Mahatru Miryala

Introduction

Humanity has set its sights on Mars for a long time now, we've landed many rovers and plan to land humans there by the 2030s. Getting to Mars offers a completely new vision for the human goal to expand outside the solar system. As a person who is very interested in the astronomical field, I felt it would be fitting if I made my project based on doing the math to get to Mars. The SLS, Space Launch System, is a spacecraft currently in development and is what NASA plans to use in future deep space missions. I will use the SLS as my rocket for this project for when I need to substitute in values. Most physics-based equations have already been made and proven but I will have to know how to use them in order to accomplish my goal. There are multiple techniques used in orbital mechanics that I must learn as well to use to model a trip to Mars. Since the actual calculations for this journey are so complex and have to take so much into account, I will have 6 assumptions during this project: the orbits of Earth and Mars are circular and centered on the Sun, Earth and Mars travel at constant speeds, Earth and Mars orbit in the same plane, the rocket can travel with instant impulses of thrust rather than extended burn times, air resistance is negligible, and there are no interfering factors such as asteroids or the rocket exploding from a production error.

Modeling the Rocket

Gathering Information

Now it's time to model the rocket based off of real information. According to NASA's website, the SLS (Space Launch System) is the rocket they will use to eventually head to Mars. The SLS is designed to use four RS-25 engines and two solid rocket boosters (SRBs). Each RS-25 uses 12,525 pounds of fuel per second and the two SRBs use 1,100 pounds per second. In total, the rocket will use 13,625 pounds of fuel per second at maximum output (1500 gallons). The bulk of the fuel will be used up by this stage, but there is still another stage with a smaller engine to be used after leaving the Earth. The engine for this second stage is called the Interim Cryogenic Propulsion Stage (ICPS). Thanks to NASA releasing some information on a previous major rocket, the Saturn V, I can use these models and some numbers that I've calculated to model mass, acceleration, and velocity functions. These are all related to each other as well. It is important to note that while these equations will be made under the assumptions stated in the beginning, I am now accounting for the fact that burn times will not be instantaneous.

Mission Plan

Before the equations are listed, it is important to understand the mission plan as there are some numbers that wouldn't match properly otherwise. Launch sequence times are based off the April 7 launch of the Odyssey rover sent to Mars and velocities of the rocket. In short, the rocket is to begin liftoff when the time, t, is 0. The SRBs are going to run out of fuel 272.7273 into the flight, and they will be dropped almost 3 seconds later when t is at 276 seconds. The core stage is expected to run out of fuel at 500 seconds, and the rocket will separate stages 3 seconds after. The stage 2 engines will engage at 511 seconds in, to enter the transfer orbit, and shut off at 878 seconds. The equations will not account for the mass lost due to dropping the SRBs and the core stage as the time progresses, but they are accounted for between the equations.

Mass Function

The rocket will be losing mass at different times because of the different stages. Because of this, multiple functions would be best to model the equation for mass. The first part of the function would include the total mass and take into account the SRBs and the core stage rocket. Since it takes about 272.7273 seconds for this to run out, the equation m(t) = -15,354t + 5,632,700 when $0 \leq \mathrm{t} \leq 272.7273$ can be used to approximate mass during takeoff. Unfortunately, information on the ICPS is scarce and information pertaining to fuel consumption and such is not available. However, while I was not able to find direct information, I found the burn time of the second stage to be 1125 seconds and the fuel mass to be 6,010 lbs. I could then divide to find fuel use in pounds per second. This equation for the third part is m(t)=-53.34t+67,700 when $0 \leq \mathrm{t} \leq 367$. Put together, the functions are:

$$m(t) = -15{,}354\mathrm{t} + 5{,}632{,}700, \quad 0 \leq \mathrm{t} \leq 272.7273$$
$$m(t) = -4354\mathrm{t} + 2{,}432{,}700, \quad 0 \leq \mathrm{t} \leq 500$$
$$m(t) = -53.34\mathrm{t} + 67{,}700, \quad 0 \leq \mathrm{t} \leq 367$$

