Evaluation of an Appearance-Preserving Mesh Simplification Scheme

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Outline

Outline

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Make Titles Informative. Use Uppercase Letters.

Subtitles are optional.

- Item 1
- Item 2

Outline

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Make Titles Informative.

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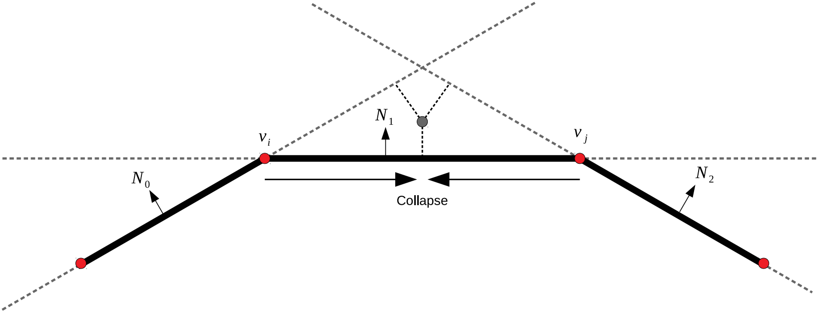
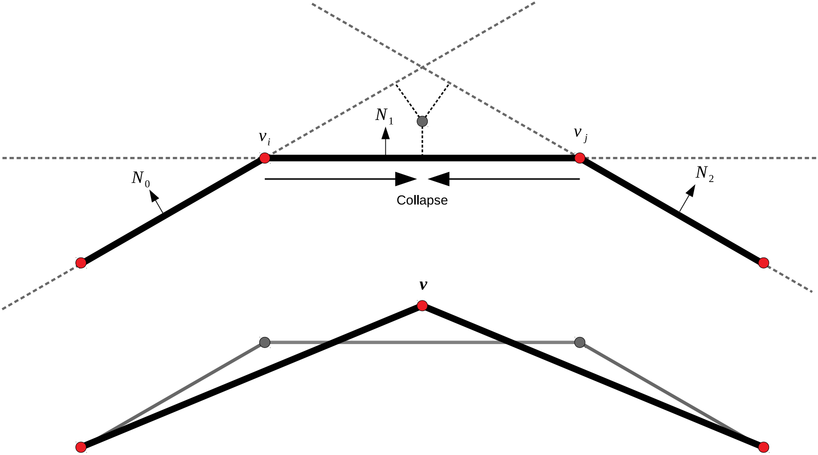
Quadric Error Metric

- Iteratively perform edge collapses $(\mathbf{v}_i, \mathbf{v}_j) \rightarrow \mathbf{v}$
- Cost based on distance to neighboring faces' planes



Quadric Error Metric



Calculating cost

Squared distance from point v to plane f

$$\mathbf{v} = [x, y, z, 1]^\mathsf{T}, \ \mathbf{f} = [a, b, c, d]^\mathsf{T}$$

$$\begin{aligned} D^2 &= (\mathbf{f}^\mathsf{T}\mathbf{v})^2 \\ &= \mathbf{v}^\mathsf{T}(\mathbf{f}\mathbf{f}^\mathsf{T})\mathbf{v} \\ &= \mathbf{v}^\mathsf{T}\mathbf{Q}\mathbf{v} \end{aligned}$$

$$\mathbf{Q} = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

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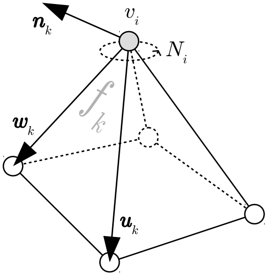
Calculating cost

Sum distances to planes f_k of triangles in neighborhood N_i of v_i

$$
\begin{aligned}
D^2 &= \sum_k \mathbf{v}_i^\mathsf{T} \mathbf{Q}_k \mathbf{v}_i \\
&= \mathbf{v}_i^\mathsf{T} \left(\sum_k \mathbf{Q}_k \right) \mathbf{v}_i \\
&= \mathbf{v}_i^\mathsf{T} \mathbf{Q}_i \mathbf{v}_i
\end{aligned}
$$



Finding Optimal Position

Optimal position $\bar{\mathbf{v}}$ after collapse $(\mathbf{v}_i, \mathbf{v}_j) \rightarrow \mathbf{v}$

$$(\mathbf{Q}_i + \mathbf{Q}_j)\bar{\mathbf{v}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \bar{\mathbf{v}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

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Overview of Algorithm

Initialization

1. Compute matrix Q for each vertex
2. Compute optimal vertex position for each edge collapse
3. Compute *cost* of each edge collapse
4. Store edge collapses in min-heap with *cost* as key

