Kalman Filter Applications In Airborne Radar Tracking

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Abstract

This paper studies the application of Kalman filtering to single-target track systems in airborne radar. An angle channel Kalman filter is configured which incorporates measures of range, range rate, and on-board dynamics. Theoretical performance results are given and a discussion of methods for reducing the complexity of the Kalman gain computation is presented. A suboptimal antenna controller which operates on the outputs of the angle Kalman filter is also described. In addition, methodological improvements are shown to exist in the design of range and range-rate trackers using the Kalman filter configuration.

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I. Introduction

This paper investigates the application of modern estimation theory to the design of single-target track systems in airborne radar. The functions of the tracking system in airborne radar are to provide estimates of variables associated with the relative range vector between the tracking aircraft and the target and to optimize the system's ability to maintain lock-on. The estimates are used for execution of the intercept mission, for instance, in an airborne interceptor for interceptor guidance and missile launch computations, and in an air-to-air missile to form the missile guidance commands.

The desired estimates are obtained from two basic systems, called the angle and range tracking systems. The angle tracking system provides estimates of angular variables, such as direction and spatial angular rate of the target line-of-sight (LOS), while the range tracking system estimates such variables as scalar range and range rate. Tracking of range rate independent of range measurements is also available with Doppler radar.

The methodology traditionally used to design these systems has been the classical servomechanism theory. For each track loop a measure of the control error provided by the radar receiver is operated on by some form of gain and compensation network to produce the loop output and feedback signals. The input characteristics of the system for design purposes are the statistics of the input noises and the expected characteristics of the input geometrical variables. For a fixed-parameter system, the compensator must be chosen so that the error is tolerable for worst-case conditions [1]. If the resulting accuracy over the total set of conditions is not adequate, the need then arises to adapt the loop structure as a function of the conditions. This situation presents the question of how this adaptation is best done and, in particular, how to make best use of measurable on-board variables from other subsystems. The Kalman filter provides one approach to answering these optimization questions.

The first sections of this paper give a description of one form of angle channel Kalman filter, along with some discussion of its performance and means for minimizing its computational complexity. A discussion is then given of a controller structure for an angle track loop using the Kalman filter. The paper concludes with a description of the design of range and velocity track loops using the Kalman approach.

A digital implementation based on the tracking filter structures described in this paper has been flight tested in an airborne tactical radar system by the Hughes Aircraft Company. The flight tests successfully demonstrated many of the performance improvements which are predicted in theory and are discussed in the paper.

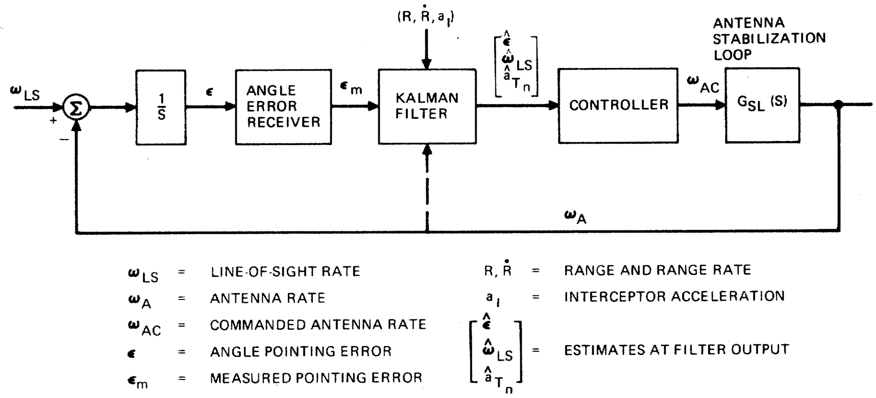


Fig. 1. Pointing error control system using Kalman estimates.

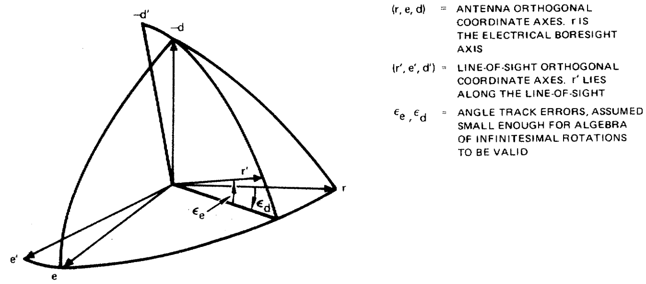


Fig. 2. Antenna and line-of-sight coordinate frames.

II. Angle Filter Structure

The general form of the angle track system to be studied is shown in Fig. 1. The integral of the difference between the line-of-sight rate ω_{LS} and the antenna rate ω_A is seen to be the pointing error ϵ, a measure of which is produced by the angle error receiver of the radar. This measurement ϵ_m is inputted to the Kalman filter, along with measures of ω_A, range and range rate, and the inertial acceleration of the interceptor, a_I. The outputs of the filter are estimates of the states ϵ, ω_{LS}, and a_{T_n}, where a_{T_n} is the component of the target acceleration normal to the line-of-sight. The filter outputs are seen to be used as the inputs to the controller, which forms the commanded antenna rate ω_{A_c}. The typical antenna stabilization loop functions as an inertial rate servo, with the input and output signals being the commanded and actual values of the antenna spatial rate ω_A. In this section we shall consider the structure of the Kalman filter portion of the loop. A discussion of the controller will be given at a later point in Section V.

In order for the techniques of Kalman filtering to be applicable for estimating the states of a system, it must be possible to represent the dynamics and outputs of the system sufficiently well by a linear vector differential state equation of the form

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{u} + \mathbf{w} \tag{1}$$

with linear measurement or output equation

$$\mathbf{y} = M\mathbf{x} + \mathbf{n} \tag{2}$$

where the values of A, $\mathbf{u}$, and M are known functions of time, and $\mathbf{w}$ and $\mathbf{n}$ are white noise vectors with known spectral densities. $\mathbf{x}$ is called the system state vector and $\mathbf{y}$ the system output vector, the latter being used as the filter input. The Kalman filter then computes the best mean-square (minimum variance [2]) estimate of $\mathbf{x}(t)$ based on $\mathbf{y}(\tau)$ for $t_0 \leqslant \tau \leqslant t$ using the Kalman filter structure parameters determined by the values of A, $\mathbf{u}$, and M, along with the statistics of $\mathbf{w}$ and $\mathbf{n}$.

