Notes on Social Security Reform in a Life-Cycle Model

Reference: J. Conesa and D. Krueger, Social Security Reform with Heterogeneous Agents, *Review of Economic Dynamics*, 2, p. 757-95.

- Question: Should the current Pay-as-you-go social security policy in the U.S. be abandoned?
- Methodology:
  - Finite dynamic programming gives decision rules which satisfy $v_t = Tv_{t+1}$ iterating off known final value function (i.e. 0).
  - Cross-sectional distribution solves $\mu_{t+1} = T^* \mu_t$, where initial distribution is known.
  - From these we can compute stationary distributions.
  - Also we can compute transition paths between stationary equilibria.
- Policy experiment: We study the welfare effects of a reform, which terminates the pay-as-you-go social security system in the U.S. We also show how social security affects agent's savings and labor-leisure choices as well as aggregate capital stock and labor supply and therefore the prices.
- There are many other papers which use a framework where age matters to consider problems such as health care provision, endogenous fertility, accumulation of human capital, etc.

1 Environment

- Each period a continuum of agents is born.
- Agents live a maximum of N periods.
- Probability of surviving to age t conditional on having survived to age $t-1$ denoted s_t.
- Population grows at rate n.
- Assume age t agents make up a constant fraction μ_t of the population at any point in time where the weights are normalized to sum to 1 with $\mu_{t+1} = \frac{s_{t+1}}{(1+n)}\mu_t$. That is, if p_t^j is the total number of age t agents alive at date j, then the total population at time j is $P^j = \sum_{t=1}^N p_t^j$. Letting $\mu_t^j = \frac{p_t^j}{P^j}$, we have

$$1 = \sum_{t=1}^{N} \frac{p_t^j}{P^j} = \sum_{t=1}^{N} \mu_t^j.$$

In steady state per capita terms, μ_t^j is independent of calendar time j. This "normalization" makes the model stationary in the presence of population growth in the same way that RBC models are stationary around a balanced growth path.

- Preferences of age 1 agents:

$$\mathbb{E}_0 \left[\sum_{t=1}^{N} \beta^t \left(\prod_{j=1}^{t} s_j \right) u(c_t, l_t) \right]$$

where $u(c_t, l_t) = \frac{(c_t^\gamma (1-l_t)^{1-\gamma})^{1-\sigma}}{1-\sigma}$.

- Labor endowment $e(z,t)$ depends on age t and idiosyncratic labor productivity shock z. The shocks follow a finite state Markov chain and are iid across agents.

- Constant returns to scale production technology $Y = F(K, L) = AK^\alpha L^{1-\alpha}$ with depreciation δ.

- Competitive labor and capital rental markets at prices $w = F_2(K, L)$ and $r = F_1(K, L) - \delta$.

- One period bonds $a' \geq \underline{a}$ pay rate r. Agent cannot hold debt in last period of life and is endowed with zero asset holdings in the first period of life.

- The government is involved in 3 activities:

  - It imposes a liner social security tax θ to finance age-dependent social security benefits b_t, where

  $$b_t = \begin{cases} 0 & \text{if } t < R \\ b & \text{if } t \geq R \end{cases}$$

    given retirement age R. Note that this social security benefit policy is a very simple approximation, which neglects how benefits are linked to earnings (this way you don't have to keep track of the earnings history in the state space).

  - It imposes a liner capital and labor tax τ to finance government spending, G, which is unproductive (i.e. wasted) in the context of this model.

  - It collects accidental bequests and redistributes them as lump-sum transfers, T, across all agents.

