

Area 1 : $\int_{-1}^{3/2} [f(y) - g(y)] dy$

Area 2 : $\int_{1}^{3/2} [h(y) - g(y)] dy$

Area total : Area 1 + Area 2.

Resolution:

$$A_1 = \int_{-1}^1 [(y-1)^3 - (2y-6)] \cdot dy$$

$$A_1 = \int_{-1}^1 (y^3 - 3y^2 + 3y - 1 - 2y + 6) dy$$

$$A_1 = \int_{-1}^1 (y^3 - 3y^2 + y + 5) dy$$

$$A_1 = \left[\frac{y^4}{4} - \frac{3y^3}{3} + \frac{y^2}{2} + 5y \right]_{-1}^1$$

$$A_1 = \frac{1}{4}[1^4 - (-1)^4] - [1^3 - (-1)^3] + \frac{1}{2}[1^2 - (-1)^2] + 5[1 - (-1)]$$

$$A_1 = \frac{1}{4}(1-1) - (1+1) + \frac{1}{2}(1-1) + 5(1+1)$$

$$A_1 = -2 + 10 = 8 \quad \langle \text{units} \rangle^2$$

$$A_2 = \int_1^{1.5} [(-6y + 6) - (2y - 6)] dy$$

$$A_2 = -4 \int_1^{1.5} (2y - 3) dy$$

$$A_2 = -4 \left[\frac{2y^2}{2} - 3y \right]_1^{3/2}$$

$$A_2 = -4 \left[\left(\frac{9}{4} - 1 \right) - 3 \left(\frac{3}{2} - 1 \right) \right]$$

$$A_2 = -4 \left[\left(\frac{9-4}{4} \right) - 3 \left(\frac{3-2}{2} \right) \right]$$

$$A_2 = -4 \left(\frac{5}{4} \right) - (-4) \cdot 3 \cdot \frac{1}{2}$$

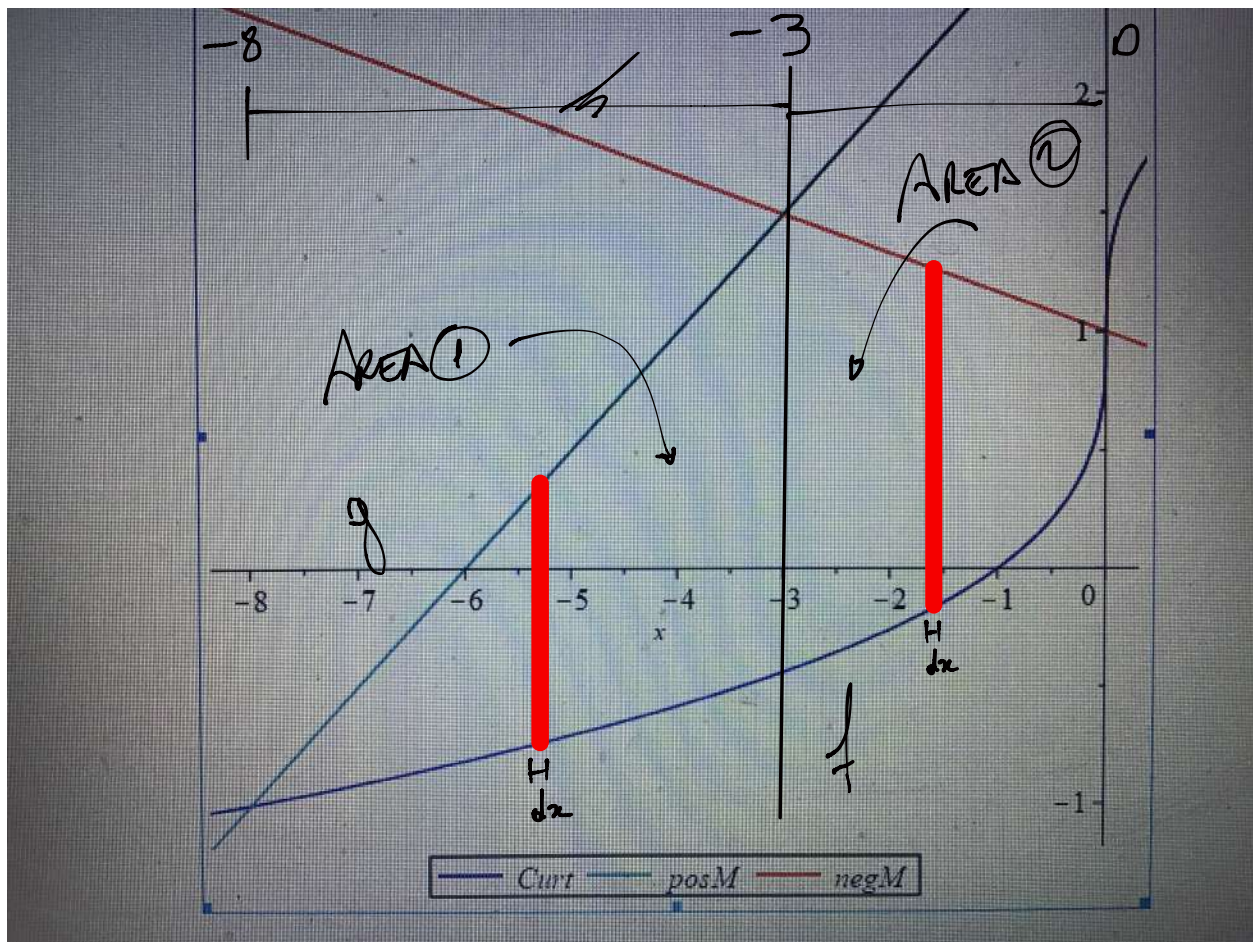
$$A_2 = -5 + 6 = 1 \quad (\text{unit})^2$$

