Deep Learning: More About LLMs

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November, 2025

Summary up to now

Defined main neural network architecture pieces, discussed training methods.

Focused on the Attention mechanism and Transformer architectures, the central building blocks of modern LLMs.

Today: present more in detail a few aspects of the LLM training pipeline.

Motivation: Towards Large Language Models

Goal: provide some insights into modern practices in LLM development to understand what has become a central piece of technology today.

We will focus on a few key aspects of LLM training pipelines, notably some architectural choices, and fine-tuning strategies.

What we will not discuss at all:

- infrastructure: distributed training, memory optimization
- precision: quantization, data types, mixed precision
- evaluation: benchmarks, metrics

and many other important aspects.

Some Case studies: SmolLM, Qwen

Two loose guiding cases:

- SmolLM3 [paper] [model]
- Qwen [paper] [model]

We will refer to these two models to illustrate the concepts discussed.

Setting: Training a chat model, with two main parts

- Pretraining with next-token prediction on large corpora of text data
- Post-training: modify the model to guide the ouptput behavior towards desired characteristics

Focus on some key aspects of the training pipeline.

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Scale and Parameter type: the Example of GPT-3

Three types of layers:

- Embedding layer: maps input tokens to vectors. Size: $V \times d$.
- Self-attention layers: L layers, each with self-attention.
- Feed-forward layers: L layers, each with two linear layers and a non-linearity.

Model	**Layers**	d_{model}	d_{ff}	**Heads**	**Total Params**
175B	96	12288	49152	96	175B

Layer Type	**Formula**	**Parameters**	**Fraction**
Self-Attention	$4d_{\text{model}}^2$	58 B	33%
Feedforward	$2d_{\text{model}}d_{\text{ff}}$	116 B	66%
LayerNorms + biases	$\approx 9d_{\text{model}}L$	< 0.01 B	$< 0.01\%$
Token Embedding	Vd_{model}	0.62 B	0.4%
Total		**$\approx$175 B**	**100%**

Table: GPT-3 175B parameter count per component.

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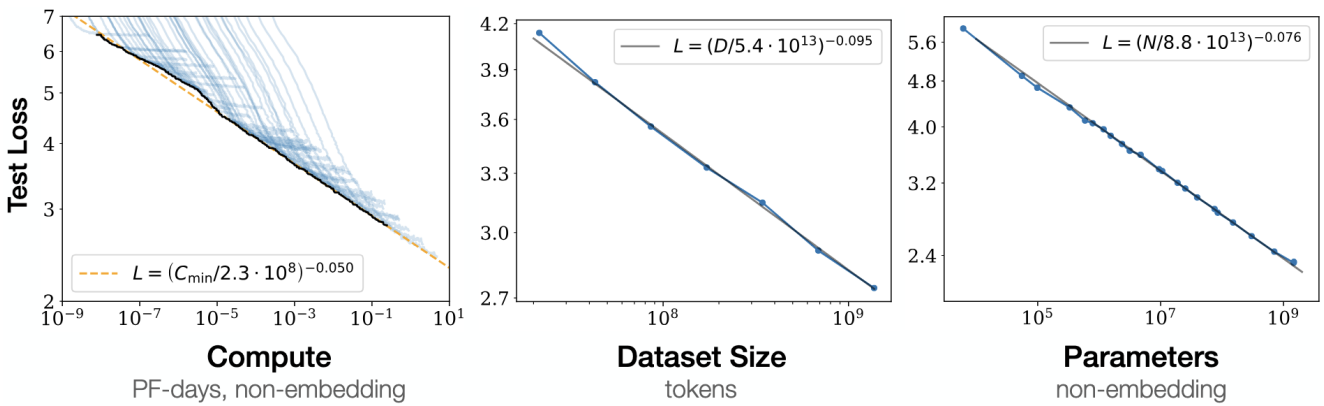
Empirical Scaling Laws

Empirical relation between loss L and parameters N, data D, compute C:
For a fixed N, increase C (or, equivalently increase D). Then at some point $C^\star(N)$ the loss decrease slows down. Denote the value of the loss at this point $L^\star(C)$.

$$L^\star(C) \propto C^{-\alpha_C} \quad \text{and} \quad L^\star(N) \propto N^{-\alpha_N} \quad \text{and} \quad L^\star(D) \propto D^{-\alpha_D}.$$



- Data is number of fresh tokens seen during training, so compute is proportional to $\approx 6N \times D$.
- Scale N is number of parameters. Model is a transformer decoder, scale is changed by increasing number of layers, hidden size, attention heads

Using Scaling Laws

Scaling laws provide:

- An estimation of optimal model size and data for a given compute budget.
- A prediction of the loss at this optimal point.

