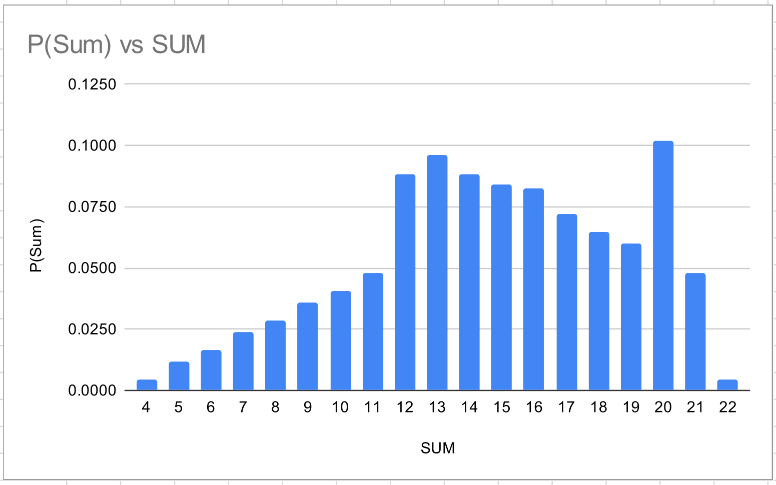
Let's consider the following arrangement of possible pairs and sums of face values. Let's count how many there are.

Qtys	Cards	2	3	4	5	6	7	8	9	10	J	Q	K	A
12	2	**4**	5	6	7	8	9	10	11	12	12	12	12	13
32	3	5	**6**	7	8	9	10	11	12	13	13	13	13	14
44	4	6	7	**8**	9	10	11	12	13	14	14	14	14	15
64	5	7	8	9	**10**	11	12	13	14	15	15	15	15	16
76	6	8	9	10	11	**12**	13	14	15	16	16	16	16	17
96	7	9	10	11	12	13	**14**	15	16	17	17	17	17	18
108	8	10	11	12	13	14	15	**16**	17	18	18	18	18	19
128	9	11	12	13	14	15	16	17	**18**	19	19	19	19	20
236	10	12	13	14	15	16	17	18	19	**20**	20	20	20	21
-	J	12	13	14	15	16	17	18	19	20	**20**	20	20	21
-	Q	12	13	14	15	16	17	18	19	20	20	**20**	20	21
-	K	12	13	14	15	16	17	18	19	20	20	20	**20**	21
256	A	13	14	15	16	17	18	19	20	21	21	21	21	**22**
	Qtys	256	236	224	220	192	172	160	272	128	-	-	-	12

Examples:
For Qty sum 5, two ways to be dealt
2, 3 or 3, 2.
each with 4 possible ways.
so 2*4*4=32
For sum 6 (and all even sums)
there is one way (same face value),
with counts 4*3, when equal cards, so
2*4*4+1*4*3 = 44

SUM	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	19	***possible sums***
QTY	**12**	**32**	**44**	**64**	**76**	**96**	**108**	**128**	**236**	**256**	**236**	**224**	**220**	**192**	**172**	**160**	**272**	**128**	**12**	***2668***	***qty of each sum***
P(Sum)	0.0045	0.0120	0.0165	0.0240	0.0285	0.0360	0.0405	0.0480	0.0885	0.0960	0.0885	0.0840	0.0825	0.0720	0.0645	0.0600	0.1019	0.0480	0.0045	1.0000	total



Let's compute PT(Sum) probability of a player's hand be involved in a TIE after the second card is dealt.

First DECISION: BET on TIES or NOT

For Ties, possible sums are calculated based on initial cards from both player and dealer, as follows:

SET A: For the person with the lower face card, possible tie-sums goes from LowerCard + 2 up to LowerCard + 11

SET B: For the person with the higher face card, possible tie-sums go from HigherCard + 2 up to HigherCard + 11

SET A n SET B: The list of possible sums goes from HigherCard + 2 up to LowerCard + 11. See examples below:

Example 1: Player gets 4, Dealer gets 9. This example gives 5 possible sums: 11, 12, 13, 14, 15 for a tie-value.

