Evaluation of an Appearance-Preserving Mesh Simplification Scheme

Rasmus Hedin

Department of Electrical Engineering
Linköpings Universitet

2018-09-10

Outline

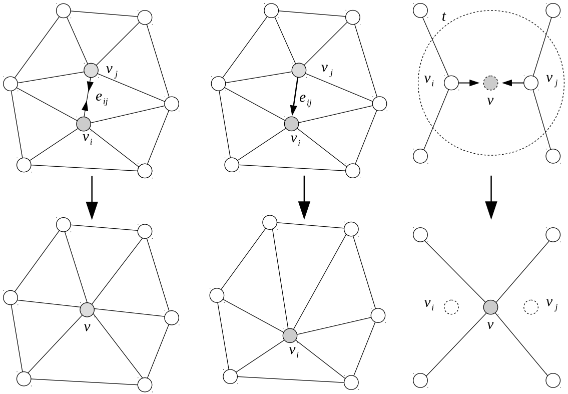
LINKÖPINGS UNIVERSITET

Problem

Only considering geometry during simplification give poor results for textured meshes

Vertex Merging

- Mesh can be simplified by merging vertices
- (a) edge collapse, (b) vertex removal, and (c) pair contraction



(a) (b) (c)

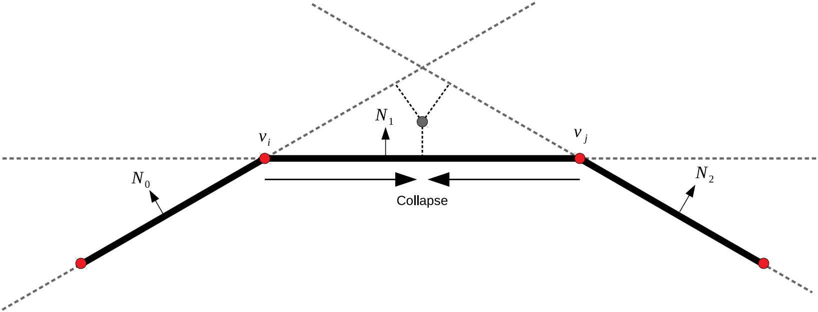
Mesh Simplification Methods

- Progressive meshes
- Quadric Error Metrics
- Appearance-preserving Simplification

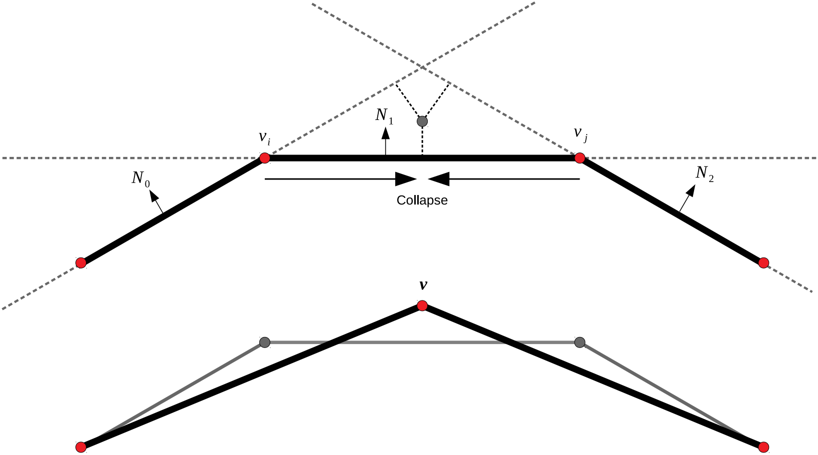
Outline

Quadric Error Metric

- Iteratively perform edge collapses $(\mathbf{v}_i, \mathbf{v}_j) \rightarrow \mathbf{v}$
- Cost based on distance to neighboring faces' planes



Quadric Error Metric



Calculating cost

Squared distance from point v to plane f

$$\mathbf{v} = [x, y, z, 1]^\mathsf{T}, \ \mathbf{f} = [a, b, c, d]^\mathsf{T}$$

$$\begin{aligned} D^2 &= (\mathbf{f}^\mathsf{T}\mathbf{v})^2 \\ &= \mathbf{v}^\mathsf{T}(\mathbf{f}\mathbf{f}^\mathsf{T})\mathbf{v} \\ &= \mathbf{v}^\mathsf{T}\mathbf{Q}\mathbf{v} \end{aligned}$$

