4.2 a)

Calculate radius if limit cycle by setting r' =0   which gives 0=mu*r-r^3 => r^2=mu => r=0 or r=sqrt(mu) or r= -sqrt(mu)
Calcualte periopd T: periodtime = 2 pi/phi'   and phi' = omega + nu *r^2
therfore T= 2 Pi /(omega + nu * mu)

4.2 b)

In[1540]:=

```
ClearAll["Global`*"]

leftDerivative1 = D[Sqrt[x1[t]^2 + x2[t]^2], t] // Simplify;
leftDerivative2 = D[ArcTan[x1[t], x2[t]], t] // Simplify;


rightHandSide1 = (Sqrt[x1[t]^2 + x2[t]^2]) * (μ - (x1[t]^2 + x2[t]^2));
rightHandSide2 = ω + ν * (x1[t]^2 + x2[t]^2);

equation1 = leftDerivative1 == rightHandSide1;
equation2 = leftDerivative2 == rightHandSide2;
solution1 = Solve[equation1, x1'[t]] // Simplify;
solution2 = Solve[equation2, x2'[t]] // Simplify;
solution2WithSubstitution = solution2 /. solution1[[1]] // Simplify;


derivativeEquation1 = x2'[t] == ν x1[t]^3 + x1[t] (ω + 2 ν x2[t]^2) +
    (x2[t] (-x1[t]^3 + ω x2[t] + ν x2[t]^3 + x1[t] (μ - 2 x2[t]^2) -
        (x2[t] (-μ x2[t] + x2[t]^3 + Derivative[1][x2][t])) / x1[t])) / x1[t];

solutionDerivative1 = Solve[derivativeEquation1, x2'[t]] // ExpandAll;

solution1WithSubstitution = solution1 /. solutionDerivative1[[1]] // ExpandAll
```

Out[1552]=

$$\left\{\left\{x1'[t] \to \mu\, x1[t] - x1[t]^3 - \omega\, x2[t] - \nu\, x1[t]^2\, x2[t] - x1[t]\, x2[t]^2 - \nu\, x2[t]^3\right\}\right\}$$