The particular system model to be studied includes states which correspond to the components of the angle tracking error and the components of the spatial rate of the line-of-sight. A Kalman filter whose mechanization is based on this system model will thus estimate the angular position and spatial angular rate of the line-of-sight, which are the geometrical variables about which information is also provided by a classically designed angle tracking system. Turning to a definition of coordinate systems, ϕ will be used throughout to designate a right-hand orthogonal coordinate frame. Let ϕ_A be the antenna coordinate frame with axes r, e, and d. The antenna electrical boresight, or forward-pointing centerline axis, will be labeled the r axis. The e and d axes are defined as being the axes about which the two components of the angle tracking error are measured by the two channels of the angle error receiver (see Fig. 2). The components of the angle tracking error about the e and d axes will be labeled ϵ_e and ϵ_d. Define a second frame, to be called the line-of-sight frame ϕ_{LS}, as that orthogonal coordinate frame which is obtained by perturbing the antenna frame ϕ_A through the small angles ϵ_e about the e axis and ϵ_d about the d axis. Let the corresponding axes of the ϕ_{LS} coordinate frame be called the r', e', and d' axes. The r' axis is then, by definition, coincident with the target line-of-sight.

The nomenclature to be used to describe the inertial (or spatial) angular rate of a coordinate frame will be ω. Let ω_A and ω_{LS} be the inertial angular rates of the frames ϕ_A and ϕ_{LS}. The components of ω_A along the axes of ϕ_A are then ω_{A_r}, ω_{A_e}, and ω_{A_d}, respectively, while the components of ω_{LS} along the axes of ϕ_{LS} are $\omega_{LS_{r'}}$, $\omega_{LS_{e'}}$, and $\omega_{LS_{d'}}$.

With the above definitions in mind, let us next examine the following set of equations:

$$y_e = k_e \epsilon_e + n_e \tag{3}$$

$$y_d = k_d \epsilon_d + n_d \tag{4}$$

$$\dot{\epsilon}_e = \omega_{LS_{e'}} - \omega_{A_e} + \omega_{A_r} \epsilon_d \tag{5}$$

$$\dot{\epsilon}_d = \omega_{LS_{d'}} - \omega_{A_d} - \omega_{A_r} \epsilon_e \tag{6}$$

$$\dot{\omega}_{LS_{e'}} = -(2\dot{R}/R)\omega_{LS_{e'}} + (1/R)(a_{T_{-d'}} + a_{I_d}) + \omega_{A_r}\omega_{LS_{d'}} \tag{7}$$

$$\dot{\omega}_{LS_{d'}} = -(2\dot{R}/R)\omega_{LS_{d'}} + (1/R)(a_{T_{e'}} - a_{I_e}) - \omega_{A_r}\omega_{LS_{e'}} \tag{8}$$

$$\dot{a}_{T_{-d'}} = -(1/\tau)a_{T_{-d'}} + \omega_{A_r} a_{T_{e'}} + w_{d'} \tag{9}$$

$$\dot{a}_{T_{e'}} = -(1/\tau)a_{T_{e'}} - \omega_{A_r} a_{T_{-d'}} + w_{e'} \tag{10}$$

Equations (3) and (4) are models for the angle error receiver outputs y_e and y_d, where k_e and k_d are the angle error receiver slopes and n_e and n_d are additive noises. The derivation of (5) through (10) is indicated in the Appendix, and is based on the assumption that the track errors are small enough for the operations of small angle transformations to be valid. Equations (5) and (6) define the dynamical interrelations between the tracking errors and the components of ω_A and ω_{LS}, while (7) and (8) represent the dynamical interrelations between the components of ω_{LS} and the inertial accelerations of the two vehicles. The quantities $a_{T_{e'}}$ and $a_{T_{-d'}}$ designate the components of the target acceleration along the e' and $-d'$ axes, while a_{I_e} and a_{I_d} are components of interceptor (or ownship) acceleration along the e and d axes. (Writing the equations in terms of $a_{T_{-d'}}$ instead of $a_{T_{d'}}$ is a notational convenience which simplifies the discussion at a later point.) The quantities R and $\dot{R}$ denote range to the target and its derivative, while τ represents the time constant of the assumed target acceleration model and $w_{e'}$ and $w_{d'}$ are independent white noises with equal spectral densities.

As discussed in the Appendix, the target acceleration state equations (9) and (10) are especially appropriate for the situation in which the magnitudes of the LOS spatial rates $\omega_{LS_{e'}}$ and $\omega_{LS_{d'}}$ do not exceed 15 or 20 degrees per second. Although the use of these models could yield adequate results for higher LOS rate cases, a more nearly optimum approach in this situation involves modeling the components of the target acceleration vector along space-stabilized axes as states [3]. A significant disadvantage of the latter approach is that it is computationally more complex and is less amenable to the method of reducing computation discussed in the next section. It is also noted that the low angular rate situation is the primary one of interest in airborne tracking, since at long ranges the LOS rates are perforce small, while at short ranges the geometric condition of interest for gun and rocket firing is the lead collision course, for which the spatial rates are again relatively small.

The above state and measurement equations, (3) through (10), form the system model on which the angle channel Kalman filter is based, with the filter thus producing estimates of the components of ϵ, ω_{LS}, and a_T. A discussion is next given of how the parameters required for the mechanization may be obtained on-line. In the context of (1) and (2), these parameters are those associated with the matrixes A, $\mathbf{u}$, and M, along with the statistics of the system driving and measurement noises. The receiver gain and noise statistics associated with the measurement equations (3) and (4) may often be determined from a priori analysis of the specific receiver being implemented. In relation to the state equations (5) through (10), it will be seen that the filter parameter requirements are well matched to the outputs of on-board sensors which are often available. Considering first the measurement of the antenna rates ω_{A_e} and ω_{A_d}, the antenna stabilization loops typically provide space stabilization of the antenna about the e and d axes using gyros mounted on these axes. These gyros are usually either of the rate or rate integrating type. If rate gyros are used, their outputs provide direct measures of the rates ω_{A_e} and ω_{A_d}. Alternately, if rate integrating gyros are employed, the input signals to the stabilization loops can be used as the measures of ω_{A_e} and ω_{A_d} in the filtering process, since these inputs are the commanded values of the rates ω_{A_e} and ω_{A_d}. Although a measure of ω_{A_r} is often not used in design, it is seen to be easily obtainable. The simplest method is to also mount a rate gyro on the r axis of the antenna. As to the remaining measureable parameters, measures of R and $\dot{R}$ are often available from range channel processing. Also, it is often possible to obtain measures of the ownship acceleration components a_{I_e} and a_{I_d}. These would be obtainable from the outputs of on-board accelerometers by first eliminating the gravity component from their outputs, and then resolving the resulting vector into antenna coordinates. Finally, the parameters which specify the statistical characteristics of the target acceleration models, namely the time constant tan and the spectral densities of w_e' and w_d', are chosen by the designer in accordance with the expected behavior of the actual target acceleration.

