# Notes on Social Security Reform in a Life-Cycle Model: Transitional dynamics

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Reference: J. Conesa, and D. Krueger (1999): Social Security Reform with Heterogeneous Agents, *Review of Economic Dynamics*, 2, p. 757-95.

- In the previous set of notes we compared the stationary equilibria with and without social security and found that there are substantial welfare gains from the reform.

- The key question, however, is whether this reform is politically feasible: Would those households, who are alive in the initial steady state, support this reform if they had to vote on it in a referendum?

- In order to answer this question, we need to understand how the reform affects the future equilibrium outcomes and the welfare of each individual alive in the initial steady state during her entire life-time. Therefore, we will have to consider transitional dynamics from the initial steady state with social security to the final steady state without social security.

- Different from the analysis of the stationary equilibria, the decision rules, the value functions and the distribution function become time-dependent.

- Conesa and Krueger (1999) find that a majority of current voters would lose along the transition and therefore favor the system with social security.

- In section **??** we describe the computational strategy to compute for the transition dynamics. We will assume that in the initial steady state the government makes an unexpected and a credible announcement to eliminate the social security system. Credibility is a strong assumption but relaxing it significantly complicates the analysis and the computational strategy.[1] In section **??** we discuss the welfare effects of eliminating social security.

[1] Corbae, D'Erasmo and Kuruscu (2009) provide a detailed description of the computational strategy to solve for the politico-economic recursive competitive equilibrium without commitment.

# 1 Computational strategy

- We assume that the initial aggregate capital stock at $t = 0$ is the steady state level associated with $\tau_0 = \tau_0^{SS}$, so that $K_0 = K^{SS}(\tau_0^{SS}) = K_0^{SS}$, where $\tau$ is the proportional labor tax rate and $SS$ stands for steady state.[2] The initial stationary distribution of agents over productivity levels, $z$, asset holdings, $a$, and age, $j$, is given by $\Gamma_0^{SS}(z, a, j; \tau_0^{SS})$. The initial value function is denoted by $V_0^{SS}(z, a, j; \tau_0^{SS})$.

- In period $t = 0$, the government makes an unexpected and a credible announcement that it is going to abolish the social security system from $t = 1$ onwards. The final steady state is then associated with $\tau_N = \tau_N^{SS} = 0$ with $K_N = K^{SS}(\tau_N^{SS}) = K_N^{SS}$ and $\Gamma_N^{SS}(z, a, j; \tau_N^{SS})$, where $N$ is the number of periods (approximately) it takes to get to the new steady state (see below). Denote the final value function by $V_N^{SS}(z, a, j; \tau_N^{SS})$.

- Thus, in order to compute transition dynamics we need to make two steps: 1) compute the initial and final stationary competitive equilibria, and 2) find the transition path of the economy from the initial to the final steady state. These steps are described in detail below.

## Step 1: Calculating the stationary competitive equilibrium

The household's problem in the steady state can be formulated as follows:

$$V(z, a, j; \tau) = \max_{\{c, a'\}} \{u(c) + \beta s_{j+1} E[V(z', a', j + 1; \tau)|z]\} \tag{1}$$

subject to:

$$\begin{aligned} c + a' &= a(1 + r) + (1 - \tau)e(z, j)w + b_j \\ & c \geq 0, a' \geq \underline{a}, a = 0 \text{ if } j = 1 \text{ and } a' = 0 \text{ if } j = J. \end{aligned}$$

**Algorithm to compute the stationary competitive equilibrium:**

1. Make initial guess of the steady state value of the aggregate capital stock $K$.

2. Compute social security benefits $b_j$ and the prices $w$ and $r$, implied by the guess.

3. Solve household's decision problem given by (**??**) by backward induction from $j = J, J - 1, ..., 1$ to find the optimal savings decision, $g^{SS}(\epsilon, a, z; \tau)$, and the value function, $V^{SS}(\epsilon, a, z; \tau)$.

