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Smart Rebalancing for Bike Sharing Systems using Quantum Approximate Optimization Algorithm

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***Abstract*—Smart Mobility is the key component of Smart City initiative that are being explored throughout the world. The bike-sharing system (BSS) aims to provide an alternative mode of Smart Mobility transportation system, and it is being widely adopted in urban areas. The use of bikes for short-distance travel helps to reduce traffic congestion, reduce carbon emissions, and decrease the risk of overcrowding. Effective bike sharing system operations requires rebalancing analysis, which corresponds to transferal of bikes across various bike stations to ensure the supply meets expected demand. In this work, we present Quantum Approximate Optimization Algorithm(QAOA), a variational hybrid quantum-classical algorithm that has shown significant computational advantages in solving combinatorial optimization problems such as bike sharing system rebalancing problem (BSS-RBP). Here, we minimize the overall distance travelled by the transport vehicle across various bike station. In this preliminary work, we demonstrate the application of QAOA using the IBM-Qiskit quantum computing simulator for rebalancing analysis across three bike locations.**

***Index Terms*—Bike sharing system, smart mobility, QAOA, Optimization, Rebalancing, IBM-qiskit, QUBO**

I. INTRODUCTION

In the recent years, the Internet of Things (IoT) has become more ubiquitous by providing promising applications in the emergence of Smart Cities. This new paradigm of IoT can be described as a sequence of inter-connected objects linked through wireless networks that can sense, simulate, actuate, and communicate with the physical world [1]. The vigorous expansion of IoT infrastructure has enabled the development of Smart Cities initiative throughout the world [2]. However, as the cities requirements tend to increase exponentially, we have to take necessary actions to mitigate the urban challenges. Problems such as traffic congestion, overcrowding, and increase of carbon emissions are some of the challenges currently faced in urban cities.

Economical and innovative modes of transportation system are required to improve efficiency and sustainability to the urban cities [3]. This transformation in urban transportation is only possible with the development of Smart Mobility, as one of the key component behind Smart Cities initiative. Hence, alternate mode of transport system such as bike sharing systems (BSS) are required to alleviate urban-mobility challenges. BSS is an ideal example of a Smart City enabled initiative service that are becoming the major mode of urban-mobility for short distance travels within densely populated cities. Bixi in Montreal, Citi-bike in New York, Hangzhou public bike system in China, London Bike share system, and Capitol bike share in Washington DC [4] are some of the well-known BSS throughout the world.

A BSS is described as a set of stations that are placed throughout the city and each station have a fixed set of docks. During, the start of every day the station is filled with predetermined set of bikes. The customers have the flexibility to pick and drop off bikes from any station and they are charged with respect to the duration being used. However, due to high frequency of bike usage it is critical for bike service providers to rebalance the bikes among the stations to ensure there is no shortage in supply of bikes. The dynamics of bike sharing mobility often leads to discrepancy in bike supply and demand [5].

Bike rebalancing analysis requires real-time analysis as it is followed by redistribution of available bikes across various stations to ensure the bike supply meets predicted demand. One approach that can be used to increase the computational performance of the analysis is by using the principles of quantum computing. Quantum computing is a new paradigm that uses the principles of quantum mechanical systems such as superposition and entanglement to improve the algorithmic computational performance when compared to classical computational analysis [6]–[9].

To satisfy the interest and requirements of solving NP-hard optimization problems such as bike rebalancing, researches have conducted many studies with respect to various quantum computing application platforms. Amongst various quantum computing techniques, we present the application of Quantum Approximate Optimization Algorithm, (QAOA) for solving bike sharing system rebalancing problem, (BSS-RBP). QAOA was introduced by Farhi et al. [10], [11] as one of the leading methods to demonstrate the quantum supremacy to solve NP-hard optimization problem to find a solution of higher quality, compared to other classical algorithms.

Furthermore, the major advantage of QAOA is that it requires only shallow circuit depth with simple repetitive structures and can be simulated in Noisy Intermediate-Scale Quantum (NISQ) circuit devices with finite tuning in error correction [12]. QAOA has also been used to solve a wide variety of combinatorial optimization problems in fields such as vehicle routing problem [13], network community detection

[14], tail assignment problem [15], and maximum graph cut [16]. Following the application of QAOA for solving combinatorial optimization in various domains, we provide the formulations for solving the BSS-RBP in QAOA framework.