The foregoing discussion indicates that implementing a Kalman filter based on the state and measurement equations (3) through (10) is viable from the point of view of obtaining the required parameters needed in the filter structure. The estimates $\hat{\epsilon}_e$ and $\hat{\epsilon}_d$ produced by the filter will be seen in a later section to be useful for antenna control. As measures of the LOS rate components are standard quantities needed for the intercept mission, the filter's estimates $\hat{\omega}_{LS_e}$, and $\hat{\omega}_{LS_d}$, can be used for this purpose. The estimates $\hat{a}_{T_e}$, and $\hat{a}_{T-d}$, of the target acceleration components are of potential use, for instance, in optimal air-to-air missile navigation [5].

It is of interest to note that several of the on-board dynamical measurements, specifically those of ω_{A_e}, ω_{A_d}, a_{I_e}/R, and a_{I_d}/R, are treated as elements of the known deterministic driving vector **u** in (1). As the Kalman filter accuracies are theoretically independent of the deterministic input **u** [4], assuming that **u** is used as an input to the filter, this implies, for the system at hand, that the filter accuracies will be relatively independent of interceptor acceleration and also the e and d components of antenna rate. This latter fact implies that if the Kalman filter were operating on the outputs of a track loop not closed through the filter, then the filter estimation accuracies could be expected to be close to independent of the loop compensation and bandwidth, provided that the receiver linearity assumptions embodied by (3) and (4) remained in force.

III. Simplified Gain Computation

A major factor contributing to the complexity of the Kalman filter discussed in the previous section is the Kalman gain computation, which, for the filter under discussion, would, in general, require the solution of a six-by-six matrix Ricatti error covariance equation. It will be seen in this section that in some cases of interest it is possible to compute the required filter gains using considerably less computation, while at the same time incurring little or no reduction in estimation accuracy. Two not atypical sets of conditions will be cited under which it would be permissible to compute the gains from the solution of a three-by-three instead of a six-by-six error covariance equation.

A first situation under which a simplified gain calculation is legitimate is specified by the following "symmetric" conditions: 1) receiver gains k_e and k_d are equal, and receiver noises n_e and n_d are statistically independent white noises with equal spectral densities; 2) the initial error covariance matrix of the states, $P(0)$, is diagonal with the diagonal elements being such that $P_{11}(0) = P_{22}(0)$, $P_{33}(0) = P_{44}(0)$, and $P_{55}(0) = P_{66}(0)$; and 3) the target acceleration state equations have the properties discussed in the Appendix, namely, that the states a_{T_e}, and a_{T-d}, are initially statistically independent and have equal initial variance, with the white noises w_e, and w_d, in (9) and (10) being statistically independent but having equal spectral densities. Condition 2) states that the initial uncertainties on the two components of ϵ are equal, as also are those on the two components of ω_{LS} and a_T.

Under the above conditions, the complexity of the gain computation can be greatly reduced since the gains can be obtained from those associated with the analogous two-dimensional problem. The equivalent two-dimensional problem is specified by the state and measurement equations of either the elevation or azimuth channel when ω_{A_r} is set to zero in (3) through (10). Define the 3×1 Kalman gain matrix associated with the planar state and measurement equations as having the elements K_{11}, K_{21}, and K_{31}. Next assume that the parameters R, $\dot{R}$, k, and τ, along with the statistics of n and w, are identical for the two- and three-dimensional cases, and, further, let the symmetrical conditions given in 1) through 3) hold for the three-dimensional case. Then, an examination of the error covariance and gain equations for the six-state filter of the three-dimensional case shows that the 6×2 Kalman gain matrix K can be written in the particularly simple form

$$K = \begin{bmatrix} K_{11} & 0 & K_{22} & 0 & K_{31} & 0 \\ 0 & K_{11} & 0 & K_{21} & 0 & K_{31} \end{bmatrix}^T \tag{11}$$

where the K_{i1} shown are the two-dimensional gains. That is, the three-dimensional gain matrix can, in this case, be computed from its two-dimensional equivalent and is independent of the antenna roll rate ω_{A_r}. The zero elements in the gain matrix (11) indicate that there is no measurement residual cross coupling between the channels, so that the filter differential equations for the elevation channel states are not driven by the measured azimuth track error, and vice versa. It is also noted that, in this case, the computed estimation error variances for each of the two components of ϵ, ω_{LS}, and a_T in the three-dimensional case are identical and equal to their equivalents in the two-dimensional solution.

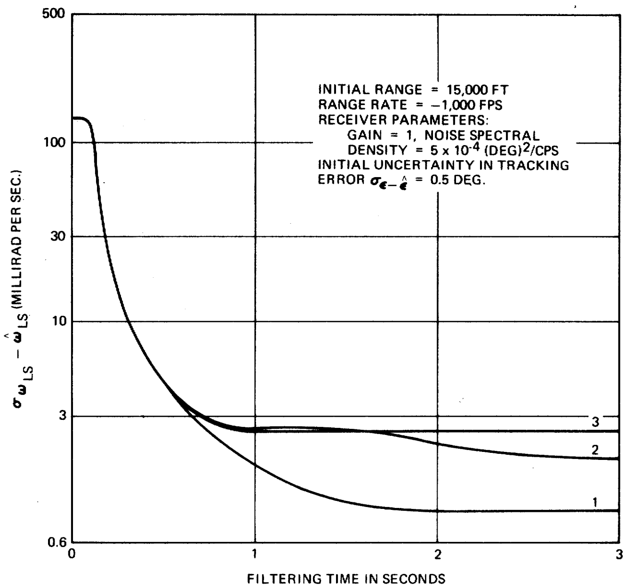
A somewhat more general set of conditions can also be cited for which the computational reduction discussed above is possible. Suppose that the conditions met in the previous case hold, except that k_e does not equal k_d, and, also, the receiver noise spectral densities are not necessarily the same. To examine the legitimacy of computing the gains for both angle channels from one two-dimensional gain computation in this case, it is first noted that the receiver inputs y_e and y_d of (3) and (4) can be multiplicatively scaled so that the two resulting receiver gains are equal. As a result of this scaling, the present case differs from the previous one only in that the spectral densities of the e and d channel receiver noises are not necessarily equal. However, if the Kalman gains are computed for the two-dimensional situation using the larger of the receiver noise spectral densities, and these gains are used in the three-dimensional filter as shown in (11), then it is easily demonstrated that the estimation error variances are upper bounded by their corresponding values associated with the two-dimensional gain solution. Consequently, if these upper bounds give sufficient accuracy, the suggested procedure can be used and provides the desired simplification in computing the three-dimensional gains.