Kaplan et al. (2020) estimate that given C, optimal N and D should scale as

$$N_{\text{opt}} \propto C^{0.73} \quad D_{\text{opt}} \propto C^{0.27} .$$

i.e., as compute budget grows, models should grow faster than new training data.

Scaling with Optimal Compute Dependence

Model

$$L(N, D) = L_\infty + aN^{-\alpha_N} + bD^{-\alpha_D}$$

Keeping compute C fixed, we have $D = C/N$, so

$$L(N, C) = L_\infty + aN^{-\alpha_N} + b\left(\frac{C}{N}\right)^{-\alpha_D}$$

so optimal N should satisfy

$$N^{-\alpha_N} \approx \left(\frac{C}{N}\right)^{-\alpha_D}$$

i.e.

$$N^\star \approx C^{\alpha_D/(\alpha_D+\alpha_N)}. \quad \text{and} \quad D^\star \approx C^{\alpha_N/(\alpha_D+\alpha_N)}.$$

And the loss will decrease as $(N^\star)^{-\alpha_N}$

Improved Measurements:

- Hoffmann et al. (2022): $\alpha_N \approx 0.34$, $\alpha_D \approx 0.28$. Then $N^\star \approx C^{0.45}$ and $D^\star \approx C^{0.55}$, and $L(C) \approx C^{-0.16}$.

Chinchilla Scaling Recommendation

Scale data proportionally to model size. Empirical rule of thumb:

Limits of scaling (laws)

Scaling laws have been a strong driver of the push for ultra-large models, providing quantitative predictions of the benefits of scaling.
As for any empirical laws, they have limitations:

- Loss will not decrease indefinitely: natural language has inherent entropy, and real-worlds models are bound by physical limitations + data is not infinite.
- Practical factors may change scaling: data quality, architecture changes, optimization practices, etc.
- Compute budget at training is not the only scarce resource and one must also consider inference cost in the loop, which favor smaller models too.

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Motivations for architectural changes

Standard transformer decoder architecture has limitations.

- **Quadratic memory** and compute cost of attention limits context length.
- Dense models have **high inference cost** for large parameter counts.
- Need for better **length generalization**, long-context handling + efficiency.

Mixture of Experts: Goal

- Large models are costly to deploy.
- **Key idea:** Activate only a subset of model parameters per input.
- **Mixture of Experts (MoE)** introduces conditional computation:

$$y = \sum_{i=1}^{M} g_i(x) f_i(x)$$

where:

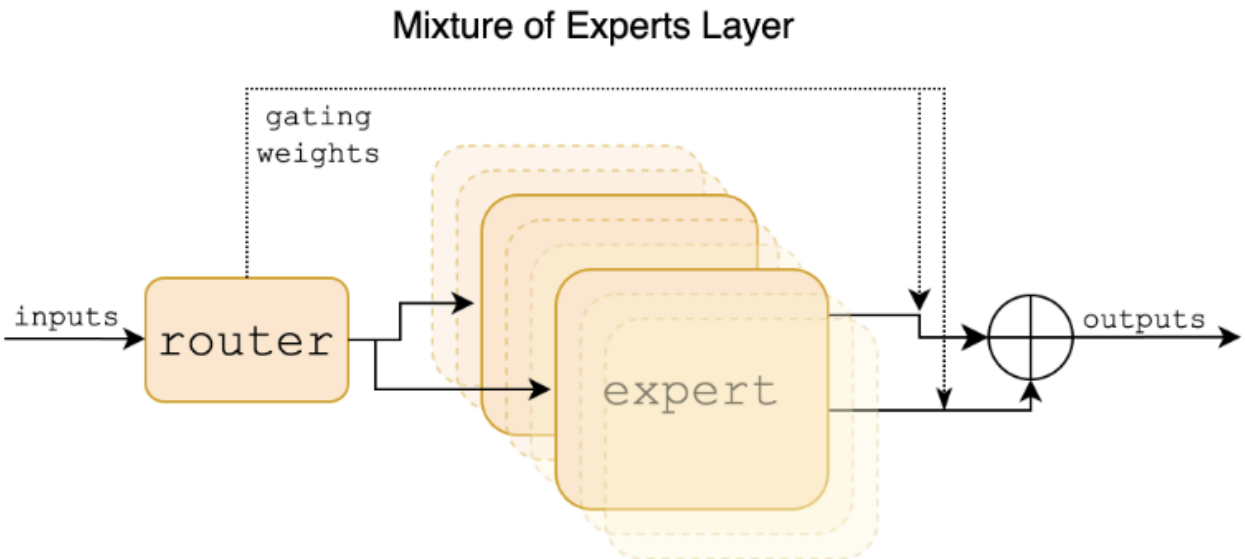
- $f_i(x)$ = output of expert i
- $g_i(x)$ = gating weight (learned function, typically softmax)

