1) (a) $\{HHH, HHT, HTH, THH$
$TTH, THH, TTT, HTT\}$

(b) $A = \{TTH, THT, HTT\}$
$B = \{TTH, THT, HTT, TTT\}$
$C = \{HHH, THH\}$

(c) $\bar{A} = \{HHH, HHT, HTH, THH, TTT\}$
$A \cup B = \{TTH, THT, HTT, TTT\}$
$A \cap B = \{TTH, THT, HTT\}$
$A \cap C = \emptyset$

2) C: Knows c++
F: Knows Fortran
B: Knows both

(a) $P(F) = 0.4$

(b) $P(\overline{F} \cap \overline{C}) = 0.2$

Know c++ Know both

$0.7 - 0.5 = 0.2$
$0.6 - 0.5 = 0.1$ Know only c++ / fortran
$+ 0.5$ Know both
0.8

Know Fortran

$1 - 0.8 = 0.2$

(c) $P(C - B) = 0.2$

3.)

$10 \cdot 65^6 \cdot 55 = 4.148 \times 10^{13}$

$= 41$ trillion

1 number

1 non-number

6 of any character or number

$26 \times 2 + 3 + 10 = 65$

lower + upper   special   numbers

4) $\binom{12}{4} = 495 \leftarrow$ Total combinations

(a) $\dfrac{\binom{7}{4}}{495} = 0.\overline{07}$

(b) $\dfrac{\binom{7}{1}\binom{4}{2}\binom{1}{1}}{495} = \dfrac{6 \cdot 7 \cdot 1}{495} = 0.0\overline{84}$

(c) $1 - \dfrac{\binom{5}{4}}{495} = \dfrac{490}{495} = 0.9\overline{89} \approx 0.99$

5) (a) $\mathbb{P}(A) = 0.7$

$\mathbb{P}(B) = 0.65$

$\mathbb{P}(A \cap B) = 0.4$

(b) $\mathbb{P}(A|B) = \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \dfrac{0.4}{0.65} = 0.615$

(c) $\mathbb{P}(B|A) = \dfrac{\mathbb{P}(A \cap B)}{P(A)} = \dfrac{0.4}{0.7} = 0.571$

(d)

If $\mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B)$

AND

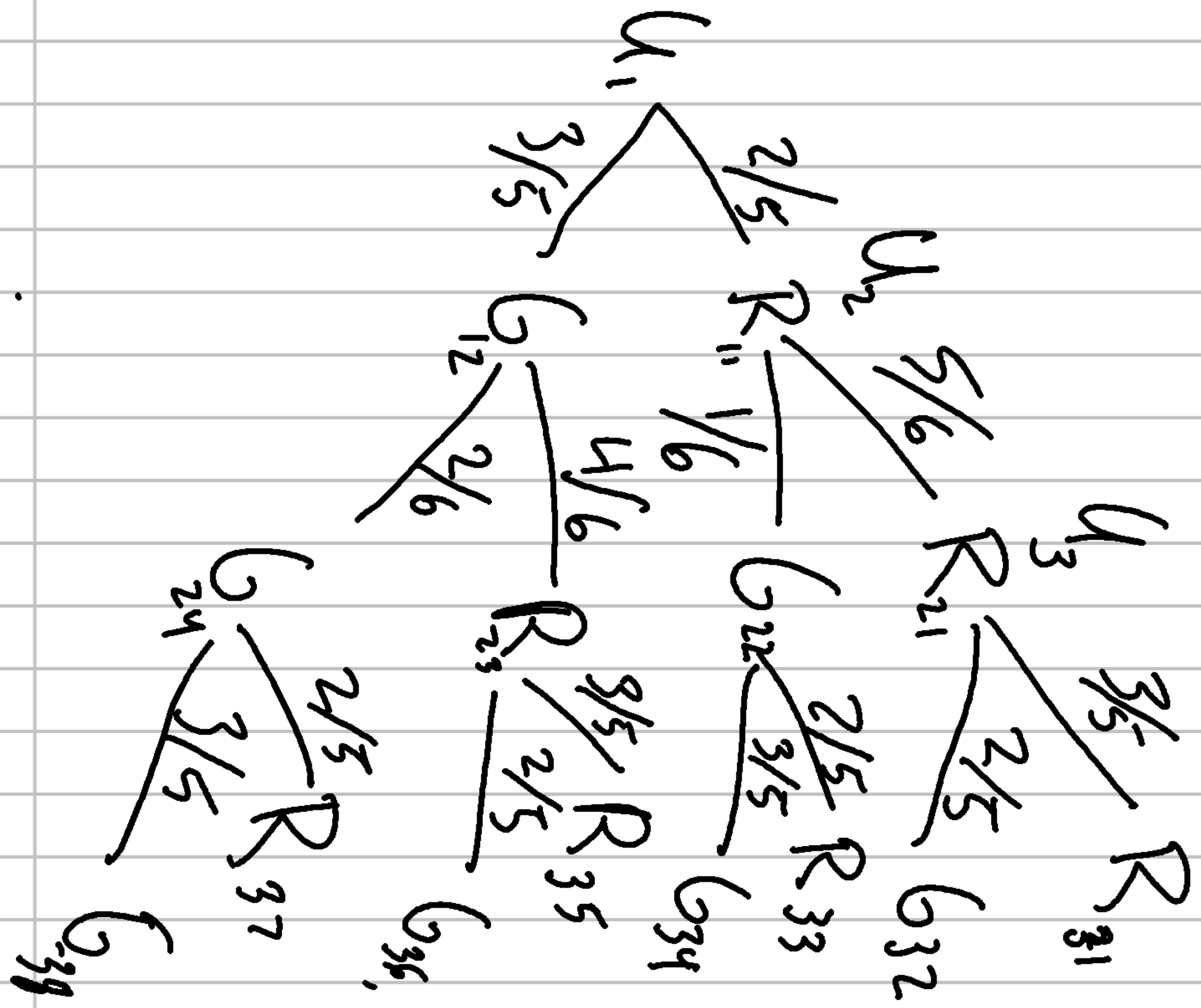
$\mathbb{P}(A|B) = \mathbb{P}(A)$ if $P(B) \neq 0$

$0.615 \neq 0.7$

then independent

Since at least the second statement is not true then A and B are not independent.

