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$$\frac{1}{b-0}\int_0^b f(x)\,dx = 9 \quad , b \neq 0$$

$$\int_0^b \left(8+10x-9x^2\right)\,dx = 9b$$

$$8x + \frac{10x^2}{2} - \frac{9x^3}{3}\Big|_0^b = 9b$$

$$8(b-0) + 5(b^2-0^2) - 3(b^3-0^3) = 9b$$

$$8b - 9b + 5b^2 - 3b^3 = 0$$

$$-3b^3 + 5b^2 - b = 0$$

$$b(-3b^2 + 5b - 1) = 0$$

$$\Rightarrow b = 0 \quad \text{or} \quad (-3b^2 + 5b - 1) = 0$$

but $b \neq 0$, thus:

$$b = \frac{-5 \pm \sqrt{25 - 4.3.1}}{-2.3}$$

$$b = \frac{-5 \pm \sqrt{13}}{-6}$$

$$\begin{cases} b = \dfrac{5 - \sqrt{13}}{6} \\ \text{or} \\ b = \dfrac{5 + \sqrt{13}}{6} \end{cases}$$

$$S = \left\{ \frac{5 - \sqrt{13}}{6} \; , \; \frac{5 + \sqrt{13}}{6} \right\}$$