

Evaluation of an Appearance-Preserving Mesh Simplification Scheme

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2018-06-15

Outline

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First Subsection

Implementation

Quadric Error Metric

Improving Texture Atlas

Evaluation

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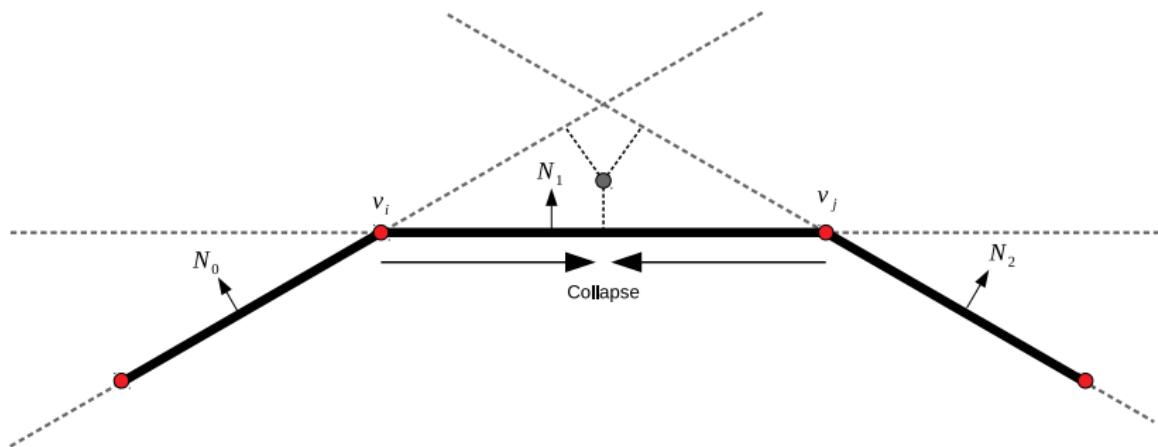
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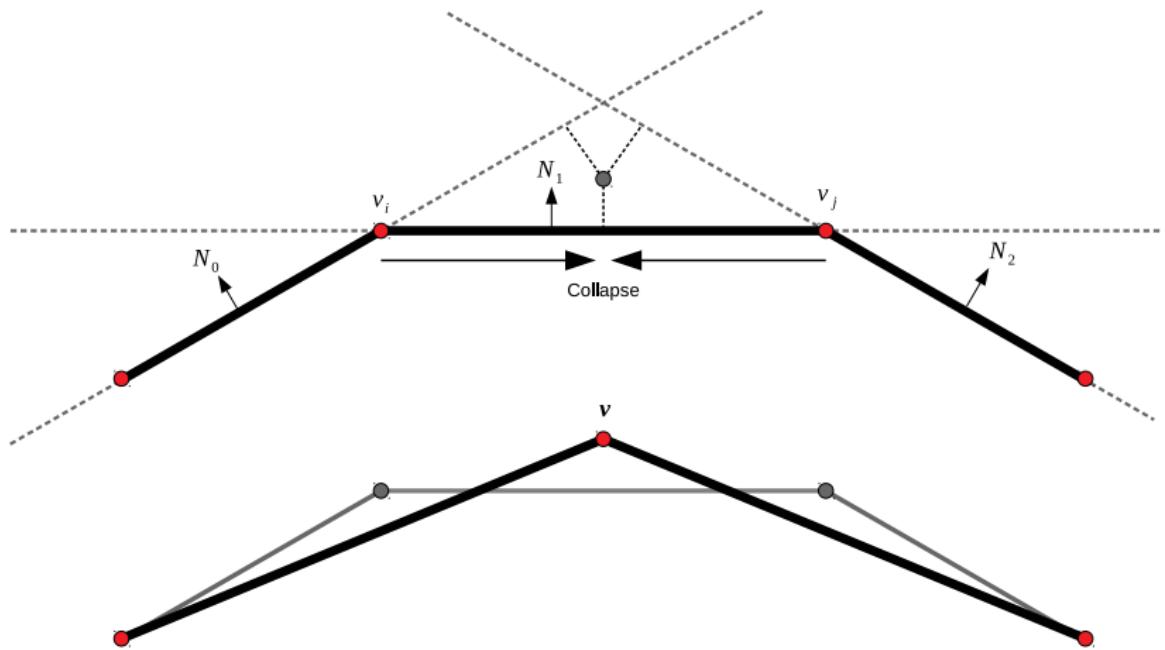
Results