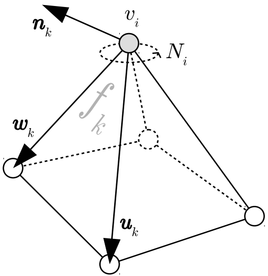
$$\mathbf{Q} = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

LINKÖPINGS UNIVERSITET

Calculating cost

Sum distances to planes f_k of triangles in neighborhood N_i of v_i

$$\begin{aligned} D^2 &= \sum_k \mathbf{v}_i^\mathsf{T} \mathbf{Q}_k \mathbf{v}_i \\ &= \mathbf{v}_i^\mathsf{T} \left(\sum_k \mathbf{Q}_k \right) \mathbf{v}_i \\ &= \mathbf{v}_i^\mathsf{T} \mathbf{Q}_i \mathbf{v}_i \end{aligned}$$



Finding Optimal Position

Optimal position $\bar{\mathbf{v}}$ after collapse $(\mathbf{v}_i, \mathbf{v}_j) \rightarrow \mathbf{v}$

$$(\mathbf{Q}_i + \mathbf{Q}_j)\bar{\mathbf{v}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Overview of Algorithm

Initialization

1. Compute matrix Q for each vertex
2. Compute optimal vertex position for each edge collapse
3. Compute *cost* of each edge collapse
4. Store edge collapses in min-heap with *cost* as key

Simplification

1. Collapse edge on top of min-heap
2. Recompute optimal positions and costs
3. Repeat

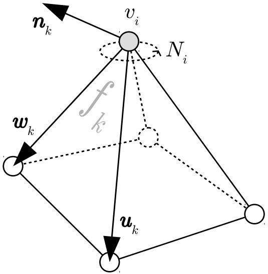
Alternative Notation

$$
\begin{aligned}
D^2 &= (\mathbf{n}^\mathsf{T}\mathbf{v} + d)^2 \\
&= \mathbf{v}^\mathsf{T}(\mathbf{n}\mathbf{n}^\mathsf{T})\mathbf{v} + 2d\mathbf{n}^\mathsf{T}\mathbf{v} + d^2
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
Q &= (\mathbf{n}\mathbf{n}^\mathsf{T}, d\mathbf{n}, d^2) \\
&= (\mathbf{A}, \mathbf{b}, c)
\end{aligned} \tag{2}
$$

$$
Q(\mathbf{v}) = \mathbf{v}^\mathsf{T}\mathbf{A}\mathbf{v} + 2\mathbf{b}^\mathsf{T}\mathbf{v} + c \tag{3}
$$

$$
\mathbf{A}\bar{\mathbf{v}} + \mathbf{b} = 0 \tag{4}
$$



Quadric Error Metric with Attributes

Extend **Q** to include attributes

$$Q = (\mathbf{A}, \mathbf{b}, c) = \left(\left[\begin{array}{c|c} \mathbf{nn}^\mathsf{T} & \ddots 0 \ddots \\ \hline \ddots 0 \ddots & \ddots 0 \ddots \end{array} \right], \left[\begin{array}{c} d\mathbf{n} \\ \hline 0 \end{array} \right], d^2 \right)$$

Quadric Error Metric with Attributes

Expected attribute value at point p:

$$\hat{s}_j(\mathbf{p}) = \mathbf{g}_j^\mathsf{T}\mathbf{p} + d_j$$

$\hat{s}_j$ interpolate the vertices of face $\left(\binom{p_1}{s_1}, \binom{p_2}{s_2}, \binom{p_3}{s_3}\right)$

$\mathbf{g}_j$ and d_j obtained by solving:

$$\begin{bmatrix} \mathbf{p}_1^\mathsf{T} & 1 \\ \mathbf{p}_2^\mathsf{T} & 1 \\ \mathbf{p}_3^\mathsf{T} & 1 \\ \mathbf{n}^\mathsf{T} & 0 \end{bmatrix} \begin{bmatrix} \\ \mathbf{g}_j \\ \\ d_j \end{bmatrix} = \begin{bmatrix} s_{1,j} \\ s_{2,j} \\ s_{3,j} \\ 0 \end{bmatrix}$$

