# A Monocentric City Model (Alonso-Mills-Muth Model)

RE420: URBAN AND REGIONAL ECONOMICS

# **Quick Review**

- Housing durability and housing supply curve
- The compounding challenges that shrinking cities face:
  - 1. City budget issue
  - 2. Labor market decline
  - 3. Vacant and abandoned home

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- How does the skyline of the city look like?

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Paris, France



TOGETHER FORWARD®

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- 2. How about the sizes of individual dwellings? (apartments/houses)
- 3. How about the housing prices?
- The monocentric city model (a.k.a. Alonso-Muth-Mills model) attempt to capture these regularities of urban spatial structure

- 1. How does the skyline of the city look like?
  - The urban center has a concentration of tall buildings
  - Building heights gradually falling with distance from the center
  - The heights of the residential buildings drop to 2-3 stories
  - Single-story houses become common in the distant suburbs
- 2. How about the sizes of individual dwellings? (apartments/houses)
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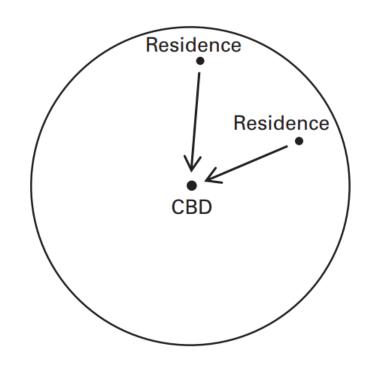
- 1. How does the skyline of the city look like?
- 2. How about the sizes of individual dwellings? (apartments/houses)
  - The dwellings in the tall residential buildings near the city center are relatively small
  - Suburban houses are much more spacious
- 3. How about the housing prices?
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- 1. How does the skyline of the city look like?
- 2. How about the sizes of individual dwellings? (apartments/houses)
- 3. How about the housing prices?
  - Rental/purchase price per sqft is much higher near the city center than in the suburbs
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#### **Basic Assumptions**

- Basic assumptions facilitate a simplified analysis while capturing essential features of cities:
  - 1. There is a single city in the world with fixed number of population
  - All the city's jobs are in the center, in an area called the "central business district" (CBD)
  - 3. The city has a dense network of radial roads
  - 4. The city's residents consume only two goods: a composite good ("bread") and housing



# Commuting Cost and Disposable Income

- Let x denote radial distance from a consumer's residence to the workplace
- The larger is the commuting distance x, the cost of commuting is higher
  - Money cost such as the transit fare
  - Opportunity cost of the time spent commuting
- Let's say t represents the per-mile cost of commuting
  - Total cost of commuting is  $t \times x$
- Each household earns income amount y
  - Total disposable income is: y tx

# Consumer Spending and Budget Constraint

- Each household decides how much bread and housing to consume
  - Denote the amounts of bread and housing consumptions as c and q, respectively.
- The price of bread is normalized to \$1 and the price of housing \$p
  - How can we arbitrarily normalize the price of bread?
- Total expenditure of bread and housing is: c + pq
- Due to the budget constraint, the consumer expenditures on bread and housing will be equal to the disposable income:

$$y - tx = c + pq$$

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- Choice variable: a decision that an economic agent can actively make, like the quantity of a good to produce
  - 1. Where to live: This choice determines the commuting distance x.
  - 2. Housing space consumption: This decision determines q.
  - 3. Bread consumption: This choice determines c.
- As a result of the individual's choices, the price of housing, p, at each location will be determined.

