Simple Models of Firm Dynamics©

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Questions

Models of Firm Dynamics have been used to address the following questions:

1. How much do firing costs affect employment of different size firms? (Hopenhayn and Rogerson (1993, JPE))
2. How much do firm borrowing constraints affect aggregate measures of TFP and misallocation? (Buera and Shin (2013, JPE))
3. What is the impact of trade liberalization on productivity? (Melitz (2003, ECMTA))
4. How does market structure affect firm investment decisions? (Ericson and Pakes (1995, RESTUD))

Data from Davis, Haltiwanger, and Schuh (1986)

Questions:

- What role do plant births and deaths play in the creation and destruction of jobs?
- More generally, how are JC and JD distributed by plant level employment growth rates?
- Does JC and JD involve mild expansions and contractions spread among a large number of plants or dramatic changes at a few plants?
- Key point of their work: Shutdowns (exit) and startups (entry) are important for understanding employment data.

Data from DHS cont.

- Need unbalanced panel in order to ask questions about entry and exit.
- Longitudinal Research Database (LRD) from the Census Bureau is constructed from 2 sources:
    1. Census of Manufactures. From every man. establishment with 1 or more employees. Collects data on labor, material, and capital inputs; output, location, legal form of organization, etc.
    2. Annual Survey of Manufactures. Collects same basic stuff as CM but also has assets, capital expenditures, rental payments, depreciation, etc.

DHS Definitions

- Job Creation (JC) employment gains summed over all plants that expand or start up from t-1 to t
- Job Destruction (JD) employment losses summed over all plants that contract or shutdown up from t-1 to t
- Job Reallocation (JR). The sum of all plant level gains and losses from t-1 to t (i.e. JR=JC+$|JD|$). An upper bound calculation.
- Minimum worker reallocation = max$\{$JC,$|JD|\}$. A lower bound calculation

DHS Concentration

- Plant level growth rates measured as:

$$g_t = \frac{EMP_t - EMP_{t-1}}{(1/2)(EMP_t + EMP_{t-1})}$$

- This way growth rates are not ∞ for startups and -1 for shutdowns (rather distribution is between -2 and $+2$)
- Symmetry and bounded makes statistical work easier.
- Results: Shutdowns account for 23% of annual JD and startups account for 16% of annual JC.

DHS Differences by Age and Size

- Large, mature plants account for most JC and JD.
- Size: Firms with ≥ 500 workers account for 69% of employment.
- Age: Firms with ≥ 10 years account for 78% of employment.

Models of Firm Dynamics

- Jovanovic (1982) "Selection and the Evolution of the Industry"
  - Firms draws a productivity shock from a type dependent distribution (high or low) which is overlapping.
  - It takes time (draws) for the firm to know which type it is.
  - High type firms stay, low types exit.
  - Thus, age is a relevant state variable.

- Hopenhayn (1992) "Entry, Exit, and Firm Dynamics in Long Run Equilibrium"
  - Firms draw persistent productivity shocks
  - Production function is decreasing returns and there are fixed operating costs.
  - Since labor is the only factor of production, employment size is the relevant state variable.

Hopenhayn Model

Methodology

- Just as the simple buffer stock model of Huggett generated an endogenous cross-sectional distribution over earnings and wealth, models of firm dynamics generate an endogenous cross-sectional distribution over firm size (typically measured by e.g. employment) but unlike Huggett where the measure was exogenously set to 1, here the “size” of the industry is endogenous and not necessarily equal to 1.

- The key technical differences from the Huggett model is that there is now a discrete choice exit problem and an endogenous entry condition.

- Perhaps surprisingly, the entry condition will endogenously determine the industry price while the market clearing condition will endogenously determine the distribution of firms.

Hopenhayn's (1992) Competitive Model Environment

- Population: Industry composed of a continuum of firms (mass not necessarily 1) which produce a homogeneous product.
- Firms behave competitively, taking prices in the output (P_t) and labor input (W_t) markets as given.
- Aggregate Demand given by the *inverse demand* function $P(Q_t)$. Assume P is continuous, strictly decreasing, and $\lim_{Q\to\infty} P(Q) = 0$. For example, let $P = \frac{1}{d+Q}$ where $d = 0.1$.
- The input price $W(N_t)$,where N_t is total industry labor demand, is assumed to be continuous, nondecreasing and strictly bounded above zero. For example, $W = 1$ (a normalization).

Hopenhayn Environment - cont.

- Technology: output of a given firm is $q_t = f(\varphi_t, n_t)$ where $\varphi_t \in [0, 1]$ is a productivity shock which follows a Markov process, independent across firms with conditional distribution $F(\varphi_t|\varphi_{t-1})$.