Research Contribution: The contributions made through this paper are: (1) a quadratic unconstrained binary optimization (QUBO) formulation for a BSS-RBP with vehicle carrier/depot constraints; (2) QAOA Circuit construction for BSS-RBP formulation; (3) demonstration of QAOA for a 3 bike-station rebalancing problem.

Paper Organization: The rest of the paper is organized as follows. Section II provides the literature review of various bike rebalancing approaches. Section III provides an overview of Quantum Approximate Optimization algorithm. Section IV discusses the the bike sharing system rebalancing problem using QUBO formulation. Section V discusses results and discussion for the 3-station case study followed by concluding remarks and future work in Section VI.

II. Related Work

Over the past few years, the application of bike-sharing system as an urban-mobility mode of transportation have received major attention and momentum within the related field of research. In the following, we provide a brief overview of various rebalancing approaches adopted for BSS.

Pan et al. [17] mentioned that BSS rebalancing approaches can be classified into three groups such as truck(vehicle)-based, bike-trailers, and user-based rebalancing approaches respectively. The above mentioned approaches most often employ static and dynamic methods to solve the bike rebalancing problem [18], [19].

BSS static rebalancing problems are usually constraint-based problems solved by mixed integer programming methods [20], [21]. Identifying optimal route for vehicle carriers with minimum distance is an extended version of travelling salesman problem, (TSP) [22] which is often regarded as bicycle routing problem, (BRP). BRP [23] is an ideal example of NP-hard problem where there is a mutual trade-off between feasibility and computational constraints of solution. Furthermore, bike rebalancing approaches also divides bike stations into clusters based on geographical locations and inventory levels [24]. Thereby, performing rebalancing on each of the cluster across the travelling routes with minimum distance.

The high problem complexity of BSS with dynamic rebalancing makes the problem more difficult to identify feasible solutions in real-time. To overcome this issue analytical models and heuristic algorithms are mostly used to solve constrained based dynamic bike rebalancing problem [25]. In addition, to the existing approaches Jan et al. [26] proposed two strategies to solve dynamic bike rebalancing problem, a long-term strategy to determine the overall estimate of the bikes required to satisfy the predicted demand, and short-term strategy that only considered demand in each of the bike stations that have to be rebalanced with a fixed set of bikes.

Based on analyzing both static and dynamic schemes of BSS rebalancing problems. We can infer, that BSS-RBP requires real-time analysis to successfully rebalance the bikes across the stations. However, this attempt is possible only by adopting advanced computation platform that enables to increase the efficiency and accuracy of the results [6]. Hence, in this paper we are introducing a variational hybrid quantum-classical algorithm, QAOA that works on principle of Adiabatic Quantum Computation (AQC), and it has shown significant computational benefits in solving combinatorial optimization problems. Since BSS-RBP is a combinatorial optimization problem, we discuss in this paper the problem formulation and a solution methodology.

III. Quantum Approximate Optimization Algorithm

A. Overview of QAOA

Quantum approximate optimization is a meta-heuristic algorithm for solving combinatorial optimization problems based on the principles of quantum mechanics [27]. Quantum approximate optimization algorithm is implemented by evolving the Hamiltonian dynamics from an initial quantum state to a final quantum state, which corresponds to the solution of an optimization problem of interest [28]. Let H_I and H_F represent the initial and final Hamiltonians of a quantum system. , then the Hamiltonian at any time, $H(t)$, between the evolution can be written as [29]:

$$H(t) = \left(1 - \frac{t}{T}\right) H_I + \left(\frac{t}{T}\right) H_F \tag{1}$$

We map the minimization problem of interest (here BSS-RBP) to the final Hamiltonian. The final Hamiltonian, and thus the final ground state, corresponds to the minimization problem of interest. A quantum system comprises of several individual qubits, and the Hamiltonian of an N-qubit Ising system can be written as:

$$H(\boldsymbol{z}) = \sum_{1 \leq i \leq N} h_i z_i + \sum_{1 \leq i < j \leq N} J_{ij} z_i z_j \tag{2}$$

where z_i represents the state of i^{th} qubit, which can be either -1 or 1, h_i and J_{ij} are the bias and interaction terms relating to individual qubits [30], and $\boldsymbol{z}$ is a vector of all qubit states.