Quadric Error Metric with Attributes

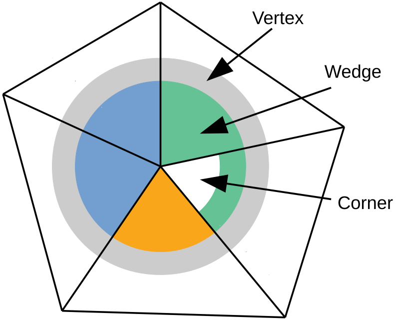
$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{n}\mathbf{n}^\mathsf{T} + \sum_j \mathbf{g}_j\mathbf{g}_j^\mathsf{T} & -\mathbf{g}_1 \cdots - \mathbf{g}_m \\ \hline -\mathbf{g}_1 & \\ \vdots & \mathrm{I} \\ -\mathbf{g}_m & \end{array}\right]$$

$$\mathbf{b} = \left[\begin{array}{c} d\mathbf{n} + \sum_j d_j\mathbf{g}_j \\ \hline -d_1 \\ \vdots \\ -d_m \end{array}\right]$$

$$c = d^2 + \sum_j d_j^2$$

Vertex with Multiple Attributes

- Associate each vertex with one or more wedges
- Store attribute values in the wedges



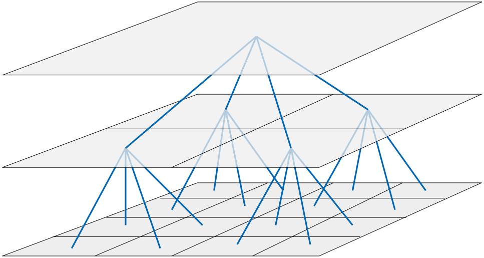
Outline

Problem with Texture Atlas

Bad texture values in seams

Improving Texture

- Use a pull-push algorithm to fill invalid pixels
- Creates a pyramid of images with decreasing resolution
- Each pixel is assigned weight w_i and color x_i



Find valid pixels (method 1)

- Create mesh with UV-coordinates as vertices
- Cast rays toward the mesh to find valid pixels

Find valid pixels (method 2)

- Could also be obtained with a threshold-filter
- Apply edge-filter to trim edges

Pull Phase

- Create lower resolution level with Gaussian blur filter

$$w_i^{r+1} = \sum_k \tilde{h}_k \min(w_k^r, 1)$$

$$x_i^{r+1} = \frac{1}{w_i^{r+1}} \sum_k \tilde{h}_k \min(w_k^r, 1) x_i^r$$

1	2	2	1
2	4	4	2
2	4	4	2
1	2	2	1

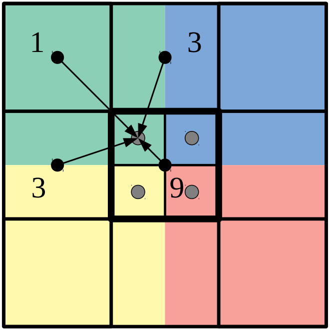
Push Phase

- Blend neighboring pixels

$$tw_i^r = \sum_k h_k \min(w_k^{r+1}, 1)$$
$$tx_i^r = \frac{1}{tw_i^r} \sum_k h_k \min(w_k^{r+1}, 1) x_i^{r+1}$$

- Blend higher resolution pixels with the computed value

$$x_i^r = tx_i^r (1 - w_i^r) + w^r x_i^r$$
$$w_i^r = tw_i^r (1 - w_i^r) + w^r$$



Pyramid in Pull Phase

Pyramid in Push Phase

Outline

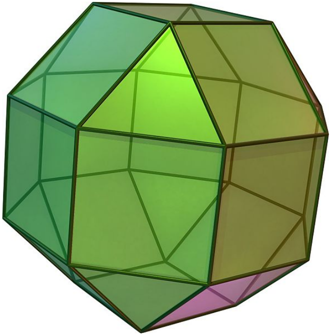
Appearance Preservation

- Two methods for model comparison
  - Image comparison
  - Distance between surfaces
- The original model is compared to four LoD:s