- While Hopenhayn assumes F is continuous, persistent and satisfies certain mixing conditions, here we will assume a two state Markov process: $\varphi \in \{\varphi_L, \varphi_H\}$ with $1 = \varphi_H > \varphi_L > 0$. $F(\varphi' = \varphi_{j \neq i}|\varphi = \varphi_i) = 1 - \theta < \frac{1}{2}$, $i, j \in \{H, L\}$. Thus, the process displays persistence provided $\theta > \frac{1}{2}$. We take $\theta = 0.9$.

- $f(\varphi, n) = \varphi n^{\alpha}$ where here $\alpha = \frac{1}{2}$.

Hopenhayn Environment - cont.

- A fixed cost $W_t c_f$ must be paid every period by an incumbent firm (for exit to take place). Here we will assume that the cost is associated with management.

- Per period profit function given by $\pi(\varphi_t, P_t, W_t = 1) = max_n P_t q_t - n_t - c_f$.

- Firms discount profits at rate $0 < \beta < 1$. E.g. $\beta = .8$ (time period is 5 years at 4%).

Hopenhayn Environment - cont.

- Firm's static optimization problem:

$$P\alpha\varphi_i n^{\alpha-1} = 1 \Longleftrightarrow n_i^* = (P\alpha\varphi_i)^{1/(1-\alpha)}, i \in \{H, L\}$$

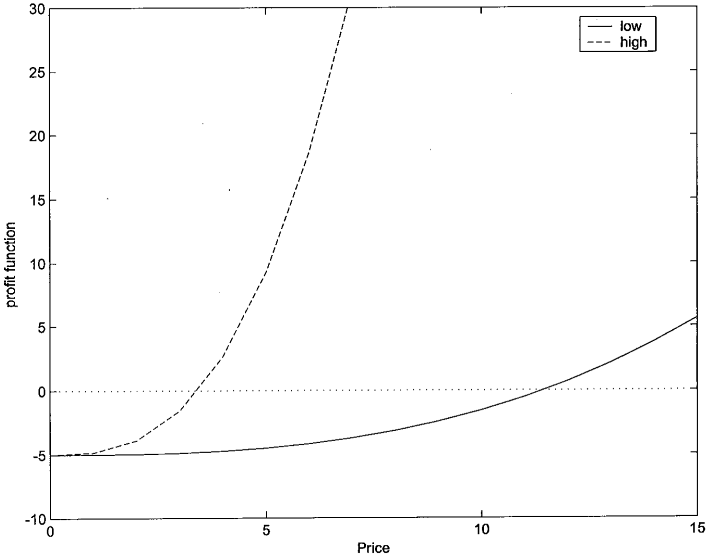
In this specific case, $n_i^* = \left(\frac{P\varphi_i}{2}\right)^2$ .Hence firm size, measured by number of workers, is increasing in productivity.

- Thus

$$\begin{aligned}\pi(\varphi_i, P) &= P\varphi_i (n_i^*)^\alpha - n_i^* - c_f \\ &= (P\alpha\varphi_i)^{1/(1-\alpha)}\left[\frac{1}{\alpha} - 1\right] - c_f\end{aligned}$$

Note that with $\alpha = 1/2$, the static profit function is increasing and convex in price (as in Varian) and productivity.

Profit Function



Hopenhayn Environment - cont.

- Persistence implies that if your profits are high (low) today, they are expected to stay high (low) in the near future. This is important for the exit decision.

- Each period, before φ_t is realized, incumbent firms may exit. An exiting firm receives 0 present value.

- In equilibrium, firms exit whenever their state φ_{t-1} falls below a reservation level x_{t-1}. Given this cutoff point, a mixing condition implies the life span of a firm is almost surely finite (i.e. there is a steady stream of exit).

Hopenhayn Environment - cont.

- An entering firm must pay a fixed cost $W_t c_e$.
- Then it draws a productivity shock φ_t from initial density function G. Here we assume $\Pr\{\varphi = \varphi_H\} = \eta = \frac{1}{2}$.

Hopenhayn Environment - cont.

Incumbent Timing is:

1. enter period t with last period's shock φ_{t-1}.
2. if $\varphi_{t-1} < x_{t-1}$,exit (to avoid future low productivity)
3. receive this period's shock φ_t from $F(\varphi_t|\varphi_{t-1})$
4. choose labor demand $n_t = g(\varphi_t; P, W)$ and pay fixed cost wc_f
5. enter period $t+1$ with φ_t

Potential Entrant Timing is:

1. Decide whether to pay fixed cost Wc_e and receive this period's shock $\varphi_t = \varphi_H$ with prob η
2. Same as 4-5 above

Hopenhayn Environment - cont.

