

Problem Set #1

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- The sequential problem can be written as follows:

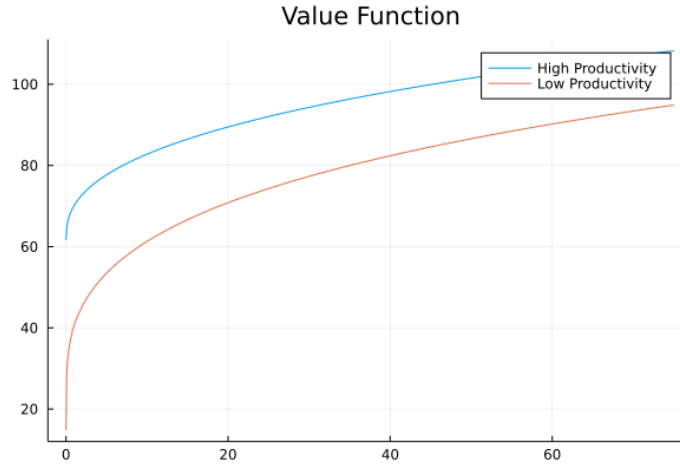
$$\begin{aligned} \max_{(k_{t+1}, c_t)} E\left[\sum_{t=0}^{\infty} \beta^t \log(c_t)\right] \\ \text{s.t. } c_t + k_{t+1} - (1 - \delta)k_t \leq z_t k_t^\theta \end{aligned}$$

Thus the Bellman equation is:

$$V(k, z) = \max_{k'} (\log(zk^\theta + (1 - \delta)k - k')) + \beta E[v(k', z')]$$

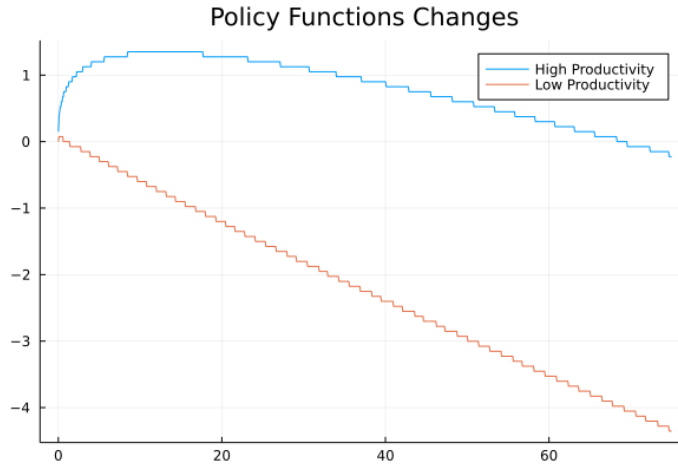
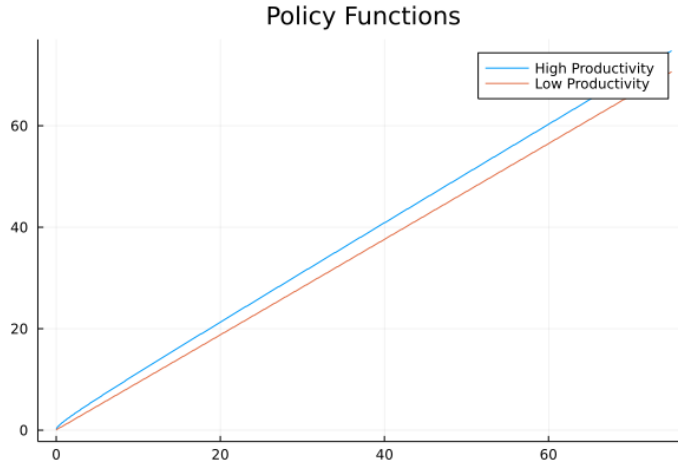
2

- As shown in the figure below, the value function over k for each state z is increasing and concave (i.e., $V(k_{i+1}, z) \geq V(K_i, z)$ for $k_{i+1} \geq k_i$ and $V(k_{i+1}, z) - V(K_i, z)$ is decreasing).



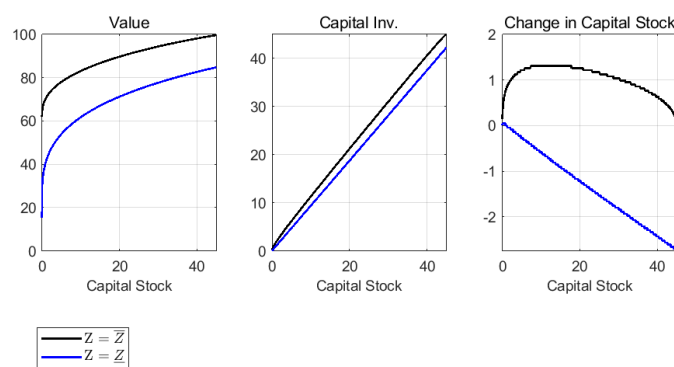
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- The decision rule is increasing in k and z (i.e., $K'(k_{i+1}, z) \geq K'(K_i, z)$). Also, saving is increasing in z (i.e., $k'(k, z) - k$ is increasing in z). With regard to k , in z_g saving is first increasing but turns to be decreasing, whereas in z_b saving is always decreasing. My argument is presented in the below two figures.



- Also, figures drawn from Fortran and Matlab are presented below.

1. Fortran



2. Matlab