LINKÖPINGS UNIVERSITET

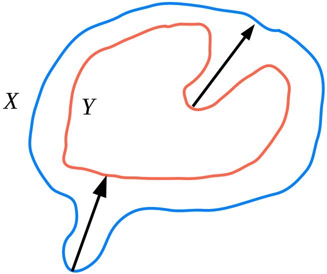
Image Comparison

- Render the models to compare from multiple camera positions
- Measure RMS of luminance between the images



Distance between surfaces

- Sample points on the surface of the model
- Measure the squared distance to the other surface
- The error is then the RMS distance
- Non-symmetric process



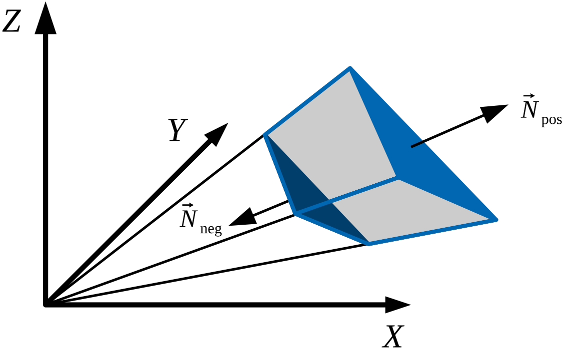
Volume Preservation

- Vertices of a polygon and the origin creates a tetrahedron
- Signed volume of a tetrahedron can be calculated with