IV. Line-of-Sight Rate Estimation Accuracies

This section presents some theoretical accuracies in the estimates of the LOS spatial rate components produced by the filter being studied and discusses their implications. The accuracies to be given are those associated with the diagonal elements of the error covariance matrix used in the filter gain computation. The symmetrical conditions discussed in the previous section have been assumed for the calculations. As indicated previously, these conditions imply that the three-dimensional filtering accuracies are identical with those arising in the corresponding two-dimensional error covariance calculation, and the latter computation has been used here for convenience.

A delineation is first made between three types of target accelerations which represent alternative target phenomena encountered in airborne tracking. The first type assumes that the target acceleration is relatively small, as exemplified by the situation in which a stationary ground target is being tracked. The second case is one in which the target acceleration is sizable, but constant or slowly varying, such as when tracking a ballistic vehicle. The third situation is that of tracking a maneuvering airborne vehicle. The general procedure for designing the filter under study in each of these cases involves the choice of an acceleration model which best represents the ensemble of expected target accelerations. The theoretical accuracies which are obtained from the error covariance matrix solution in the gain computation then represent an averaging of the filter's performance over the statistics of the target acceleration model. Since the filter parameters associated with the A and M matrixes, as well as the deterministic input $\mathbf{u}$ and the noise statistics, are assumed known in the computation used here, the actual filter performance will be somewhat degraded relative to the curves, with the amount of degradation depending on the degree of modeling error, that is, on the accuracy with which the parameters such as R and $\dot{R}$ are known and the extent to which the target acceleration violates the assumed acceleration model.

Fig. 3 indicates the computed accuracies in estimating each of the line-of-sight rate components, assuming representative models for the three target acceleration classes given above. The parameters specifying the assumed target acceleration models are given in the figure. Typical values of range, range rate, receiver characteristics, and initial conditions are assumed for the computations and are also given in the figure. Curves 1, 2, and 3 in the figure represent, respectively, the situations in which the target acceleration is relatively small, large but slowly varying, and large and rapidly varying. A trend which would be expected a priori is seen in comparing the curves, namely, that more uncertainty in the estimate of target acceleration implies decreased accuracy in the line-of-sight rate estimates.

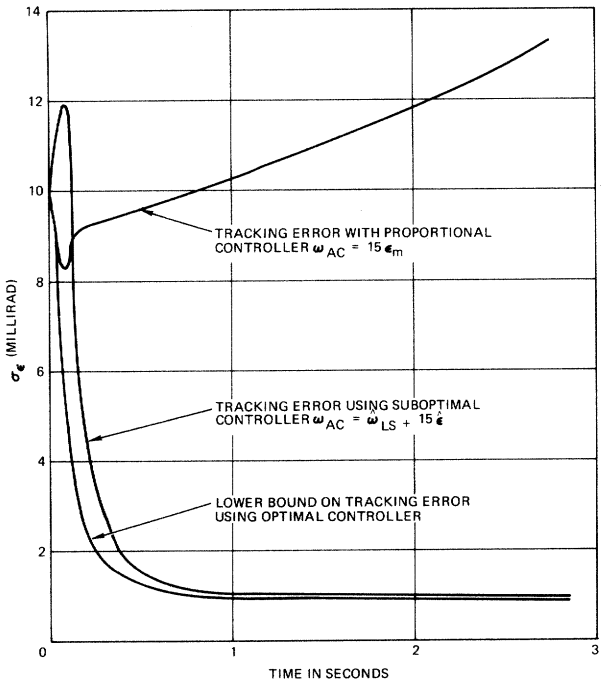


TARGET ACCELERATION MODELS ASSUMED:

CURVE	σ_{a_T} (FT/SEC^2)	τ_{a_T} (SEC)
1	10	3
2	64	10
3	64	1

Fig. 3. Comparison of LOS estimation accuracies.

Fig. 3 may be used as a reference point for the following brief summary of some of the main advantages of using the Kalman filter approach in place of the classical technique. It is first seen that a priori knowledge about the characteristics of the target acceleration may be included in the design. Secondly, it is recalled that the accuracy of these estimates can be made independent of the accelerations of the interceptor to the extent that the quantities a_{I_e} and a_{I_d} can be measured and used to drive the filter. In the same vein, the accuracies can be made relatively independent of the antenna motion or closed-loop tracking behavior, since the antenna rate components ω_{A_e} and ω_{A_d} enter in as deterministic driving terms, and, for the symmetrical conditions discussed in the previous section and assumed for the above computations, the accuracies are also theoretically independent of the antenna roll rate ω_{A_r} about the boresight axis. This latter effect is a decided departure from the noticeable effect of roll rates on the performance of the classical design. A final type of advantage is that the filtering computations adapt themselves to R and $\dot{R}$, as well as the receiver gains and noise statistics, to the extent that these quantities can be measured and inputted to the calculations. The above effects taken together imply that the angle channel Kalman filter which has been described offers the flexibility for providing increased accuracies in estimating the angular variables needed in airborne radar tracking.



PARAMETERS SPECIFYING INPUT GEOMETRY: $E(\omega_{LS}(t_0)) = 0$
$\sigma_{\omega_{LS}}(t_0) = 0.133$ RAD/SEC, INITIAL RANGE, 15,000 FT
RANGE RATE = −1000 FPS
RECEIVER PARAMETERS: GAIN = 1, NOISE SPECTRAL DENSITY = 5×10^{-4} (DEG)^2/CPS
STABILIZATION LOOP MODEL: $G_{SL}(S) = \omega_m^2/(S^2 + 2\zeta\omega_m S + \omega_m^2)$
$\omega_m = 125,\ \zeta = 0.5$

Fig. 4. Tracking error performance using the suboptimal controller.

V. Simplified Angle Loop Controller Design

This section demonstrates a simplified angle loop controller structure with close to optimal properties under some conditions. To simplify the discussion, the development will primarily be concerned with the two-dimensional version of the control problem.

Assuming that the control loop is of the form shown in Fig. 1, the particular suboptimal controller of interest is suggested by the following arguments. A heuristic approach for reducing the angle tracking error to a minimum would be to drive the antenna boresight axis at a spatial rate equal to the target line-of-sight rate ω_{LS}, and also to null out any angular differences between the two axes. An implementation of this appraoch based on the filter outputs would be to let the antenna rate command be

$$\omega_{A_c} = \hat{\omega}_{LS} + \lambda\hat{\epsilon} \qquad (12)$$

where the quantities with a caret are the filter outputs and λ is a constant. The first term $\hat{\omega}_{LS}$ drives the antenna at the Kalman estimated spatial rate of the line-of-sight, and the second term has the effect of driving the antenna in the direction needed to null the Kalman estimated tracking error. It is noted that the use of the LOS rate estimate $\hat{\omega}_{LS}$ in the controller equation is akin to a technique called "aided tracking" in classical loop design, in which an externally provided measure of ω_{LS}, if available, is added to the antenna rate command signal. With the Kalman filter in-the-loop approach, aided tracking is seen to be implicitly available in the form of the estimate $\hat{\omega}_{LS}$.