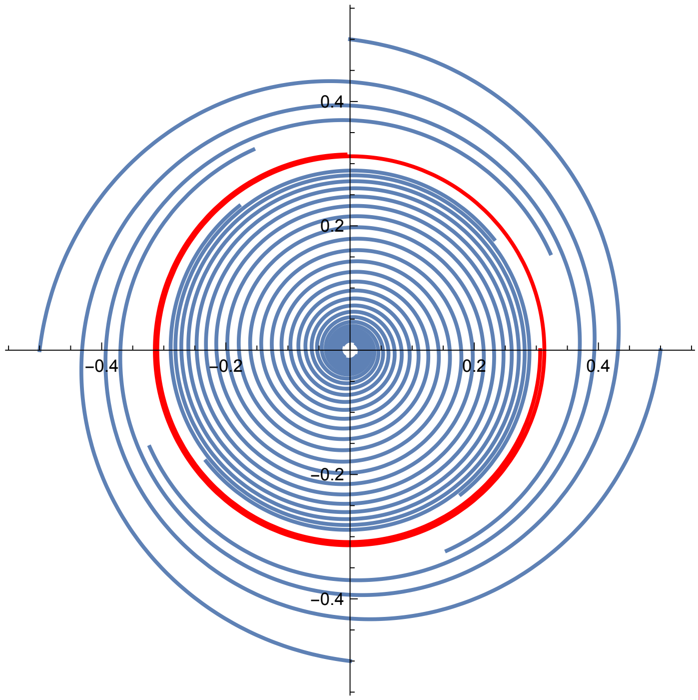
In[1553]:=

```
ClearAll["Global`*"]
```

```
muValue = 1 / 10;
nuValue = 1;
omegaValue = 1;
eqX1 = x1'[t] == muValue * x1[t] + nuValue * x1[t] ^ 2 * x2[t] -
    x1[t] * x2[t] ^ 2 - x1[t] ^ 3 + nuValue * x2[t] ^ 3 + omegaValue * x2[t];
eqX2 = x2'[t] == -nuValue * x1[t] ^ 3 - nuValue * x1[t] * x2[t] ^ 2 -
    x1[t] ^ 2 * x2[t] - omegaValue * x1[t] + muValue * x2[t] - x2[t] ^ 3;

system1 = {eqX1, eqX2};
fixedPoint1 = {0, 0};

initialTime = 0;
timeMax1 = 40;
timeMax2 = 5;
timeMax3 = 10;

distance1 = 0.01;
distance2 = 0.5;
distance3 = Sqrt[muValue] - 0.01;

initialConditions1 =
  {{x1[0] == fixedPoint1[[1]] + distance1, x2[0] == fixedPoint1[[2]] + distance1},
   {x1[0] == fixedPoint1[[1]] - distance1, x2[0] == fixedPoint1[[2]] + distance1},
   {x1[0] == fixedPoint1[[1]] + distance1, x2[0] == fixedPoint1[[2]] - distance1},
   {x1[0] == fixedPoint1[[1]] - distance1, x2[0] == fixedPoint1[[2]] - distance1}};

initialConditions2 = {{x1[0] == fixedPoint1[[1]] + distance2, x2[0] == fixedPoint1[[2]]},
   {x1[0] == fixedPoint1[[1]] - distance2, x2[0] == fixedPoint1[[2]]},
   {x1[0] == fixedPoint1[[1]], x2[0] == fixedPoint1[[2]] + distance2},
   {x1[0] == fixedPoint1[[1]], x2[0] == fixedPoint1[[2]] - distance2}};

initialConditionsLimitCycle =
  {x1[0] == fixedPoint1[[1]] + distance3, x2[0] == fixedPoint1[[2]]};

sol1 = NDSolve[{system1, #}, {x1, x2}, {t, initialTime, timeMax1}] & /@
   initialConditions1;
sol2 = NDSolve[{system1, #}, {x1, x2}, {t, initialTime, timeMax2}] & /@
   initialConditions2;
sol3 = NDSolve[{system1, initialConditionsLimitCycle},
   {x1, x2}, {t, initialTime, timeMax3}];


plot1 =
  ParametricPlot[Evaluate[{x1[t], x2[t]} /. #], {t, initialTime, timeMax1}] & /@
   sol1;
```

```
plot2 =
  ParametricPlot[Evaluate[{x1[t], x2[t]} /. #], {t, initialTime, timeMax2}] & /@
   sol2;
plot3 = ParametricPlot[Evaluate[{x1[t], x2[t]} /. sol3],
   {t, initialTime, timeMax3}, PlotStyle → Red];

Show[plot1, plot2, plot3, FrameLabel → {"t", "Solution"}, PlotRange → Full]
```

Out[1577]=



4.2 c)

from b [1/10,1,1]

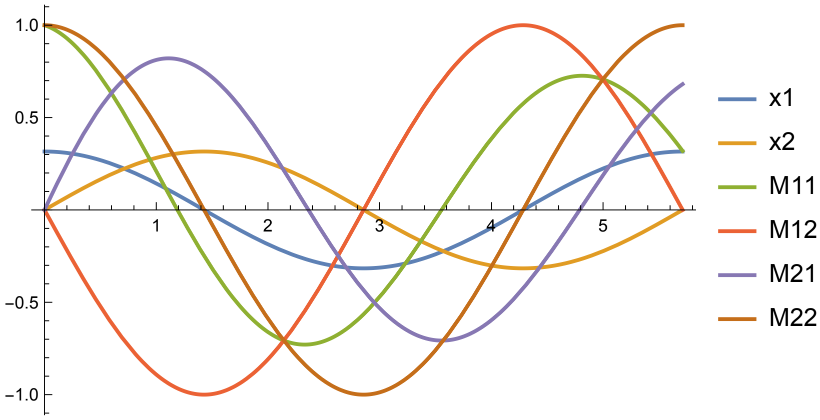
4.2 d)