Acceleration Function

Acceleration is also a vector quantity. It focuses on the change of velocity, whether it be increasing or decreasing. Acceleration will be affected by the mass equations as the burning of the rocket fuel will be what moves the rocket. An equation for the forces acting on the rocket can be $\sum F = \frac{\Delta m \times v}{\Delta t} - \frac{GM \times m(t)}{r(t)^2}$ for when the rocket is escaping Earth. However, r(t) cannot be calculated without further knowledge of the rocket and the associated math. Thus, I will make an equation for acceleration where Earth's gravity is not needed to be taken into account - this is when the rocket will accelerate to enter the Hohmann orbit. By rewriting the force equation:

$$a = \frac{m'(t) \times v_{exhaust}}{m(t)} = \frac{-53.34 \times (-14.15)}{67{,}700 - 53.34t} = \frac{754.761}{67{,}700 - 53.34t}, \quad 0 \leq t \leq 367$$

Velocity Function

Velocity is the speed at which an object moves. It is a vector, meaning that it includes direction. Since the acceleration of the rocket is due to the fuel being burned, the velocity will be directly related to mass and acceleration. An equation for velocity is $v_{final} = v_{initial} + at$. Plugging in the proper values for this equation results in:

$$v_f = 11{,}200 + t\left(\frac{754.761}{67{,}700 - 53.34t}\right)$$

This equation for velocity can be calculated with the restraints of $0 \leq t \leq 367$, just like with the acceleration function. After that, the velocity will remain constant throughout the transfer until it reaches Mars where it will be affected by gravity once again.

Escape Velocity

Put simply, finding the escape velocity will be to find the speed at which the rocket must be traveling to not be stopped and returned to the Earth by its gravity. Obviously, this velocity gets weaker the farther away you are from the object and that kinetic energy gets converted into potential energy. This can be described as a limit: as the distance the rocket is from Earth reaches infinity, the velocity will infinitely reach 0. The rocket will stop when it reaches an infinite distance from Earth, but it won't be able to fall back. To derive the equation for the speed, calculus can be used (it is important to note that I am assuming air resistance is negligible). In the equations, G represents the gravitational constant, M represents the mass of the Earth, g is the acceleration due to gravity (9.8 m/s), m is the mass of the rocket, and r is the distance (in meters) between the rocket and the center of the Earth. Force can be modeled as $F = \frac{GmM}{r^2}$. Since Work is given by Force times Distance, the Work needed to move a Distance of dr is given by $dW = F \times dr = -\frac{GmM}{r^2} dr$. It's negative since the Force is acting in the opposite direction of dr. To sum up the total work:

$$W = \int_{r_0}^{\infty} -\frac{GmM}{r^2} dr = -\frac{GmM}{r_0} = -mgr_0$$

This fits in with the equation for conservation of kinetic energy, $K + W = 0$ which expands to $\frac{1}{2} mv_E^2 - \frac{GmM}{r_0} = 0$, giving $v_E = \sqrt{\frac{2GM}{r_0}} = \sqrt{2gr_0}$. Plugging in the proper values, $v_E = \sqrt{2(9.8)(6378100)} = 11180.8211 \ \approx 11.2 \text{ km/s}$. Thus, the escape velocity needed to leave Earth if air resistance is negligible is approximately 11.2 km/s.

The Hohmann Transfer

Values and Velocity

Now, the next goal is to find the extra velocity that the rocket needs to get to Mars from Earth. The most efficient method of travelling between orbits is known as the Hohmann transfer, as it uses the least energy. The premise of it is that the spacecraft can use this method to move from a lower circular orbit to a higher one and vice versa. In order to increase its orbit, the spacecraft must add energy. If done properly, the new elliptical orbit should intersect Mars's orbit at its vertex.

In order to perform the calculations, I'll need the proper values. $r_1$will be the distance of Earth from the Sun and $r_2$will be the distance of Mars to the Sun. GM is a constant known as the standard gravitational parameter, which is the gravitational constant multiplied by the mass of the Sun. $r_1 = 149{,}600{,}000$ km, $r_2 = 227{,}920{,}000$ km, $GM = 1.327 \times 10^{11} \frac{km^3}{s^2}$.

