Home Assignment 1

André Hedesand

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1 Solutions

1. (a) *"Recursive systems are never stable"*
   False. Recursive systems are IIR systems e.g.

   $$y(n) = \frac{1}{2}y(n-1) + x(n)$$

   If we by stability mean BIBO-stability we can simply prove that the system above is stable. The Z-transorm of the system above is

   $$Y(z) = \frac{1}{2}z^{-1}Y(z) + X(z) \Rightarrow Y(z)(1 - \frac{1}{2}z^{-1}) = X(z) \Rightarrow$$

   $$Y(z) = \frac{1}{1-\frac{1}{2}z^{-1}}X(z) = H(z)X(z) \Rightarrow H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \Rightarrow$$

   $$h(n) = (\frac{1}{2})^n u(n)$$

   By definition the system above is stable if $\sum_{n=0}^{\infty} |(\frac{1}{2})^n| < \infty$. This is true because $(\frac{1}{2})^n$ goes to zero as n goes to ∞.

   (b) *"A convolution of two sequences in the time domain corresponds to a multiplication of the Z transforms of the signals."*
   True. This is true by definition.

   (c) *"An FIR filter is always stable."*
   True. A FIR filter can be discribed with a difference equation:

   $$y(n) = \sum_{k=0}^{N} a_k x(n-k)$$

   The system is stable as all $a_n < \infty$.

   (d) *A first order IIR filter is stable iff the absolute value of the value of the factor in front of $y(n-1)$ is greater than 1.*
   False. The opposite is true. We can prove this by contradiction: If we assume the statement above is true we get:

   $$y(n) = Ay(n-1) + x(n), A > 1$$

   $$\textit{and}$$

   $$\sum_k |h(k)| < \infty$$

   Lets do the same Z-transform as in *1a)*:

   $$y(n) = Ay(n-1) + x(n) \Rightarrow h(n) = A^n u(n)$$

At last we get

$$\sum_k |h(k)| < \infty \Rightarrow \sum_{n=0}^{\infty} |A^n| < \infty$$