2 Equilibrium

- Household's problem is

$$V(z, a, t) = \max_{\{c,l,a'\}} u(c, l) + \beta s_{t+1} \mathbb{E} \left[V(z', a', t+1) \mid (z, a, t) \right] \qquad (1)$$

s.t.

$$\begin{aligned} c + a' &= a(1 + r(1-\tau)) + (1-\theta-\tau)e(z,t)wl + T + b_t \\ c &\geq 0,\ 0 \leq l \leq 1,\ a' \geq \underline{a},\ a = 0 \text{ if } t = 1 \text{ and } a' \geq 0 \text{ if } t = N. \end{aligned}$$

- Distribution of agents in the population defined over age, asset holdings, and earnings status. Specifically, let $x = (z, a)$ and let $(X, \mathcal{B}(X), \psi_t)$ be a probability space where $\psi_t(B_0)$ is the fraction of age t agents whose state x lies in set B_0 as a proportion of all age t agents with initial distribution ψ_1. With zero initial wealth, ψ_1 is just the cross-sectional initial distribution of earnings in the first period of life. These agents make up a fraction $\psi_t(B_0)\mu_t$ of all agents in the economy.
- The distribution across agents at age $t = 1, ..., N-1$ is given recursively as

$$\begin{aligned} \psi_{t+1}(B_0) &= (T^*\psi_t)(B_0) = \int_X P(x,t,B_0)\psi_t(dx), \forall B_0 \in \mathcal{B}(X) \quad (2) \\ &= \int_{Z_0,A_0} \left\{ \int_{Z,A} \chi_{\{a'=g(z,a,t)\}} \pi(z'|z)\psi_t(dz,da) \right\} dz'da' \end{aligned}$$

  where $P(x,t,B)$ is a transition function which gives the probability that an age t agent transits to the set B next period given the agent's current state is x.[1]
- Definition. A stationary equilibrium is $(c(x,t), g(x,t), l(x,t), r, w, K, L, T, G, \tau, \theta, b)$ and distributions $\{\psi_1, \psi_2, ..., \psi_N\}$ such that:

1. $c(x,t)$, $l(x,t)$ and $g(x,t)$ solve the hh decision problem (1);
2. $w = F_2(K,L)$ and $r = F_1(K,L) - \delta$ in competive input markets;
3. Markets clear:
   (a) goods:

$$\sum_t \mu_t \int_X [c(x,t) + g(x,t)]\, d\psi_t = F(K,L) + (1-\delta)K - G$$

   (b) rental capital

$$K' = \sum_t \mu_t \int_X g(x,t) d\psi_t$$

   (c) rental labor

$$\sum_t \mu_t \int_X l(x,t)e(z,t)d\psi_t = L$$

[1] That is, $P(x,t,B_0) = prob((g(x,t),z') \in B_0|z)$.

4. $\{\psi_1, \psi_2, ..., \psi_N\}$ is consistent with individual behavior via (2);

5. the govt budget constraint is satisfied: $G = \tau(rK + wL)$;

6. social security (pay-as-you-go) feasibility: $\theta w L = b\left(\sum_{t=R}^{N} \mu_t\right)$;

7. transfer wealth equals accidental bequests:

$$T = \left[\sum_{t=1}^{N-1} \frac{\mu_t(1 - s_{t+1})}{(1+n)} \int_X g(x,t)(1 + r(1-\tau))d\psi_t\right].$$

   Note that $\mu_t(1 - s_{t+1})/(1+n)$ is the fraction of people who do not survive to the next cohort.

3 Computation of the stationary equilibrium

1. Make initial guesses of the steady state values of the aggregate capital stock K, aggregate labor N and government transfers T. Compute social security benefits b and government spending G implied by these guesses.

2. Compute the prices w and r, which solve firm's problem.

3. Compute the household's decision functions by backward induction.

4. Compute the optimal path for savings and labor for the new born generation by forward induction given that the initial capital stock of newborns is 0.

5. Compute the aggregate capital stock, aggregate labor supply and government transfers.

6. Update K, N and T and return to step 2 until convergence.

4 Directions for Future Research - Endogenous Earnings

- Even though earnings depended on age, they were exogenous.

- Lochner and Monge (2011) allow for an endogenous human capital choice in a life-cycle model with borrowing constraints which allow for a private default option.[2]

- They consider the response of human capital investments to three interesting experiments:

[2]Lochner, L. and A. Monge (2011) " The Nature of Credit Constraints and Human Capital", *American Economic Review*, 101, p. 2487-2529.