This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(11)+P(12)+P(13)+P(14)+P(15)= 0.4048

Ties	P\D	2	3	4	5	6	7	8	9	10	J	Q	K	A
	2								11					
	3	lower than the initial dealer's card							12					
	4	~~6~~	~~7~~	~~8~~	~~9~~	~~10~~	11	12	13	14	14	14	14	15
	5								14					
	6								15					
	7								~~16~~					
	8								~~17~~					
	9								~~18~~					
	10		not reachable by player's second card {						~~19~~					
	J								~~19~~					
	Q								~~19~~					
	K								~~19~~					
	A								~~20~~					

Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value

This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (10, 3) will be P(12)+P(13)+P(14)= 0.2729

Ties	P\D	2	3	4	5	6	7	8	9	10	J	Q	K	A
	2		~~5~~											
	3		~~6~~											

4		~~7~~											
5		~~8~~		} lower than the initial player's card									
6		~~9~~											
7		~~10~~											
8		~~11~~											
9		12											
10		13											
J		13				not reachable by player's second card							
Q	12	13	14	~~15~~	~~16~~	~~17~~	~~18~~	~~19~~	~~20~~	~~20~~	~~20~~	~~20~~	~~21~~
K		13											
A		14											

General table where each cell represents a probability of a tie for each pair of first cards Player x Dealer

This way, in the table the point P(Player's 1st. Card, Dealer's 1st Card) :

Ties	P\D	2	3	4	5	6	7	8	9	10-K	A
	2	0.3943	0.3898	0.3778	0.3613	0.3373	0.3088	0.2729	0.2324	0.1844	0.0960
	3	0.3898	0.4783	0.4663	0.4498	0.4258	0.3973	0.3613	0.3208	0.2729	0.1844
	4	0.3778	0.4663	0.5502	0.5337	0.5097	0.4813	0.4453	0.4048	0.3568	0.2684
	5	0.3613	0.4498	0.5337	0.6162	0.5922	0.5637	0.5277	0.4873	0.4393	0.3508
	6	0.3373	0.4258	0.5097	0.5922	0.6642	0.6357	0.5997	0.5592	0.5112	0.4228
	7	0.3088	0.3973	0.4813	0.5637	0.6357	0.7001	0.6642	0.6237	0.5757	0.4873
	8	0.2729	0.3613	0.4453	0.5277	0.5997	0.6642	0.7241	0.6837	0.6357	0.5472
	9	0.2324	0.3208	0.4048	0.4873	0.5592	0.6237	0.6837	0.7856	0.7376	0.6492
	10-K	0.1844	0.2729	0.3568	0.4393	0.5112	0.5757	0.6357	0.7376	0.7856	0.6972
	A	0.0960	0.1844	0.2684	0.3508	0.4228	0.4873	0.5472	0.6492	0.6972	0.7016

Based on this table, our strategy will always bet for TIEs as the gain is 10:1, except when the initial initial pair of card is (A,2) or (2, A), which will be diregarded, and the bet for Tie will always happen.

Second DECISION is to double or to keep origial bet for winning.

For doubling decision, we will evaluate the odds of winning also based on the first cards dealt, as follows:

Fact: For the person with the face card, the possible maximun total hand is Card + 11

Example 1: Player gets 4, Dealer gets 9. This example gives 5 possible sums: 11, 12, 13, 14, 15 for a tie-value.

This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(MinCard+11+1) + ... P(LargerCard + 11), to Dealer

Ties	P\D	2	3	4	5	6	7	8	9	10	J	Q	K	A
	2								~~11~~					
	3	lower than the initial dealer's card							~~12~~					
	4	~~6~~	~~7~~	~~8~~	~~9~~	~~10~~	~~11~~	~~12~~	~~13~~	~~14~~	~~14~~	~~14~~	~~14~~	~~15~~
	5								~~14~~					
	6								~~15~~					
	7								16					
	8								17					
	9								18					
	10			advantage of dealer over player {					19					
	J								19					
	Q								19					
	K								19					
	A								20					

Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value

This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(MinCard+11+1) + ... P(LargerCard + 11), to Player