4. Find the stationary distribution of agents, $\Gamma^{SS}(\epsilon, a, z; \tau)$, associated with the optimal decision rules. Compute the optimal path for savings for the new born generation by forward induction given that the initial capital stock of newborns is 0.

[2] To simplify notation, we assume in these notes that labor supply is exogenous.

5. Check that the aggregate capital stock is consistent with household' decisions.

6. Update $K$ and return to step 2 until convergence.

## Step 2: Solving for the transition path

This step solves for a transition path from the initial steady state with $\tau_0 = \tau_0^{SS}$ at $t = 0$ to the final steady state with tax rate $\tau_N = \tau_N^{SS}$ at $t = 1, 2, ....$ In order to compute the transition, we need to specify a path dependent version of the household problem in (**??**) for $t = 0, ..., N$. Given an initial guess for the sequence of capital stocks, $\{K_t\}_{t=0}^N$, and therefore prices, $\{r_t, w_t\}_{t=0}^N$, and pension benefits,$\{b_{jt}\}_{t=0}^N$, the algorithm starts at $t = N$ and then shoots backwards.

### Household's problem for a given sequence of prices

$$V_t(z, a, j; \tau_t) = \max_{\{c,a'\}} \{u(c) + \beta s_{j+1} E [V_{t+1}(z', a', j+1; \tau_{t+1})|z]\} \tag{2}$$

subject to:

$$\begin{aligned} c + a' &= a(1 + r(K_t)) + (1 - \tau_t)e(z, j)w(K_t) + b_{jt}(K_t) \\ & c \geq 0, a' \geq \underline{a}, a = 0 \text{ if } j = 1 \text{ and } a' = 0 \text{ if } j = J. \end{aligned}$$

Note that now the value function also depends on time. Observe that from step 1, we already know $V_N(z, a, j; \tau_N^{SS})$ and therefore can solve the problem backwards.

Also note that since the change in the policy is effective from $t = 1$ onwards, households solve in period $t = 0$:

$$V_0(z, a, j; \tau_0^{SS}, \tau_N^{SS}) = \max_{\{c,a'\}} \left\{u(c) + \beta s_{j+1} E \left[V_1(z', a', j+1; \tau_N^{SS})|z\right]\right\}$$

subject to

$$c + a' = a(1 + r(K_0)) + (1 - \tau_0^{SS})e(z, j)w(K_0) + b_{jt}(K_0).$$

### Algorithm to compute for the transition path

1. Choose $N$ – the number of periods to reach the final steady state. Start with $N = 30$.

2. Make an initial guess for the sequence $\{K_t\}_{t=0}^N$ with $K_0 = K_0^{SS}$ and $K_N = K_N^{SS}$. A good guess could simply be a linear function from $K_0^{SS}$ to $K_N^{SS}$ given by $K_t = K_0^{SS} + \triangle$, where $\triangle = [K_N^{SS} - K_0^{SS}]/N$.

3. Starting in period $N - 1$ with current capital $K_{N-1}$ and future capital $K_N$, solve for the value function, $V_{N-1}(z, a, j; \tau_N^{SS})$, and savings functions, $g_{N-1}(z, a, j; \tau_N^{SS})$, for all $j = J, J-1, ..., 1$ by solving problem (**??**) backwards, with $V_N(z, a, j; \tau_N^{SS})$ given by the solution of problem (**??**)

with $\tau = \tau_N^{SS}$. You don't need to store the path of value functions, since this is very costly in terms of computational memory, but make sure to store $V_0(z, a, j; \tau_0^{SS}, \tau_N^{SS})$ because we will need it to quantify the welfare effects of the reform for the initial generation.

4. Continue backwards until period 0, i.e. until we obtain the sequence of value functions and decision rules that solve the problem (**??**) for $t = 0, ..., N$.