  - changes in the enforcement institutions underlying private lending,
  - changes in the extent of GSL programs,
  - changes in government subsidies.

- They took the initial distribution of assets to be exogenously given. In an online appendix, they consider parental transfers, but take the parents initial wealth as exogenously given.

- If the initial wealth distribution affects human capital accumulation because of borrowing constraints and human capital accumulation affects earnings and earnings affect the wealth distribution (which is a property of the Huggett 1996 paper), then wealth inequality can be transferred across generations.

- To address this, we need to find a fixed point where the steady state wealth distrbution that an individual starts the period with is consistent with the wealth distribution from which his/her parents make their transfers from.

- This is taken up in Gallipoli, Meghir, and Violante (2008, GMV).[3]

- While GMV is quite complicated, one could consider a simplified version.

  - Individulals are one of two ability types $\gamma \in \{H, L\}$.
  - In the first period of life ($t = 1$), individuals make a discrete human capital choice $h \in \{0, 1\}$ where $h = 1$ implies a college choice and $h = 0$ implies no college.
  - Earnings depend on both ability and education $e^h_\gamma(z, t) * w$, with e increasing in both γ and h.
  - If parental college contribution is denoted $\Omega \geq 0$ (i.e. children cannot be forced to make a transfer to the parents) and the cost of college is denoted Λ, then to fund a college education, the $t = 1$ borrowing constraint is denoted

$$a_2 = \Omega + T - \Lambda - c_1 \geq \underline{a}.$$

  - There is no labor/leisure choice (different from GMV).
  - In the period (call it ζ) where the parent's child makes his/her college choice, the one time transfer of funds generates utility

$$u(c) + \omega V^h_\gamma(z, \Omega, 1)$$

    where $V^h_\gamma(z, \Omega, 1)$ is the value function of the parent's child following their college choice h.

[3] Gallipoli, G., C. Meghir, G. Violante (2008) "Equilibrium Effects of Education Policies: A Quantitative Evaluation", mimeo.

– The college decision solves

$$V_\gamma^h(z,\Omega,1) = \max_h \left\{ V_\gamma^1(z,\Omega,1), V_\gamma^0(z,\Omega,1) \right\}$$

where

$$V_\gamma^1(z,\Omega,1) = \max_{a_2 \geq \underline{a}} u(\Omega + T - \Lambda - a_2) + \beta E_{z'} V_\gamma^1(z', a_2, 2)$$

and

$$V_\gamma^0(z,\Omega,1) = \max_{a_2 \geq \underline{a}} u(\Omega + T + (1-\theta-\tau) e_\gamma^0(z,1) \cdot w - a_2) + \beta E_{z'} V_\gamma^0(z', a_2, 2).$$

– The parental transfer decision solves

$$\begin{aligned} V_\gamma^h(z, a_\zeta, \zeta) &= \max_{\Omega \geq 0, a_{\zeta+1} \geq \underline{a}} u((1 + r(1-\tau)) \cdot a_\zeta + (1-\theta-\tau) e_\gamma^h(z,\zeta) \cdot w + T - a_{\zeta+1} - \Omega) \\ &\quad + \omega \cdot V_\gamma^h(\widetilde{z}, \Omega, 1) + \beta E_{z'} V_\gamma^h(z', a_{\zeta+1}, \zeta+1). \end{aligned}$$

– The production function needs to be amended to include both college and non-college hours in efficiency units. In particular,

$$Y = AK^\alpha L^{1-\alpha}.$$

Hours are aggregated via a CES aggregator function

$$L = \left[\sum_h \left(L^h \right)^\rho \right]^{1/\rho}$$

where L^h is the integral over all working age individuals within each each group h of individual hours 1 times their respective efficiency units $e_\gamma^h(z,t)$. Specifically, if we expand the definition of the state $x = (z, a, \gamma, h)$, then

$$L^h = \sum_t \mu_t \int e_\gamma^h(z,a) \psi_t(dz, da, d\gamma, h).$$

Unlike the case above, this is not exogenous since the h choice is endogenous.