- Let the measure $\mu_t(A)$ summarize the number of firms that have $\varphi_t \in A \subset S$. Due to entry and exit, total mass of firms $\mu_t(S)$ is not necessarily 1.
- Notice that under the timing above, even if firms with $\varphi_{t-1} \leq x_{t-1}$ are exiting, the distribution $\mu_t(\varphi_t)$ includes new entrants who may draw low φ_t from the distribution function $G(\varphi_t)$.
- Aggregate output supply and input demand given by

$$Q_t^s(\mu_t; P, W) = \int q_t(\varphi_t; P, W)\mu_t(d\varphi_t) \tag{1}$$

$$N_t^d(\mu_t; P, W) = \int n_t(\varphi_t; P, W)\mu_t(d\varphi_t) \tag{2}$$

Equilibrium in Hopenhayn's Environment

- Given that all the uncertainty is idiosyncratic and there is a continuum of firms, we will be considering stationary equilibria where $\{P_t, W_t\} = \{P, 1\}$.
- The problem of an incumbent firm at stage 4 in the above timing is really just a decision on exit next period (note that the static labor demand problem has already been solved implicitly in the profit function):

$$v(\varphi; P) = \pi(\varphi, P) + \beta \max\left\{0, E\left[v(\varphi'; P)|\varphi\right]\right\} \tag{3}$$

where $E\left[h(\varphi')|\varphi\right] = \int_{\varphi' \in [0,1]} h(\varphi') F(d\varphi'|\varphi)$.

Competitive Equilibrium -cont.

- The solution to problem (3) is a cutoff rule. Let

$$x = \inf\{\varphi \in S : E\left[v(\varphi'; P)|\varphi\right] \geq 0\} \tag{4}$$

  Thus any firm with $\varphi < x$ will exit.

- Prop. 1. v is a bounded, continuous, and strictly increasing function of φ. Note: present discounted profits are increasing in productivity follows from properties of π and F.

Competitive Equilibrium -cont.

- Since all firms are ex-ante identical, entry occurs provided

$$\int v(\varphi; P)\eta(d\varphi) \geq c_e.$$

- Let M_t denote the mass of entrants in period t. An equilibrium with free entry requires

$$\int v(\varphi; P)\eta(d\varphi) \leq c_e, \tag{5}$$

  with equality if $M_t > 0$ (in which case we are free to "choose" any measure M_t we want).

Competitive Equilibrium -cont.

- Entry/exit implies an evolution for the state of the industry

$$\mu_{t+1}([0,\overline{\varphi}']) = \int_{\varphi' \in [0,\overline{\varphi}']} \int_{\varphi \geq x} dF(\varphi'|\varphi)\mu_t(d\varphi)d\varphi' + M_{t+1}G(\overline{\varphi}') \tag{6}$$

  The first term considers only those firms which do not exit.

- Could write this in terms of Transition function

$$T'(\varphi, A) = \begin{cases} \int_{s' \in A} dF(s'|\varphi)ds' & \text{if } \varphi \geq x \\ 0 & \text{otherwise} \end{cases}$$

  so that (6) given by $\mu_{t+1} = T'\mu_t + M_{t+1}\eta$.

- Importantly, μ_{t+1} is homogeneous of degree 1 in μ_t and M_{t+1} (which allows separation between determination of P and M_{t+1}).

Competitive Equilibrium -cont.

Definition

A **stationary competitive industry equilibrium** is a list $\{P^*, Q^*, N^*, M^*, x^*, \mu^*\}$ such that

1. $P^* = D(Q^*)$, $Q^* = Q^s(\mu^*, P^*)$, $N^* = N^d(\mu^*, P^*)$ (markets clear)
2. x^* satisfies (4) (optimal exit)
3. (5) satisfied at P^* with equality if $M^* > 0$ (free entry)
4. $\mu_t = \mu^*$ is a fixed point of (6)

Competitive Equilibrium -cont.

- To prove existence, find a fixed point of
  $\mu \to (p(\mu)) \to (v, g) \to \mu$.

- Theorem 2. A SS equilibrium exists (there are many possible equilibria).

- Theorem 3. A SS equilibrium with entry/exit exists iff entry costs are sufficiently low.

- Theorem 4. Conditions for uniqueness include production functions of the form $f(\varphi, n) = \varphi n^{\alpha}$, $\alpha \in (0, 1)$.

- An important step in Theorem 2 is existence of a unique invariant measure μ^* which follows from mixing conditions.

