

2.1a)

In[539]:=

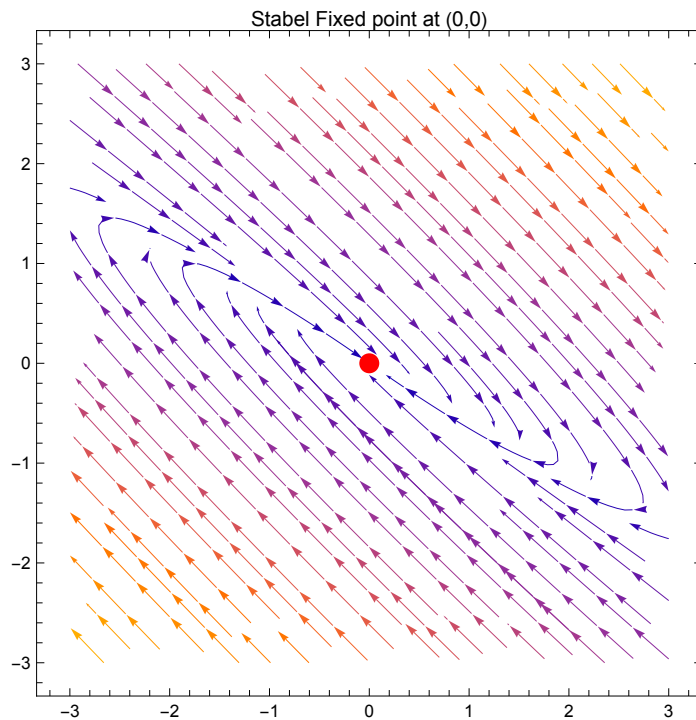
```
sigma = -1
stream = StreamPlot[{(sigma + 3) x + 4 y, (-9 / 4) x + (sigma - 3) y},
  {x, -3, 3}, {y, -3, 3}, PlotLabel -> "Stabel Fixed point at (0,0)"];
fp = Graphics[{Red, PointSize[0.03], Point[{0, 0}]}],
  Axes -> True, AxesOrigin -> {0, 0}];
Show[stream, fp]
f[x_, y_] := (2) x + 4 y
g[x_, y_] := (-9 / 4) x + (-4) y

j = D[{f[x, y], g[x, y]}, {{x, y}}];
Eigenvalues[j]
```

Out[539]=

- 1

Out[542]=



Out[546]=

{-1, -1}

This is a stable fixed point at $(0,0)$ because eigenvalues real part negative

In[557]:=

```

sigma = 0
stream =
  StreamPlot[{(sigma + 3) x + 4 y, (-9 / 4) x + (sigma - 3) y}, {x, -3, 3}, {y, -3, 3},
    PlotLabel -> "Line of fixed points along the eigen vector, only flow
      in one direction"];
f[x_, y_] := (3) x + 4 y
g[x_, y_] := (-9 / 4) x + (-3) y

j = D[{f[x, y], g[x, y]}, {{x, y}}];
Eigenvalues[j]
eVector = Eigenvectors[j];
eVector[[1, 2]];
fp =
  Graphics[{Red, Thickness[0.01], Line[{{-3, -3 eVector[[1, 2]] / eVector[[1, 1]]},
    {3, 3 eVector[[1, 2]] / eVector[[1, 1]]}}]};

Show[stream, fp]

