12.12.

№6 $x^3 y''' + 7x^2 y'' + 4xy' - 4y =$

$= x^2(18\ln x + 81)$

$x = e^t$

$y' = e^{-t}\dfrac{dy}{dt}$

$y'' = e^{-2t}\left(\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt}\right)$

$\cancel{y''' = e^{-2t}dy - e^{-t}\dfrac{dy}{dt} + e^{-t}\dfrac{d^2y}{dt^2}}$

$\cancel{y'' = \dfrac{dy'}{dx} = \dfrac{dy'/dt}{e^t}}$

$y''' = \dfrac{dy''/dt}{dx/dt} = e^{-t}\left(-e^{-2t}\left(\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt}\right) +\right.$

$\left.+ e^{-2t}\left(\dfrac{d^3y}{dt^3} - \dfrac{d^2y}{dt^2}\right)\right) = \cancel{e^{-3t}\left(\dfrac{d^3y}{dt^3} \dfrac{dy}{dt}\right.}$

$= e^{-3t}\left(\dfrac{d^3y}{dt^3} - 2\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt}\right)$

$$\frac{c' \sqrt{p}\sqrt{p-1}}{(p-1)^2} = \frac{3\sqrt{p}}{2p(1-p)}$$

$$c' = \frac{3(1-p)}{2p\sqrt{p-1}}$$

$$c' = -\frac{3}{2}\,\frac{\sqrt{p-1}}{p}$$

2) $c\,ctg\,y$ ...

$\cdot\ n\cos^2 y \cos 2y \quad \sin y$

$y = x(2(y')^2 - y') + 2(y')^{\frac{3}{2}}$

$y' = p$

$y = x(2p^2 - p) + 2p^{\frac{3}{2}}$

$p\,dx = (2p^2 - p)\,dx + x(4p-1)\,dp + 3\sqrt{p}\,dp$

$(2p - 2p^2)\,dx = x \cdot (4p-1)\,dp + 3\sqrt{p}\,dp$

1) $y = 0$ – реш.

2) $p = 1$: $y + c = x + 2 \to y = x + 2$ – реш.

$$\frac{dx}{dp} = x \cdot \frac{4p-1}{2p(1-p)} + \frac{3\sqrt{p}}{2p(1-p)}$$

1) $$\frac{dx}{dp} = -x\,\frac{4p-1}{2p(p-1)}$$

$$\frac{dx}{x} = \frac{1}{2p(p-1)} - \frac{2}{p-1}$$

$$\frac{dx}{x} = \frac{1}{2}\left(\frac{1}{p} + \frac{1}{p-1}\right) - \frac{2}{p-1}$$

$$\ln|x| = \ln|\sqrt{p}| + \ln|\sqrt{p-1}| + \ln\left|\frac{C}{(p-1)^2}\right|$$

$$x = C \cdot \frac{\sqrt{p}\cdot\sqrt{p-1}}{(p-1)^2} = \frac{C\cdot\sqrt{p^2-p}}{(p-1)^2}$$

2) $$c'\,\frac{\sqrt{p}\sqrt{p-1}}{(p-1)^2} + \cancel{c\left(\frac{\sqrt{p^2-p}}{(p-1)^2}\right)' = x\cdot\frac{4p-1}{2p(1-p)}} + \frac{3\sqrt{p}}{2p(1-p)}$$

$$y'' = 2Ae^{2t} + 2Ae^{2t} + 4Ate^{2t} + 4Be^{2t} =$$

$$= 4Ae^{2t} + 4Ate^{2t} + 4Be^{2t}$$

$$y''' = 4Ae^{2t} + 4Ae^{2t} + 8Ate^{2t} +$$

$$+4Ae^{2t} + 8Be^{2t} = 12Ae^{2t} + 8Ate^{2t} + 8Be^{2t}$$

$$12Ae^{2t} + 8Ate^{2t} + 8Be^{2t} +$$

$$+20Ae^{2t} + 20At\cdot e^{2t} + 20Be^{2t} -$$

$$-2Ae^{2t} - 4tAe^{2t} + 4Be^{2t} = 18t\cdot e^{2t} + 81e^{2t}$$

$t\cdot e^{2t}$:

$$8A + 20A - 4A = 18$$

$$24A = 18 \qquad A = \frac{3}{4}$$

e^{2t}:

$$12A + 8B + 20A + 20B - 2A + 4B =$$

$$= 81$$

$$30A + 32B = 81$$

$$B = \frac{81-30A}{32} = \frac{234}{128} = \frac{117}{64}$$

$$e^{\ln x} = e^{\ln \frac{\tilde{C}(x)}{p^{\frac{1}{2}}(p-2)^{\frac{7}{2}}}}$$

$$x = \frac{\tilde{C}(x)}{p^{\frac{1}{2}}(p-2)^{\frac{7}{2}}}$$

$$\cancel{\ln x} \qquad x' = \frac{\tilde{C}'(x)\, p^{\frac{1}{2}} \cdot (p-2)^{\frac{7}{2}} - \left(\frac{1}{2} p^{-\frac{1}{2}} (p-2)^{\frac{7}{2}} + p^{\frac{1}{2}} \cdot \frac{7}{2} (p-2)^{\frac{5}{2}}\right)}{p\,(p-2)^{7}}$$

2) $c \cdot ctg y - \frac{}{\sin^2 y} \quad \frac{}{2\sin y \cos y}$

$\cdot \ \ \cos^2 y \cos 2y \quad \sin y$

$$y = x(2(y')^2 - y') + 2(y')^{\frac{3}{2}}$$

$$y' = p$$

$$y = x(2p^2 - p) + 2p^{\frac{3}{2}}$$

$$p\,dx = (2p^2 - p)\,dx + x(4p-1)\,dp + 3\sqrt{p}\,dp$$

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1) $y = 0$ — реш.