Simplification

1. Collapse edge on top of min-heap
2. Recompute optimal positions and costs
3. Repeat

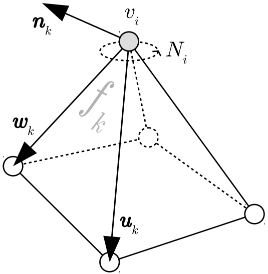
More convenient notation

$$D^2 = (\mathbf{n}^\mathsf{T}\mathbf{v} + d)^2 = \mathbf{v}^\mathsf{T}(\mathbf{n}\mathbf{n}^\mathsf{T})\mathbf{v} + 2d\mathbf{n}^\mathsf{T}\mathbf{v} + d^2 \quad (1)$$

$$Q = (\mathbf{n}\mathbf{n}^\mathsf{T}, d\mathbf{n}, d^2) = (\mathbf{A}, \mathbf{b}, c) \quad (2)$$

$$Q(\mathbf{v}) = \mathbf{v}^\mathsf{T}\mathbf{A}\mathbf{v} + 2\mathbf{b}^\mathsf{T}\mathbf{v} + c \quad (3)$$

$$\mathbf{A}\bar{\mathbf{v}} + \mathbf{b} = 0 \quad (4)$$



Quadric Error Metric with Attributes

Extend **Q** to include attributes

$$Q = (\mathbf{A}, \mathbf{b}, c) = \left(\left[\begin{array}{c|c} \mathbf{n}\mathbf{n}^{\mathsf{T}} & \ddots 0 \ddots \\ \hline \ddots 0 \ddots & \ddots 0 \ddots \end{array} \right], \left[\begin{array}{c} d\mathbf{n} \\ \hline 0 \end{array} \right], d^2 \right)$$

Quadric Error Metric with Attributes

Expected attribute value at point p:

$$\hat{s}_j(\mathbf{p}) = \mathbf{g}_j^\mathsf{T}\mathbf{p} + d_j$$

$\hat{s}_j$ interpolate the vertices of face $\left(\binom{p_1}{s_1}, \binom{p_2}{s_2}, \binom{p_3}{s_3}\right)$

$\mathbf{g}_j$ and d_j obtained by solving:

$$\begin{bmatrix} \mathbf{p}_1^\mathsf{T} & 1 \\ \mathbf{p}_2^\mathsf{T} & 1 \\ \mathbf{p}_3^\mathsf{T} & 1 \\ \mathbf{n}^\mathsf{T} & 0 \end{bmatrix} \begin{bmatrix} \\ \mathbf{g}_j \\ \\ d_j \end{bmatrix} = \begin{bmatrix} s_{1,j} \\ s_{2,j} \\ s_{3,j} \\ 0 \end{bmatrix}$$

Quadric Error Metric with Attributes

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{n}\mathbf{n}^T + \sum_j \mathbf{g}_j\mathbf{g}_j^T & -\mathbf{g}_1 \cdots - \mathbf{g}_m \\ \hline -\mathbf{g}_1 & \\ \vdots & \mathrm{I} \\ -\mathbf{g}_m & \end{array}\right]$$

$$\mathbf{b} = \left[\begin{array}{c} d\mathbf{n} + \sum_j d_j\mathbf{g}_j \\ \hline -d_1 \\ \vdots \\ -d_m \end{array}\right]$$

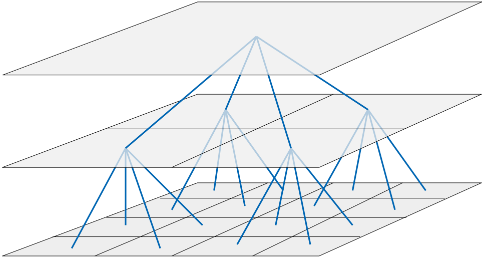
$$c = d^2 + \sum_j d_j^2$$

Problem with Texture Atlas

Bad texture values in seams

Improving Texture

- Use a pull-push algorithm to fill invalid pixels
- Creates a pyramid of images with decreasing resolution
- Each pixel is assigned weight w_i and color x_i



Find valid pixels

- Create mesh with UV-coordinates as vertices
- Cast rays toward the mesh to find valid pixels