Generally, quadratic unconstrained binary optimization (QUBO) problems can be mapped to the quantum Ising Hamiltonian shown in Eq. (1). Therefore, we will need to reformulate the optimization problem of interest into a QUBO formulation. In the presence of constraints, a commonly used approach to obtain a QUBO formulation is the penalty method [31]. For example, if $f(x)$ and $c(x) \leq 0$ represent the objective function and constraint respectively, then an equivalent cost function $q(x)$ for an unconstrained formulation can be written as $q(x) = f(x) + \gamma g(c(x))$, where γ is the penalty term, and g is a function defined over the constraint.

Farhi et al. [10] introduced the Quantum Approximate Optimization Algorithm, (QAOA), to solve NP-hard combinatorial optimization problem on a gate model quantum computer. A gate refers to a mathematical operation perform on one or more qubits. The gate set $G = \{H, CNOT, X, Z\}$ forms a universal set of gates for quantum computing based on a famous theorem [6]. Any n-qubit unitary operation, represented by a $2^n \times 2^n$ matrix, can be approximated up to an arbitrary precision ϵ by a sequence of gates consisting of gates from the set G [32].

The $CNOT$ (Controlled-NOT) gate is a two-qubit gate, where one qubit controls the state of the other qubit (target) to determine if a NOT gate (Pauli-X-gate) should act on target. The following matrices illustrate various gates using the computational basis of $|0\rangle$ and $|1\rangle$:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \text{CNOT} = \begin{bmatrix} I_{2\times 2} & 0 \\ 0 & X \end{bmatrix}$$

$$\text{(Pauli) X} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \text{Z} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Here, $I_{2\times 2}$ is a 2×2 identity matrix. The quantum state of the target only changes by a Pauli X gate, when the control is in state $|1\rangle$. To understand how CNOT acts on the quantum states, imagine a 2-qubit system $|q_0 q_1\rangle$ in the quantum state $|\Psi_0\rangle$ where $|\Psi_0\rangle = a_0|00\rangle + a_1|01\rangle + a_2|10\rangle + a_3|11\rangle$. By applying a CNOT gate such that q_0 is the control and q_1 the target qubit, $CNOT \otimes |\Psi_0\rangle = |\Psi_1\rangle$ where $|\Psi_1\rangle = a_0|00\rangle + a_1|01\rangle + a_2|11\rangle + a_3|10\rangle$. As you observe, when the control qubit is $|0\rangle$, $|00\rangle$ and $|01\rangle$ remain unchanged while $|10\rangle$ and $|11\rangle$ changed to $|11\rangle$ and $|10\rangle$ respectively because the X gate is applied when the control qubit is $|1\rangle$.

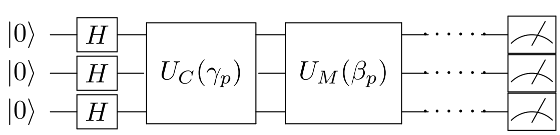


Fig. 1: Illustrative quantum circuit of QAOA analysis

Fig. 1 provides an schematic quantum circuit of QAOA. H represents the Hadamard gate, $U_C(\gamma_p)$ and $U_M(\beta_p)$ represent the cost and mixer operators (gates) as detailed later in Section III-B. Here, p is a postive integer. It is shown that when the limit p $\rightarrow \infty$, QAOA is capable of finding the global optimum value of the any classical combinatorial optimization problem [10]. Depending on the problem complexity, a lower value of p can also result in the correct solution.