Competitive Equilibrium -cont.

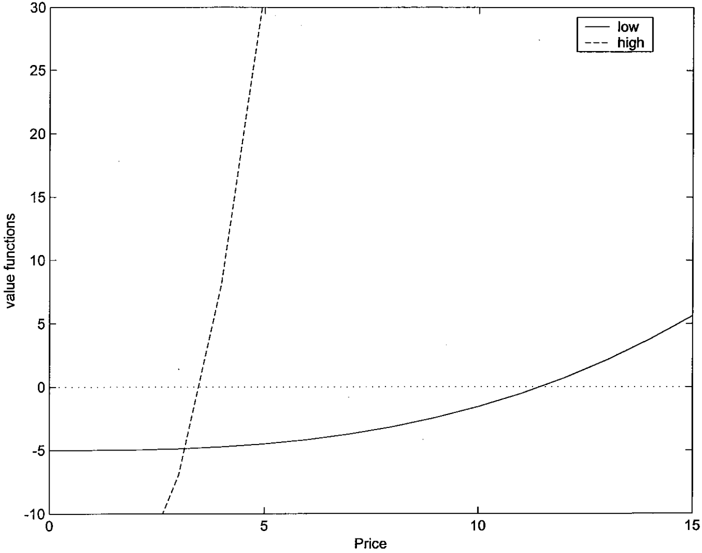
For the 2 state markov process we have:

- In general, the two steady state value functions indexed by φ_i are

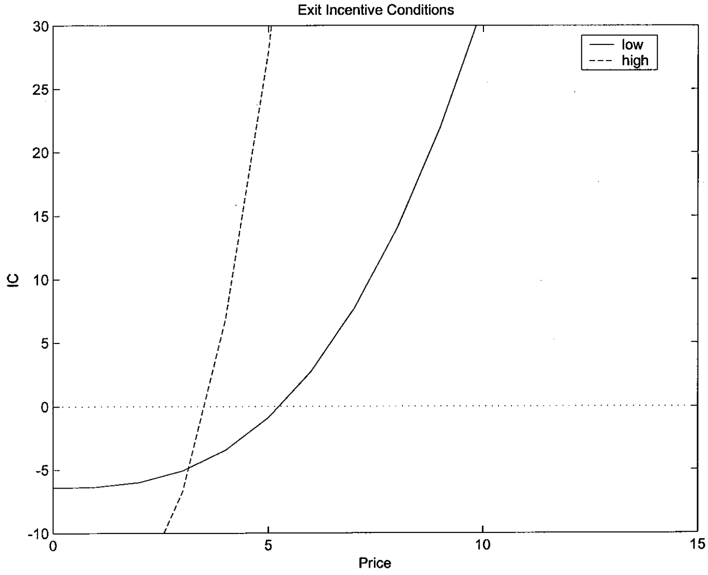
$$v(\varphi_i; P) = \frac{(P\varphi_i)^2}{4} - c_f + \beta \max\{0, \theta v(\varphi_i; P) + (1-\theta)v(\varphi_{j\neq i}; P)\}$$

- A firm will exit if $\theta v(\varphi_i; P) + (1- \theta)v(\varphi_{j\neq i}; P) < 0$.

Value Functions



Exit Decision



Competitive Equilibrium -cont.

Rather than prove existence of equilibrium, we will "construct" an equilibrium.

- If a firm receives a low shock, it will exit (i.e. $\theta v(\varphi_L; P) + (1- \theta)v(1; P) < 0)$
- If it receives a high shock, it will not exit (i.e. $\theta v(1; P) + (1- \theta)v(\varphi_L; P) > 0)$.
- It may be possible to satisfy these two conditions with $v(1; P) > v(\varphi_L; P)$ since $\theta > \frac{1}{2}$.

Competitive Equilibrium -cont.

- Conjecture this exit strategy is optimal and then check under what conditions on parameters it is optimal. Under the construction,

$$v(\varphi_L; P) = \frac{(P\varphi_L)^2}{4} - c_f + \beta \cdot 0 \tag{7}$$

$$v(1; P) = \frac{P^2}{4} - c_f + \beta \left\{\theta v(1; P) + (1-\theta) v(\varphi_L; P)\right\} \tag{8}$$

- That is, 2 linear equations in 2 unknowns.
- Note timing is the reason that $v(\varphi_L; P) \neq 0$.

Competitive Equilibrium -cont.

- Since we have exit, in order to ensure a stationary equilibrium we need positive entry.
- If $\eta v(1, P) + (1 - \eta) v(\varphi_L, P) = c_e$ then since firms are indifferent between entering and staying out of the market, we are free to choose how many firms enter (i.e. M).