The next step is to find the orbital periods (in seconds) of Earth and Mars, or the time it takes for them to revolve around the sun. $p_1$will be Earth's period and $p_2$will be Mars's period. $p_1 \approx 365.26$ days $\times$ $86{,}400 = 31{,}558{,}464$ s $p_2 \approx 686.68$ days $\times$ $86{,}400 = 59{,}329{,}152$ s.

Next, I have to find the semi-major axis of the orbit, denoted by a. Since the purpose of this mission is to go to Mars, it can be calculated with:

$$a = \frac{r_1 + r_2}{2} = \frac{(149.6 \times 10^6) + (227.92 \times 10^6)}{2} = 188{,}760{,}000 \text{ km.}$$

The period of the transfer orbit can be found with Kepler's third law. By plugging in the previous values, $p_t = \sqrt{\frac{4\pi^2 a^3}{GM}} = 44731077.52$ s.

Earth's velocity is simply $v_1 = \frac{(2\pi \times r_1)}{p_1} \approx 29.78 \frac{km}{s}$. Mars's velocity is $v_2 = \frac{(2\pi \times r_2)}{p_2} \approx 24.14 \frac{km}{s}$.

The spacecraft will have an uneven velocity due to its elliptical orbit. The point where the spacecraft will be closest to the Sun, called the perihelion, will also be where it has the most velocity. The velocity at its aphelion, or the point farthest from the Sun (and where it will intersect Mars's orbit), has the least velocity. Both velocities can be found with their formulas:

$$v_p = \left(\frac{2\pi \times a}{p_t}\right)\left(\sqrt{\frac{2a}{r_1} - 1}\right) \approx 32.73 \frac{km}{s}$$

$$v_a = \left(\frac{2\pi \times a}{p_t}\right)\left(\sqrt{\frac{2a}{r_2} - 1}\right) \approx 21.48 \frac{km}{s}$$

The final step is to find Δv_1 and Δv_2 - which are the changes in velocity needed to enter the transfer orbit and leave it, respectively. Δv_2 is not needed since the spacecraft will meet Mars at exactly half of its transfer orbit.