Farhi mentioned that the parameters of the optimization algorithm are the times in which each Hamiltonian is applied during each stage of iteration. The output from the final Hamiltonian is estimated based on the optimized time sequence during each stage of the model [33]. The p-level QAOA algorithm has 2p parameters (γ_i and β_i, $i = 1, 2, 3 \ldots p$), that correspond to the angles of the final Hamiltonian and mixing Hamiltonian that are applied during each stage of iteration. The performance of the p-level QAOA increases gradually with increase in number of layers. A QAOA circuit of fixed level, is practically feasible for small value of p, as the level increases the complexity of optimization also increases [33].

B. QAOA workflow

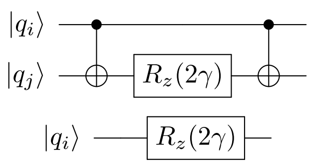
Here, we describe the workflow of QAOA analysis from problem formulation to quantum circuit construction to parameter estimation to obtaining the final result.

1) Define an objective function, $f(\mathbf{x})$, defined over n-bit binary strings $\mathbf{x} = (x_1, x_2, \ldots x_n)$
2) As discussed in Section III-A, the binary strings need to take the values $\{-1, 1\}$ to use the quantum Ising Hamiltonian approach. Therefore, we perform a linear transformation to convert the x variables with values 0,1 to another set of z variables, which take the values -1,1 using the transformation $x_i = \frac{1+z_i}{2}$. This results in the objective function in terms of z variables, $f(\mathbf{z})$
3) The optimal value of the objective function can be obtained from the eigen values of the cost Hamiltonian, denoted as H_C; this encodes the solution of the classical optimization problem. We define a cost operator to be able to represent the cost Hamiltonian on a quantum circuit. The cost operator has the parameter γ described in Fig. 1.
$$\mathrm{H}_C |z\rangle = f(z)|z\rangle \tag{3}$$
$$\mathrm{U}_C(\gamma) = e^{-i\gamma(H_C)} \tag{4}$$
4) Following the cost operator in Fig. 1, we define a mixer Hamiltonian and a corresponding mixer operator. The Mixer operator Hamiltonian, H_M is defined in Eq. 5. Where, σ_j^x is Pauli- X operator and n is the number of qubits in the system and β is the mixing operator parameter.
$$\mathrm{H}_M = \sum_{j=1}^{n} X_j \tag{5}$$
$$\mathrm{U}_M(\beta) = e^{-i\beta(H_M)} \tag{6}$$
5) The quantum state evolution begins from an uniform superposition of all computational states $|+\rangle^{\otimes n}$. The evolution is implemented by applying a sequence of p alternating operators for parameters γ, β. Where, U_C and U_M are cost and mixer operators respectively.
$$|s\rangle = |+\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}} \sum_z |\mathbf{z}\rangle \tag{7}$$
$$|\psi(\gamma, \beta)\rangle = U_M(\beta_p)U_C(\gamma_p)...U_M(\beta_1)U_C(\gamma_1)|s\rangle \tag{8}$$
6) In order to perform quantum evolution, we need to estimate the parameters, γ_i and β_i; these parameters are typically estimated through s classical optimizer. QAOA is a quantum-classical hybrid algorithm. Here, classical analysis correspond to the estimation of γ_i and β_i parameters, and quantum analysis refers to the estimation of lowest energy state dependent on the values of γ_i and β_i. The key role of classical optimizer is to determine the variational parameters γ_i, β_i to optimize the expected value of the cost Hamiltonian. The optimal value of

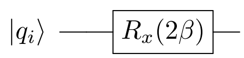
the objective function, f is dependant on the optimal parameters γ_i,$\beta_i \in [0, 2\pi]$, $\gamma_i \in [0, 2\pi]$ [10]. If $\psi(\gamma, \beta)$ refers to the final quantum state after implementing p cost and mixer operators, then the expected cost value can be calculated as shown in Eq. 9.

$$\langle H_c \rangle = \langle \psi(\gamma, \beta) | H_C | \psi(\gamma, \beta) \rangle = \langle q(x) \rangle \tag{9}$$