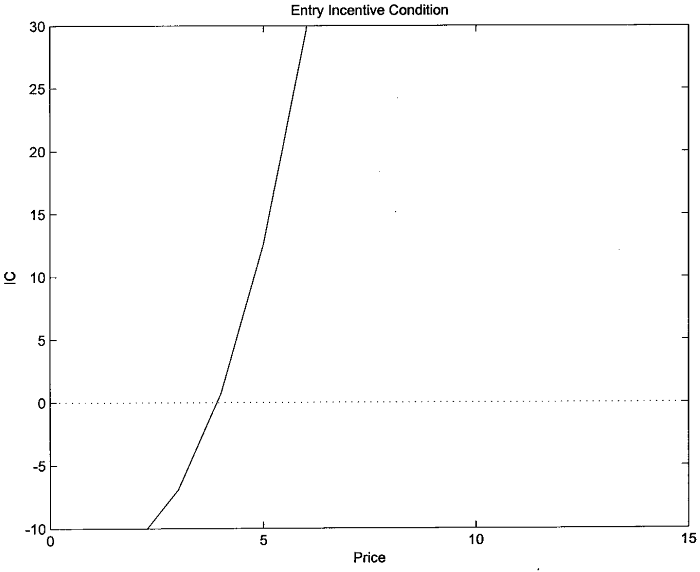
Competitive Equilibrium -cont.

- Given $v(1; P)$ and $v(\varphi_L, P)$ under our conjecture, this entry condition can be written

$$\eta\left[\frac{\frac{P^2}{4}\left[1+\beta(1-\theta)\varphi_L^2\right]-c_f\left[1+\beta(1-\theta)\right]}{(1-\beta\theta)}\right]$$
$$+(1-\eta)\left[\frac{(P\varphi_L)^2}{4}-c_f\right]=c_e$$

which is one equation in one unknown P. The lhs is continuously increasing in P,is negative if $P=0$, is ∞ as $P\to\infty$ and is independent of c_e. Thus a solution P^* exists.

Entry Decision



Competitive Equilibrium -cont.

- Note from the Figure on exit incentive conditions that a necessary condition for the conjectured equilibrium to exist is that P^* lies somewhere between 4 and 5.

- We can choose c_e to make that happen. Another way to think of this is that if we have data on the equilibrium price in the market P^*, this can help us identify c_e which we may not have data on.

Competitive Equilibrium -cont.

- Next we need the firm size distribution.
- We have

$$\begin{aligned} \mu_{t+1}(1) &= \mu_t(1)\theta + M_{t+1}\eta \\ \mu_{t+1}(\varphi_L) &= \mu_t(1)(1-\theta) + M_{t+1}(1-\eta) \end{aligned} \tag{9}$$

- That is, the mass of firms at the beginning of next period at the high productivity level are incumbents who remain high and new entrants who receive a high productivity level.
- Further, the mass of firms at the beginning of next period (before exit) at the low productivity level are incumbents who were high but received a low productivity draw and new entrants who receive a low productivity draw.

Competitive Equilibrium -cont.

- From (9), we can solve for the invariant distribution

$$\begin{aligned} \mu(1) &= \frac{\eta M}{(1-\theta)} \\ \mu(\varphi_L) &= \eta M + M(1-\eta) = M \end{aligned} \tag{10}$$

  which makes clear the dependence of the distribution on the mass of new entrants M.
- Note that the invariant distribution is linear in M. We will make use of this when computing an equilibrium.

Competitive Equilibrium -cont.

- Output given by

$$Q^s = \mu(1)\left(n_H^*\right)^\alpha + \mu(\varphi_L)\varphi_L\left(n_L^*\right)^\alpha \tag{11}$$

  That is there are some incumbent firms and new entrants who produce in a given period t before they exit in $t+1$ (if they receive a bad shock in t).

- In that case, goods market clearing implies $Q^s = Q$ from (11) and the inverse demand function

$$P = 1/(d + Q^s). \tag{12}$$

- But we know P from the entry condition so the lhs is given, implying an equation which pins down M.

Competitive Equilibrium -cont.

- In summary, it may seem counterintuitive, but the free entry condition effectively pins down prices (P) and the market clearing condition pins down the mass of new entrants (M).

Models with Imperfect Competition

- Ericson and Pakes (1995, especially the example in section IV) consider investment dynamics in an oligopolistic industry with idiosyncratic productivity shocks.

- Key difference:
  - With a finite number of firms, one firm's action affects others (with a continuum of firms each firm's action is negligible).

  - Equilibrium concept changes: Strategic Markov Perfect Eq.

