SKKU 2024 Challenge: Dicke State Preparation

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Introduction

An n qubit Dicke state $|D_{nk}\rangle$ is the equal superposition of all n qubit states with Hamming weight = k. The Hamming weight of a quantum state is simply the number of ones in the state. For example, the $|D_{32}\rangle$ state is shown below:

$$|D_{32}\rangle = (|110\rangle + |101\rangle + |011\rangle)/\sqrt{3}.$$

Dicke states can be used in Topological data analysis (TDA) and optimization problems with constraints. This exercise introduces some of the steps needed to prepare such states.

Exercises

1. Quantum phase estimation (QPE) can be used to measure the Hamming weight of a quantum state and prepare Dicke states. [Easy]
    a) Write down the Hamiltonian whose eigenvalues can distinguish the Hamming weight of an n qubit quantum state.
    b) For an n qubit state, what is the minimum number of bits needed to measure the hamming weight of the state? Hint: The Hamming weight of the state is in the range {0,1,..,n}.

2. Using qiskit, create a function that takes in a quantum state in vector format as input and returns a quantum circuit that measures the Hamming weight of the state. Hint: see ref [1]. You can use qiskit's **initialize** module to initialize the states. [Easy – ☆]

3. Use the function created in part 2 above to measure the Hamming weight of the following states: [Medium]
    a.) $|\Psi\rangle = |11\rangle$.
    b.) $|\Psi\rangle = |01\rangle$.
    c.) $|\Psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$.
    d.) $|\Psi\rangle = (|011\rangle + |101\rangle + |110\rangle)/\sqrt{3}$.
    e.) Explain the result in part c) above.

4. Explain how the function in part 2 above can be used to prepare a Dicke state. [Medium - ☆]

5. Suppose we want to prepare the Dicke state with n = 100 and k = 50. [Hard – ☆]
    a.) What is the probability that the state will be measured?
    b.) What is the minimum number of measurements required?
    c.) How many gates are required?

References

1. Rethinasamy, S., LaBorde, M.L., Wilde, M.M.: Logarithmic-depth quantum circuits for hamming weight projections. arXiv preprint arXiv:2404.07151 (2024)