- A **gating network** selects one or few experts for each input.
- Each **expert** is a small feedforward network (FFN or MLP).
- Typical MoE block replaces the Transformer feed-forward layer.

Goal

Scale model capacity without increasing inference cost per token.

Architecture Overview



Gating layer outputs logits over experts. Only activate the top-k experts. Compute their outputs and combine them using the gate probabilities.

Issue with argmax is non-differentiability.

Gumbel-Softmax Trick

Theorem: Gumbel-Softmax

If g_i are logits, and G_i are i.i.d. Gumbel(0,1) random variables i.e. if $U_i \sim \text{Uniform}(0, 1)$ and $G_i = -\log(-\log(U_i))$, then

$$\arg\max_i(g_i + G_i) \sim \text{softmax}(g)$$

Idea: use this to have argmax in forward pass and softmax in backward pass, so that the gradient flows coherently through the softmax approximation, while the forward and backward are consistent (in distribution).

In practice, MoEs can collapse to using only a few experts: add a **load balancing loss** that encourages uniform expert usage.

MoE saves inference compute

Since the FF layers account for ∼66% of parameters and compute in GPT-3, replacing them with MoE layers can greatly increase parameter count without increasing inference cost.

e.g for known MoE models and their active parameter usage per token.

Model	**Total Params**	**Experts**	k	**Active Params / Token**
Switch Transformer	∼1.5T	2048	1	∼ 1.4B
Mixtral 8×7B	46.7B	8	2	12.9B
Qwen3-235B-A22B	235B	128	2	22B
Qwen3-30B-A3B	30B	–	–	3B

Positional Encoding: Motivation

- The Transformer architecture has no inherent notion of order.
- Self-attention is permutation-invariant: shuffling tokens yields the same output.
- We must inject positional information into token embeddings.

Posisitional crucially affects **context-length generalization**, i.e., the ability of models to handle sequences longer than those seen during training.
Recall the Transformer architecture can handle variable-length sequences, and has a priori no limit on context length. With positional encoding, we hard code some notion of order and sequence length in the model.

Formulation

Let $x_i \in \mathbb{R}^d$ be the embedding of token i.
We add a position-dependent vector p_i:

$$z_i = x_i + p_i$$

The model learns to use these p_i to infer order relationships (who attends to whom in attention).

$$PE_{(pos,2i)} = \sin\left(\frac{pos}{10000^{2i/d}}\right),$$
$$PE_{(pos,2i+1)} = \cos\left(\frac{pos}{10000^{2i/d}}\right)$$

Each dimension corresponds to a sinusoid of a different frequency.