Models with Imperfect Competition - cont.

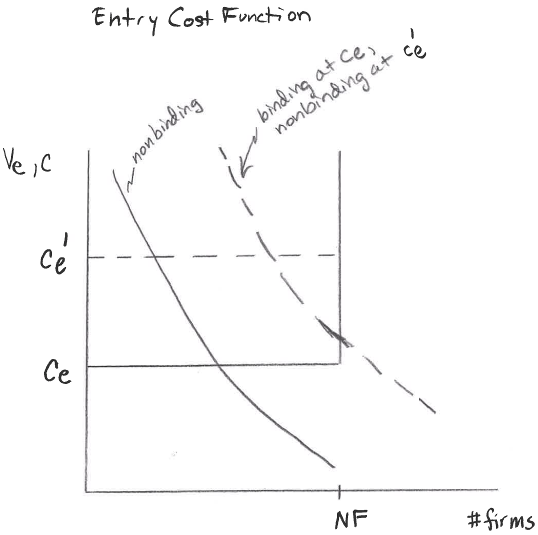
Minor Differences

- Ericson and Pakes (1995) assume a scrap value instead of fixed operating costs.
- Linear inverse demand function $P(Q) = D - Q$ with $D = 5$.
- Entering firm must pay fixed cost c_e one period in advance (time to build). For simplicity, we assume the following cost structure:

$$c_e = \begin{cases} c & \text{if number of incumbent firms} \leq NF \\ \infty & \text{if number of incumbent firms} > NF \end{cases}$$

  We take $c_e = 3.0$. In Hopenhayn, $NF = \infty$.
- This structure allows us to control the number of firms in the market and then we check if the "constraint" is binding.
- Timing: Firms receive their productivity shock *before* their exit decision.

Entry Cost Function



Monopoly Example ($NF = 1$)

- In this case, there is "trivial" aggregate uncertainty due to the idiosyncratic shocks the monopolist receives.

- The monopolist's static optimization problem:

$$n_i^m = \arg\max_n \left[D - \varphi_i n^\alpha\right] \varphi_i n^\alpha - n - c_f \qquad (13)$$

Monopoly Example -cont.

- The monopolist FOC is

$$P\alpha\varphi_i\,(n^m)^{\alpha-1} - \varphi_i^2\alpha(n^m)^{2\alpha-1} = 1, i \in \{H, L\} \tag{14}$$

- Recall the competitive firm's FOC:

$$P\alpha\varphi_i n^{\alpha-1} = 1, i \in \{H, L\} \tag{15}$$

- The monopolist recognizes that its quantity choice will affect the price it faces.

- For a given price, the MB of labor for a monopolist (the lhs of (14)) is lower than the MB for a competitive firm (15)) while the MC (the rhs of the equations) is identical $\Rightarrow Q^m < Q^c$.

Monopoly Example -cont.

- The two steady state value functions indexed by φ_i are

$$v^m(\varphi_i) \quad = \quad \max\left\{0, \pi^m(\varphi_i) + \beta\left[\theta v^m(\varphi_i) + (1-\theta)v^m(\varphi_{j\neq i})\right]\right\}$$

- The monopolist will choose to exit (i.e. $x(\varphi_i) = 1$) if $\pi^m(\varphi_i) + \beta\left[\theta v^m(\varphi_i) + (1-\theta)v^m(\varphi_{j\neq i})\right] < 0$.

- Since $\pi^m > \pi^c$, it is possible to "construct" an equilibrium where the monopolist always stays in the market for the same parameter values while a firm in a competitive market would not.

- The only condition we need to check is that exit is not profitable in any state. It is sufficient to simply check $v^m(\varphi_L) \geq 0$.

Monopoly Example -cont.

Comparing Monopoly vs Competitive Eq for same parameter values:

Monopoly Example
$P^m(0.6) = 4.338,\ Q^m(0.6) = 0.662$
$P^m(1.0) = 3.750,\ Q^m(1.0) = 1.250$
$n^m(0.6) = 1.217,\ \pi^m(0.6) = -0.246, v^m(0.6) = 9.073$
$n^m(1.0) = 1.563,\ \pi^m(1.0) = 1.225, v^m(1.0) = 15.412$

Competitive Example
$P = 3.296,\ Q = 1.704,\ M = 0.301$
$n(0.6) = 0.994,\ \pi(0.6) = -0.922, v(0.6) = 0$ (o-e-p)
$n(1.0) = 1.284,\ \pi(1.0) = 0.551, v(1.0) = 4.0498$

- Note that aggregate output and prices are state dependent in the monopoly example (no law of large numbers).