Fig. 4 indicates the performance of the suboptimal controller introduced above under some typical operating conditions. A second-order model is used to approximate the antenna stabilization loop dynamics. The figure gives the rms value of the tracking error ϵ averaged over the receiver noise and over an ensemble of input LOS rates statistically generated as shown in the figure. The parameters used in the state equations which generate the input ω_{LS} process are also given in the figure and assumed to be known to the filter. Two additional curves are given in the figure for comparison purposes. One of the curves makes it possible to compare the performance of the specified suboptimal controller with that of an equivalent classical configuration, as it indicates the rms pointing error under identical conditions, except that the antenna rate command ω_{A_c} is formed as $\lambda\epsilon_m$ where ϵ_m is the angle error receiver output and λ is the same controller gain value used in the suboptimal controller. The third curve in the figure shows a lower bound on the pointing error accuracy which could be obtained if the controller were designed using the principles of optimal control theory, and indicates that the suboptimal accuracy lies close to the lower bound after an initial transient period. The lower bound is based on [6], where it is shown that, in the feedback solution to the optimal linear stochastic control problem obtained using the "separation principle" [7], [8], the mean-square values of the system states being controlled are lower-bounded by the mean-square errors in the Kalman estimates of these states. In the present context, the system being controlled would be the planar version of the state equations given by (5) through (10), plus a set of linear state equations representing the antenna stabilization loop (see Fig. 1), which relates the control variable or antenna rate command ω_{A_c} to the actual antenna rate ω_A. An optimal feedback control minimizing a quadratic functional in ϵ would be a linear function of the estimated states, and application of the discussion given in [6] shows that the resulting variance on the track error would be lower-bounded by the variance on the Kalman estimate of ϵ. Further, the Kalman error variance, assuming the system described above, is itself lower-bounded by the error variance of the Kalman estimate when the antenna rate ω_A is assumed known, for which case the states associated with the stabilization loop are eliminated in the filter structure. The square root of the latter Kalman error variance is plotted in Fig. 4.

The suboptimal controller described above is of the simplest generic aided tracking form. However, additional simulation has also shown that its accuracies fall close to the lower bound under a fairly wide set of encounter conditions specified by R, $\dot{R}$, and the receiver characteristics, assuming that the stabilization loop model shown in the figure is used. For general purposes, a major factor in the question of whether the simplified suboptimal controller assumed above is adequate for a given situation is the bandwidth of the stabilization loop. If the stabilization loop bandwidth is sufficiently high, as in the case of the model shown in Fig. 4, then the performance of the simple suboptimal controller is close to the lower bound and also notable constant over a sizable range of values of the controller gain λ. The upper bound on this range of values is specified by loop stability considerations, while the lower bound occurs when the tracking loop begins to be sluggish. If the bandwidth of the stabilization loop to be used is reduced, the performance of the loop with the baseline suboptimal controller becomes increasingly degraded relative to the high bandwidth case. In these situations it is thus desirable to consider modifying the controller structure to account for the stabilization loop dynamics. Although the design of such a controller would appear to be most optimally approached using the principles of optimal control, an alternate suboptimal technique is to modify (12) by first passing $\hat{\epsilon}$ through a lead lag compensator. For an excellent study of additional aided tracking concepts from the modern viewpoint, see [9].

The extension of the foregoing discussion to the three-dimensional situation implies the use of the suboptimal controller type given in (12) for each of the angle control channels. The primary modification in the form of each of the channels relative to Fig. 1 becomes the cross-coupling terms between the channels caused by antenna roll motion about the boresight axis [see (5) and (6)]. Roll cross-coupling terms have, however, close to negligible effect for systems with low antenna roll rates, so that in these cases the planar analysis given above remains meaningful for each of the angle control channels. Examples of systems with low antenna roll rates are roll-stabilized missiles and interceptor radars with roll-stabilized antennas.

A very practical situation in which the controller type discussed above has additional appeal is that in which it is necessary to optimally extrapolate through blind zones, or periods in which no valid receiver data is available. The estimator/controller system described in this section poses an optimal type solution for this environment. First, it makes best use of the available receiver and on-board dynamical information to determine the Kalman estimates of $\epsilon(t)$ and $\omega_{LS}(t)$ which are to be used to control the antenna. During the blind zones the filter is informed that the angle error receiver slopes are zero, and it automatically goes into an extrapolation which forms the best prediction of the angular motion of the line-of-sight. The controller then ensures that the antenna boresight remains closely aligned with the best predicted direction of the line-of-sight. Such a system provides significant methodological improvements over the antenna extrapolation schemes which have been used in conjunction with classical designs.

VI. Range and Range-Rate Estimation and Control

This section discusses the application of Kalman filtering to range and range-rate tracking in airborne radar. The types of airborne radar systems which will be of interest are those whose fundamental measures are R, $\dot{R}$, or, perhaps, both R and $\dot{R}$. Examples of these are, respectively, pulse, pulsed-Doppler, and range-gated pulsed-Doppler type radars [1]. In a range type radar, the estimate of $\dot{R}$ must be obtained by taking the derivative of a filtered measure of R. In a pulsed-Doppler radar, whose primary measurement is $\dot{R}$, it is sometimes possible to obtain a direct measure of R using range gating. When this is not possible, a measure of R may be obtained by some less direct means, such as FM ranging [1].

