# Battlefield – 2P81 Group project

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## **Objective**

The objective of the game is to have a higher sum of cards than the dealer.

## **Components**

- One player and dealer (minimum)
- 52 card deck
- Cash value chips

#### **General Rules**

Cards 2-10 are equal to the value listed on each specific card. Face cards are equal to 10 and aces are equal to 11. No sum of cards can be less than 4 or greater than 22. The dealer must always deal counterclockwise. In the event of a tie, the players original cards become dead and are not included in the new sum. The maximum number of cards that can be dealt to a player during the game is four.

#### Setup

The dealer will begin by asking the player(s) to place their bets in cash value chips. After the initial bet is placed, it cannot be changed into a lower amount. The dealer will then deal each player one individual card face up while finishing with themselves. The player(s) will receive their second card at a later time.

#### **Starting The Game**

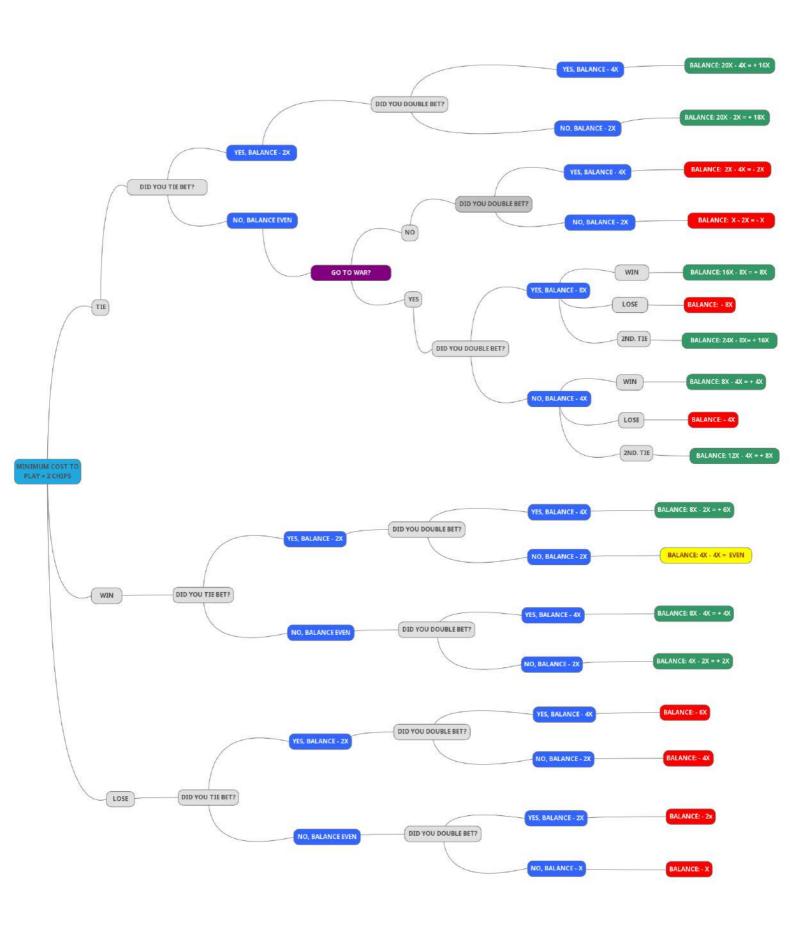
Once the player(s) received their first card they have three options. The player(s) can either keep their initial bet the same or double it. The player(s) can also place an independent side bet of the same value of their initial bet that their sum of cards will equal the sum of the dealer.

## Winning and Losing The Game

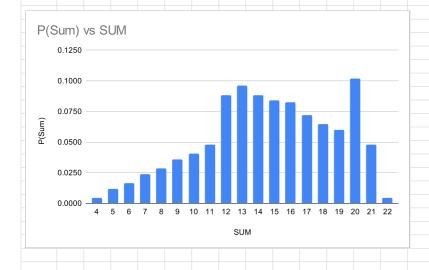
After the second round of potential bets are placed, the dealer deals the player(s) their second card. If the dealer has a higher sum of cards than the player(s) then the dealer wins their total bets placed. If the dealer has a lower sum of cards than the player(s) then the player(s) wins twice fold their total bets placed. If the player(s) bet on a tie, they win tenfold their side bet. In the event of a tie in which the player(s) didn't bet on a tie, they will have the option to take half their bet and leave half for the dealer or double their total bets placed. They are no longer able to bet on a tie.

#### **Second Turn**

If the player(s) chooses to double after a tie, the dealer deals them two new cards face up and then themselves two new cards. The same rules apply if the dealer has a lower or higher sum than the player(s). In the event of another tie, the player wins threefold their total bets.