Quadric Error Metric

- ▶ Iteratively perform edge collapses $(\mathbf{v}_i, \mathbf{v}_j) \rightarrow \mathbf{v}$
- ▶ Cost based on distance to neighboring faces' planes



Quadric Error Metric



Calculating cost

Squared distance from point v to plane f

$$\mathbf{v} = [x, y, z, 1]^T, \mathbf{f} = [a, b, c, d]^T$$

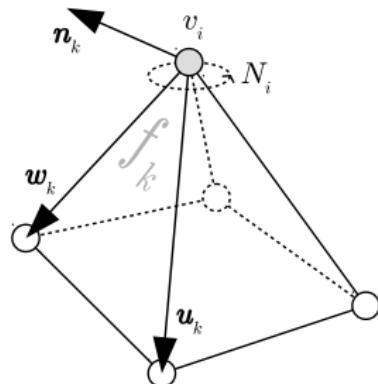
$$\begin{aligned} D^2 &= (\mathbf{f}^T \mathbf{v})^2 \\ &= \mathbf{v}^T (\mathbf{f} \mathbf{f}^T) \mathbf{v} \\ &= \mathbf{v}^T \mathbf{Q} \mathbf{v} \end{aligned}$$

$$\mathbf{Q} = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

Calculating cost

Sum distances to planes f_k of triangles in neighborhood N_i of v_i

$$\begin{aligned} D^2 &= \sum_k \mathbf{v}_i^T \mathbf{Q}_k \mathbf{v}_i \\ &= \mathbf{v}_i^T \left(\sum_k \mathbf{Q}_k \right) \mathbf{v}_i \\ &= \mathbf{v}_i^T \mathbf{Q}_i \mathbf{v}_i \end{aligned}$$



Finding Optimal Position

Optimal position $\bar{\mathbf{v}}$ after collapse $(\mathbf{v}_i, \mathbf{v}_j) \rightarrow \mathbf{v}$

$$(\mathbf{Q}_i + \mathbf{Q}_j)\bar{\mathbf{v}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \bar{\mathbf{v}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Overview of Algorithm

Initialization

1. Compute matrix Q for each vertex
2. Compute optimal vertex position for each edge collapse
3. Compute *cost* of each edge collapse
4. Store edge collapses in min-heap with *cost* as key

Simplification

1. Collapse edge on top of min-heap
2. Recompute optimal positions and costs
3. Repeat

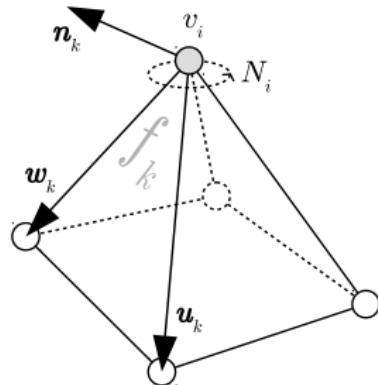
More convenient notation

$$\begin{aligned} D^2 &= (\mathbf{n}^\top \mathbf{v} + d)^2 \\ &= \mathbf{v}^\top (\mathbf{n}\mathbf{n}^\top) \mathbf{v} + 2d\mathbf{n}^\top \mathbf{v} + d^2 \end{aligned} \tag{1}$$

$$\begin{aligned} Q &= (\mathbf{n}\mathbf{n}^\top, d\mathbf{n}, d^2) \\ &= (\mathbf{A}, \mathbf{b}, c) \end{aligned} \tag{2}$$

$$Q(\mathbf{v}) = \mathbf{v}^\top \mathbf{A} \mathbf{v} + 2\mathbf{b}^\top \mathbf{v} + c \tag{3}$$