- Smoothly encodes positions; nearby positions have similar encodings.
- Enables extrapolation to unseen sequence lengths
- Hard-codes absolute position in the sequence

Rotary Positional Encoding (RoPE)

Recent models use **Rotary Positional Encoding** on keys and queries:

$$\text{RoPE}(q_p) = R(p)q_p \qquad \text{RoPE}(k_p) = R(p)k_p$$

where $R(p)$ is a rotation matrix:

$$R_p = \begin{pmatrix} R(\theta_1(p)) & 0 & 0 & \cdots & 0 \\ 0 & R(\theta_2(p)) & 0 & \cdots & 0 \\ 0 & 0 & R(\theta_3(p)) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & R(\theta_{d/2}(p)) \end{pmatrix}, \qquad R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

and

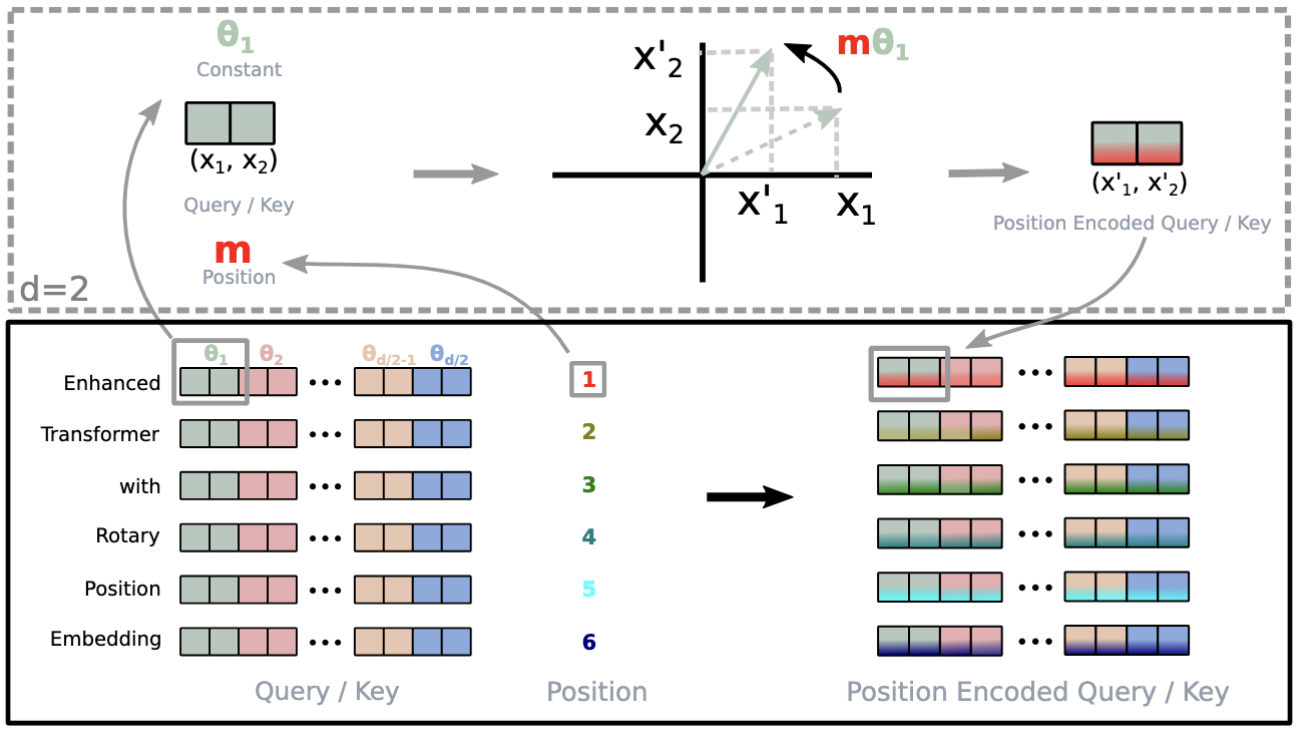
$$\theta_i(p) = \frac{p}{10000^{2i/d}}$$

Then when computing attention between tokens i and j:

$$\langle \text{RoPE}(q_p), \text{RoPE}(k_{p'}) \rangle = \langle q_p, R_{p-p'} k_{p'} \rangle$$

- Encodes relative position via phase differences.
- Supports extrapolation and efficient attention computation.

RoPE


θ1
Constant
(x1, x2)
Query / Key
m
Position
d=2
x'2
x2
x'1
x1
mθ1
(x'1, x'2)
Position Encoded Query / Key
θ1
θ2
θd/2-1
θd/2
Enhanced
Transformer
with
Rotary
Position
Embedding
1
2
3
4
5
6
Query / Key
Position
Position Encoded Query / Key

Other design choices that matter

Attention variants: no need to have different weights for keys and queries for different heads in the same layer, Grouped-Query Attention (GQA) or Multi-Query attention share the weights.