	Let's c	onside	er the f	ollowin	ıg arrar	gemer	t of po	ssible	pairs a	nd sum	s of fa	ce valu	es. Let	's cour	nt how	many th	ere are.				
Qtys	Cards	2	3	4	5	6	7	8	9	10	J	Q	K	A		Exampl	les:				
12	2	4	5	6	7	8	9	10	11	12	12	12	12	13		For Qty	sum 5,	two ways	to be d	ealt	
32	3	5	6	7	8	9	10	11	12	13	13	13	13	14		2, 3 or 3	3, 2.				
44	4	6	7	8	9	10	11	12	13	14	14	14	14	15		each wi	ith 4 pos	sible way	s.		
64	5	7	8	9	10	11	12	13	14	15	15	15	15	16		so 2*4*	4=32				
76	6	8	9	10	11	12	13	14	15	16	16	16	16	17		For sun	n 6 (and	all even	sums)		
96	7	9	10	11	12	13	14	15	16	17	17	17	17	18		there is	one wa	y (same f	ace valı	ue),	
108	8	10	11	12	13	14	15	16	17	18	18	18	18	19		with cou	unts 4*3	, when ec	ual car	ds, so	
128	9	11	12	13	14	15	16	17	18	19	19	19	19	20		2*4*4+	1*4*3 =	44			
236	10	12	13	14	15	16	17	18	19	20	20	20	20	21							
-	J	12	13	14	15	16	17	18	19	20	20	20	20	21							
-	Q	12	13	14	15	16	17	18	19	20	20	20	20	21							
-	K	12	13	14	15	16	17	18	19	20	20	20	20	21							
256	Α	13	14	15	16	17	18	19	20	21	21	21	21	22							
	Qtys	256	236	224	220	192	172	160	272	128	-	-	-	12							
SUM	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	19	possible sums
QTY	12	32	44	64	76	96	108	128	236	256	236	224	220	192	172	160	272	128	12	2668	qty of each sum
P(Sum)	0.0045	0.0120	0.0165	0.0240	0.0285	0.0360	0.0405	0.0480	0.0885	0.0960	0.0885	0.0840	0.0825	0.0720	0.0645	0.0600	0.1019	0.0480	0.0045	1.0000	total



Let's compute PT(Sum) probability of a player's hand be involved in a TIE after the second card is dealt.

First DECISION: BET on TIES or NOT

For Ties, possible sums are calculated based on initial cards from both player and dealer, as follows:

SET A: For the person with the lower face card, possible tie-sums goes from LowerCard + 2 up to LowerCard + 11

SET B: For the person with the higher face card, possible tie-sums go from HigherCard + 2 up to HigherCard + 11

SET A n SET B: The list of possible sums goes from HigherCard + 2 up to LowerCard + 11. See examples below:

Example 1: Player gets 4, Dealer gets 9. This example gives 5 possible sums: 11, 12, 13, 14, 15 for a tie-value. This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(11)+P(12)+P(13)+P(14)+P(15)=

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s	P\D	2	3	4	5	6	7	8	9	10	J	Q	K	Α				
	2								11									
	3	lower th	an the in	itial de	aler's	card			12									
	4	6	7	8	9	10	11	12	13	14	14	14	14	15				
	5								14									
	6								15									
	7							Ī	16									
	8								17									
	9								18									
	10		not reach	able by	player':	s second	card {		<del>19</del>									
	J								<del>19</del>									
	Q								<del>19</del>									
	K								19									
	Α								20									
mple	2: Play	er gets	10, Dea	aler ge	ts 3. Th	nis exa	mple giv	ves 3 p	ossible	sums	12, 13	, 14 for	a tie-v	alue				
									ard) = (						0.2729			