5. Update the path of capital as follows: Start with $\Gamma_0^{SS}(z, a, j; \tau_0^{SS})$ and use the sequence of decision rules $g_t$ to obtain $\Gamma_t$ and $K_t$ for $t = 1...N$ using:

$$\Gamma_{t+1}(z', a', j; \tau_{t+1}) = H(\Gamma_t(z, a, j; \tau_t))$$

and

$$K_{t+1} = \sum_{j=1}^{J} \sum_{z} \int_a g_t(z, a, j; \tau_t)\Gamma_t(z, a, j; \tau_t)da.$$

6. If the new sequence of capital is the same as the old sequence, we have found a candidate equilibrium. If not, slowly update the sequence of aggregate capital as a convex combination of the new one and the old one and go back to step (3).

7. There is nothing that assures us that this candidate sequence takes us to the steady state before $N$. We must check if it does. If we are within an acceptable tolerance, we are done. If not, increase $N$.

## 2 Results of the policy experiment

Now we discuss the results of the reform, which abolishes the social security system. The results follow Conesa and Krueger (1999).

#### Evolution of macroeconomic variables

- Figure **??** plots the evolution of macroeconomic aggregates. Under the reform, the economy reaches its final steady state after about 30 years. The elimination of the labor income tax makes working more attractive. Therefore, directly after the reform is implemented there is a sharp increase in hours worked and hence aggregate labor supply. Since the aggregate capital stock is predetermined from the period before the reform, the capital-labor ratio drops sharply, resulting in a substantial initial increase in the real interest rate and a decrease in the pre-tax wages.

- We can see from the figure that most of the action in macroeconomic variables takes place during the very first periods. This will be crucial when we evaluate the welfare consequences of the reform for the *initial* generation.

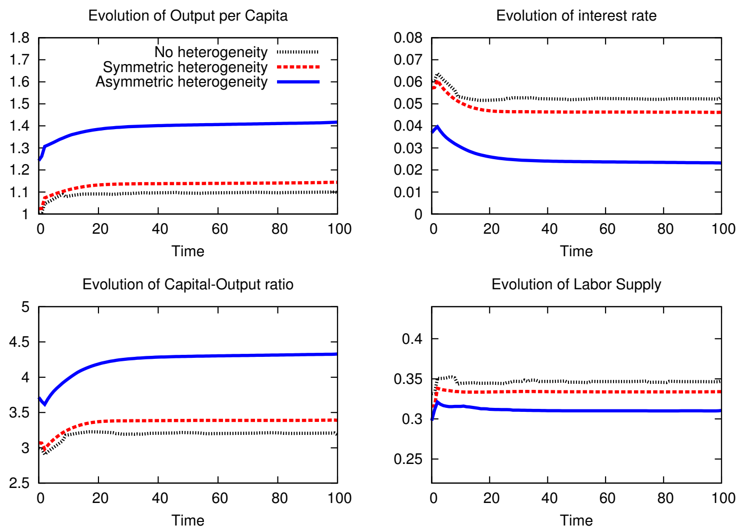


Figure 1: Evolution of macroeconomic aggregates

### Votes in favor of the reform

- We quantify the welfare change of a given policy reform for an individual of type $(z, a, j)$ in the initial steady state by asking: By how much (in percent) this individual's consumption has to be increased in all future periods and contingencies (keeping leisure constant) in the initial steady state with $V_0^{SS}(z, a, j; \tau_0^{SS})$, so that her expected future utility equals that under the reform, $V_0(z, a, j; \tau_0^{SS}, \tau_N^{SS})$. Given the form of the utility function[3],
$$u(c, l) = \frac{(c^{\gamma}(1-l)^{1-\gamma})^{1-\sigma}}{1-\sigma},$$
these welfare measures are given by
$$EV(z, a, j) = \left(\frac{V_0(z, a, j; \tau_0^{SS}, \tau_N^{SS})}{V_0^{SS}(z, a, j; \tau_0^{SS})}\right)^{\frac{1}{\gamma(1-\sigma)}}.$$
- For example, an $EV(z, a, j)$ of 1.10 implies that if social security is abolished, then an individual of type $(z, a, j)$ in the initial steady state will experience an increase in welfare due to the reform equivalent to receiving 10% higher consumption in the initial steady state in all future nodes of her event tree. Thus, if $EV(z, a, j) \geq 1.0$, the agent would vote for the reform; she would vote against the reform otherwise. The total mass of

[3] As in Conesa and Krueger (1999), labor supply is endogenous.