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1) $$\frac{dx}{dp} = \cancel{x} \cdot -x\,\frac{4p-1}{2p(p-1)}$$

$$\frac{dx}{x} = \frac{1}{2p(p-1)} - \frac{2}{p-1}$$

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$$\ln|x| = \ln|\sqrt{p}| + \ln|\sqrt{p-1}| + \ln\left|\frac{C}{(p-1)^2}\right|$$

$$x = C \cdot \frac{\sqrt{p} \cdot \sqrt{p-1}}{(p-1)^2} = \frac{C \cdot \sqrt{p^2 - p}}{(p-1)^2}$$

2) $$c' \cdot \frac{\sqrt{p}\,\sqrt{p-1}}{(p-1)^2} + \cancel{c\left(\frac{\sqrt{p^2-p}}{(p-1)^2}\right)'} = \cancel{x \cdot \frac{4p-1}{2p(1-p)}} + \frac{3\sqrt{p}}{2p(1-p)}$$

$$d\tilde{C}(x) = -\frac{3}{2}\frac{\sqrt{p-1}}{p}dp$$

$$\tilde{C}(x) = 3\operatorname{arctg}(\sqrt{p-1}) - 3\sqrt{p-1} + c$$

$$x = (3\operatorname{arctg}(\sqrt{p-1}) - 3\sqrt{p-1} + c)\cdot$$

$$\cdot\frac{\sqrt{p}\cdot\sqrt{p-1}}{(p-1)^2} \rightarrow y = x(2p^2 - p) + 2p^{\frac{3}{2}}$$

$$6\lambda^2 - 2\lambda - 4$$
$$- \quad 6\lambda^2 - 6\lambda$$

$$4\lambda - 4$$
$$- \quad 4\lambda - 4$$
$$0$$

$$y = y_{adin} + y_{4}$$

$$A\,\cancel{t}\,e^{2t} + B\,e^{2}$$

$$y' =$$

y_{adn}

y

№1 $y = x(2(y')^2 - y') + 2(y')^{\frac{3}{2}}$ $\quad y' = p$

$$p\,dx = dx(2p^2 - p) + x(4p-1)dp +$$

$$+ \quad 3p^{\frac{1}{2}}dp$$

$$\frac{(2p - 2p^2)dx}{dp} = x(4p-1)\cancel{dp} + 3p^{\frac{1}{2}}$$

$$x'(2p - 2p^2) - x(4p-1) = 3p^{\frac{1}{2}}$$

$$x'(2p - p^2) - x(4p-1) = 0$$

$$(2p - p^2)\frac{dx}{dp} = x(4p-1)$$

$$\int \frac{dx}{x} = \int \frac{(4p-1)dp}{(2p - p^2)}$$

$$\ln x = -\frac{\ln \cancel{p}}{2} - \frac{7\ln|\cancel{p}-2|}{2} + \tilde{C}_{(x)}$$

$$\left(\frac{d^3y}{dt^3} - 2\frac{d^2y}{dt^2} + \frac{dy}{dt}\right) + 7\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) +$$

$$+ 4\frac{dy}{dt} - 4y = e^{2t}(18 \cdot \ln e^t + 81)$$

$$\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 4y = e^{2t}(18t + 81)$$

$$\lambda^3 + 5\lambda^2 - 2\lambda - 4 = 0$$

$$\begin{array}{r|l} \lambda^3 + 5\lambda^2 - 2\lambda - 4 & \lambda - 1 \\ \lambda^3 - \lambda^2 & \lambda^2 + 6\lambda + 4 \\ \hline 6\lambda^2 - 2\lambda - 4 & \\ 6\lambda^2 - 6\lambda & \\ \hline 4\lambda - 4 & \\ 4\lambda - 4 & \\ \hline 0 & \end{array}$$

$$(\lambda - 1)(\lambda^2 + 6\lambda + 4) = 0$$

$$\left[\begin{array}{l} \lambda = 1 \\ \lambda = -3 \pm 2\sqrt{5} \end{array}\right.$$

$$y = y_{одн} + y_ч \qquad y_{одн} = C_1 e^x + C_2 e^{(-3-2\sqrt{5})x} + C_3 e^{(-3+2\sqrt{5})x}$$

$$A \cancel{t} \cdot e^{2t} + B e^{2t} = 18e^{2t}t + 81e^{2t}$$

$$y' = A(e^{2t} + 2te^{2t}) + 2Be^{2t}$$

the end.