Ties	P\D	2	3	4	5	6	7	8	9	10	J	Q	K	A
	2		~~5~~											
	3		~~6~~											
	4		~~7~~											
	5		~~8~~		} lower than the initial player's card									
	6		~~9~~											
	7		~~10~~											
	8		~~11~~											
	9		~~12~~											
	10		~~13~~											
	J		~~13~~				advantage of player over the dealer							
	Q	~~12~~	~~13~~	~~14~~	15	16	17	18	19	20	20	20	20	21
	K		~~13~~											
	A		~~14~~											

Adv.	P\D	2	3	4	5	6	7	8	9	10-K	A
	2	0.0000	-0.0885	-0.1724	-0.2549	-0.3268	-0.3913	-0.4513	-0.5532	-0.6012	-0.6057
	3	0.0885	0.0000	-0.0840	-0.1664	-0.2384	-0.3028	-0.3628	-0.4648	-0.5127	-0.5172
	4	0.1724	0.0840	0.0000	-0.0825	-0.1544	-0.2189	-0.2789	-0.3808	-0.4288	-0.4333
	5	0.2549	0.1664	0.0825	0.0000	-0.0720	-0.1364	-0.1964	-0.2984	-0.2744	-0.3508

Legend:

Advantage to Player

Advantage to Dealer

No advantage

6	0.3268	0.2384	0.1544	0.0720	0.0000	-0.0645	-0.1244	-0.2264	-0.2744	-0.2789
7	0.3913	0.3028	0.2189	0.1364	0.0645	0.0000	-0.0600	-0.1619	-0.2099	-0.2144
8	0.4513	0.3628	0.2789	0.1964	0.1244	0.0600	0.0000	-0.1019	-0.1499	-0.1544
9	0.5532	0.4648	0.3808	0.2984	0.2264	0.1619	0.1019	0.0000	-0.0480	-0.0525
10-K	0.6012	0.5127	0.4288	0.3463	0.2744	0.2099	0.1499	0.0480	0.0000	-0.0045
A	0.6057	0.5172	0.4333	0.3508	0.2789	0.2144	0.1544	0.0525	0.0045	0.0000

General strategy will be to double everytime the first cards are greater than the dealers' first card.

In the case of tie for first cards Player vs Dealer, we will assume no doubling will be made.

As we will always bet for Tie, as per our strategy, if cards are the same, we will ALSO Double when it's tie below:

Decision:	**P\D**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10-K**	**A**
Double	**2**	TIE/DD	KP	KP	KP	KP	KP	KP	KP	KP	KP
or Not	**3**	DD	TIE/DD	KP	KP	KP	KP	KP	KP	KP	KP
	4	DD	DD	TIE/DD	KP	KP	KP	KP	KP	KP	KP
	5	DD	DD	DD	TIE/DD	KP	KP	KP	KP	KP	KP
	6	DD	DD	DD	DD	TIE/DD	KP	KP	KP	KP	KP
	7	DD	DD	DD	DD	DD	TIE/DD	KP	KP	KP	KP
	8	DD	DD	DD	DD	DD	DD	TIE/DD	KP	KP	KP
	9	DD	DD	DD	DD	DD	DD	DD	TIE/DD	KP	KP
	10-K	DD	DD	DD	DD	DD	DD	DD	DD	TIE/DD	KP
	A	DD	DD	DD	DD	DD	DD	DD	DD	DD	TIE/DD

Legend:

TIE	meas equal changes
DD	means double initial bet
KP	means keep original bet

Let's evaluate possible outcomes from different strategies and simulate a large number of rounds

For each possible dollar outcome, an histogram will show the weight of each dollar outcome.

Initial bet will always be 2 (even number) $value chips for standard reference

See attached earning_paths.png

Functions: p(y), F(Y), E(Y), sd(Y), plots f(Y) and F(Y)

So far, I cound not find out how to calculate the probability of each outcome, as there are so many paths.

This way, I will do the reverse way: create a program to run this game millions of times, and register the p(y) for each possible outcome.

I will attach the code to be used on our report.