Home Assignment 2

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Solutions

1. *"A rotor on a helicopter has four blades and rotates with 4 500 revolutions/minute, counter clockwise, (note, revolutions/minute). The rotor is filmed with a digital camera that takes 50 pictures/second. What will the perceived rotation speed be when watching the recorded film sequence? What will be the resulting direction of rotation? How does it look if the rotor goes slightly slower, with 4 499 revolutions/minute?"*

   Let the frequency of the helicopters' rotor be $4500rpm = \frac{4500}{60}Hz = 75Hz$. The video camera does the sampling with $F_s = 50Hz$. This means we get the relative frequency $f = \frac{75Hz}{50Hz} = 1.5 = 0.5+1$.

   Since we are under-sampling, the video of the rotor is only going to show $f = 0.5$(half a turn). When really the rotors are spinning 1.5 turns per picture.

   If we then can't distinguish between the four rotor blades, it will look like the rotorblades are static in place.

   Lets now assume the helicopters' rotor is spinning at $4499rpm = 74.98Hz$. With this we get $f = 1.499 = 0.499 + 1$. Now the video camera shows 0.499 revolutions per minute. This will give the effect of a slow movement *clockwise*(The rotor is actually spinning *counter clockwise*).

   We could also take an extra look at the rotor: Because the four rotor blades are seated in a +-sign, only rotations less than $\frac{\pi}{2}rad$ can be distinguished. This means the f in the first case can be written as $f = 1.5 = 0 + 0.25 * 6$ which means that the rotor blades appears to stand still as we've seen before. The second case can be written as $f = 1.499 = (-0.001) + 0.25 * 6$ where the blade appears to have a relative frequency of -0.001.

2. *"The following discrete-time signals are given:"*

$$x_1(n) = \begin{bmatrix}1 & \underline{-3} & -1 & 3\end{bmatrix}, x_2(n) = \begin{bmatrix}\underline{-1} & -1 & 2 & -2 & 1 & -2\end{bmatrix}$$

   "Determine the following: (3 out of 4 correct answers gives full points.)"

   (a) *"The linear convolution of the sequences, that is, $y(n) = x_1(n) * x_2(n)$."*

$$y(n) = x_1(n) * x_2(n) = \begin{bmatrix}1 & \underline{-3} & -1 & 3\end{bmatrix} * \begin{bmatrix}\underline{-1} & -1 & 2 & -2 & 1 & -2\end{bmatrix}$$

$$= \begin{array}{c|cccc} x_1 * x_2 & 1 & \underline{-3} & -1 & 3 \\ \hline \underline{-1} & -1 & 3 & 1 & -3 \\ -1 & -1 & 3 & 1 & -3 \\ 2 & 2 & -6 & -2 & 6 \\ -2 & -2 & 6 & 2 & -6 \\ 1 & 1 & -3 & -1 & 3 \\ -2 & -2 & 6 & 2 & -6 \end{array} \implies y(n) = \begin{bmatrix} \underline{-1} & 2 & 6 & -10 & 2 & 3 & -1 & 5 & -6 \end{bmatrix}$$

(b) *"The circular convolution modulo 4, that is, $y(n) = x1(n) *_4 x2(n)$."*

$$x_1(n) * x_2(n) = \begin{array}{c|ccccccccc} x_1 *_4 x_2 & 3 & 1 & -3 & -1 & 3 & 1 & \underline{-3} & -1 & 3 \\ \hline \underline{-1} & -3 & -1 & 3 & 1 & -3 & -1 & 3 & 1 & -3 \\ -1 & -3 & -1 & 3 & 1 & -3 & -1 & 3 & 1 & -3 \\ 2 & 6 & 2 & -6 & -2 & 6 & 2 & -6 & -2 & 6 \\ -2 & -6 & -2 & 6 & 2 & -6 & -2 & 6 & 2 & -6 \\ 1 & 3 & 1 & -3 & -1 & 3 & 1 & -3 & -1 & 3 \\ -2 & -6 & -2 & 6 & 2 & -6 & -2 & 6 & 2 & -6 \end{array} \implies y(n) = \begin{bmatrix} -5 & \underline{5} & 5 & -5 \end{bmatrix}$$