Duopoly Example ($NF = 2$)

- The two firms plan a non-cooperative Cournot game in quantity choices.
- Each firm chooses the quantity of goods produced taking into account the production and exit strategies of other firms.
- The state space is now doubled. The set of all possible states is given by $\{(\varphi_L, \varphi_L), (\varphi_L, \varphi_H), (\varphi_H, \varphi_L), (\varphi_H, \varphi_H), (\varphi_L, \varnothing), (\varphi_H, \varnothing), (\varnothing, \varnothing)\}$.
- In a symmetric duopoly equilibrium there are 3 aggregate states (i.e. boom, bust, and middle road).

Duopoly Example - cont.

A bit of messy notation :-)

- Let any function f (which could be a value function or decision rule or distribution of firms) be written $f(\varphi_s^i, \overline{\varphi}_{-i})$ where
    - $\varphi_s^i \in \{\varnothing, \varphi_L, \varphi_H\}$ is the productivity shock of firm i (and if $\varphi_s^i = \varnothing$ it is understood that firm i is contemplating entry)
    - $\overline{\varphi}_{-i} \in \{\varnothing, \varphi_L, \varphi_H\}$ is the vector of all other possible productivity shocks $\overline{\varphi}_{-i}$ of potential competitors (with $NF = 2$, $\overline{\varphi}_{-i}$ is simply a scalar, not a vector).

- E.g. $\pi^d(\varphi_H, \varphi_L)$ is the profit of a firm with own productivity φ_H whose competitor received φ_L.

- E.g. $e^d(\varnothing, \varphi_s)$ denotes the entry decision of a potential firm when there is an incumbent firm with productivity φ_s.

Duopoly Example - cont.

- If only one firm is active, the (static) optimal labor choice is given by the solution to the monopolist problem.
- If two firms are active, firm i solves

$$\pi^d(\varphi_i, \varphi_j) = \max_{n_i} \left[D - \varphi_i (n_i)^\alpha - \varphi_j (n_j)^\alpha \right] \varphi_i (n_i)^\alpha - n_i - c_f \tag{16}$$

- The FOC is

$$D\alpha\varphi_i (n_i^d)^{\alpha-1} - 2\varphi_i^2 \alpha \left(n_i^d\right)^{2\alpha-1} - \varphi_j n_j^\alpha \alpha \varphi_i \left(n_i^d\right)^{\alpha-1} = 1 \tag{17}$$

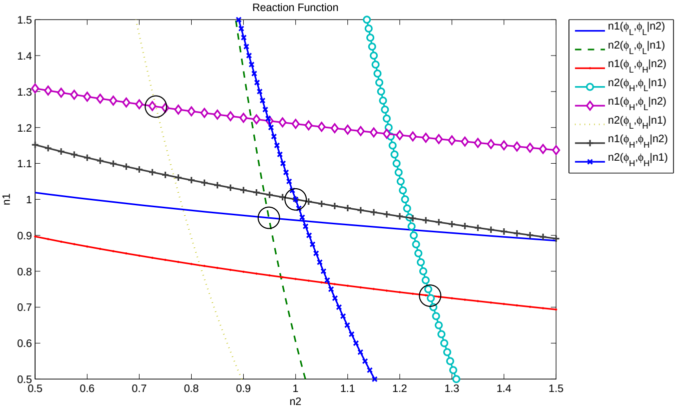
- This gives the labor demand policy function (given the second firm labor choice) of firm i, $n_i^d(\varphi_i, \varphi_j | n_j)$.

Duopoly Example - cont.

- Note that in the foc (17), firm i takes as given its competitor's actions n_j.
- A Cournot-Nash equilibrium requires consistency; that is, firm $i's$ belief about n_j in (17) is consistent with optimal actions taken by firm j.
- Conditional on holding firm j's output constant (i.e. n_j and φ_j), labor at firm i rises if productivity there rises.
- The decision rules $n_i^d(\varphi_i, \varphi_j | n_j^d)$ and $n_j^d(\varphi_j, \varphi_i | n_i^d)$ can also be thought of as *reaction functions*. The slope of the reaction function of firm i is given by

$$\frac{dn_i^d}{dn_j} = -\left(\frac{\varphi_i \left[D - \varphi_j n_j^{1/2} \right]}{2\left[1 + \varphi_i^2\right]} \right) \frac{\varphi_i \left[\varphi_j n_j^{-1/2} \right]}{2\left[1 + \varphi_i^2\right]} < 0.$$

Reaction Functions



- Any reaction function that has a vector where $\varphi_i \neq \varphi_j$ is not consistent with behavior on-the-equilibrium path

Duopoly Example - cont.