When creating a quantum circuit, we replace each of the z variables -1,1 with the Z gate described in Section III-A. In such a case, the cost Hamiltonian H_C will be of the form $\sum_{1 \leq i \leq N} h_i Z_i + \sum_{1 \leq i < j \leq N} J_{ij} Z_i Z_j$). Given the cost Hamiltonian, the cost operator is defined as $U_C(\gamma) = e^{-i\gamma(H_C)}$, as discussed in step 3 in the above QAOA workflow. Therefore, $U_C(\gamma) = e^{-i\gamma(\sum_{1 \leq i \leq N} h_i Z_i + \sum_{1 \leq i < j \leq N} J_{ij} Z_i Z_j))}$. Expanding this term, we have $e^{-i\gamma J_{ij}(Z_i Z_j)}$ and $e^{-i\gamma h_i Z_i}$ terms. In general, e$^{-i\gamma(Z_i Z_j)}$ and e$^{-i\gamma(Z_i)}$ are represented in a quantum circuit as follows:



Here, $|q_i\rangle$ and $|q_j\rangle$ represent the set of qubits on which the e$^{-i\gamma(Z_i Z_j)}$ is applied. Similarly, the mixer term e$^{-i\beta(X_i)}$ can be implemented as



The next step is in the estimation of variational parameters, γ_i and β_i using classical optimizers. Several gradient-based and gradient-free approaches (COBYLA, Nelder-Mead) and Bayesian optimization have commonly been used for parameter estimation. In this paper, we adopt a two-stage approach for optimizing the variational parameters.

1) We employ design of experiments(DOE) techniques such as Latin Hypercube Sampling (LHS) to explore the parameter space. At each of the parameter values, we calculated the optimum value of the expected cost function, using Eq. 8.
2) We use the best parameter value-combination from the first step, and use it as the initial solution for local optimizers such as COBYLA and Nelder-Mead.

Using the optimized values of the variational parameters, we can re-run the quantum circuit to obtain the optimum solution (values of the optimization decision variables). Fig. 2 shows the flow of analysis.

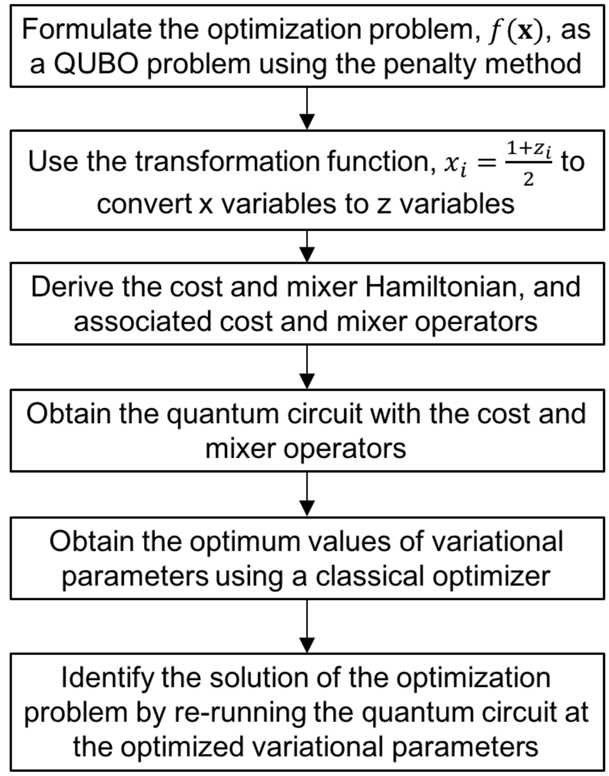


Fig. 2: QAOA Flow

IV. BSS - RBP Formulation

Given a BSS-RBP with K number of vehicles, x_{ij} is the vehicle traveling from source i to location j, q_j is the demand location at demand bike location j. We consider the following assumptions in our BSS-RBP formulations:

1) At the beginning of BSS-RBP analysis, we have a set of supply stations (have more bikes than estimated demand) and demand stations (less bikes than estimated demand).
2) The decision-making agent creates an initial route plan for various vehicles considering the estimated bike demand.