(4)



Quadric Error Metric with Attributes

Extend **Q** to include attributes

$$Q = (\mathbf{A}, \mathbf{b}, c) = \left(\begin{array}{c|c} \mathbf{n}\mathbf{n}^T & \cdots 0 \cdots \\ \hline \cdots 0 \cdots & \cdots 0 \cdots \end{array} \right), \left[\frac{d\mathbf{n}}{0} \right], d^2 \right)$$

Quadric Error Metric with Attributes

Expected attribute value at point p :

$$\hat{s}_j(\mathbf{p}) = \mathbf{g}_j^T \mathbf{p} + d_j$$

\hat{s}_j interpolate the vertices of face $((\frac{p_1}{s_1}), (\frac{p_2}{s_2}), (\frac{p_3}{s_3}))$

\mathbf{g}_j and d_j obtained by solving:

$$\begin{bmatrix} \mathbf{p}_1^T & 1 \\ \mathbf{p}_2^T & 1 \\ \mathbf{p}_3^T & 1 \\ \mathbf{n}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{g}_j \\ d_j \end{bmatrix} = \begin{bmatrix} s_{1,j} \\ s_{2,j} \\ s_{3,j} \\ 0 \end{bmatrix}$$

Quadric Error Metric with Attributes

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{n}\mathbf{n}^T + \sum_j \mathbf{g}_j\mathbf{g}_j^T & -\mathbf{g}_1 \cdots -\mathbf{g}_m \\ \hline -\mathbf{g}_1 & \\ \vdots & \mathbf{I} \\ -\mathbf{g}_m & \end{array} \right]$$

$$\mathbf{b} = \left[\begin{array}{c} d\mathbf{n} + \sum_j d_j \mathbf{g}_j \\ \hline -d_1 \\ \vdots \\ -d_m \end{array} \right]$$

$$c = d^2 + \sum_j d_j^2$$

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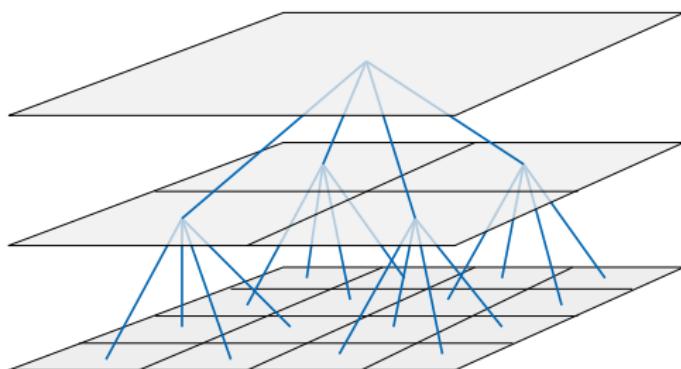
Problem with Texture Atlas

Bad texture values in seams



Improving Texture

- ▶ Use a pull-push algorithm to fill invalid pixels
- ▶ Creates a pyramid of images with decreasing resolution
- ▶ Each pixel is assigned weight w_i and color x_i



Find valid pixels

- ▶ Create mesh with UV-coordinates as vertices
- ▶ Cast rays toward the mesh to find valid pixels



Find valid pixels

- ▶ Could also be obtained with a threshold-filter
- ▶ Apply edge-filter to trim edges