Tics   PP   2   3   4   5   6   7   8   9   10   J   Q   K   A	1100	P\D	2	3	4	5	6	7	8	9	10	J	Q	K	Α							
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Second Decision   Second Dec																						
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10		8		11																		
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3 0.3880 0.473 0.4850 0.4850 0.4850 0.3850 0.3873 0.3810 0.3810 0.2728 0.1844 4 0.377 0.4850 0.5320 0.5320 0.372 0.570 0.4810 0.4850 0.4850 0.4850 6 0.337 0.4280 0.5320 0.4850 0.5277 0.5997 0.6842 0.527 0.475 0.475 8 0.272 0.2850 0.4850 0.5277 0.5997 0.6842 0.5727 0.4875 0.475 8 0.272 0.2850 0.4850 0.5277 0.5997 0.6842 0.5727 0.4875 0.475 8 0.272 0.2850 0.4850 0.5727 0.5997 0.6842 0.5727 0.4757 0.4757 0.4757 9 0.225 0.2550 0.4850 0.5727 0.5997 0.6842 0.5727 0.4757 0.4757 0.4757 0.4757 1 0.4840 0.2729 0.3990 0.4850 0.4872 0.4857 0.5757 0.4875 0.4875 0.4872	Ties	P\D	2	3	4	5	6	7	8	9	10-K	Α										
4 0.377 0.0485 0.5826 0.5827 0.5870 7.0813 0.4853 0.0486 0.5869 0.2084     5 0.3013 0.4286 0.3015 0.4805 0.3015 0.4815 0.5820 0.5815 0.3015 0.4228     7 0.3086 0.3715 0.4815 0.5827 0.5875 0.7013 0.4842 0.2012 0.4723     9 0.3234 0.3200 0.4815 0.6475 0.5927 0.5875 0.7013 0.4842 0.2021 0.4723     9 0.3234 0.3200 0.4845 0.6475 0.5922 0.5827 0.5827 0.5827 0.5827 0.472     9 0.3234 0.3200 0.4845 0.6475 0.5922 0.5827 0.5827 0.5827 0.5827 0.472     10 K 0.344 0.2722 0.5286 0.4325 0.4326 0.5827 0.5827 0.5827 0.5827 0.5827 0.472     A 0.5900 0.1844 0.2024 0.2030 0.4222 0.4221 0.4827 0.5827 0.7927 0.4927 0.7924     Based on this table, court stategy will always bet for TES as the gains to 11, except when the initial initial pair of ones (s. A.) 27 0.4, which will be disagrated for judyer - celestry = 9     Second DECISION is to double or to keep original bet for winning.   For doubling decision, we will avaluate the odds of winning also based on the first cards dealt, as follows: Fact: For the person with the face card, the possible maximum total hand is Card + 11     Example 1: Player gets 4, Dealer gets 9. This example gives 5 possible sums: 11, 12, 13, 14, 15 for at its-value.   This way, in the table the point [Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be PMiniCard+11+1) + P(LargerCard + 11), to Dealer 1st 0.5		2	0.3943	0.3898	0.3778	0.3613	0.3373	0.3088	0.2729	0.2324	0.1844	0.0960										
\$ 0.00010 0.00			0.3898	0.4783	0.4663	0.4498	0.4258	0.3973	0.3613	0.3208												
6 0.3371 0.4296 0.5197 (0.592) 0.5942 0.6542 0.6527 0.6527 0.6528 0.510 0.4225 0.4425																						
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10-K   1.1544   0.2726   0.3566   0.1545   0.5717   0.2717   0.7826   0.4972   0.6972   0.7972		-																				
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Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value  Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value  Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value  Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value  Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value  This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(MinCard+11+1) + P(LargerCard + 11), to Player  Ties PID 2 3 4 5 6 7 8 9 10 J Q K A  3 6 4 7 7 8 9 10 J Q K A  5 8 8 ) lower than the initial player's card  6 9 9 42 13 44 15 16 17 18 19 20 20 20 20 20 21	This way	y, in the	table t	he poi	nt (Play	er's 1s	t. Card	nple gi	ves 5 p r's 1st	ossible Card) =	e sums = (4, 9) \	: 11, 12 will be	, 13, 14 P(MinC	ard+11	+1) +		gerCar	rd + 11	), to D	)ealer		
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Second Process   Seco	This way	y, in the P\D 2 3 4 5	table t	he poil	nt (Play	/er's 1s 5 dealer's	t. Card 6	mple gi <sup>,</sup> , Deale 7	ves 5 p r's 1st 8	ossible Card) = 9 41 42 43 14	e sums = (4, 9) · 10	: 11, 12 will be	, 13, 14 P(MinC Q	ard+11 K	+1) + A		rgerCar	rd + 11	), to D	)ealer		
10	This way	y, in the P\D 2 3 4 5	table t	he poil	nt (Play	/er's 1s 5 dealer's	t. Card 6	mple gi <sup>,</sup> , Deale 7	ves 5 p r's 1st 8	ossible Card) = 9 11 12 13 14 15	e sums = (4, 9) · 10	: 11, 12 will be	, 13, 14 P(MinC Q	ard+11 K	+1) + A		gerCai	rd + 11	), to D	ealer		
Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value  Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value  This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(MinCard+11+1) + P(LargerCard + 11), to Player  Ties PID 2 3 4 5 6 7 8 9 10 J Q K A  2 5 6 7 8 9 10 J Q K A  3 6 4 7 7 40 5 8 ) lower than the initial player's card 6 9 7 40 8 11 9 12 9 12 9 12 9 12 9 12 9 12 9 12	This way	y, in the P\D 2 3 4 5 6 7	table t	he poil	nt (Play	/er's 1s 5 dealer's	t. Card 6	mple gi <sup>,</sup> , Deale 7	ves 5 p r's 1st 8	ossible Card) = 9 11 12 13 14 15	e sums = (4, 9) · 10	: 11, 12 will be	, 13, 14 P(MinC Q	ard+11 K	+1) + A		gerCar	rd + 11	), to D	ealer		
Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value  This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(MinCard+11+1) + P(LargerCard + 11), to Player  Ties PID 2 3 4 5 6 7 8 9 10 J Q K A  2 5 6 7 8 9 10 J Q K A  3 6 4 7 7 8 9 10 J Q K A  5 8 8 ) lower than the initial player's card 6 9 9 10 10 10 10 10 10 10 10 10 10 10 10 10	This way	y, in thee P\D 2 3 4 5 6 7	table t	he poil	nt (Play	/er's 1s 5 dealer's	t. Card 6	mple gi <sup>,</sup> , Deale 7	ves 5 p r's 1st 8	ossible Card) = 9 41 42 43 44 45 16	e sums = (4, 9) · 10	: 11, 12 will be	, 13, 14 P(MinC Q	ard+11 K	+1) + A		gerCar	rd + 11	), to D	ealer		
Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value	This way	y, in thee P\D 2 3 4 5 6 7 8	table t	he poil	nt (Play	yer's 1s 5 dealer's	card 40	mple gir , Deale 7	ves 5 p r's 1st 8	ossible Card) = 9 41 42 43 44 45 16 17	e sums = (4, 9) · 10	: 11, 12 will be	, 13, 14 P(MinC Q	ard+11 K	+1) + A		gerCa	rd + 11	), to D	Dealer		
Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value  This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(MinCard+11+1) + P(LargerCard + 11), to Player  Ties P\D 2 3 4 5 6 7 8 9 10 J Q K A  2 5 6 3 6 9 10 J Q K A  5 8 0) lower than the initial player's card  6 9 7 40 10 10 143 11 11 11 11 11 11 11 11 11 11 11 11 11	This way	y, in the P\D 2 3 4 5 6 7 8 9	table t	he poil	nt (Play	yer's 1s 5 dealer's	card 40	mple gir , Deale 7	ves 5 p r's 1st 8	ossible Card) = 9 41 42 43 44 45 16 17 18	e sums = (4, 9) · 10	: 11, 12 will be	, 13, 14 P(MinC Q	ard+11 K	+1) + A		rgerCar	rd + 11	), to D	Dealer		
Example 2: Player gets 10, Dealer gets 3. This example gives 3 possible sums: 12, 13, 14 for a tie-value  This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(MinCard+11+1) + P(LargerCard + 11), to Player  Ties P\D 2 3 4 5 6 7 8 9 10 J Q K A  2 6 3 6 9 9 9 10 9 10 9 10 9 10 9 10 9 10 9 1	This way	y, in the P\D 2 3 4 5 6 7 8 9 10 J	table t	he poil	nt (Play	yer's 1s 5 dealer's	card 40	mple gir , Deale 7	ves 5 p r's 1st 8	ossible Card) = 9 41 42 43 44 45 16 17 18 19	e sums = (4, 9) · 10	: 11, 12 will be	, 13, 14 P(MinC Q	ard+11 K	+1) + A		gerCai	rd + 11	), to D	)ealer		
This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(MinCard+11+1) + P(LargerCard + 11), to Player  Ties P\D 2 3 4 5 6 7 8 9 10 J Q K A  2 5 6 3 6 9 4 7 7 5 8 ) lower than the initial player's card 6 9 9 7 40 8 11 9 9 12 9 10 9 12 9 10 9 10 9 10 9 10	This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J	table t	he poil	nt (Play	yer's 1s 5 dealer's	card 40	mple gir , Deale 7	ves 5 p r's 1st 8	ossible Card) = 9 41 42 43 44 45 16 17 18 19 19	e sums = (4, 9) · 10	: 11, 12 will be	, 13, 14 P(MinC Q	ard+11 K	+1) + A		gerCar	d + 11	), to D	)ealer		
This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(MinCard+11+1) + P(LargerCard + 11), to Player  Ties PID 2 3 4 5 6 7 8 9 10 J Q K A  2 5 6	This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J	table t	he poil	nt (Play	yer's 1s 5 dealer's	card 40	mple gir , Deale 7	ves 5 p r's 1st 8	ossible Card) = 9 11 12 13 14 15 16 17 18 19 19	e sums = (4, 9) · 10	: 11, 12 will be	, 13, 14 P(MinC Q	ard+11 K	+1) + A		gerCar	d + 11	), to E	Dealer		
This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(MinCard+11+1) + P(LargerCard + 11), to Player  Ties PID 2 3 4 5 6 7 8 9 10 J Q K A  2 5 6	This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J	table t	he poil	nt (Play	yer's 1s 5 dealer's	card 40	mple gir , Deale 7	ves 5 p r's 1st 8	ossible Card) = 9 11 12 13 14 15 16 17 18 19 19	e sums = (4, 9) · 10	: 11, 12 will be	, 13, 14 P(MinC Q	ard+11 K	+1) + A		gerCai	d + 11	), to E	Dealer		