A. Problem Parameters:

Here, we define the parameters that will later be used in the optimization formulation.

x_{ij}: A binary variable, which is equal to 1 if demand bike station j is served from supply bike station i

x_{ik}: A binary variable, which is equal to 1 if vehicle carrier travels from one supply station i to another supply station k

x_{jh}: A binary variable, which is equal to 1 if vehicle travels from one demand station j to another demand station h

μ_i: Is the first supply bike station served from parking lot

η_i: Is the last demand bike station served before reaching the parking lot

D_{ij}: Distance between the supply (pick-up) and demand (drop-off) bike stations i and j

D_{ik}: Distance distance between one supply bike station i to another supply bike station k

D_{jh}: Distance distance between one demand bike station j to another demand bike station h

i, k: indices to represent supply bike stations

j, h: indices to represent demand bike stations

B. Objective function and Constraints:

Objective function: We consider minimization of distance traveled by the vehicle carrier across all the supply and demand bike stations as the objective function. Mathematically, it can be written as:

$$\begin{aligned}\text{Min} \quad & \sum_{\substack{i\in I\\ j\in J}} D_{ij}x_{ij} + \sum_{\substack{i\neq k\\ i,k\in I}} D_{ik}x_{ik} \\ & + \sum_{\substack{j\neq h\\ j,h\in J}} D_{jh}x_{jh} + \sum_{i\in I} D_i\mu_i + \sum_{j\in J} D_j\eta_j\end{aligned} \tag{10}$$

Constraint 1: From the supply bike station the number of bikes picked up must be equal to the number of bikes dropped at the demand bike stations.

$$\sum_{j\in J} x_{ji} + \sum_{\substack{k\neq i\\ i,k\in I}} x_{ki} = \sum_{j\in J} x_{jh} + \sum_{\substack{i\neq k\\ i,k\in I}} x_{ik} \tag{11}$$

Constraint 2: From the demand bike station the number of bikes dropped must be equal to the number of bikes picked at the supply bike stations.

$$\sum_{i\in I} x_{ij} + \sum_{\substack{j\neq h\\ j,h\in J}} x_{hj} = \sum_{i\in I} x_{ji} + \sum_{\substack{j\neq h\\ j,h\in J}} x_{jh} \tag{12}$$

Constraint 3: Each trip should start and end at the parking lot. The variable $\mu_i = 1$ indicates that the first bike station served by vehicle carrier after starting from the parking lot. Similarly, $\eta_j = 1$ indicates that the last demand bike station served by vehicle carrier before returning to the parking lot.

$$\sum_{i\in I} \mu_i = 1 \tag{13}$$

Constraint 4: Each vehicle should end at its parking lot from only a single demand bike station location.

$$\sum_{i\in I} \eta_j = 1 \tag{14}$$

Constraint 5: Each vehicle carrier should visit every single demand bike station only once.

$$\sum_{i\neq j} x_{ij} + \sum_{j\neq h} x_{jh} = 1 \tag{15}$$

C. QUBO Formulation of BSS-RBP:

The Hamiltonian term, H_O that corresponds to the objective function is simply the objective function in Eq. 4.

$$\begin{aligned}H_O = \text{Min} & \sum_{\substack{i\in I\\ j\in J}} D_{ij}x_{ij} + \sum_{\substack{i\neq k\\ i,k\in I}} D_{ik}x_{ik} \\ & + \sum_{\substack{j\neq h\\ j,h\in J}} D_{jh}x_{jh} + \sum_{i\in I} D_i\mu_i + \sum_{j\in J} D_j\eta_j\end{aligned} \tag{16}$$

Minimization of distance covered by the vehicles from all the supply and demand bike stations and the Hamiltonian equation for BSS-RBP is given by:

$$\begin{aligned}Overall_H = H_O + H_{C_1} + H_{C_2} + H_{C_3} + \\ H_{C_4} + H_{C_5}\end{aligned} \tag{17}$$

The Hamiltonian terms that correspond to various constraints in Section IV are given below. Let $C_i, i = 1 \dots 5$ represent the five constraints, and let $H_{C_i}, i = 1 \dots 5$ represent the Hamiltonian terms corresponding to the constraints.