Pull Phase

Create lower resolution level
with gaussian blur filter

$$w_i^{r+1} = \sum_k \tilde{h}_k \min(w_k^r, 1)$$

$$x_i^{r+1} = \frac{1}{w_i^{r+1}} \sum_k \tilde{h}_k \min(w_k^r, 1) x_i^r$$

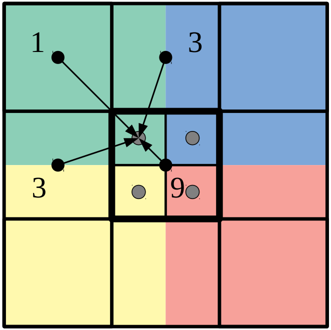
1	2	2	1
2	4	4	2
2	4	4	2
1	2	2	1

Figure: Weights of filter $\tilde{h}$

Push Phase

- Blend neighboring pixels

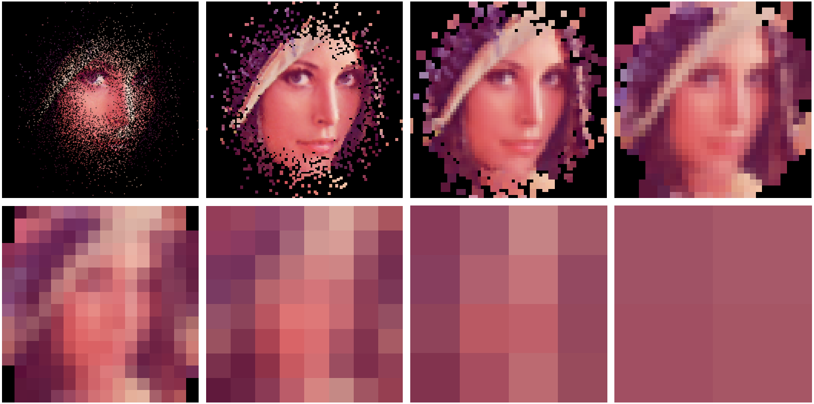
$$tw_i^r = \sum_k h_k \min(w_k^{r+1}, 1)$$
$$tx_i^r = \frac{1}{tw_i^r} \sum_k h_k \min(w_k^{r+1}, 1) x_i^{r+1}$$



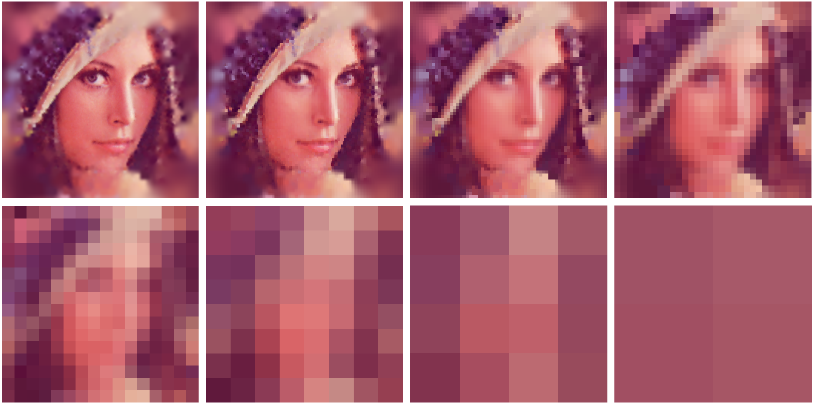
- Blend higher resolution pixels with the computed value