The form of the Kalman filter which can be used for the R and $\dot{R}$ tracking operation is next described. Consider the following state space description, in which the states are $x_1 = R$, $x_2 = \dot{R}$, and $x_3 = a_{T_{r'}}$, where $a_{T_{r'}}$ is the component of the target acceleration along the LOS:

$$\dot{x}_1 = x_2 \tag{13}$$

$$\dot{x}_2 = \omega_{LS}^2 x_1 + x_3 - a_{I_{r'}} \tag{14}$$

$$\dot{x}_3 = -(1/\tau)x_3 + w_3 \tag{15}$$

$$y_1 = k_1 x_1 + n_1 \tag{16}$$

$$y_2 = k_2 x_2 + n_2. \tag{17}$$

Equation (14) is derived in the Appendix. The parameter ω_{LS}^2 designates the sum of the squares of the LOS angular rate components which are measured traditionally by the angle tracking system, and which may be estimated using Kalman filtering, as described in Section II. Either of these measures may be used to compute an estimate of the quantity $\omega_{LS}{}^2$, which may then be considered for use in the role of a known system parameter. The quantity $a_{I_{r'}}$ is the component of the interceptor acceleration along the LOS. A measure of its value can be obtained by resolving interceptor acceleration measurements into the antenna axes, and may be treated as a deterministic input. Equation (15) is a first-order Markov state equation model for the target acceleration component $a_{T_{r'}}$, with w_3 being white noise. Finally, (16) and (17) form the observation structure associated with the foregoing state equations. The parameters k_1 and k_2 will be seen to correspond to the discriminator slopes, and the white noises n_1 and n_2 represent the additive noises in the discriminator outputs.

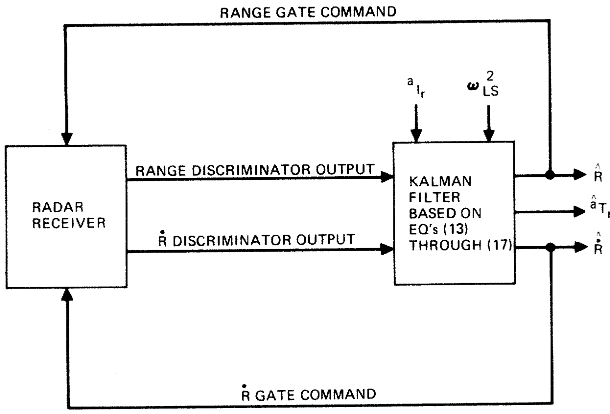
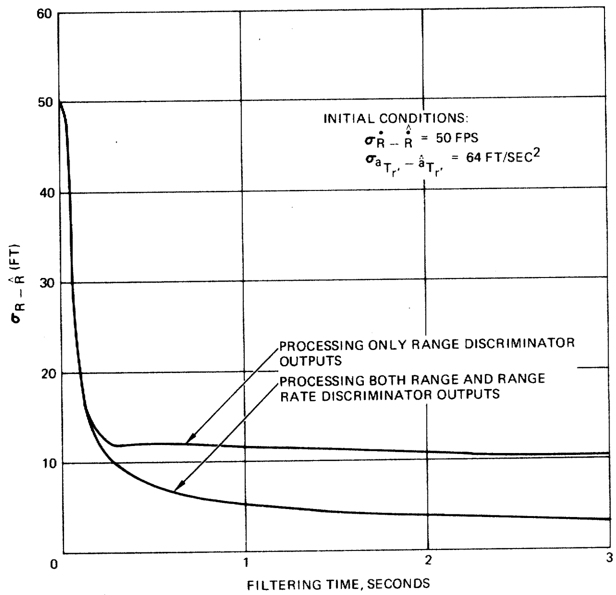


Fig. 5. Range/range-rate tracking using Kalman filtering.



RECEIVER PARAMETERS: RANGE DISCRIMINATOR GAIN = 1
RANGE RATE DISCRIMINATOR GAIN = 1 OR 0
RANGE NOISE SPECTRAL DENSITY = 25 (FT)^2/CPS
$\dot{R}$ NOISE SPECTRAL DENSITY = 1 (FPS)^2/CPS

GEOMETRY PARAMETERS: ω_{LS}^2 = 0.018
TARGET ACCELERATION MODEL: $\sigma_{a_{T_r'}}$ = 64 FT/SEC^2
$\tau_{a_{T_r'}}$ = 3 SEC

Fig. 6. Typical accuracy of the range tracking Kalman filter.

Fig. 5 indicates how a Kalman filter based on the state and measurement equations (13) through (17) would be used to provide the R and $\dot{R}$ tracking functions. It is seen that the filter's estimates of R and $\dot{R}$ are used not only as the tracker outputs, but also to position the range and velocity gates; in other words, to center the discriminators. Assuming that the track errors are small enough for the discriminators to be operating in their linear region, the discriminator outputs are of the form

$$Y_1 = k_1'(R - \hat{R}) + n_1 \tag{18}$$

$$Y_2 = k_2'(\dot{R} - \hat{\dot{R}}) + n_2 \tag{19}$$

where k_1' and k_2' are the discriminator slopes, n_1 and n_2 are the additive receiver noises, and $\hat{R}$ and $\hat{\dot{R}}$ are the range-rate estimates produced by the filter. Equations (16) and (17) show that the discriminator outputs Y_1 and Y_2 may be used to exactly mechanize the residuals required for the Kalman filter, and that the discriminator gains k_1' and k_2', together with the discriminator noise statistics, would then be those associated with the measurement structure, (16) and (17), in computing the Kalman gains.

The general form of the system indicated in Fig. 5 applies to situations in which range and/or range-rate R and $\dot{R}$ estimates produced by the filter. Equations (16) cases when only range or only range-rate are available may be obtained respectively by setting k_2 and k_1 to zero in (17) and (16). In the case when only $\dot{R}$ measurements are to be available, a particular question of interest is whether an accurate estimate of range is produced by the filter. This will depend on the magnitude of the centripetal acceleration $\omega_{LS}^2 R$ relative to the uncertainty in the radial target acceleration $a_{T_r'}$. For instance, if a stationary ground target is being tracked by an airborne vehicle flying overhead, then $a_{T_r'}$ is small and close to being known, and, for sufficiently low altitudes, $\omega_{LS}^2 R$ is sizable. Under these conditions a meaningful estimate of range will be provided by the filter. At the other extreme, if the range is large, so that $\omega_{LS}^2 R$ is small, and the uncertainties in $a_{T_r'}$ are relatively large, as, for instance, when tracking a maneuvering airborne vehicle, then $\omega_{LS}^2 R$ will be small relative to the uncertainties in $a_{T_r'}$ and will be indistinguishable from $a_{T_r'}$. In such cases, an accurate estimate of range cannot be obtained and there will be essentially no loss of accuracy in estimating R using a filter which is configured with two states instead of three, the two states being $\dot{R}$ and the sum of $\omega_{LS}^2 R$ and $a_{T_r'}$.

The benefits of using the Kalman approach to R and $\dot{R}$ tracking are similar to those discussed in connection with the angle channel. The approach makes systematic use of parameters such as ω_{LS}^2 and a_{I_r} which may be available from other on-board subsystems, and optimally adapts to on-line changes in the discriminator gains and noise statistics if these changes can be measured and provided to the filter. An important example in the latter category is that the system provides optimal capability for extrapolating through radar blind zones, thus maximizing the probability that the target will reappear in the range and velocity gates when the signal returns.