$$\begin{aligned}H_{C_1} = B\Bigg(\Bigg(\sum_{j\in J} x_{jh} + \sum_{\substack{i\neq k\\ i,k\in I}} x_{ik}\Bigg) - \\ \Bigg(\sum_{j\in J} x_{ji} + \sum_{\substack{k\neq i\\ i,k\in I}} x_{ki}\Bigg)\Bigg)^2\end{aligned} \tag{18}$$

$$\begin{aligned}H_{C_2} = B\Bigg(\Bigg(\sum_{i\in I} x_{ji} + \sum_{\substack{j\neq h\\ j,h\in J}} x_{jh}\Bigg) - \\ \Bigg(\sum_{i\in I} x_{ij} + \sum_{\substack{h\neq j\\ j,h\in J}} x_{hj}\Bigg)\Bigg)^2\end{aligned} \tag{19}$$

$$H_{C_3} = B\Bigg(1 - \Bigg(\sum_{i\in I} \mu_i\Bigg)\Bigg)^2 \tag{20}$$

$$H_{C_4} = B\Bigg(1 - \Bigg(\sum_{i\in I} \eta_j)\Bigg)\Bigg)^2 \tag{21}$$

$$H_{C_5} = B\Bigg(1 - \Bigg(\sum_{i\neq j} x_{ij} + \sum_{j\neq h} x_{jh}\Bigg)\Bigg)^2 \tag{22}$$

In Eqs. 18-22, B is a large positive constant that corresponds to the penalty incurred when the constraints are violated. After obtaining Hamiltonian terms that correspond to the objective function and various constraints, the overall Hamiltonian $Overall_H$ is equal to the sum of the individual Hamiltonian terms.

V. Results & Discussion

A. BSS-RBP Network

Case-study description: For the case-study of BSS-RBP in the Fig. 3, a small scale (up to 3 bike stations) has been considered for solving the optimization problem to determine optimal route covering the target station with minimal distance using QAOA. The case study model consist of two supply nodes : $[S_1, S_2]$ and one demand node $[D_3]$ respectively. The bike stations with excess number of bikes in supply node is denoted, E_i and the bike stations which needs to be rebalanced is denoted as target, T_j. The excess bikes available in supply node is given as $[E_1 = 3, E_2 = 2]$ and target nodes are deficient of $[T_3 = -5]$ bikes are needed for satisfying the demand. All the routes starts and end at the parking lot of the bikes. Hence, we have considered a distance matrix that covers all the supply node and demand nodes from the parking lot.

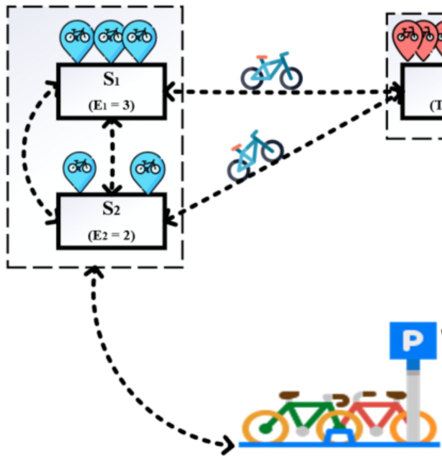


Fig. 3: Schematic network flow of BSS-RBP

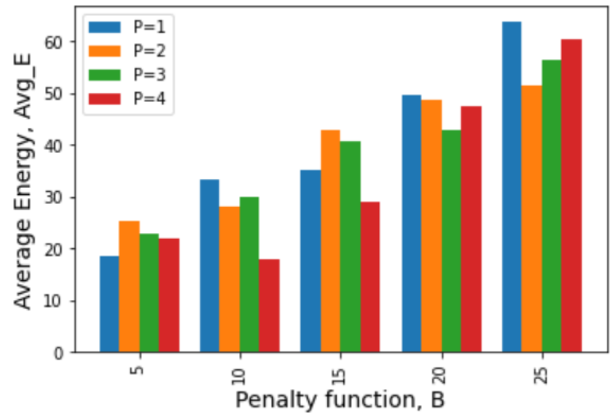


Fig. 4: Average energy output for all p & B

The distances are measured in miles and the distance matrix is given by:

$$D = \begin{bmatrix} 0 & 3 & 3 & 4 \\ 3 & 0 & 2 & 4 \\ 3 & 2 & 0 & 2 \\ 4 & 4 & 2 & 0 \end{bmatrix}$$

Let μ_1 and μ_2 represent the random variables from parking lot to each of the two supply stations. x_{12} and x_{21} are two random variables that represent trips between the two supply stations. Finally, we have x_{13} and x_{23}, which represent trips between each of the two supply stations and the demand station. In total, we have six random variables. Therefore, we use six qubits to represent each of the six decision variables.