This way, in the table the point (Player's 1st. Card, Dealer's 1st Card) = (4, 9) will be P(MinCard+11+1) + P(LargerCard + 11), to Player  Ties P\D 2 3 4 5 6 7 8 9 10 J Q K A  2 5 6 3 6 9 4 7 7 5 8 ) lower than the initial player's card 6 9 9 7 40 8 11 9 9 12 9 10 9 12 9 10 9 10 9 10 9 10	This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J	table t	he poil	nt (Play	yer's 1s 5 dealer's	card 40	mple gir , Deale 7	ves 5 p r's 1st 8	ossible Card) = 9 11 12 13 14 15 16 17 18 19 19	e sums = (4, 9) · 10	: 11, 12 will be	, 13, 14 P(MinC Q	ard+11 K	+1) + A		rgerCai	d + 11	), to E	Pealer		
Ties PND 2 3 4 5 6 7 8 9 10 J Q K A  2 5 6	This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K	table t 2 lower th 6	3 3 annan the 2 7	nt (Play 4 4 initial ( 8	dealer's 9	card 40	mple gir , Deale 7 41	ves 5 p r's 1st 8 8 42	ossible (Card) = 9	9 sums F (4, 9) 1 10	: 11, 12 will be J	, 13, 14 P(MinC Q 14	44	+1) + A		'gerCai	d + 11	), to D	dealer		
2	This way Ties	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A	table t  2  lower title  6	3 annan the: 7	nt (Play 4  initial d 8  advantaj	yer's 1s 5 5 5 dealer's 9 9	card 40	mple gir , Deale 7 41 +	ves 5 p r's 1st 8 8 42	ossible Card) = 9	e sums f (4, 9) 1 10 44	: 11, 12 will be J 44	, 13, 14 P(MinC Q 14	r a tie-	+1) + A	P(La						
3 6 4 7 5 6 8 3 lower than the initial player's card 6 9 9 7 140 8 141 9 9 142 10 10 143 14 15 16 17 18 19 20 20 20 20 21 1	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A	table t  2  lower tit  6  //er gets table t	s 10, Do	nt (Play 4 initial d 8 advantaj	yer's 1s 5 dealer's 9 ge of de	card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  8  42  (	ossible (Card) = 9	e sums fr (4, 9) 10 10 110 110 110 110 110 110 110 110	: 11, 12 will be J J 44	, 13, 14 P(MinC Q 14	r a tie- ard+11	+1) + A  45	P(La						
4 7 5 8 3 lower than the initial player's card 6 9 7 40 8 41 9 42 13 14 15 16 17 18 19 20 20 20 20 21 K 13 14 15 16 17 18 19 20 20 20 20 21	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A	table t  2  lower tit  6  //er gets table t	s 10, Do	nt (Play 4 initial d 8 advantaj	yer's 1s 5 dealer's 9 ge of de	card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  8  42  (	ossible (Card) = 9	e sums fr (4, 9) 10 10 110 110 110 110 110 110 110 110	: 11, 12 will be J J 44	, 13, 14 P(MinC Q 14	r a tie- ard+11	+1) + A  45	P(La						
5 8 } lower than the initial player's card 6 9 9 7 140 8 141 9 142 10 143 J 143   advantage of player over the dealer Q 142 143 144 15 16 17 18 19 20 20 20 20 21 K 143	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A	table t  2  lower tit  6  //er gets table t	s 10, Do	nt (Play 4 initial d 8 advantaj	yer's 1s 5 dealer's 9 ge of de	card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  8  42  (	ossible (Card) = 9	e sums fr (4, 9) 10 10 110 110 110 110 110 110 110 110	: 11, 12 will be J J 44	, 13, 14 P(MinC Q 14	r a tie- ard+11	+1) + A  45	P(La						
6 9 9 10 10 10 10 10 10 10 10 10 10 10 10 10	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A	table t  2  lower tit  6  //er gets table t	s 10, Do	nt (Play 4 initial d 8 advantaj	yer's 1s 5 dealer's 9 ge of de	card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  8  42  (	ossible (Card) = 9	e sums fr (4, 9) 10 10 110 110 110 110 110 110 110 110	: 11, 12 will be J J 44	, 13, 14 P(MinC Q 14	r a tie- ard+11	+1) + A  45	P(La						
7	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A	table t  2  lower tit  6  //er gets table t	s 10, Do	nt (Play 4 initial d 8 advantaj	yer's 1s 5 dealer's 9 ge of de	card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  8  42  (	ossible (Card) = 9	e sums fr (4, 9) 10 10 110 110 110 110 110 110 110 110	: 11, 12 will be J J 44	, 13, 14 P(MinC Q 14	r a tie- ard+11	+1) + A  45	P(La						
8	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A	table t  2  lower tit  6  //er gets table t	s 10, Do  s 10, Do  s 10, Bo  s 10,	nt (Play 4 initial d 8 advantaj	yer's 1s 5 dealer's 9 ge of de	card 6 card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  8  42  {	ossible (Card) = 9	e sums fr (4, 9) 10 10 110 110 110 110 110 110 110 110	: 11, 12 will be J J 44	, 13, 14 P(MinC Q 14	r a tie- ard+11	+1) + A  45	P(La						
9	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A 2: Play y, in thee P\D 2 3 4 5 6	table t  2  lower tit  6  //er gets table t	s 10, Do  s 10, Do  s 10, Do  s 20, So  s 10, Do  s 20, So  s 20, So  s 20, So  s 30, So  s 30, So  s 40, So  s 50, So  s 50, So  s 80, So  s 90, So  s 90, So  s 10,	nt (Play 4 initial d 8 advantaj	yer's 1s 5 dealer's 9 ge of de	card 6 card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  8  42  {	ossible (Card) = 9	e sums fr (4, 9) 10 10 110 110 110 110 110 110 110 110	: 11, 12 will be J J 44	, 13, 14 P(MinC Q 14	r a tie- ard+11	+1) + A  45	P(La						
10	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A 2: Play y, in thee P\D 2 3 4 5 6 7	table t  2  lower tit  6  //er gets table t	s 10, Do  s 10,	nt (Play 4 initial d 8 advantaj	yer's 1s 5 dealer's 9 ge of de	card 6 card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  8  42  {	ossible (Card) = 9	e sums fr (4, 9) 10 10 110 110 110 110 110 110 110 110	: 11, 12 will be J J 44	, 13, 14 P(MinC Q 14	r a tie- ard+11	+1) + A  45	P(La						
J     43     advantage of player over the dealer       Q     42     43     14     15     16     17     18     19     20     20     20     20     21       K     43	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A 2: Play y, in thee P\D 2 3 4 5 6 7 8	table t  2  lower tit  6  //er gets table t	s 10, Do  s 10, Do  s 10, Do  the points  f 2  s 10, Do  the points  f 3  f 6  f 7  g 8  g 9  the points  f 10  f	nt (Play 4 initial d 8 advantaj	yer's 1s 5 dealer's 9 ge of de	card 6 card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  8  42  {	ossible (Card) = 9	e sums fr (4, 9) 10 10 110 110 110 110 110 110 110 110	: 11, 12 will be J J 44	, 13, 14 P(MinC Q 14	r a tie- ard+11	+1) + A  45	P(La						
Q 12 13 14 15 16 17 18 19 20 20 20 21 K 13	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A 2: Play y, in thee P\D 2 3 4 5 6 7 8 9 9	table t  2  lower tit  6  //er gets table t	s 10, Do  s 10, Do  s 10, Do  the points  f 2  s 10, Do  the points  f 3  f 6  f 7  g 8  g 9  the points  f 10  the poin	nt (Play 4 initial d 8 advantaj	yer's 1s 5 dealer's 9 ge of de	card 6 card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  8  42  {	ossible (Card) = 9	e sums fr (4, 9) 10 10 110 110 110 110 110 110 110 110	: 11, 12 will be J J 44	, 13, 14 P(MinC Q 14	r a tie- ard+11	+1) + A  45	P(La						
K 13	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A 2: Play y, in thee P\D 2 3 4 5 6 7 8 9 10 10 10 10 10 10 10 10 10 10 10 10 10	table t  2  lower tit  6  //er gets table t	s 10, Do  s 10, Do  s 10, Do  the point  s 9  the point  the point	nt (Play 4 initial d 8 advantaj	yer's 1s 5 dealer's 9 ge of de	card 6 card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  42  {	ossible Card) = 9	e sums fr (4, 9) 10 10 110 110 110 110 110 110 110 110	: 11, 12 will be J  14	, 13, 14 P(MinC Q 14	r a tie- ard+11	+1) + A  45	P(La						
	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A 5 6 6 7 8 9 10 J Q K A 5 6 6 7 8 9 10 J J 5 6 6 7 7 8 9 10 J J J J J J J J J J J J J J J J J J	table t  2  lower th  6  //er gets  table t  2	s 10, Do  s 10, Do  s 10, Do  the point  s 9  10  11  12  13  13	advanta	yer's 1s 5  dealer's 5  9  ge of dealers 3. T yer's 1s 5  } lower	card 6 card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  42  {  l player'  8	ossible Card) = 9	e sums (4, 9) 1 10 44 44 10 10 10 10 10 10 10 10 10 10 10 10 10	: 11, 12 will be J 44  44  J J J J J J J J J J J J J J	3, 14 for Q	r a tie- ard+11 K	+1) + A  45	P(La						
A 14	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A 5 6 7 8 9 10 J J G G G G G G G G G G G G G G G G G	table t  2  lower th  6  //er gets  table t  2	s 10, Do  s 10, Do  s 10, Do  the point  s 9  the point  s 40  the point  s 42  the point  the point  s 40	advanta	yer's 1s 5  dealer's 5  9  ge of dealers 3. T yer's 1s 5  } lower	card 6 card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  42  {  l player'  8	ossible Card) = 9	e sums (4, 9) 1 10 44 44 10 10 10 10 10 10 10 10 10 10 10 10 10	: 11, 12 will be J 44  44  J J J J J J J J J J J J J J	3, 14 for Q	r a tie- ard+11 K	+1) + A  45	P(La						
	This way Ties  Example This way	y, in thee P\D 2 3 4 5 6 7 8 9 10 J Q K A 5 6 7 8 9 10 J Q K A 5 6 7 8 9 10 J G K A 5 6 7 8 9 10 J K 6 6 7 7 8 9 10 J K 6 6 7 7 8 9 10 J K 6 9	table t  2  lower th  6  //er gets  table t  2	s 10, Do  s 10, Do  the point  s 6  7  8  9  10  11  12  13  13  13	advanta	yer's 1s 5  dealer's 5  9  ge of dealers 3. T yer's 1s 5  } lower	card 6 card 40 aler over	mple gir, Deale 7	ves 5 p r's 1st 8  42  {  l player'  8	ossible Card) = 9	e sums (4, 9) 1 10 44 44 10 10 10 10 10 10 10 10 10 10 10 10 10	: 11, 12 will be J 44  44  J J J J J J J J J J J J J J	3, 14 for Q	r a tie- ard+11 K	+1) + A  45	P(La						

