COMS 4770 Homework 10

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1. (15 pts)

Consider a battery with a completely unknown voltage (initial covariance $P_0 = \infty$). Two independent measurements of the voltage are taken to estimate the voltage, the first with a variance of 1, and the second with a variance of 4.

(a) (2 pts) Write the weighted least squares voltage estimate in terms of the two measurements y_1 and y_2.

The weighted least squares estimate for the voltage $\hat{V}$ using two measurements y_1 and y_2 with variances $\sigma_1^2 = 1$ and $\sigma_2^2 = 4$, respectively, is given by:

$$\hat{V} = \frac{w_1 y_1 + w_2 y_2}{w_1 + w_2}$$

where the weights w_1 and w_2 are inversely proportional to the variances:

$$w_1 = \frac{1}{\sigma_1^2} = 1, \quad w_2 = \frac{1}{\sigma_2^2} = \frac{1}{4}$$

thus,

$$\hat{V} = \frac{y_1 + \frac{1}{4} y_2}{1 + \frac{1}{4}} = \frac{4y_1 + y_2}{5}$$

(b) (10 pts) If weighted least squares is used to estimate the voltage, what is the variance of the voltage estimate after the first measurement? What is the variance of the voltage estimate after the second measurement?

Variance after the First Measurement:

$$P_1 = \left(\frac{1}{P_0} + \frac{1}{\sigma_1^2}\right)^{-1} = (0+1)^{-1} = 1$$

Variance after the Second Measurement:

The updated variance P_2 after incorporating the second measurement y_2 with variance $\sigma_2^2 = 4$ is:

$$P_2 = \left(\frac{1}{P_1} + \frac{1}{\sigma_2^2}\right)^{-1} = \left(1 + \frac{1}{4}\right)^{-1} = \left(\frac{5}{4}\right)^{-1} = \frac{4}{5}$$

(c) (8 pts) If the voltage is estimated as $(y_1 + y_2)/2$, an unweighted average of the measurements, what is the variance of the voltage estimate?

The variance of this estimate is given by:

$$\text{Var}(\hat{V}) = \frac{1}{2}^2 \text{Var}(y_1) + \frac{1}{2}^2 \text{Var}(y_2)$$

Substitute the given variances $\text{Var}(y_1) = 1$ and $\text{Var}(y_2) = 4$:

$$\text{Var}(\hat{V}) = \frac{1}{4} \cdot 1 + \frac{1}{4} \cdot 4 = \frac{1}{4} + 1 = \frac{5}{4}$$

Thus, the variance of the unweighted average is:

$$\text{Var}(\hat{V}) = \frac{5}{4}$$