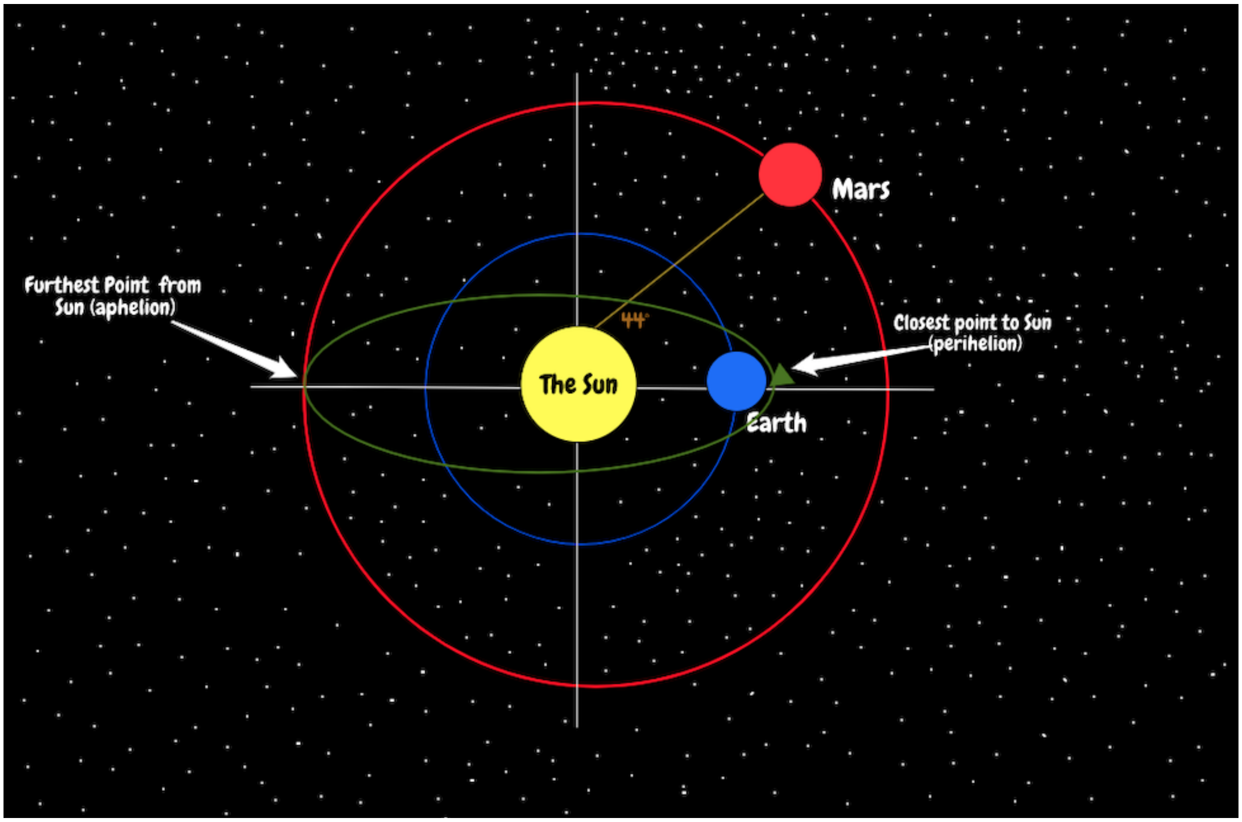
$\Delta v_1 = v_p - v_1 \approx 32.73 - 29.78 = 2.95 \ \frac{km}{s}$ $\Delta v_2 = v_2 - v_a \approx 24.14 - 21.48 = 2.66 \ \frac{km}{s}$

<u>Travel Time</u>

The next step is to find out how long it will take for the rocket to complete half of its rotation around the orbit. This is found by just multiplying ½ with p_t, giving $t = 22{,}365{,}538.76$ s. Dividing by 86,400 results in approximately 258.86 days and dividing that by 30.417 results in an estimated 8.51 months.

<u>Angular Alignment</u>

When should the rocket be launched? This is a problem since the rocket will move while Mars moves on its orbit; they have to be at the same place at the same time. In order to find the optimal relative position of Earth and Mars, finding the distance each planet travels per day is a good start. Mars completes one revolution around the Sun ($360°$) every 686.68 days. $360 \div 686.68 \approx 0.524$, so, Mars travels $0.524°$ per day. The spacecraft, as established previously, will take almost 258.86 days to complete half of the orbit. This means that Mars will have traveled $135.64264°$ by the time the rocket completes its journey. The spacecraft is planned to start the transfer when it's at its perihelion and meet Mars at its aphelion. All that is needed then is: $180° - 135.64264° = 44.35736°$. According to the earlier stated assumptions, if Mars and Earth are revolving in the same plane, Mars must be approximately $44°$ ahead of Earth in orbit when the Hohmann orbit is started.



This image is a visual representation of the orbits and flight path of the spacecraft, Earth, and Mars at the beginning of the journey.

Conclusion

While I did have to learn a lot of new physics and math, the project overall reached a satisfactory conclusion. I calculated the escape velocity of Earth, travel time, a Hohmann transfer, and where Mars had to be in its orbit in relation to Earth. While I did have a good start on this project, I think that there is substantial room for progress still. I can add in more details and account for more factors such as how Jupiter's gravity affects the spacecraft once you start getting closer to Mars and the air resistances and gravities of Earth, the Moon, the Sun, and Mars. A major topic that I wanted to address but was far too complicated to understand was how air resistance would change while the rocket flies through the atmosphere. The assumptions I stated allowed the project to be easier and more understandable, but it isn't advanced enough to be applied to a real rocket launch.

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