A numerical example is given next which demonstrates the advantage of processing range and velocity discriminator outputs simultaneously, when both are available, as opposed to separately, which is typical in classical servo design. Fig. 6 shows the rms tracking error in range using the above described filter for a typical geometry, and compares the case of processing only range measurements with that of simultaneously processing range and range-rate measurements. The curves were obtained by solving the Kalman error covariance equation associated with the two filters, and thus represent theoretical or computed accuracies which assume no mismatch in the parameters used. The initial covariance matrix was assumed to be diagonal with the values on the diagonal as given in the figure. The relative difference between the curves indicates that a significant improvement in range tracking accuracy is possible for the case in which range and velocity data are processed simultaneously, as opposed to separately.

VII. Additional Comments

In Section II it was seen that the system dynamical equations associated with the angular motion of the LOS were linear in the components of ϵ, ω_{LS}, and a_{T_n}. This made it possible to consider the application of linear estimation to this problem by choosing these components as states, and using measurements of R and $\dot{R}$ in the role of known system parameters. Also, in the previous section it was seen that the system dynamics associated with R, $\dot{R}$, and a_{T_r}, were likewise linear in these variables, so that the Kalman structure could again be applied if a measure of $\omega_{\mathrm{LS}}{}^2$ obtained from the angle channel were used as a known system parameter. If these two channels are considered as one system, and the states of the system are chosen to be R, $\dot{R}$, the components of ϵ and ω_{LS}, and the states associated with the target acceleration vector, the resulting system model is seen to be nonlinear, so that nonlinear filtering techniques could as well be applied to the estimation of the states with a potential improvement in performance. An important disadvantage of this approach is, of course, the increased computational requirement, as the gain computation for the nine-state filter would be considerably more complex than when using the linearization in the foregoing sections.

In a more general vein, in considering the application of the above discussed filters to a particular radar tracking system, simulation studies must be made to ascertain the effect on performance of the fact that the parameters used in the filtering computations are not known exactly. For the angle filter these include range, range-rate, and the antenna rate components, while for the range channel filter, the measured LOS rate is used as an input. Additional parameters for both filters are the interceptor acceleration components along the antenna axes and the parameters associated with the receiver characteristics. If both range and angle channel filters are used, a simulation encompassing both is appropriate for specifying the system performance. Effects of general interest in modeling the receiver gains and additive noises include thermal receiver noise, clutter, angle, range, and amplitude scintillation effects, radome error, and receiver nonlinearities which occur if the tracking errors are large [1].

Implementation of the filters described would generally require the use of a digital computer due to the complexity and time-varying nature of the solution. The filter design in these cases would proceed by forming a discretized version of the state and measurement equations with respect to which the discrete version of the Kalman filter would be implemented in the computer.

VIII. Conclusions

It has been shown that the Kalman filter provides a natural framework for configuring the estimation processes required in airborne tracking and that the resulting system has features not as readily available with the classical servo approach. These features, obtained by espousing the methods of optimal estimation, include adaptivity of the processing to the specific encounter conditions, minimizing the effect of ownship acceleration and antenna dynamics on the estimation accuracies, and accounting for variations in receiver characteristics when these are measurable and included as parameters in the filter. The states estimated by the filters are important quantities needed for solution of the intercept problem and are natural variables for optimizing the feedback signals which drive the antenna and range and velocity gates.

Appendix

Summary of Derivation of State Equations

The derivation of the state equations (3) through (10) and (12) is indicated in this Appendix. The state equations for the track errors ϵ_e and ϵ_d are considered first.

Recall that the antenna and line-of-sight frames, ϕ_A and ϕ_{LS}, have angular rate vectors ω_A and ω_{LS}, respectively, and that the direction cosine matrix between ϕ_A and ϕ_{LS} is a function of the track errors ϵ_e and ϵ_d. Let E be this direction cosine matrix expressed as a skew symmetric matrix function of ϵ_e and ϵ_d [9], and let W_A and W_{LS} be the skew symmetric angular rate matrixes involving the components of ω_A and ω_{LS}. Then a well known differential property of the direction cosine matrix specifies that $\dot{E} = W_{\mathrm{LS}}E - EW_A$. Examination of the appropriate elements of this matrix equation provides the desired state equations, (3) and (4), for ϵ_e and ϵ_d.

The state equations for the variables ω_{LS_e}, and ω_{LS_d}, are next considered. Let ϕ_I be an inertial frome, ϕ_{LS}, again, be the line-of-sight frame, and let $\mathbf{R}$ be the relative position vector. Then the twice application of the Coriolis equation [9] gives

$$(d^2\mathbf{R}/dt^2)(\phi_I) = (d^2\mathbf{R}/dt^2)(\phi_{\mathrm{LS}}) + 2\omega_{\mathrm{LS}}(d\mathbf{R}/dt)(\phi_{\mathrm{LS}})$$

$$+ (d\omega_{\mathrm{LS}}/dt)(\phi_I)\,\mathbf{R} + \omega_{\mathrm{LS}}(\omega_{\mathrm{LS}} \times \mathbf{R}). \quad (20)$$

If each vector in (20) is expressed in terms of its components in the ϕ_{LS} frame, and the vector products are evaluated, (20) will then be expressed in terms of its components along the r', e', and d' axes of the ϕ_{LS} coordinate system. These three component equations may be arranged as

$$\ddot{R} = (\omega^2_{\mathrm{LS}_{e'}} + \omega^2_{\mathrm{LS}_{d'}})R + a_{T_{r'}} - a_{I_{r'}} \quad (21)$$

$$\dot{\omega}_{\mathrm{LS}_{e'}} = (2\dot{R}/R)\omega_{\mathrm{LS}_{e'}} - (1/R)(a_{T_{d'}} - a_{I_{d'}}) + \omega_{\mathrm{LS}_{r'}}\omega_{\mathrm{LS}_{d'}} \quad (22)$$

$$\dot{\omega}_{\mathrm{LS}_{d'}} = -(2\dot{R}/R)\omega_{\mathrm{LS}_{d'}} + (1/R)(a_{T_{e'}} - a_{I_{e'}}) - \omega_{\mathrm{LS}_{r'}}\omega_{\mathrm{LS}_{e'}} \quad (23)$$

where R, $\dot{R}$, and $\ddot{R}$ are scalar range and its derivatives, and where the a_T and a_I variables are the components of the target and interceptor inertial acceleration vectors along the ϕ_{LS} axes. Equation (21) is used as a state equation in the discussion of range tracking, and (22) and (23) are the dynamical relationships from which the state equations for $\omega_{\mathrm{LS}_{e'}}$ and $\omega_{\mathrm{LS}_{d'}}$ are derived, as discussed in the following.