Masking: many variants are possible in masking on top of causal masking: local attention, sliding window, etc.

Tokenizer: the way text is broken into tokens greatly affects performance, in hard to predict ways. Vocabulary size for Qwen3 and SmolLM3: $\sim$150k tokens.

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Instruction Fine-tuning

You have a model that is pretrained on a large corpus of data to complete text.
You want to make it answer in a specific way, e.g., follow instructions, answer questions.

Build a chat template that will define the input prompt structure.
(Possibly with a 'system prompt' prepended to every input.)

Example:

```
<|im_start|>user
What's the weather like in New York?<|im_end|>
<|im_start|>assistant
The weather in New York is currently sunny with a temperature of
22C. The humidity is at 45\% with a wind speed of 10 km/h.  It's
a great day to be outside!<|im_end|>
```

Train model to produce the assistant response given the user prompt.

Supervised Fine-Tuning (SFT)

Goal: Adapt a pretrained language model to follow instructions and align with human intent.

- Uses labeled **instruction-response pairs**: (x, y^*).
- Trains via standard cross-entropy loss:

$$\mathcal{L}_{\text{SFT}} = -\mathbb{E}_{(x,y^*)} \sum_t \log p_\theta(y_t^* \mid y_{<t}^*, x)$$

  with teacher-forcing.
- Shifts model distribution toward structured outputs.

SFT Data

Examples of SFT data:

Question:
How do you simplify $(u^4\ v^3/(u^2\ v^{-1}\ )^4)^0$ and write it using only positive exponents?

Answer:
To simplify the expression $(u^4\ v^3/(u^2\ v^{-1}\ )^4)^0$ and rewrite it using only positive exponents, we start by evaluating the expression from the innermost operation outward.

First, consider the exponent of zero on the entire fraction:
Any expression raised to the power of zero is equal to 1 (provided the base is not zero). *Therefore*, $(u^4\ v^3/(u^2\ v^{-1}\ )^4)^0 = 1$

This simplification makes the other calculations unnecessary because raising any non-zero expression to the power of zero will always result in 1.

So, the simplified expression is simply: 1

Why SFT Works

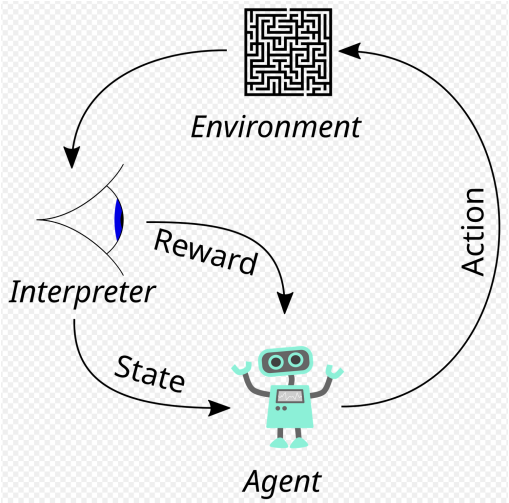
- Pretrained models learn general language priors.
- SFT teaches **task conditioning**: “Given this instruction, produce a coherent, relevant output.”
- Provides initial alignment before reinforcement learning (RLHF, DPO, etc.).
- Acts as a regularizer: keeps responses fluent but steers toward desired behaviours.

SFT narrows the model’s output manifold to align with human-like task formatting.

In pracitce, SFT is often the first step in a multi-stage alignment pipeline. We **still need to make sure the outputs are ’good/safe’ enough.** (e.g. the model can still produce harmful or untruthful outputs after SFT.)

Fine-tuning as a Reinforcement Learning Problem

Reinforcement Learning: State, Actions, Rewards.



Policy: maps states to actions.
Given a reward function that scores outputs, we can formulate fine-tuning as an RL problem:

- **State:** input prompt and conversation history.
- **Action:** next token
- **Reward:** reward of the output.

The **policy is the model**: a map from states (prompts) to actions (tokens).
Two flavors of RL: RLHF and RLVR.

RLHF (Reinforcement Learning from Human Feedback)

Idea: Use human preference data to train a *reward model* and fine-tune the base model via reinforcement learning.

