable t. Therfore the period is stated by 2 pi/sqrt(5)

In[ ]:=
```
result = 2 Pi / Sqrt[5]
```

Out[ ]=

$$\frac{2\pi}{\sqrt{5}}$$

2.2 e)

```
ClearAll["Global`*"];
sigma = 0;
solution = DSolve[{ X'[t] == (sigma + 1) X[t] + 3 Y[t],
    Y'[t] == -2 X[t] + (sigma - 1) Y[t], X[0] == u, Y[0] == v}, {X[t], Y[t]}, t];
u = 1;
v = 1;
solX = X[t] /. solution[[1]];
solY = Y[t] /. solution[[1]];

(*Define the distance function from the origin*)
r[t_] := Sqrt[(solX) ^2 + (solY) ^2];

dr = D[r[t], t]
extremPoints = Solve[dr == 0, t];
extremeR = r[t] /. extremPoints;

min1 = Min[extremeR];
max1 = Max[extremeR];
ratio = max1 / min1 // Simplify
```

Solve : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Solve : Unable to decide whether numeric quantities

$\{(70 + 210\,i) - 14\sqrt{-20 + 10\,i}\,\sqrt{11 - 2\,i},\ (10 + 30\,i) - 2\sqrt{-20 + 10\,i}\,\sqrt{11 - 2\,i},\ (-5 - 15\,i) + \sqrt{-20 + 10\,i}\,\sqrt{11 - 2\,i},\ (70 - 210\,i) - 14\sqrt{-20 - 10\,i}\,\sqrt{11 + 2\,i},\ (10 - 30\,i) - 2\sqrt{-20 - 10\,i}\,\sqrt{11 + 2\,i},\ (-5 + 15\,i) + \sqrt{-20 - 10\,i}\,\sqrt{11 + 2\,i}\}$ are equal to zero. Assuming they are.

Min : Internal precision limit \$MaxExtraPrecision = 50.` reached while evaluating

$$\sqrt{\frac{1}{25}(-5\,\text{Power}[\ll 2\gg] - 4\,\text{Power}[\ll 2\gg])^2 + \frac{1}{25}(-5\,\text{Power}[\ll 2\gg] + 3\,\text{Power}[\ll 2\gg])^2} - \sqrt{\frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2 + \frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2}.$$

Min : Internal precision limit \$MaxExtraPrecision = 50.` reached while evaluating

$$\sqrt{\frac{1}{25}(5\,\text{Power}[\ll 2\gg] - 4\,\text{Power}[\ll 2\gg])^2 + \frac{1}{25}(5\,\text{Power}[\ll 2\gg] + 3\,\text{Power}[\ll 2\gg])^2} - \sqrt{\frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2 + \frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2}.$$

Min : Internal precision limit \$MaxExtraPrecision = 50.` reached while evaluating

$$\sqrt{\frac{1}{25}(5\,\text{Power}[\ll 2\gg] - 4\,\text{Power}[\ll 2\gg])^2 + \frac{1}{25}(5\,\text{Power}[\ll 2\gg] + 3\,\text{Power}[\ll 2\gg])^2} - \sqrt{\frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2 + \frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2}.$$

General : Further output of Min::meprec will be suppressed during this calculation.

Max : Internal precision limit \$MaxExtraPrecision = 50.` reached while evaluating

$$-\sqrt{\frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2 + \frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2} + \sqrt{\frac{1}{25}(-5\,\text{Power}[\ll 2\gg] - 3\,\text{Power}[\ll 2\gg])^2 + \frac{1}{25}(-5\,\text{Power}[\ll 2\gg] + 4\,\text{Power}[\ll 2\gg])^2}.$$

Max : Internal precision limit \$MaxExtraPrecision = 50.` reached while evaluating

$$-\sqrt{\frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2 + \frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2} + \sqrt{\frac{1}{25}(5\,\text{Power}[\ll 2\gg] - 3\,\text{Power}[\ll 2\gg])^2 + \frac{1}{25}(5\,\text{Power}[\ll 2\gg] + 4\,\text{Power}[\ll 2\gg])^2}.$$

Max : Internal precision limit \$MaxExtraPrecision = 50.` reached while evaluating

$$-\sqrt{\frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2 + \frac{1}{25}(\text{Times}[\ll 2\gg] + \text{Times}[\ll 2\gg])^2} + \sqrt{\frac{1}{25}(5\,\text{Power}[\ll 2\gg] - 3\,\text{Power}[\ll 2\gg])^2 + \frac{1}{25}(5\,\text{Power}[\ll 2\gg] + 4\,\text{Power}[\ll 2\gg])^2}.$$

General : Further output of Max::meprec will be suppressed during this calculation.

Out[ ]=

$$\frac{1}{2}\left(1 + \sqrt{5}\right)$$

2.2f)

In[506]:=

```
ClearAll["Global`*"];
sigma=0;
solution=DSolve[{ X'[t]==(sigma+1)X[t]+3Y[t],Y'[t]== -2 X[t]+(sigma-1) Y[t],X[0]==u,Y[0]
u=1;
v=1;
solX=X[t]/. solution[[1]];
solY=Y[t]/. solution[[1]];

(*Define the distance function from the origin*)
r[t_]:=Sqrt[(solX)^2+(solY)^2];
dr=D[r[t],t];
ddr=D[dr[t],t];

extremPoints=Solve[dr==0,t];
extremPoints[[1]];//Simplify

t=t/. extremPoints[[1]]

solX=X[t]/. solution[[1]];
solY=Y[t]/. solution[[1]];

r[t_]:=Sqrt[(solX)^2+(solY)^2];

xValue=solX/.{u→1,v→1};
yValue=solY/. {u→1,v→1};
vector={xValue,yValue};

normVector=-Normalize[vector]// Simplify

(*To be able to copy paste to Open TA*)
equationString=ToString[normVector,InputForm];
equationModifiedString=ToLowerCase[StringReplace[equationString,{"["→"(","]"→")"}]];
Export["test.txt",{equationModifiedString}];
```

Solve : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[518]=

$$-\frac{\text{ArcCos}\left[-\sqrt{\frac{1}{14}\left(7-3\sqrt{5}\right)}\right]}{\sqrt{5}}$$

Out[525]=

$$\left\{\frac{5\sqrt{7-3\sqrt{5}}+4\sqrt{5\left(7+3\sqrt{5}\right)}}{7\sqrt{5\left(5+\sqrt{5}\right)}},\frac{5\sqrt{7-3\sqrt{5}}-3\sqrt{5\left(7+3\sqrt{5}\right)}}{7\sqrt{5\left(5+\sqrt{5}\right)}}\right\}$$