Adv.	P\D	2	3	4	5	6	7	8	9	10-K	Α										
	2	0.0000	-0.0885	-0.1724	-0.2549	-0.3268	-0.3913	-0.4513	-0.5532	-0.6012	-0.6057		Legen	d:							
	3	0.0885	0.0000	-0.0840	-0.1664	-0.2384	-0.3028	-0.3628	-0.4648	-0.5127	-0.5172			Advan	tage to	Player					
	4	0.1724	0.0840	0.0000	-0.0825	-0.1544	-0.2189	-0.2789	-0.3808	-0.4288	-0.4333			Advan	tage to	Dealer					
	5	0.2549	0.1664	0.0825	0.0000	-0.0720	-0.1364	-0.1964	-0.2984	-0.2744	-0.3508			No ad	vantag	Э					
	6	0.3268	0.2384	0.1544	0.0720	0.0000	-0.0645	-0.1244	-0.2264	-0.2744	-0.2789										
	7	0.3913	0.302	0.2189	0.1364	0.0645	0.0000	-0.0600	-0.1619	-0.2099	-0.2144										
	8	0.4513	0.3628	0.2789	0.1964	0.1244	0.0600	0.0000	-0.1019	-0.1499	-0.1544										
	9	0.5532	0.4648	0.3808	0.2984	0.2264	0.1619	0.1019	0.0000	-0.0480	-0.0525										
	10-K	0.6012	0.5127	0.4288	0.3463	0.2744	0.2099	0.1499	0.0480	0.0000	-0.0045										
	Α	0.6057	0.5172	0.4333	0.3508	0.2789	0.2144	0.1544	0.0525	0.0045	0.0000										
General s	strateg	y will b	e to do	uble e	verytin	ne the f	irst car	ds are	greate	than t	he dea	lers' fir	st card								
In the ca	se of ti	e for fi	st card	is Play	er vs D	ealer, v	ve will	assum	e no do	ubling	will be	made.									
As we wi	ill alwa	ys bet 1	for Tie,	as per	our st	rategy,	if cards	are th	e same	e, we w	ill ALS	O Doub	le whe	n it's ti	e belov	v:					
Decision	:	P\D	2	3	4	5	6	7	8	9	10-K	Α									
Double		2	TIE/DD	KP	KP	KP	KP	KP	KP	KP	KP	KP		Legen	d:						
or Not		3	DD	TIE/DD	KP	KP	KP	KP	KP	KP	KP	KP		TIE	meas	equal ch	anges				
		4	DD	DD	TIE/DD	KP	KP	KP	KP	KP	KP	KP		DD	means	double	initial be	t			
		5	DD	DD	DD	TIE/DD	KP	KP	KP	KP	KP	KP		KP	means	keep o	iginal be	t			
		6	DD	DD	DD	DD	TIE/DD	KP	KP	KP	KP	KP									
		7	DD	DD	DD	DD	DD	TIE/DD	KP	KP	KP	KP									
		8	DD	DD	DD	DD	DD	DD	TIE/DD	KP	KP	KP									
		9	DD	DD	DD	DD	DD	DD	DD	TIE/DD	KP	KP									
		10-K	DD	DD	DD	DD	DD	DD	DD	DD	TIE/DD	KP									
		Α	DD	DD	DD	DD	DD	DD	DD	DD	DD	TIE/DD									
Strategy f	for whe	ther or	not goir	ng to wa	ar after	a TIE (r	non-bet)	will de	pend u	opon th	ere was	a doub	le-bet.	DD =>	NO WA	R					
	Let's e	valuate	possibl	le outco	omes fro	om diffe	rent stra	ategies	and sin	nulate a	large i	number	of roun	ds							
	For ea	ch poss	ible dol	lar outo	come, a	n histog	gram wi	I show	the wei	ght of e	ach do	lar outo	ome.								
	Initial	bet will	always	s be 2 (	even n	umber	\$valu	e chips	for sta	andard	refere	nce									
	See at	tached	earnin	g_path	s.png																
	Function	ons: p(y	), F(Y),	E(Y), s	sd(Y), p	lots f(Y)	and F(	Y)													
	This ga	ame will	do the	reverse	e way:	create a	progra	m to ru	n this g	ame 10	million	s of time	es, and	registe	r the p(	y) for ea	ch poss	ble outo	ome.		
	Check	attache	d the c	ode to l	be used	d on dat	a-genei	ated re	port.												
	https://	ann mir	ndmun (	com/ma	ap/ free	/2021/1	11/56a2	a1b048	3811ec	9708eb	12a2b8	37af4									