Pipeline:

1. Collect human preference pairs (y^+, y^-) for the same prompt x.
2. Train a reward model $r_\phi(x, y)$ so that $r_\phi(x, y^+) > r_\phi(x, y^-)$.
3. Fine-tune the language model π_θ using RL (often PPO) to maximize expected reward:

$$\max_\theta \ \mathbb{E}_{x,y\sim\pi_\theta}[\, r_\phi(x, y)\,] - \beta\, \mathrm{KL}(\pi_\theta || \pi_{\mathrm{SFT}})$$

4. β controls deviation from the SFT model.

Goal: Align model behavior with human preference while preserving fluency and diversity.
Issue: Instability, reward overfitting ("reward hacking"), expensive human annotation.

RL with Verifiable Rewards

Idea: Get rid of model and human preferences: Train on problems that have an objective solution.

e.g. math problems, coding problems, etc.

DPO (Direct Preference Optimization)

Motivation: Avoid explicit RL training and reward model sampling.
Key idea: Optimize model directly on human preference pairs (x, y^+, y^-).

$$L_{\text{DPO}}(\theta) = -\mathbb{E}_{x,y^+,y^-}\Big[\log \sigma\Big(\beta\big(\log \pi_\theta(y^+|x) - \log \pi_\theta(y^-|x) - \log \pi_{\text{ref}}(y^+|x) + \log \pi_{\text{ref}}(y^-|x)\big)\Big)\Big]$$

- π_{ref}: reference (typically SFT) model.
- β: inverse temperature controlling preference strength.
- Equivalent to optimizing for the same fixed point as RLHF under a KL constraint.

Benefits: No RL loop, stable gradients, easier implementation. Used in Qwen3 post-training.

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LoRA: Motivation: Finetuning Large Models

Finetuning all parameters of large pretrained models is expensive.

- Billions of parameters $\Rightarrow$ huge memory and compute cost.
- Storing separate copies for each task is impractical.

Goal: Adapt models efficiently with minimal trainable parameters.

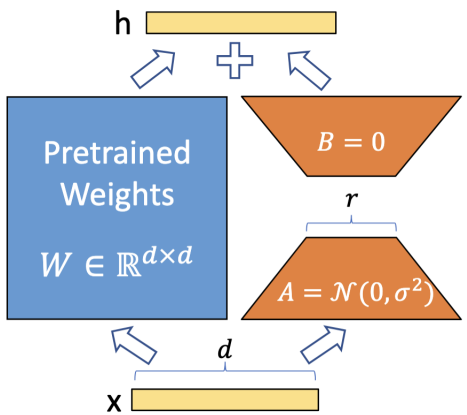
Idea of LoRA (Low-Rank Adaptation)

Instead of updating all weights $W_0 \in \mathbb{R}^{d\times k}$, we **freeze** W_0 and learn a low-rank update:

$$W = W_0 + \Delta W, \qquad \Delta W = AB, \quad A \in \mathbb{R}^{d\times r},\ B \in \mathbb{R}^{r\times k},$$

where $r \ll \min(d, k)$.

- Train only A, B (few million parameters).
- Keeps inference cost identical to original model.



Implementation in Practice

For a linear layer $y = W_0 x$, LoRA modifies the forward pass:

$$y = W_0 x + \alpha\, A(Bx),$$

where α is a scaling factor (often α/r).

- W_0 frozen, gradients flow only through A, B.
- Drop-in replacement for linear/attention projection layers.
- Often applied to query/key/value projections in Transformers.

Concretely: take a model, pick some layers. Replace the forwards by a wrapped LoRA layer with the same input-output sizes.

Why (?) It Works

- Neural networks often lie in a low-intrinsic-rank subspace.
- Adaptation directions can be well-approximated by low-rank updates.

Benefits:

- *Parameter-efficient:* train 1% of weights.
- *Composable:* merge LoRA adapters for multiple tasks.
- *Deployable:* inference identical to base model.

Can use LoRA at any stage of post-training: SFT and RL.

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Current + Future practices

More optimization: extra long context, scaling RL

Reasoning: train the models to use extra tokens to dynamically allocate extra computations before answering.

Multimodality: handle sound and images in addition to text (remember CLIP)

Agents: handle actions, tools use.

Beyond single LM systems: memory, retrieval, modular systems.

Small models: distillation, compression, efficient architectures.