(c) *"The linear correlation of the sequences, that is, $r_{x_1 x_2}(n) = x_1(n) * x_2(-n)$."*

$$x_1(n) = \begin{bmatrix} 1 & \underline{-3} & -1 & 3 \end{bmatrix}, x_2(-n) = \begin{bmatrix} -2 & 1 & -2 & 2 & -1 & \underline{-1} \end{bmatrix}$$

$$x_1(n) * x_2(-n) = \begin{array}{c|cccc} r_{x_1 x_2} & 1 & \underline{-3} & -1 & 3 \\ \hline -2 & -2 & 6 & 2 & -6 \\ 1 & 1 & -3 & -1 & 3 \\ -2 & -2 & 6 & 2 & -6 \\ 2 & 2 & -6 & -2 & 6 \\ -1 & -1 & 3 & 1 & -3 \\ \underline{-1} & -1 & 3 & 1 & -3 \end{array} \implies y(n) = \begin{bmatrix} -2 & \underline{7} & -3 & 1 & -2 & -6 & 10 & -2 & -3 \end{bmatrix}$$

(d) *"The circular modulo 5 correlation of the sequences, that is, $r_{x_1 x_2} = x_1(n) *_5 x_2(-n)$"*

We will begin with padding the shorter signal to the length 5:

$$x_1(n) = \begin{bmatrix} 1 & \underline{-3} & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & \underline{-3} & -1 & 3 & 0 \end{bmatrix}$$

$$x_1(n) *_5 x_2(-n) = \begin{array}{c|cccccccccc} x_1 *_5 x_2 & 1 & -3 & -1 & 3 & 0 & 1 & \underline{-3} & -1 & 3 & 0 \\ \hline -2 & -2 & 6 & 2 & -6 & 0 & -2 & 6 & 2 & -6 & 0 \\ 1 & 1 & -3 & -1 & 3 & 0 & 1 & -3 & -1 & 3 & 0 \\ -2 & -2 & 6 & 2 & -6 & 0 & -2 & 6 & 2 & -6 & 0 \\ 2 & 2 & -6 & -2 & 6 & 0 & 2 & -6 & -2 & 6 & 0 \\ -1 & -1 & 3 & 1 & -3 & 0 & -1 & 3 & 1 & -3 & 0 \\ \underline{-1} & -1 & 3 & 1 & -3 & 0 & -1 & 3 & 1 & -3 & 0 \end{array} \implies$$

$$y(n) = \begin{bmatrix} -8 & \underline{17} & -5 & -2 & -2 \end{bmatrix}$$

3. *"Signals are sampled, down sampled or up sampled and reconstructed ideally according to the items below. Determine what the resulting signal will be."*

   (a) *"The signal $cos(2\pi 450t)$ is sampled using $F_s = 1000Hz$, and then down sampled by a factor 3 (that is, only every third sample value is kept). The resulting signal is then ideally reconstructed (using $F_s = 1000Hz$)."*

$$cos(2\pi{*}450t) \xrightarrow{Sampling\mathbf{1kHz}} cos(2\pi{*}\frac{450}{1000}n) = \{n = 3n\}\, cos(2\pi{*}\frac{1350}{1000}n) = cos(2\pi{*}(\frac{350}{1000}{+}k)n)$$

$$\xrightarrow{Reconstruction\mathbf{1kHz}} cos(2\pi * 350t)$$

   (b) *"The signal $cos(2\pi 1680t)$ is sampled with $F_s = 600Hz$, up-sampled (that is, after every sample value two zeroes are inserted), and then ideally reconstructed with a new sample rate, $Fs = 500Hz$"*

$$cos(2\pi * 1680t) \xrightarrow{Sampling\mathbf{600Hz}} cos(2\pi * \frac{1680}{600}n) = cos(2\pi * \frac{28}{10}n) = cos(2\pi * (\frac{8}{10} + k)n) =$$

$$\left\{f = \frac{f}{3}\right\} cos(2\pi * (\frac{8}{30} + k)n) \xrightarrow{Reconstruction\mathbf{500Hz}} cos(2\pi * \frac{400}{3}t)$$