Pull Phase

- ▶ Create lower resolution level with Gaussian blur filter

$$w_i^{r+1} = \sum_k \tilde{h}_k \min(w_k^r, 1)$$

$$x_i^{r+1} = \frac{1}{w_i^{r+1}} \sum_k \tilde{h}_k \min(w_k^r, 1) x_i^r$$

1	2	2	1
2	4	4	2
2	4	4	2
1	2	2	1

Push Phase

- ▶ Blend neighboring pixels

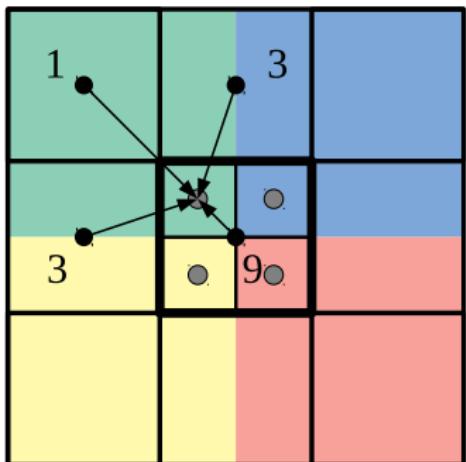
$$tw_i^r = \sum_k h_k \min(w_k^{r+1}, 1)$$

$$tx_i^r = \frac{1}{tw_i^r} \sum_k h_k \min(w_k^{r+1}, 1) x_i^{r+1}$$

- ▶ Blend higher resolution pixels with the computed value

$$x_i^r = tx_i^r(1 - w_i^r) + w^r x_i^r$$

$$w_i^r = tw_i^r(1 - w_i^r) + w^r$$



Pyramid in Pull Phase



Pyramid in Push Phase



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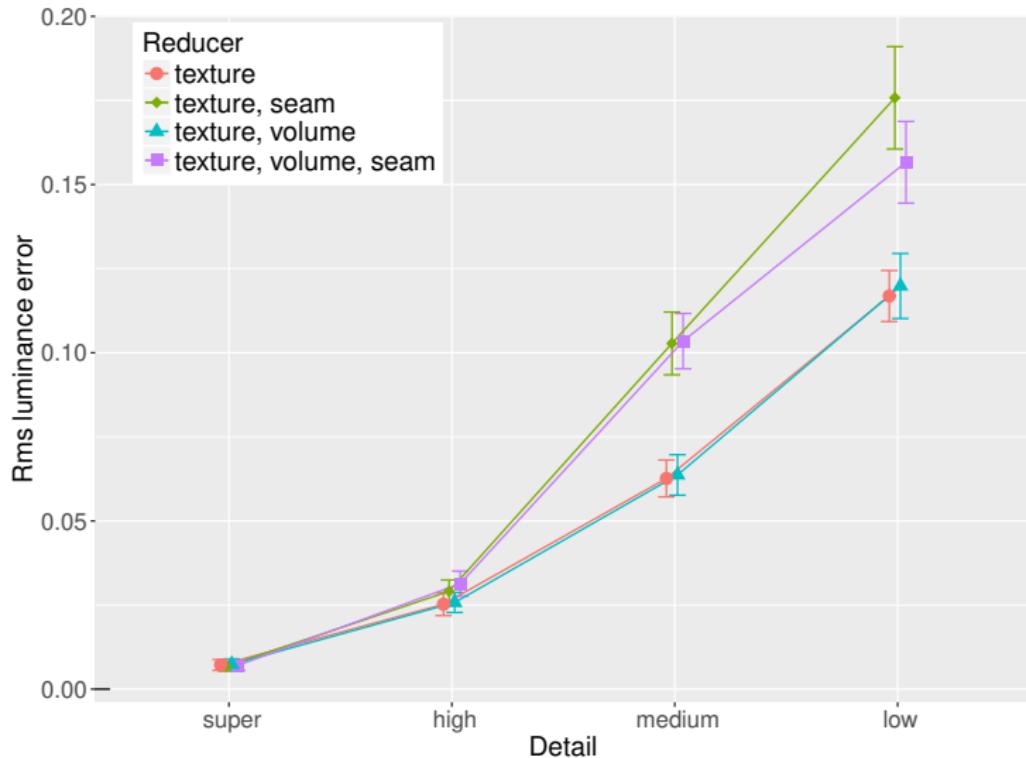
Quadric Error Metric