```

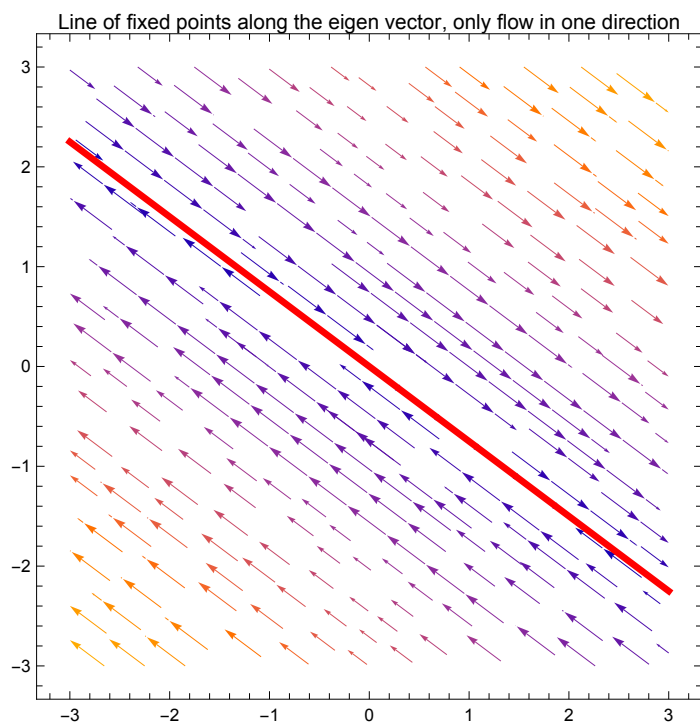
Out[557]=

0

Out[562]=

{0, 0}

Out[566]=



Line of fix points

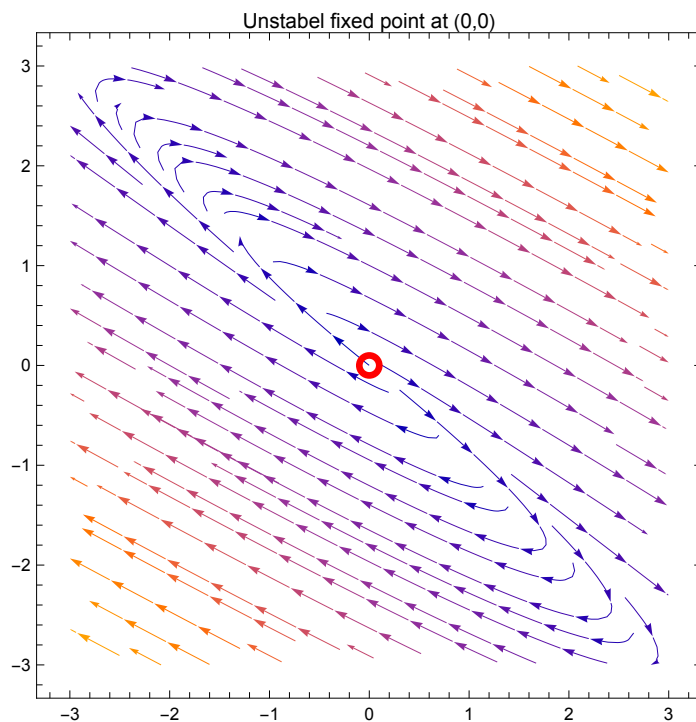
In[567]:=

```
sigma = 1
stream = StreamPlot[{(sigma + 3) x + 4 y, (-9 / 4) x + (sigma - 3) y},
  {x, -3, 3}, {y, -3, 3}, PlotLabel -> "Unstabel fixed point at (0,0)"];
fp = Graphics[{Red, Thickness[0.01], Circle[{0, 0}, 0.1]}];
Show[stream, fp]
```

Out[567]=

1

Out[570]=



```
In[ ]:= f[x_, y_] := (4) x + 4 y
g[x_, y_] := (-9 / 4) x + (-2) y
```

```
j = D[{f[x, y], g[x, y]}, {{x, y}}];
Eigenvalues[j]
```

Out[]:=

```
{1, 1}
```

This is a unstable fixed point at (0,0) because eigenvalues real part positive

2.1 b)

```
matrix = {{σ + 3, 4}, {-9 / 4, σ - 3}}
```

```
In[*]:= matrix = {{σ + 3, 4}, {-9 / 4, σ - 3}};
```

2.1c)

```
In[*]:= vectors = Eigenvectors[{{3 + σ, 4}, {-9 / 4, -3 + σ}}]
```

```
Out[*]=
```

$$\left\{ \left\{ -\frac{4}{3}, 1 \right\}, \{0, 0\} \right\}$$

```
In[*]:= eigenVector = vectors[[1]]
```

```
Out[*]=
```

$$\left\{ -\frac{4}{3}, 1 \right\}$$

```
In[*]:= magnitude = Sqrt[Total[eigenVector^2]];
```

```
In[*]:= normalizedVector = -eigenVector / magnitude
```

```
Out[*]=
```

$$\left\{ \frac{4}{5}, -\frac{3}{5} \right\}$$

2.1 d)

```
In[*]:= Inverse[matrix]
```

```
Out[*]=
```

$$\left\{ \left\{ \frac{-12 + 4 \sigma}{4 \sigma^2}, -\frac{4}{\sigma^2} \right\}, \left\{ \frac{9}{4 \sigma^2}, \frac{3 + \sigma}{\sigma^2} \right\} \right\}$$

12.1 e)

when $\sigma = 0$ inverse of A goes to inf therefor $\sigma = 0$ implies that invers of A dosent exist in this particular case ($\sigma = 0$)

2.1 f)

In[529]:=

```
matrixGeneral={{σ-cd,d^2},{-c^2,σ+cd}}
matrixSigmaMinus={{-1-cd,d^2},{-c^2,-1+cd}}
```

Out[529]=

$$\left\{ \left\{ -cd + \sigma, d^2 \right\}, \left\{ -c^2, cd + \sigma \right\} \right\}$$

Out[530]=

$$\left\{ \left\{ -1 - cd, d^2 \right\}, \left\{ -c^2, -1 + cd \right\} \right\}$$

In[*]:= Eigensystem[matrixSigmaMinus]

Out[*]=

$$\left\{ \left\{ -1 - \sqrt{cd^2 - c^2 d^2}, -1 + \sqrt{cd^2 - c^2 d^2} \right\}, \left\{ \left\{ -\frac{cd - \sqrt{cd^2 - c^2 d^2}}{c^2}, 1 \right\}, \left\{ -\frac{cd + \sqrt{cd^2 - c^2 d^2}}{c^2}, 1 \right\} \right\} \right\}$$

In[*]:= Eigenvalues[matrixSigmaMinus]

Out[*]=

$$\left\{ -1 - \sqrt{cd^2 - c^2 d^2}, -1 + \sqrt{cd^2 - c^2 d^2} \right\}$$

In[*]:= Eigenvectors[matrixSigmaMinus]

Out[*]=

$$\left\{ \left\{ -\frac{cd - \sqrt{cd^2 - c^2 d^2}}{c^2}, 1 \right\}, \left\{ -\frac{cd + \sqrt{cd^2 - c^2 d^2}}{c^2}, 1 \right\} \right\}$$

Eigenvalues should be the same as previous [sigma,sigma] (sigma=-1) therefore $\sqrt{cd^2 - c^2 d^2} = 0$. Given this the eigenvector is $[d/c, 1]$.