6)



$$\mathbb{P}(R) = \left(\mathbb{P}(R_{11}) \cdot \mathbb{P}(R_{21} \mid R_{11}) \cdot \mathbb{P}(R_{31} \mid R_{21}\right) +$$

$$\left(\mathbb{P}(R_{11}) \cdot \mathbb{P}(G_{22} \mid R_{11}) \cdot \mathbb{P}(R_{33} \mid G_{22}\right) +$$

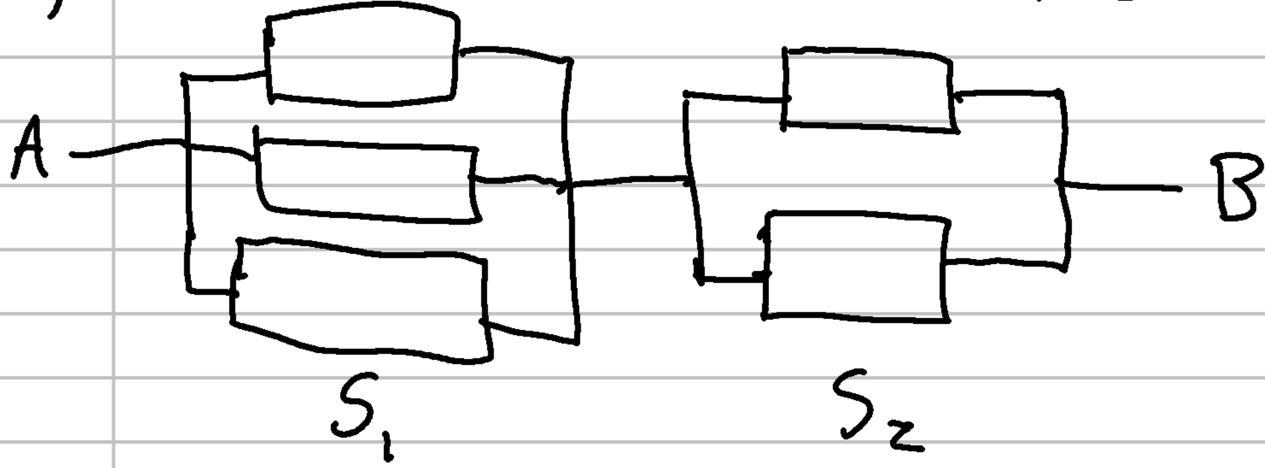
$$\left(\mathbb{P}(G_{12}) \cdot \mathbb{P}(R_{23} \mid G_{12}) \cdot \mathbb{P}(R_{35} \mid R_{23}\right) +$$

$$\left(\mathbb{P}(G_{12}) \cdot \mathbb{P}(G_{24} \mid G_{12}) \cdot \mathbb{P}(R_{37} \mid G_{24}\right) =$$

$$\left(\frac{2}{5} \cdot \frac{5}{6} \cdot \frac{3}{5}\right) + \left(\frac{2}{5} \cdot \frac{1}{6} \cdot \frac{2}{5}\right) + \left(\frac{3}{5} \cdot \frac{4}{6} \cdot \frac{3}{5}\right) +$$

$$\left(\frac{3}{5} \cdot \frac{2}{6} \cdot \frac{2}{5}\right) = 0.54\overline{6} \approx 0.55$$

7) fail prob: 0.3



$P(\text{system works}) = P(1 \text{ component in } S_1 \text{ works}) \cdot P(1 \text{ component in } S_2 \text{ works})$

$P(1 \text{ component in } S_1 \text{ work}] =$

$1 - P(\text{all components fail}) = \prod_{j=1}^{3} P(c_j \text{ fails}) =$

$1 - (0.3 \cdot 0.3 \cdot 0.3) = 0.973$

$P(1 \text{ component in } S_2 \text{ works}) = \prod_{j=1}^{2} P(c_j \text{ fails}) =$

$1 - (0.3 \cdot 0.3) = 0.91$

$P(\text{system works}) = 0.88543$

8)

(a) $P(\text{play high}) \cdot P(\text{lose} \mid \text{play high}) =$

$0.15 \cdot 0.65 = 0.0975$

(b) $P(\text{win}) = \sum_{j=1}^{3} P(\text{win} \mid \text{Rank}) P(\text{Rank})$

$P(\text{win} \mid \text{novice}) P(\text{novice})$
$0.75 \cdot 0.3 = 0.225$

$P(\text{win} \mid \text{mid}) P(\text{mid})$
$0.5 \cdot 0.55 = + 0.275$

$P(\text{win} \mid \text{high}) P(\text{high})$
$0.1 \cdot 0.15 = + 0.015$

$P(\text{win}) = 0.515$

(c) $P(\text{high}) \cdot P(\text{win} \mid \text{high})$
$0.1 \cdot 0.015 = 0.015$