$$x_i^r = tx_i^r (1 - w_i^r) + w^r x_i^r$$
$$w_i^r = tw_i^r (1 - w_i^r) + w^r$$

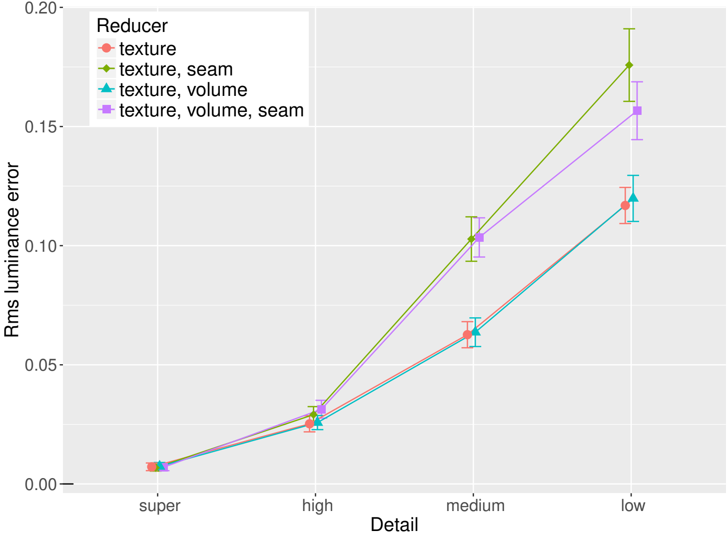
Pyramid in Pull Phase



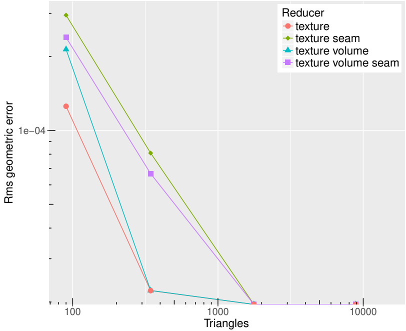
Pyramid in Push Phase



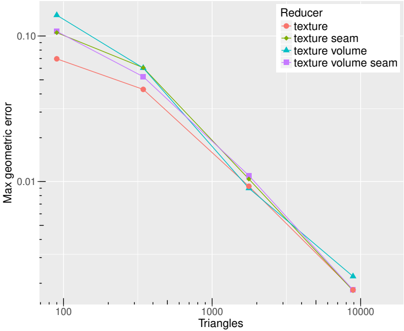
Rms Luminance Error



Geometric Error

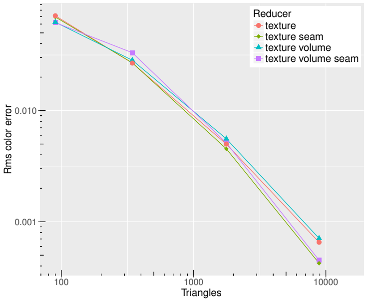


Rms geometric error

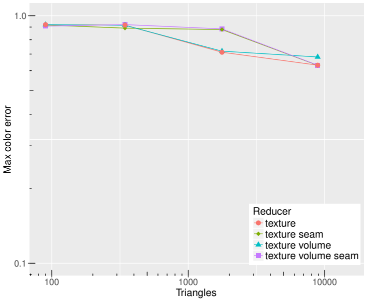


Max geometric error

Color error

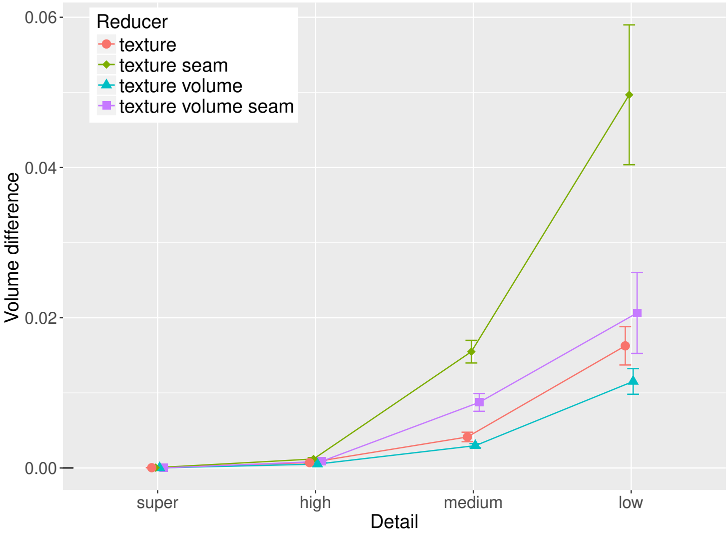


Rms color error



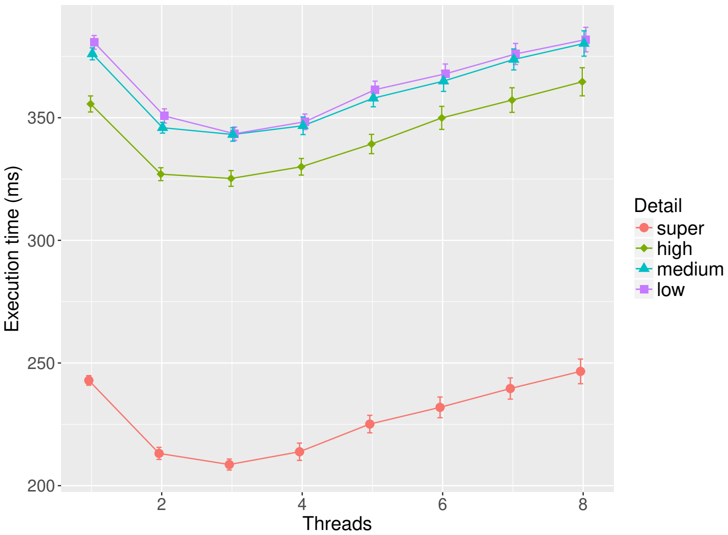
Max color error

Volume



Reducer
texture
texture seam
texture volume
texture volume seam
0.06
0.04
0.02
0.00
Volume difference
super
high
medium
low
Detail

Execution Time



Improving Texture

Original

Bound

Improved

Improving Texture

LoD:s (only geometry)

Super High Medium Low

LoD:s (geometry and texture)

Super | High | Medium | Low

LoD:s (geometry and texture)

Super High Medium Low

Summary

- The first main message of your talk in one or two lines.
- The second main message of your talk in one or two lines.
- Perhaps a third message, but not more than that.

- Outlook
  - Something you haven't solved.
  - Something else you haven't solved.

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