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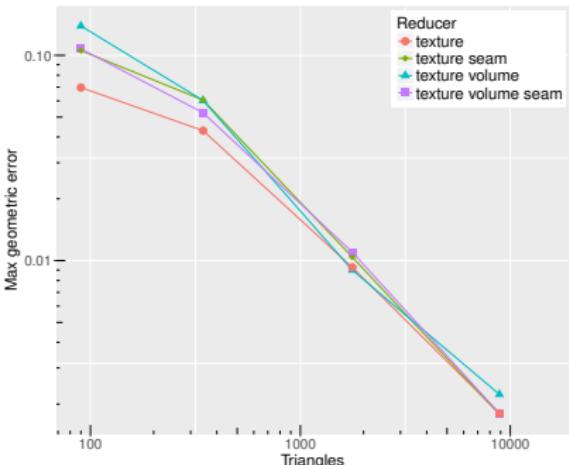
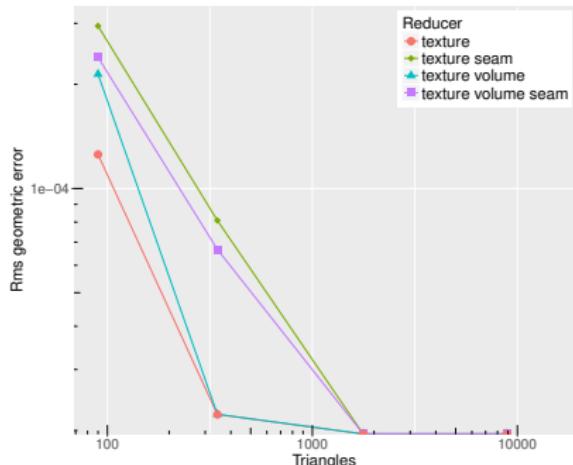
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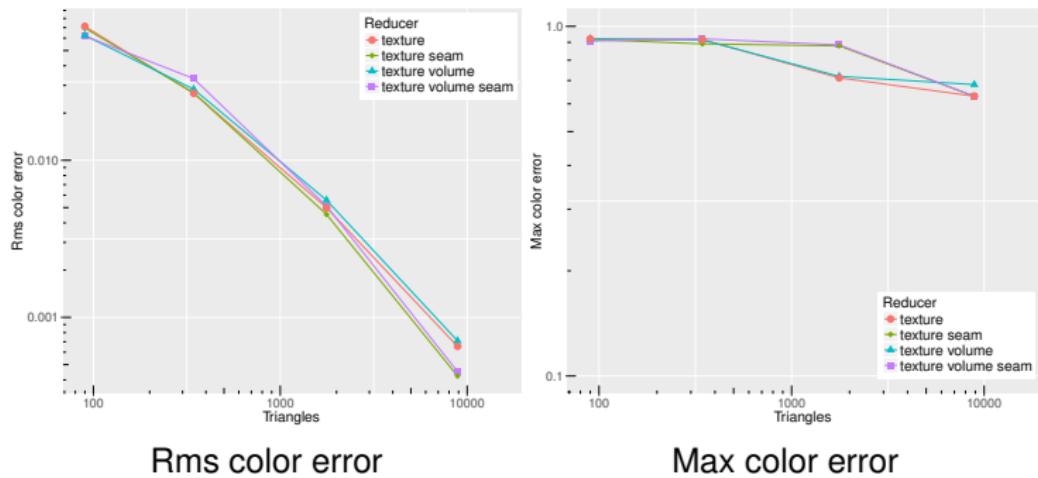
Rms Luminance Error