Data set as hist	togram per	\$2 initial be	et.									
Number of time	s a possible	e result was	actually o	btained per	row-seque	nce of million	on rounds					
Result: Y	-6	-4	-2	2	4	6	8	16	18		# Games	
	141401	317868	5926	77788	20	380140	5770	46794	24293		10^6	
	141369	317897	5958	78563	12	379532	5834	46253	24582		10^6	
	141427	317654	6009	78888	13	378858	5910	46560	24681		10^6	
	141512	317990	5926	78768	20	378975	5882	46248	24679		10^6	
	140840	317173	5924	78912	15	379892	5845	46707	24692		10^6	
	141958	316405	5949	78557	16	380274	5912	46307	24622		10^6	
	141520	317324	5829	78220	17	379877	5894	46667	24652		10^6	
	140995	317419	5851	78460	12	280579	5893	46305	24486		10^6	
	142133	317544	5867	78388	15	379433	5873	46086	24661		10^6	
	141936	317125	5990	78228	14	379512	5800	46542	24853		10^6	
Average:	141509	317440	5923	78477	15	369707	5861	46447	24620		1000000	
У	-6	-4	-2	2	4	6	8	16	18			
py(y)	0.14151	0.31744	0.00592	0.07848	0.00002	0.36971	0.00586	0.04645	0.02462		1.00000	
y * py(y)	-0.849054	-1.269759	-0.0118458	0.1569544	0.0000616	2.2182432	0.0468904	0.7431504	0.4431618	(dot prod) =	1.4778	E(Y)
# of Paths												Average
total_win	463693	463926	463653	463623	464647	464741	463989	464931	463691	463540		464043
total_lose	465146	465169	465036	465373	463892	464255	464624	464209	465489	465003		464820
total_tie	71161	70905	71311	71004	71461	71004	71387	70860	70820	71457		71137
total_war	35	34	31	41	29	43	39	33	33	30		35
total_nowar	39	36	39	36	33	32	29	36	40	32		35
total_win2	24	12	13	20	15	16	17	12	15	10		15
total_lose2	10	19	15	19	12	25	20	20	15	16		17
total_tie2	1	3	3	2	2	2	2	1	3	4		2
myGrandBala	1536896	1531844	1536278	1529828	1550184	1540960	1543256	1541866	1526984	1539930		1537803
winPerGame	1.5369	1.5318	1.5363	1.5298	1.5502	1.5410	1.5433	1.5419	1.5270	1.5399	Actual	1.5378