B. Empirical analysis

In this section, we analyze the case study model for various p levels :{1,2,3,4}, Penalty function, B-values. Since results from quantum circuits are probabilistic in nature, we run each quantum circuit (for a given p and B) with 4096 shots.

For a given value of p, we have $2p$ number of parameters. To reduce the computational complexity of the classical optimization analysis over $2p$ variables, we assumed that all the $\gamma_i = \gamma$ values are equal and all the $\beta_i = \beta$ values are equal, resulting in two parameters. The ranges of γ and β are between 0 and 2π. We generated 50 Latin Hypercube samples and identified the best solution of γ and β. This best solution is used as the initial solution for the second-stage optimization analysis using the COBYLA optimizer.

Fig. 4 shows the average cost function (sometimes referred to as average energy) values across various values of p and for various values of B, the penalty term. At each value of p and B, we calculated the best solution, and compared this solution to the true solution. Since this is a low-dimensional problem, an analytical solution can be obtained by analyzing all possible combinations of the decision variables. We haven't obtained the best solution for any value of B at p=1 but we obtained the correct solution at p=2 and B=25. The results of γ and β are given in Table I.

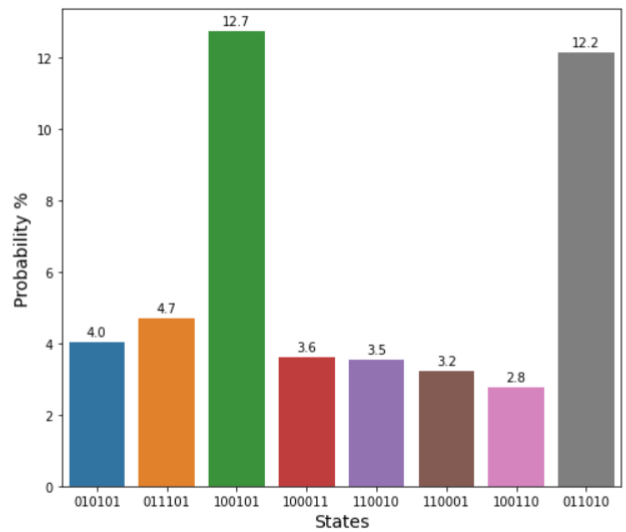


Fig. 5: Histogram of state probabilities for p = 2, B = 25

For p=2 and B=25, Fig. 5 provides the probabilities of the top 8 combinations. We see that $|100101\rangle$ and $|011010\rangle$ have the highest probabilities as these two solutions are the two valid solutions for this problem.

This preliminary work demonstrates the applicability of QAOA for bike sharing system - rebalancing problem.

TABLE I: Best values of γ & β

P-level	B-value	γ	β
2	25	3.12439307	1.85312208

VI. Conclusion Future Work

This paper discussed the quantum approximate optimization algorithm using quadratic unconstrained binary optimization (QUBO) formulations of BSS-RBP that can be solved on the quantum simulator such as IBM-qiskit. We have reduced the total decision variable to six to reduce the problem complexity in terms of the number of total decision variables. We have also provided a step-by-step solution framework to solve the associated QAOA formulations on the quantum computing hardware. The methods that we discussed in this paper are applicable to combinatorial optimization problems in other domains such as resource planning in inventory, warehouses, and micro-grids.

As future work, we will implement the proposed approach at various bike stations in New York City using the New York bike dataset, and we will also investigate scalability of QAOA for multiple demand and supply stations. We will also compare the overall computational performance of classical algorithms and also the proposed mixed classical-quantum approach.

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