## Simplified Version

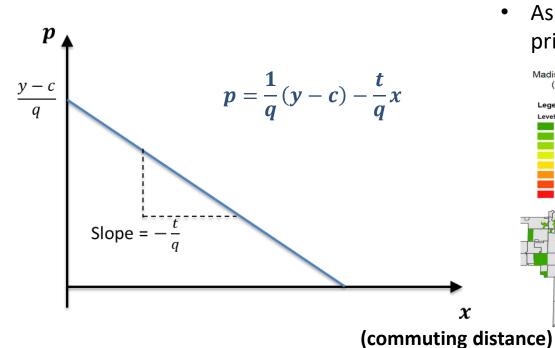
- The most simplified way to understand the model is to assume the consumption amounts, c and q, do not change.
- Then, the only variable that individuals choose vary is the commuting distance, x.
- Arrange the terms to see the relationship between x and p:

$$y - tx = c + pq$$

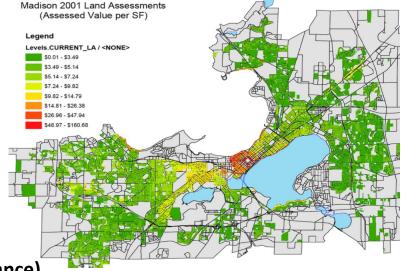
$$\Rightarrow pq = (y - c) - tx$$

$$\Rightarrow p = \frac{1}{q}(y - c) - \frac{t}{q}x$$

## Simplified Version



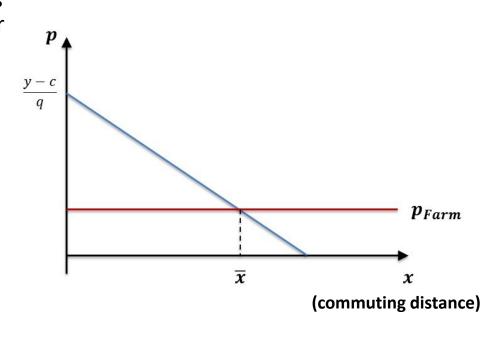
 As commuting distance gets longer, the price goes down



< Land values in Madison >

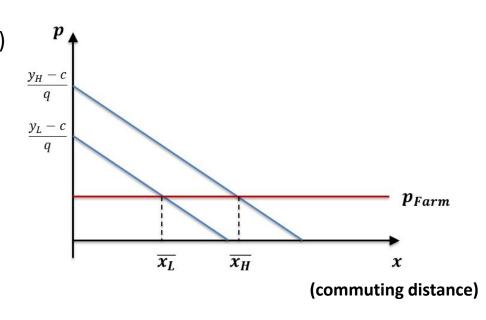
# Simplified Version: Determination of City's Edge

- From urban developers' perspective, selling price per unit goes down as the unit further away from CBD
- However, from farmers' perspective, land price per unit should be equal (=  $p_{Farm}$ ) regardless of distance from CBD.
  - The same size of land yields the same amount of corn, regardless of its proximity to CBD.
- Therefore, the city is developed up to the point where developers' selling price per unit is greater than  $p_{Farm}$



# Simplified Version: Determination of City's Edge

- Consider two cities: one with a high wage
   (city H) and another with a low wage (city L)
- How do the different wages affect the size of two cities?
- What happens to the size of cities in a developing country, if the country has relatively cheaper farmland while urban wages are relatively high?
  - Can this explain any of the stylized facts of cities?

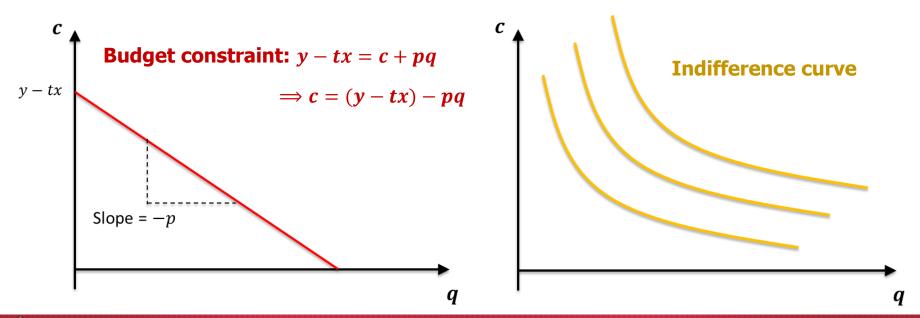




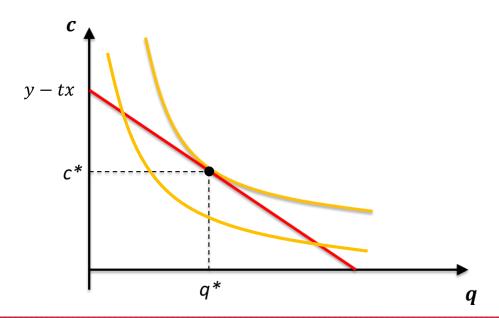
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• Now allow consumers can choose their own consumption amounts c and q.