ET WINNII	NGS:										
	Actual	Average	Winnings	per Game:	1.5378						
t's suppos	e that the cas	ino knows	that the ac	tual net earn	ning per \$2	chip is \$1.54	, which is i	n the play	er's favor		
	in order to	bring the a	advantage	back to them	n, it is char	jed a \$8 fee s	so you can	play with	\$10-value	chips.	
	Y = 5X - 8										
	E(Y) = E(5	*X - 8) = 5	*E(X) - 8 =	5* 1.47 - 8 =	= 7.7 - 8 <b>=</b> -	0.65					
	Var(Y) = V	′ar(5X - 8)	= 25 Var(>	() , where Va	r(X) = E(X')	^2) - E(X)^2					
	sd(Y) = sq	rt(Var(Y)									

```
# Double-War Data Generator # Note: time to run 10^6 rounds is about 1 min
# Code made available at:
https://github.com/hedravBands/2P81 Probability with R/tree/main/JackWar
# Global variables
max_tries <- 10^6
total_win <- 0
total_lose <- 0
total_tie <- 0
total_war <- 0
total_nowar <- 0
total_win2 <- 0
total_lose2 <- 0
total_tie2 <- 0
MasterDeck <- c(2:10, 10, 10, 10, 11, 2:10, 10, 10, 10, 11, 2:10, 10, 10, 11,
2:10, 10, 10, 10, 11)
         \leftarrow c(1:28)*0 #histogram using indexes as results: where 10 is even, 11 is
results
1x gain, 1 is -1x loss
# Balance:
myGrandBalance
               <- 0
myOriginalBalance <- 0
myOriginalTieBet <- 0
myOriginalBet
                 <- 2
# Strategy
onTieDifference <- 8
# Util functions
newDeck <- function(){</pre>
  \# face values: 2:10, J, K, Q, A times 4 = 52 cards
  return(MasterDeck)
}
# Draw card at any time during a game
drawCard <- function(){</pre>
  idx \leftarrow sample(1:52, 1)
  # re-sample until card is valid (value not 0)
  while (WorkingDeck[idx] == 0) {
    idx \leftarrow sample(1:52, 1)
  card <- WorkingDeck[idx]</pre>
  WorkingDeck <- replace(WorkingDeck, idx, 0) #set card as drawn (value 0)</pre>
  return(card)
}
# Decision-Maker 1: Absolute difference less than or equal to onTieDifference
shouldBetOnTie <- function(h, d){</pre>
  return((abs(h[1]-d[1]) <= onTieDifference))
}
# Decision-Maker 2: Player's first card is greater than or equal to Dealer's
```