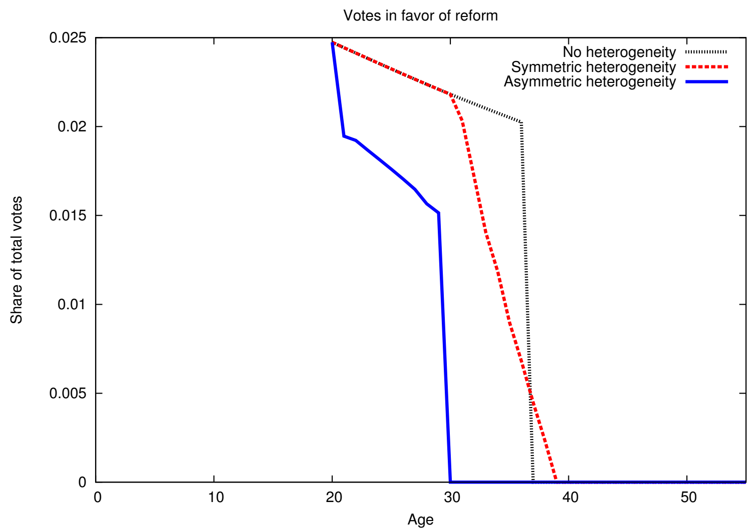


Figure 2: Votes in favor of abolishing social security

agents in the initial steady state who support the reform is then given by:

$$\sum_{j=1}^{J}\sum_{z}\int_{a}\Gamma_0^{SS}(z,a,j)\times 1_{EV(z,a,j)\geq 1.0}da,$$

where 1 is an indicator function.

- Conesa and Krueger (1999) find that the reform will not gain absolute majority of the individuals alive in the initial steady state.
- In particular, in the case of no idiosyncratic risk, only 40% of individuals would favor the reform, while support is smaller in the presence of idiosyncratic risk: 36% in the symmetric case and 21% in the asymmetric case (see figure **??**).

### Welfare effects by age and asset holdings

We can study the welfare effects along other dimensions of heterogeneity, such as asset holdings (figure **??**).

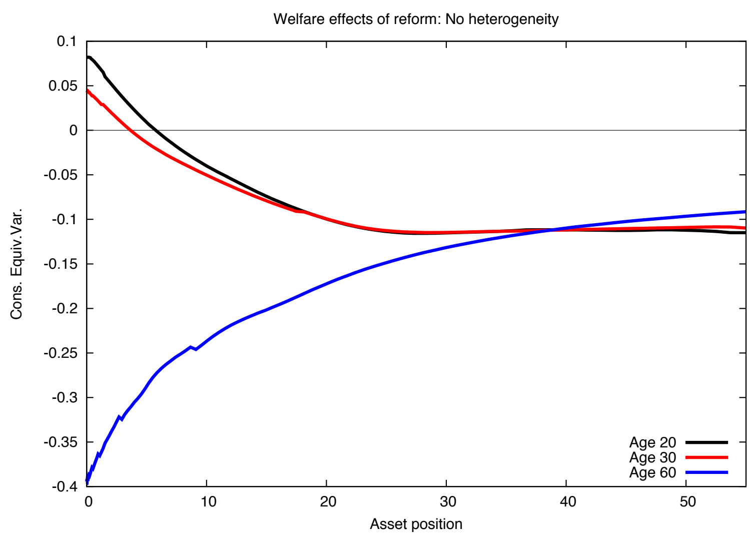


Figure 3: Welfare effects by age and asset holdings