- If two firms are in the market there are four possible states of the world (φ_L, φ_L), (φ_L, φ_H), (φ_H, φ_L) and (φ_H, φ_H).

- The value function is given by:

$$v(\varphi_i, \varphi_{-i}) = \max_{x \in \{0,1\}} (1 - x) \Big\{ \pi(\varphi_i, \varphi_{-i}) + \beta E \left[v(\varphi_i', \varphi_{-i}') \right] \Big\}$$

which yields exit decision rules $x(\varphi_i, \varphi_{-i})$.

Duopoly Example - cont.

We will construct an equilibrium with the following characteristics:

- If two firms are active:

(φ_i, φ_j)	Exit Decision
(φ_L, φ_L)	$(stay, stay)$
(φ_L, φ_H)	$(exit, stay)$
(φ_H, φ_L)	$(stay, exit)$
(φ_H, φ_H)	$(stay, stay)$

- If only one firm is active it always stays in the market (i.e. $x(\varphi_i, \varnothing) = 0$ for all $\varphi_i \in \{\varphi_L, \varphi_H\}$).
- As for entry, a firm enters only if a monopolist receives φ_L (since $NF = 2$, there is no entry with two active incumbents).
- If there are no incumbents (possible off-eq-path), then two potential entrant are randomly selected to enter.

Duopoly Example - cont.

- In this equilibrium, the economy will fluctuate between duopoly and monopoly of random lengths.
- For instance, if there is a realization of shocks such that there is asymmetry in the duopoly case, then we enter a monopoly phase until the monopolist receives a low shock then a duopoly will arise in the following period.

Duopoly Example - cont.

- Since there are 6 states, there are 6 value functions to define and you simply solve a system of 6 equations in 6 unknowns.
- Given $v(\phi_i, \phi_j)$, you can pin down the highest and the lowest value of the entry cost, $(\underline{c}, \overline{c})$, such that the equilibrium exists.
- We must check that no firm has an incentive to deviate from what we conjectured in the above table.

Duopoly Example - cont.

The most important condition to check is Row 2.

- Along the eq path, exit for an incumbent firm with φ_L when the other incumbent firm has φ_H implies $v(\varphi_L, \varphi_H) = 0$.

- The value to an incumbent firm with φ_L from a one-shot deviation of exiting in this state is given by

$$v^{dev}(\varphi_L, \varphi_H) = \pi(\varphi_L, \varphi_H)$$
$$+\beta \left\{ \begin{array}{c} \theta \left[\theta \cdot 0 + (1-\theta) v(\varphi_L, \varphi_L)\right] \\ +(1-\theta)\left[\theta v(\varphi_H, \varphi_H) + (1-\theta) v(\varphi_H, \varphi_L)\right] \end{array} \right\}$$

- Thus, an incumbent firm does not benefit from a one-shot deviate provided $v^{dev}(\varphi_L, \varphi_H) < 0 = v(\varphi_L, \varphi_H)$.

Comparing All Cases

	competitive	monopoly	duopoly
prices	$P = 3.296$	$P^m(0.6) = 4.3$	$P^d(0.6, 0.6) = 3.8$
		$P^m(1.0) = 3.7$	$P^d(1.0, 1.0) = 3.0$
quanty	$Q = 1.704$	$Q^m(0.6) = 0.66$	$Q^d(0.6, 0.6) = 1.17$
		$Q^m(1.0) = 1.25$	$Q^d(1.0, 1.0) = 2.00$
profits	$\pi(0.6) = -0.9(\text{oep})$	$\pi^m(0.6) = -0.3$	$\pi^d(0.6, 0.6) = -0.6$
	$\pi(1.0) = 0.551$	$\pi^m(1.0) = 1.23$	$\pi^d(1.0, 1.0) = 0.1$
values	$v(0.6) = 0\ (\text{oep})$	$v^m(0.6) = 9.0$	$v^d(0.6, 0.6) = 1.2$
	$v(1.0) = 4.05$	$v^m(1.0) = 15.4$	$v^d(1.0, 1.0) = 4.3$

where duopoly with one incumbent reverts to monopoly allocation.

Implications

One can then check whether the model generates procylical entry, countercylical exit, and countercylical markups (see Jaimovich and Floetotto (2008, Journal of Monetary Economics) for a DSGE model with imperfect competition with these properties).

- Obviously, the competitive case cannot generate this unless we add aggregate uncertainty.
- The monopoly case does not have entry and exit, but the markup $P^m(\varphi) - 1$ is countercyclical.
- The duopoly case also has countercyclical markups, countercyclical exit, but predicts countercylical entry.