

Problem Set #2

UW–Madison | Economics 871, Fall 2019

1. **The Melitz-Chaney Model.** The economy is made up of two countries indexed by i , each endowed with L_i units of labor and μ_i potential firms. There is a homogenous good (good 0) and a continuum of goods that are differentiated by firm. The homogenous good is produced according to

$$y_0 = l_0. \quad (1)$$

The technology for producing homogenous goods is identical across countries.¹ A differentiated good produced in country i for sale in country j is produced according to

$$y_{ij}(\omega) = x(\omega)l_{ij}(\omega). \quad (2)$$

$x(\omega)$ is the productivity of the firm that produces good ω . There exists a fixed cost, κ_{ij} , to sell goods into market j . The fixed costs are paid in units of labor. In addition to the fixed costs of production, there are variable “iceberg” costs of transporting a good between i and j , $\tau_{ij} > 1$.

The problem of the consumer in country i can be written as

$$\max (1 - \alpha) \log(c_{i0}) + \alpha \log \left[\int_0^m c_i(\omega)^\rho d\omega \right]^{1/\rho} \quad (3)$$

$$\text{s.t. } p_{i0}c_{i0} + \int_0^m p_i(\omega)c_i(\omega)d\omega = w_iL_i + \Pi_i \quad (4)$$

where Π_i is the aggregate profit generated by the firms located in country i , w_i is the wage rate in country i and $0 < \rho < 1$.

- a. **Pricing.** In this economy firms are monopolistic competitors. Assume that the producer of good ω takes the price of all other goods ω' as given. Consider the problem of a firm in country i selling to country j . Write out the firm’s profit maximization problem, and solve for the firm’s pricing rule, conditional on selling to market j .
- b. **Equilibrium.** Assume that the mass of firms in each country is μ and that firm productivity in each country is distributed by

$$G(x) = 1 - (\underline{x}/x)^\gamma, \quad (5)$$

where \underline{x} is the lower bound on the support of x . Note that this is a “Chaney” type of model: The mass of firms is fixed, so there will be aggregate profits in equilibrium which are returned lump-sum to the consumer.

Compute the equilibrium of the 2 country model in which $\mu = 1$, $\rho = 0.8$, $\alpha = 0.4$, $\underline{x} = 1$, $\gamma = 6.5$, $L_1 = L_2 = 1$, $\kappa_{11} = \kappa_{22} = 0.1$, $\kappa_{12} = \kappa_{21} = 0.2$, $\tau_{11} = \tau_{22} = 1.0$, $\tau_{12}^H = \tau_{21}^H = 1.15$.

In particular, report

¹This is a trick used to pin down the wage in the countries. You see this a lot when people are shooting for analytical solutions to a model. It is not really necessary in this case, since we will be using a computer to solve the model.

- i. the productivity levels of the cutoff domestic-only producer and exporter,
 - ii. the mass of differentiated imported goods consumed,
 - iii. the mass of differentiated domestic goods consumed,
 - iv. the CES aggregate price index for differentiated goods,
 - v. the f.o.b. value of imports, and
 - vi. the value of consumption from domestic firms.
- c. **Gains from trade.** Let's liberalize trade: Keeping all the other parameters as they are in part b., set $\tau_{12}^L = \tau_{21}^L = 1.0$.

Recompute the equilibrium (the 'free trade equilibrium') and report

- i. the productivity levels of the cutoff domestic-only producer and exporter,
- ii. the mass of differentiated goods (both imports and domestic) consumed,
- iii. the CES aggregate price index for differentiated goods,
- iv. the f.o.b. value of imports, and
- v. the value of consumption from domestic firms.

Compared to the economy with tariffs, how has the mass of differentiated goods consumed in a country changed? Through what channel do the gains from liberalization occur?

- d. **Nonlinearities.** Let's liberalize trade again: Keeping all the other parameters as they are in part b., set $\tau_{12}^L = \tau_{21}^L = 1.14999$. Compute the equilibrium.

Compute the elasticity of imports with respect to tariffs for country 1,

$$\sigma = [(X_{21}^L/X_{11}^L)/(X_{21}^H/X_{11}^H) - 1] / [\tau_{21}^L/\tau_{21}^H - 1] + 1, \quad (6)$$

where X_{ij}^k is f.o.b. trade from i to j relative to "trade" from j to j in regime $k = H, L$,

$$X_{ij} = \frac{\int_{\hat{x}_{ij}}^{\infty} p_{ii}(x) c_{ij}(x) dG(x)}{\int_{\hat{x}_{jj}}^{\infty} p_{jj}(x) c_{jj}(x) dG(x)}. \quad (7)$$

To which parameter in the model does this elasticity correspond?

Recompute the above elasticity with $\tau_{12}^L = \tau_{21}^L = 1.05$ and $\tau_{12}^L = \tau_{21}^L = 1.0$. Plot the 4 elasticities you have computed against the change in tariffs.