$$V_i = \frac{v_0 \cdot (v_1 \times v_2)}{6}$$

- Volume of a mesh is then the sum of the signed volumes

$$V_{total} = \sum_i V_i$$



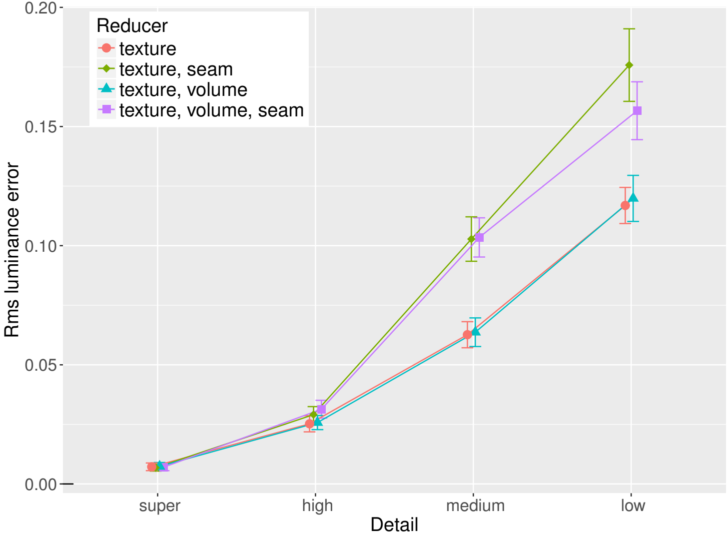
Execution Time

- Measure time multiple times to get average
- Measured for 1 to 8 threads

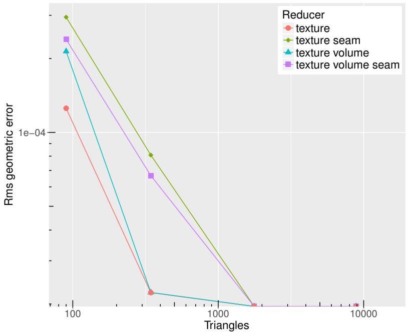
LINKÖPINGS UNIVERSITET

Outline

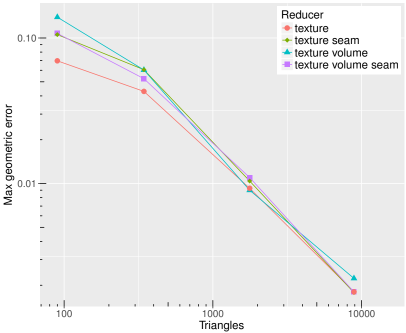
Rms Luminance Error



Geometric Error

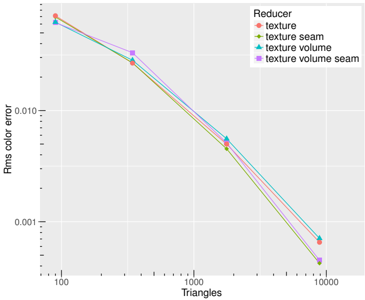


Rms geometric error

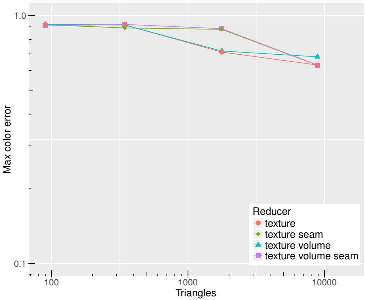


Max geometric error

Color error

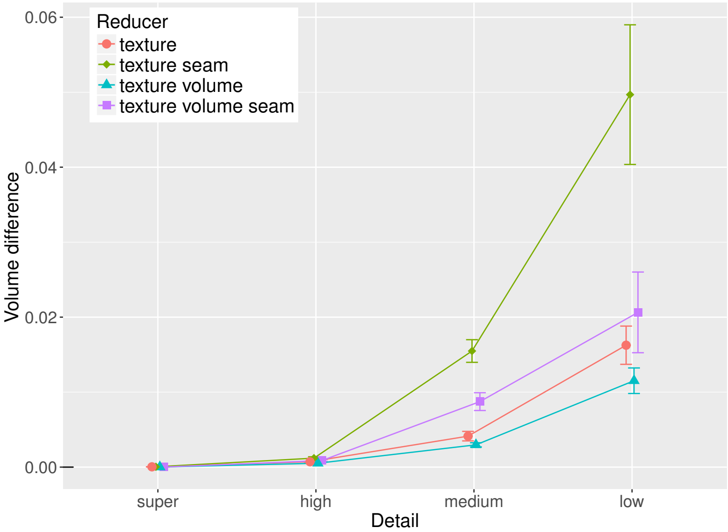


Rms color error



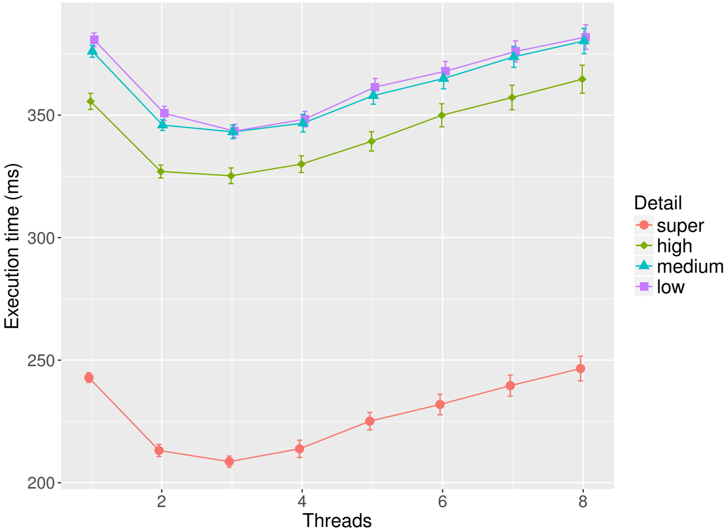
Max color error

Volume



Reducer
texture
texture seam
texture volume
texture volume seam
0.06
0.04
0.02
0.00
Volume difference
super
high
medium
low
Detail

Execution Time



LINKÖPINGS UNIVERSITET

Improving Texture


Original


Bound


Improved

Improving Texture

LoD:s (only geometry)

Super High Medium Low

LoD:s (geometry and texture)

Super High Medium Low

LoD:s (geometry and texture)

Super High Medium Low

LINKÖPINGS UNIVERSITET

Summary

- The appearance can be preserved by simplifying with extended QEM
- Using a pull-push method the texture can be improved

- Future work
  - Submeshes with different materials
  - Initial computations on GPU