COMS 4770 - Homework 5

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For the programming problem, you must use a general-purpose programming language. Do not use built-in routines from mathematical software such as Mathematica, Matlab, or Maple. You need to turn in your code, its runtime results, and your answers to the other problems in a zipped file named `Firstname_Lastname_HW5.zip` on Canvas.

Problem 1

(a) (7 pts) What is the result of rotating the point $(4, -11, 2)$ about the vector $(1, 2, 1)$ through an angle of $\frac{\pi}{6}$?

Rodrigues' Rotation Matrix:

$$R_{\hat{l}}(\theta) = \begin{pmatrix} l_x^2(1-\cos\theta)+\cos\theta & l_xl_y(1-\cos\theta)-l_z\sin\theta & l_xl_z(1-\cos\theta)+l_y\sin\theta \\ l_xl_y(1-\cos\theta)+l_z\sin\theta & l_y^2(1-\cos\theta)+\cos\theta & l_yl_z(1-\cos\theta)-l_x\sin\theta \\ l_xl_z(1-\cos\theta)-l_y\sin\theta & l_yl_z(1-\cos\theta)+l_x\sin\theta & l_z^2(1-\cos\theta)+\cos\theta \end{pmatrix}$$

$$\hat{l} = (1, 2, 1)$$

$$v = (4, -11, 2)$$

$$R_{\hat{l}}(\frac{\pi}{6}) = \begin{pmatrix} 1^2(1-\cos\frac{\pi}{6})+\cos\frac{\pi}{6} & 1*2(1-\cos\frac{\pi}{6})-1\sin\frac{\pi}{6} & 1*1(1-\cos\frac{\pi}{6})+2\sin\frac{\pi}{6} \\ 1*2(1-\cos\frac{\pi}{6})+1\sin\frac{\pi}{6} & 2^2(1-\cos\frac{\pi}{6})+\cos\frac{\pi}{6} & 2*1(1-\cos\frac{\pi}{6})-1\sin\frac{\pi}{6} \\ 1*1(1-\cos\frac{\pi}{6})-2\sin\frac{\pi}{6} & 2*1(1-\cos\frac{\pi}{6})+1\sin\frac{\pi}{6} & 1^2(1-\cos\frac{\pi}{6})+\cos\frac{\pi}{6} \end{pmatrix}$$

$$R_{\hat{l}}(\frac{\pi}{6}) = \begin{pmatrix} 1^2(1-\cos\frac{\pi}{6})+\cos\frac{\pi}{6} & 1*2(1-\cos\frac{\pi}{6})-1\sin\frac{\pi}{6} & 1*1(1-\cos\frac{\pi}{6})+2\sin\frac{\pi}{6} \\ 1*2(1-\cos\frac{\pi}{6})+1\sin\frac{\pi}{6} & 2^2(1-\cos\frac{\pi}{6})+\cos\frac{\pi}{6} & 2*1(1-\cos\frac{\pi}{6})-1\sin\frac{\pi}{6} \\ 1*1(1-\cos\frac{\pi}{6})-2\sin\frac{\pi}{6} & 2*1(1-\cos\frac{\pi}{6})+1\sin\frac{\pi}{6} & 1^2(1-\cos\frac{\pi}{6})+\cos\frac{\pi}{6} \end{pmatrix}$$

$$R_{\hat{l}}(\frac{\pi}{6}) = \begin{pmatrix} 1 & (2-\sqrt{3})-\frac{1}{2} & (1-\frac{\sqrt{3}}{2})+1 \\ (2-\sqrt{3})+\frac{1}{2} & 4-2\sqrt{3} & (2-\sqrt{3})-\frac{1}{2} \\ -\frac{\sqrt{3}}{2} & (2-\sqrt{3})+\frac{1}{2} & 1 \end{pmatrix}$$

$$v' = \begin{pmatrix} 1 & (2-\sqrt{3})-\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ (2-\sqrt{3})+\frac{1}{2} & 4-2\sqrt{3} & (2-\sqrt{3})-\frac{1}{2} \\ -\frac{\sqrt{3}}{2} & (2-\sqrt{3})+\frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} 4 \\ -11 \\ 2 \end{pmatrix}$$

$$v' = \begin{pmatrix} 8.82 \\ -3.29 \\ -9.91 \end{pmatrix}$$

(b) (8 pts) Suppose the rotation is followed by a rotation about the vector $(-1, 1, 0)$ through an angle of $\frac{\pi}{3}$. Give the quaternion that describes the composite rotation of the two rotations.

Problem 2

(10 pts) A transformation in $\mathbb{R}^3$ starts with a rotation about the axis $(-1, 1, 1)^\top$ through an angle of $\frac{\pi}{3}$ and follows with a translation by $(0, 10, -5)^\top$. Locate the screw axis for this spatial displacement in Plücker coordinates. You are required to use quaternions to perform rotations whenever needed.

[Hint: Make use of Rodrigues' rotation formula from the lecture notes titled "Rotations in the Space".]

There is also a programming problem