Equations (22) and (23) may be written, to close approximation [3], as

$$\dot{\omega}_{\mathrm{LS}_{e'}} = -(2\dot{R}/R)\omega_{\mathrm{LS}_{e'}} - (1/R)(a_{T_{d'}} - a_{I_d}) + \omega_{A_r}\omega_{\mathrm{LS}_{d'}} + [(\epsilon_e a_{I_r})/R + (\epsilon_d \omega_{A_e} - \epsilon_e \omega_{A_d})\omega_{\mathrm{LS}_{d'}}] \quad (24)$$

$$\dot{\omega}_{\mathrm{LS}_{d'}} = -(2\dot{R}/R)\omega_{\mathrm{LS}_{d'}} + (1/R)(a_{T_{e'}} - a_{I_e}) - \omega_{A_r}\omega_{\mathrm{LS}_{e'}} + [(\epsilon_d a_{I_r})/R - \epsilon_e \omega_{A_d})\omega_{\mathrm{LS}_{e'}}]. \quad (25)$$

The $\omega_{\mathrm{LS}_{e'}}$ and $\omega_{\mathrm{LS}_{d'}}$ state equations used in the main text are (24) and (25) with the bracketed terms neglected. Although the bracketed expressions can be included in the state equations, giving rise to nonlinear terms in filtering process, their inclusion is not expected to be necessary [3] in typical airborne tracking cases. The reason for this stems from the fact that neglecting these terms is equivalent to introducing errors in the measured interceptor acceleration components a_{I_e} and a_{I_d}. Typically, the tracking errors ϵ_e and ϵ_d are sufficiently close to zero for the magnitude of these equivalent acceleration measurement errors to be inappreciable. Also, a slight change in nomenclature is made in (24) when used in the main text; namely, (24) is, for later convenience, written in terms of $a_{T\_d'}$ instead of $a_{T_{d'}}$, where $a_{T\_d'} = -a_{T_{d'}}$.

The remainder of this appendix indicates the derivation for the target acceleration state equations, (9) and (10). It is assumed that an adequate general approach for modeling the target acceleration vector $\mathbf{a}_T$ is to model its component a_{T_x} along any fixed or slowly varying direction x in space by a first-order Markov process, that is, by $\dot{a}_{T_x} = -(1/\tau)a_{T_x} + w$, where w is white noise. Models for the target acceleration components along any two orthogonal axes in space will further be assumed statistically independent.

Define a coordinate frame (r', e_1, d_1) such that r' is along the line-of-sight, and such that the frame is inertially nonrotating about the r' axis. Since the e_1 and d_1 axes have zero inertial rate about the r' axis, their only angular motion is that about the d_1 and e_1 axes, respectively. Assume next that the angular rates about these axes are less than 15 or 20 degrees per second. In this case, the e_1 and d_1 axes can be taken as slowly varying directions in space, and $a_{T_{e_1}}$ and $a_{T_{d_1}}$ modeled as statistically independent and identical first-order Markov processes. Assuming this to be the case, the desired acceleration components whose state equations are next to be found are those along the e' and d' axes. But, the derivatives of $a_{T_{e'}}$ and $a_{T_{d'}}$ may be related to those of $a_{T_{e_1}}$ and $a_{T_{d_1}}$ using the Coriolis equation. Applying this property results in the following differential equations for $a_{T_{e'}}$ and $a_{T_{d'}}$:

$$\dot{a}_{T_{e'}} = -(1/\tau)a_{T_{e'}} + \omega_{\mathrm{LS}_{r'}}a_{T_{d'}} + w_{e'} \quad (26)$$

$$\dot{a}_{T_{d'}} = -(1/\tau)a_{T_{d'}} - \omega_{\mathrm{LS}_{r'}}a_{T_{e'}} + w_{d'} \quad (27)$$

where $w_{e'}$ and $w_{d'}$ are again statistically independent white noises with equal spectral density, and $\omega_{\mathrm{LS}_{r'}}$ is the rate of the ϕ_{LS} frame about the r' axis. The state equations for the target acceleration components used in the main text are obtained from (26) and (27) by approximating $\omega_{\mathrm{LS}_{r'}}$ by ω_{A_r} and by rewriting the equations given here in terms of $a_{T\_d'}$ instead of $a_{T_{d'}}$.

References

[1] D.J. Povejsil, R.S. Raven, and P. Waterman, *Airborne Radar*. Princeton, N.J.: Van Nostrand, 1961.

[2] R.E. Kalman, "New methods in Wiener filtering theory," *Proc. 1st Symp. of Engrg. Application of Random Function Theory and Probability*. New York: Wiley, 1963, pp. 270-388.

[3] J.B. Pearson, III, "Basic studies in airborne radar tracking systems," Ph.D. Dissertation, University of California at Los Angeles, September 1970.

[4] H.W. Sorenson, "Kalman filtering techniques," in *Advances in Control Systems*, vol. 3, C.T. Leondes, Ed. New York: Academic Press, 1966.

[5] A.E. Bryson, Jr., "The 1966 Martin Minta Lecture, 'Applications of optimal control theory in aerospace engineering'," *J. Spacecraft and Rockets*, vol. 4, pp. 545-553, May 1968.

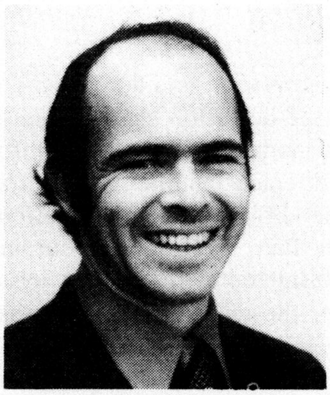
[6] A.E. Bryson, Jr., and Y.C. Ho, *Applied Optimal Control*. Waltham, Mass.: Blaisdale, 1969, p. 417.

[7] T.L. Gunkel and G.F. Franklin, "A general solution for linear sampled-data control," *J. Basic Engrg.*. vol. 85D, pp. 197-201, 1963.

[8] P. Joseph and J.T. Tou, "On linear control theory," *AIEE Trans.*, vol. 80, pp. 193-196, 1961.

[9] J.M. Fitts, "Aided tracking applied to high accuracy pointing systems," *IEEE Trans. Aerospace and Electronic Systems*, vol. AES-9, pp. 350-368, May 1973.

[10] H. Goldstein, *Classical Mechanics*. Cambridge, Mass.: Adison-Wesley, 1950.



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