Therefore, Area R is A_T ,

$$A_T = A_1 + A_2$$

$$A_T = 8 + 1$$

$$A_T = 9 \quad \text{< unit >}^2$$



$$A_1 = \int_{-8}^{-3} [g(x) - f(x)] \cdot dx$$

$$A_2 = \int_{-3}^0 [h(x) - f(x)] dx$$

Resolution:

$$A_1 = \int_{-8}^{-3} \left[\left(\frac{x}{2} + 3 \right) - (x^{\frac{1}{3}} + 1) \right] \cdot dx$$

$$A_1 = \int_{-8}^{-3} \left(\frac{x}{2} - x^{\frac{1}{3}} + 2 \right) dx$$

$$A_1 = \left[\frac{1}{2} \cdot \frac{x^2}{2} - \frac{3}{4} x^{\frac{4}{3}} + 2x \right]_{-8}^{-3}$$

$$A_1 = \frac{1}{4} \left[(-3)^2 - (-8)^2 \right] - \frac{3}{4} \left[(-3)^{\frac{4}{3}} - (-8)^{\frac{4}{3}} \right] + 2 \left[-3 - (-8) \right]$$

$$A_1 = \frac{1}{4} (9 - 64) - \frac{3}{4} \left((-3)(-3)^{\frac{1}{3}} - (-8)(-2) \right) + 2(5) \cdot \frac{4}{4}$$

$$A_1 = -\frac{55}{4} + \frac{9(-3)^{\frac{1}{3}}}{4} + \frac{3}{4} \cdot 16 + \frac{1}{4} \cdot 40$$

$$A_1 = \frac{33}{4} - \frac{9(-3)^{\frac{1}{3}}}{4} \quad \text{<UNIT>}^2$$

$$A_2 = \int_{-3}^0 \left[\left(-\frac{x}{6} + 1 \right) - \left(x^{1/3} + 1 \right) \right] dx$$

$$A_2 = - \int_{-3}^0 \left(\frac{x}{6} + x^{1/3} \right) dx$$

$$A_2 = - \left(\frac{1}{6} \cdot \frac{x^2}{2} + \frac{3}{4} x^{4/3} \right) \Big|_{-3}^0$$

$$A_2 = - \left[\frac{1}{12} \cdot \left[0^2 - (-3)^2 \right] + \frac{3}{4} \left(0^{4/3} - (-3)^{4/3} \right) \right]$$

$$A_2 = - \left[-\frac{1}{12} \cdot 9 + \frac{3}{4} \cdot 3(-3)^{1/3} \right]$$

$$A_2 = \frac{3}{4} + \frac{9 \cdot (3)^{1/3}}{4}$$

Therefore

$$A_t = A_1 + A_2$$

$$A_t = \frac{33}{4} - \frac{9(3)^{\frac{1}{3}}}{4} + \frac{3}{4} + \frac{9(3)^{\frac{1}{3}}}{4}$$

$$A_t = \frac{36}{4}$$

$$A_t = 9 \quad \langle \text{unit} \rangle^2$$