In[1578]:=

```
ClearAll["Global`*"]

(*Parameters*)
muValue = 1 / 10;
omegaValue = 1;
nuValue = 1;

(*Define functions*)
```

```
f1[x1_, x2_] := muValue * x1[t] - nuValue * x1[t] ^ 2 * x2[t] -
   x1[t] * x2[t] ^ 2 - x1[t] ^ 3 - nuValue * x2[t] ^ 3 - omegaValue * x2[t];
f2[x1_, x2_] := nuValue * x1[t] ^ 3 + nuValue * x1[t] * x2[t] ^ 2 -
   x1[t] ^ 2 * x2[t] + omegaValue * x1[t] + muValue * x2[t] - x2[t] ^ 3;

(*Calculate partial derivatives*)
J11 = D[f1[x1, x2], x1[t]];
J12 = D[f1[x1, x2], x2[t]];
J21 = D[f2[x1, x2], x1[t]];
J22 = D[f2[x1, x2], x2[t]];

(*Define system of differential equations*)
eqM11 = M11'[t] == J11 * M11[t] + J12 * M21[t];
eqM12 = M12'[t] == J11 * M12[t] + J12 * M22[t];
eqM21 = M21'[t] == J21 * M11[t] + J22 * M21[t];
eqM22 = M22'[t] == J21 * M12[t] + J22 * M22[t];

eqx1 = x1'[t] == muValue * x1[t] - nuValue * x1[t] ^ 2 * x2[t] -
    x1[t] * x2[t] ^ 2 - x1[t] ^ 3 - nuValue * x2[t] ^ 3 - omegaValue * x2[t];
eqx2 = x2'[t] == nuValue * x1[t] ^ 3 + nuValue * x1[t] * x2[t] ^ 2 -
    x1[t] ^ 2 * x2[t] + omegaValue * x1[t] + muValue * x2[t] - x2[t] ^ 3;

system = {eqM11, eqM12, eqM21, eqM22, eqx1, eqx2};
initialConditions = {x1[0] == Sqrt[muValue],
   x2[0] == 0, M11[0] == 1, M12[0] == 0, M21[0] == 0, M22[0] == 1};


initialTime = 0;
timeMax = 2 * Pi / (omegaValue + nuValue * muValue);

sol = NDSolve[{system, initialConditions},
   {x1, x2, M11, M12, M21, M22}, {t, initialTime, timeMax}];

{x1Values, x2Values, M11Values, M12Values, M21Values, M22Values} =
  {x1[t], x2[t], M11[t], M12[t], M21[t], M22[t]} /. sol[[1]];

(*Plot the trajectories*)
plotTrajectories =
  Plot[Evaluate[{x1[t], x2[t], M11[t], M12[t], M21[t], M22[t]} /. sol[[1]]],
   {t, initialTime, timeMax}, PlotLegends →
    {"x1", "x2", "M11", "M12", "M21", "M22"}, FrameLabel → {{"Values", None},
     {"Time", "Trajectories of x1, x2, M11, M12, M21, M22"}}, PlotRange → All];

Show[plotTrajectories]
```

Out[1601]=



4.2 e)

```
M11TMax = M11Values /. t → timeMax;
M12TMax = M12Values /. t → timeMax;
M21TMax = M21Values /. t → timeMax;
M22TMax = M22Values /. t → timeMax;
```

```
resultMatrix = Round[{{M11TMax, M12TMax}, {M21TMax, M22TMax}}, 0.01] // MatrixForm
```

Out[1606]//MatrixForm=

$$\begin{pmatrix} 0.32 & 0. \\ 0.68 & 1. \end{pmatrix}$$

4.2 f)

In[1607]:=

```
solMat = {{M11TMax, M12TMax}, {M21TMax, M22TMax}};

eigenValues = Eigenvalues[solMat];

solTilde =
 Round[{1 / timeMax * Log[eigenValues[[2]]], 1 / timeMax * Log[eigenValues[[1]]]}, 0.01]
```

Out[1609]=

```
{-0.2, 0.}
```

4.2 g)

```
ClearAll["Global`*"]

J11 = D[Sqrt[x1^2 + x2^2], x1];
J12 = D[Sqrt[x1^2 + x2^2], x2];
J21 = D[ArcTan[x1, x2], x1];
J22 = D[ArcTan[x1, x2], x2];

JG = {{J11, J12}, {J21, J22}};
JGInv = Inverse[JG];

JakobiPol = {{μ - 3 * r^2, 0}, {2 * ν * r, 0}};
JakExp = MatrixExp[JakobiPol * T];
T = 2 * Pi / (ω + ν * μ);
x1 = Sqrt[μ];
x2 = 0;
ω = 1;
ν = 1;
μ = 1 / 10;
r = Sqrt[x1^2 + x2^2];

solM = JGInv.JakExp.JG // Simplify
```

Out[1626]=

$$\left\{\left\{e^{-4\pi/11}, 0\right\}, \left\{1 - e^{-4\pi/11}, 1\right\}\right\}$$

4.2 h)

In[1296]:=

```
solm = Eigenvalues[solM];

Round[{1 / T * Log[solm[[2]]], 1 / T * Log[solm[[1]]]}, 0.1]
```

Out[1297]=

```
{-0.2, 0.}
```