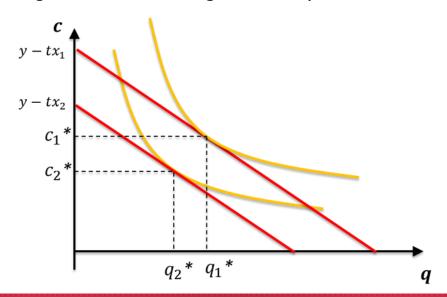
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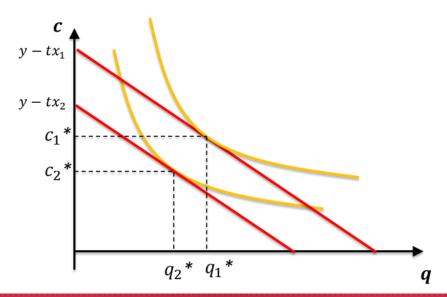
 Household consumption amount will be determined at the tangent point of indifference curve and budget constraint line.



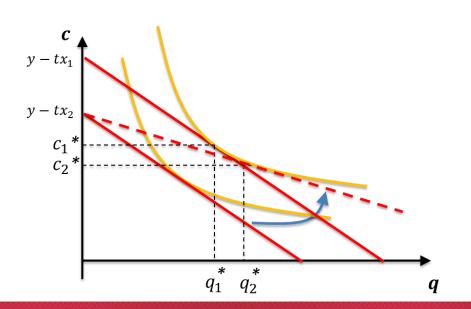
- Two locations: one closer to CBD  $(x_1)$  and another farther away from CBD  $(x_2)$
- If housing prices in the two locations are equal, the household living closer will consume more for both bread and housing than the household living farther ( $c_1^* > c_2^*$ ,  $q_1^* > q_2^*$ )
  - The household living closer will attain a greater utility



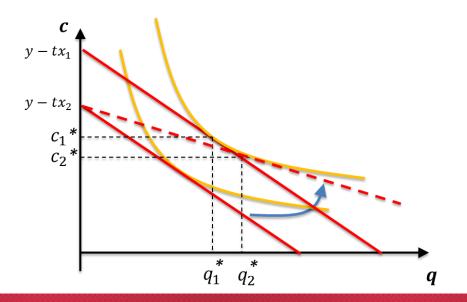
- **Spatial Equilibrium:** Consumers must achieve equal levels of utility across different locations. Otherwise, they will migrate to the location that offers a higher utility.
- Therefore, in the world with a spatial equilibrium, it is *impossible* for the household living closer to attain a greater utility



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  - 1. Housing prices closer to the CBD will be higher
  - 2. Households further from the CBD will consume more units for housing
  - 3. Since the same sized lot is more expensive near the city center, developers builds taller structures around the CBD.
- Regularities 1, 2, 3 can be explained by the monocentric model.
- How realistic the model assumptions?
  - Are there really people who commute much longer for cheaper and larger housing?

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# Video Clip

The Struggle Is Real for Super Commuters (2:30)



# Key Takeaways

- Understand the three regularities of urban spatial structure that the monocentric city model can capture.
- Understand the assumptions in the monocentric model.
- Understand the spatial equilibrium concept.
- Optional Readings:
  - Jan K. Brueckner, *Lectures on Urban Economics*. Chapter 2, Chapter 3
  - Alonso, William. 1964. Location and Land Use: Toward a General Theory of Land Rent. Harvard University Press Cambridge, MA.
  - Mills, Edwin S. 1972. Studies in the Structure of the Urban Economy. ERIC.
  - Muth, Richard F. 1975. Urban Economic Problems. HarperCollins Publishers.



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