shouldDoubleBet <- function(h, d){</pre>

```
return((h[1] - d[1]) >= 0)
}
# Decision-Maker 3: Decision whether or not go to WAR: DD => no war
shouldGoToWar <- function(dd){</pre>
  return(!dd)
# CHECK VICTORY FOR 1ST ROUND, 1 WIN, 0 TIE, -1 LOSE
isThisAVictory <- function(h, d){
  if ((h[1]+h[2]) - (d[1]+d[2]) \rightarrow \emptyset) {return(1)}
  if ((h[1]+h[2]) - (d[1]+d[2]) == 0) {return(0)}
  if ((h[1]+h[2]) - (d[1]+d[2]) < 0) {return(-1)}
*******************************
## PLAY SEVERAL TIMES
for(i in 1:max_tries){
# since only 1 deck is available, every round needs a new deck
WorkingDeck <- newDeck()</pre>
# MONEY RELATED
myBalance <- myOriginalBalance
myBet
            <- myOriginalBet</pre>
myTieBet <- myOriginalTieBet</pre>
# GET INITIAL CARDS
        <- c(drawCard(), 0)</pre>
dealer <- c(drawCard(), 0)</pre>
# DECISION-MAKER 1: BET ON TIE?
isBetOnTie = shouldBetOnTie(hand, dealer)
if (isBetOnTie) {myTieBet <- myBet}</pre>
# DECISION-MAKER 2: DOUBLE ORIGINAL BET?
isDoubleBet <- shouldDoubleBet(hand, dealer)</pre>
if (isDoubleBet) {myBet <- 2*myBet}</pre>
# Play for ROUND 1
hand[2] <- drawCard()</pre>
dealer[2] <- drawCard()</pre>
# DECISION-MAKER 3: GO TO WAR? Strategy: DD => NO WAR
isThisWar <- shouldGoToWar(isDoubleBet)</pre>
result <- isThisAVictory(hand, dealer)
# EVALUATE EACH POSSIBLE RESULT: WIN, LOSE, TIE, NO_WAR, WAR: WIN2, LOSE2, TIE2
if (result == 1) {
  total_win <- total_win + 1
  myBalance <- myBet*2 - myTieBet</pre>
  myGrandBalance <- myGrandBalance + myBalance</pre>
  index <- 10 + myBalance
  results <- replace(results, index, results[index] + 1)
}
else if (result == -1){
```

```
total_lose <- total_lose + 1
  myBalance <- result*(myBet + myTieBet)</pre>
  myGrandBalance <- myGrandBalance + myBalance</pre>
  index <- result*myBalance</pre>
  results <- replace(results, index, results[index] + 1)
}
else{
      total_tie <- total_tie + 1
      #cases for tie 1st. round
      if (isBetOnTie) {
        myBalance <- myTieBet*10 - myBet</pre>
        myGrandBalance <- myGrandBalance + myBalance</pre>
        index <- 10 + myBalance
        results <- replace(results, index, results[index] + 1)
      } #end win by tie result
      else{
             if (!isThisWar) {
            total_nowar <- total_nowar + 1</pre>
            myBalance \leftarrow as.integer((-1)*myBet/2) #will become index: integer
            myGrandBalance <- myGrandBalance + myBalance</pre>
             index \langle -(-1)*myBalance
            results <- replace(results, index, results[index] + 1)
             } #end not-war
            else {
                     total_war <- total_war + 1
                     myBet <- 2*myBet
                     # GET ANOTHER PAIR OF CARDS (AND DISCART THE OLD PAIRs)
                               <- c(drawCard(), drawCard())</pre>
                     dealer2 <- c(drawCard(), drawCard())</pre>
                     # GET NEW RESULT AND ANALYSE
                     result2 <- isThisAVictory(hand2, dealer2)</pre>
                     # ANALYSIS FOR THE SECOND ROUND, AFTER WAR: WIN2,LOSE2,TIE2
                     if (result2 == 1) {
                       total_win2 <- total_win2 + 1
                       myBalance <- myBet
                       myGrandBalance <- myGrandBalance + myBalance</pre>
                        index <- 10 + myBalance
                       results <- replace(results, index, results[index] + 1)
                     } else if (result2 == -1){
                       total_lose2 <- total_lose2 + 1
                       myBalance <- result2*(myBet)</pre>
                       myGrandBalance <- myGrandBalance + myBalance</pre>
                        index <- result2*myBalance</pre>
                       results <- replace(results, index, results[index] + 1)</pre>
                     } else {
                       total_tie2 <- total_tie2 + 1
                       myBalance <- myBet*2
                       myGrandBalance <- myGrandBalance + myBalance</pre>
                        index <- 10 + myBalance
                       results <- replace(results, index, results[index] + 1)
                     }
             }
```

```
} #end tie-cases
} # end for-loop
winPerGame = myGrandBalance/max_tries
############
total_win
total_lose
total_tie
total_war
total_nowar
total_win2
total_lose2
total_tie2
myGrandBalance
winPerGame
results
# total_win
# [1] 463540
# > total_lose
# [1] 465003
# > total_tie
# [1] 71457
# > total_war
# [1] 30
# > total_nowar
# [1] 32
# > total_win2
# [1] 10
# > total_lose2
# [1] 16
# > total_tie2
# [1] 4
# > myGrandBalance
# [1] 1539930
# > winPerGame
# [1] 1.53993
# >
#
    >
  > results
# [1]
               5990
                         0 317125
                                       0 141936
                                                      0
                                                                           0
                                                                                  0
           0
                                                             0
78228
                                       0
           0
                 10
                         0 379512
                                           5804
                                                      0
            0
# [22]
                                    46542
                                                  24853
# >
  > ###############
```