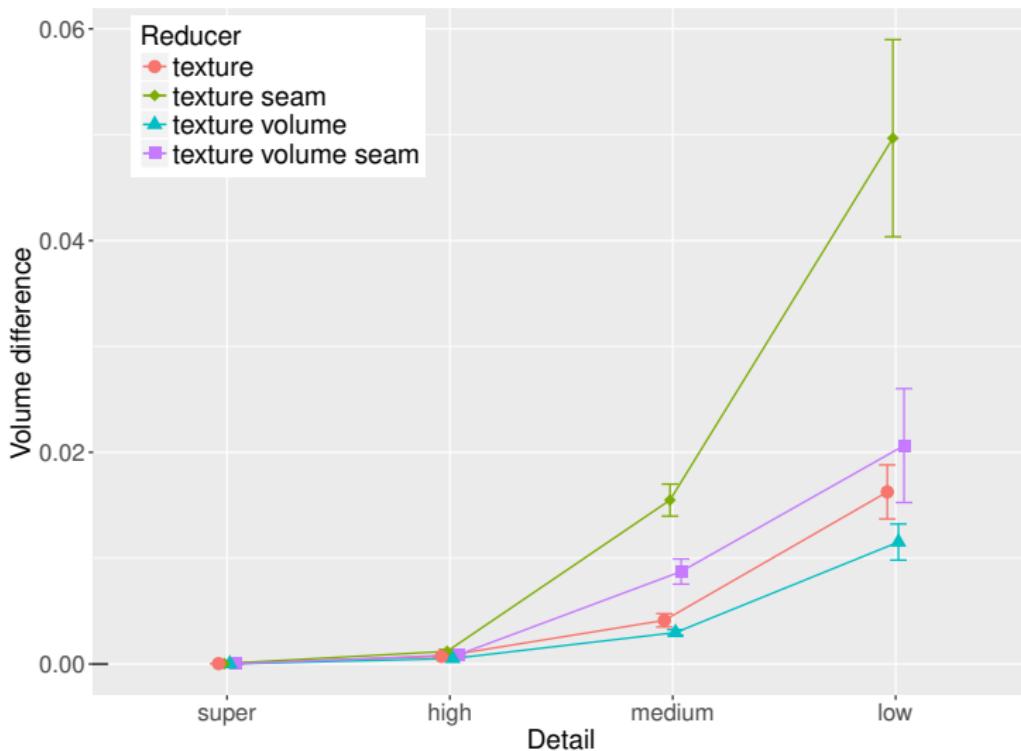
Geometric Error



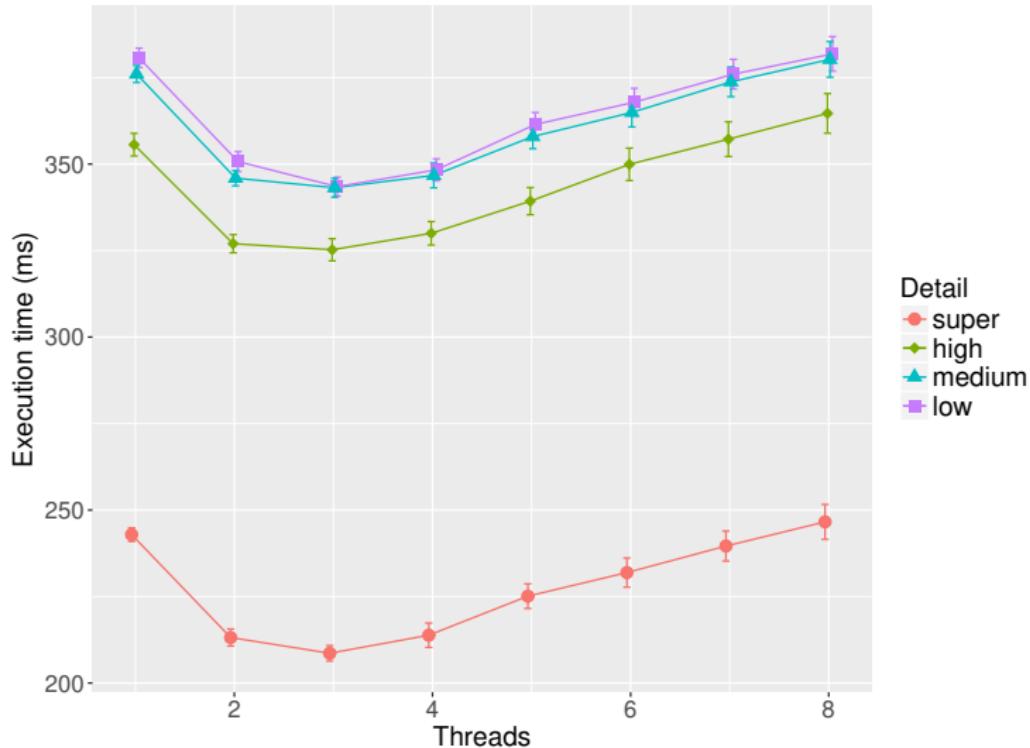
Color error



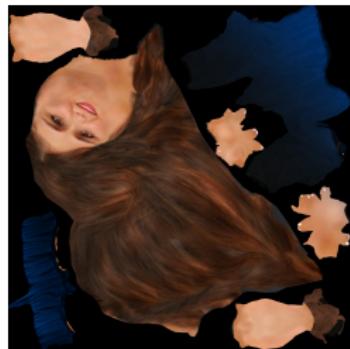
Volume



Execution Time



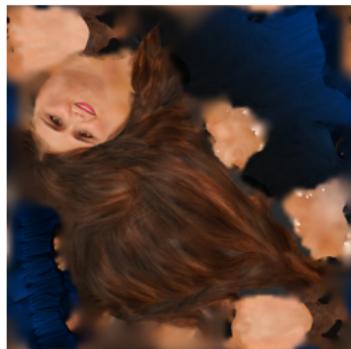
Improving Texture



Original



Bound

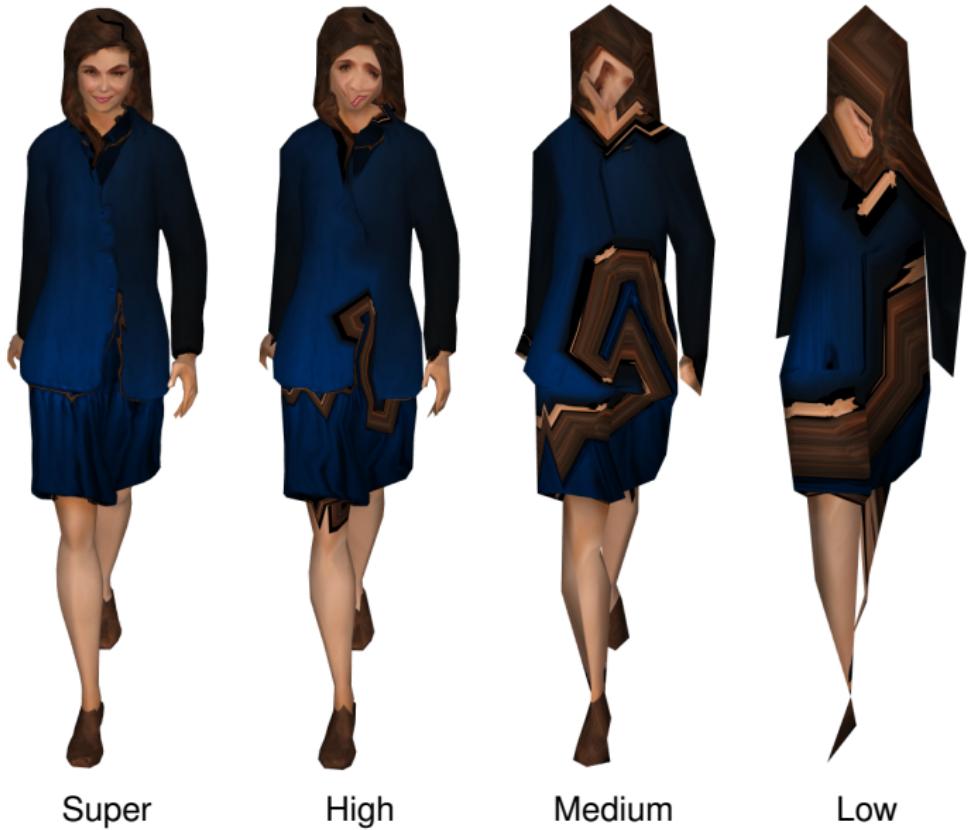


Improved

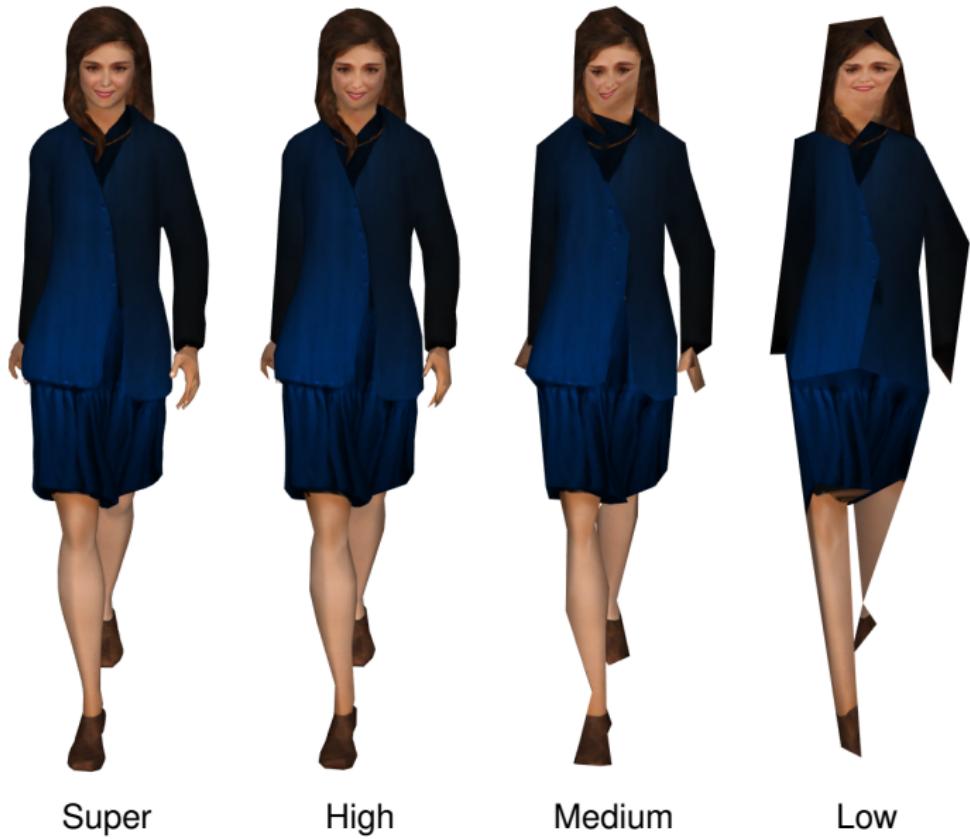
Improving Texture



LoD:s (only geometry)



LoD:s (geometry and texture)



LoD:s (geometry and texture)



Super



High



Medium



Low

Summary

- ▶ The **first main message** of your talk in one or two lines.
- ▶ The **second main message** of your talk in one or two lines.
- ▶ Perhaps a **third message**, but not more than that.

- ▶ Outlook
 - ▶ Something you haven't solved.
 - ▶ Something else you haven't solved.