

Image Filtering

CPS592 – Visual Computing and Mixed Reality

Filter Operations



The name “filter” is used because these signal-processing elements typically “pass” or amplify certain frequency components of the signal, while they “stop” or attenuate others.

Image Filtering

- Modify the pixels in an image based on some function of a local neighborhood of each pixel

10	5	3
4	5	1
1	1	7

Local image data

Some function

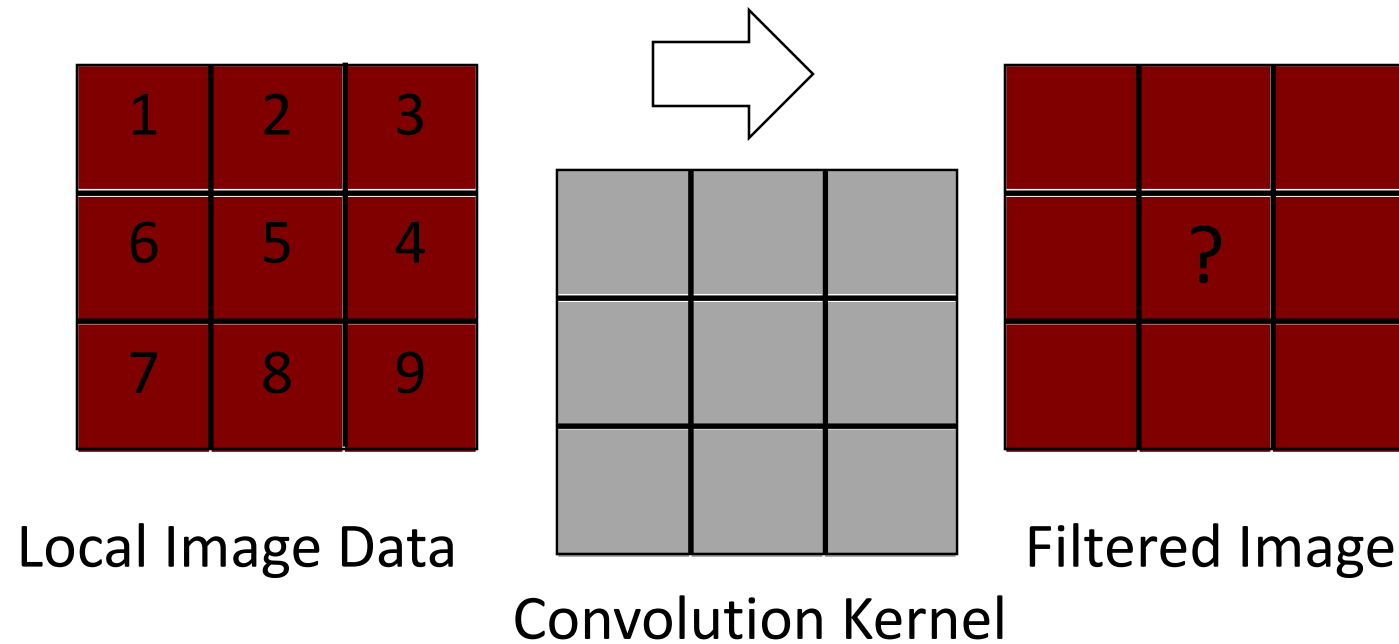


	?	

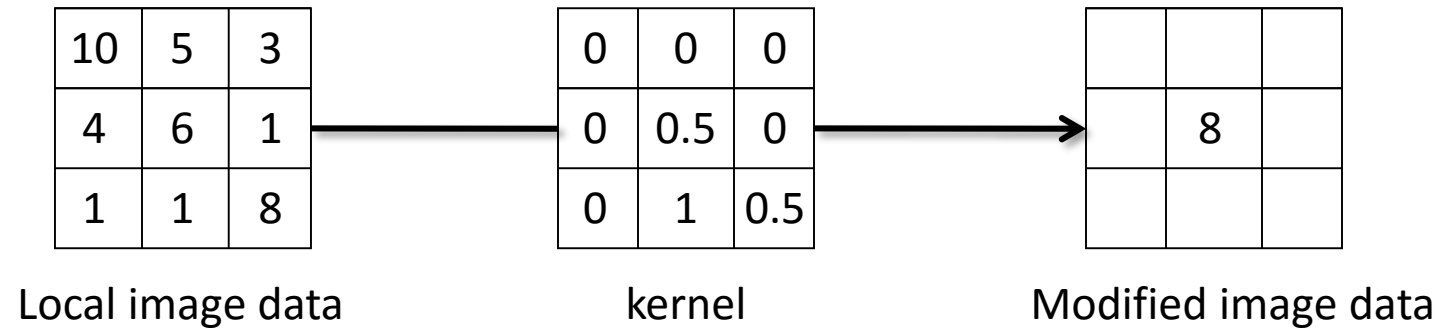
Modified image data

Convolution

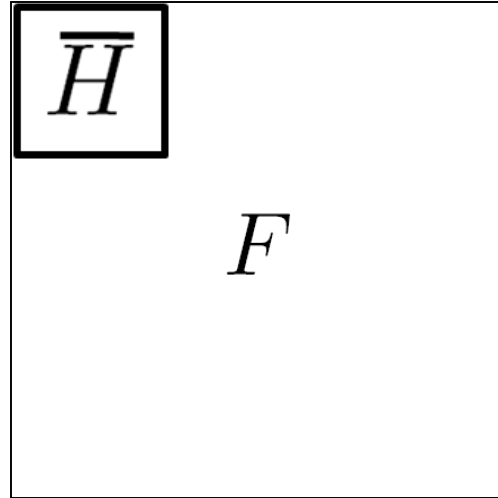
- The prescription for the linear combination is called the “convolution kernel”.



Example



Example



Mean filtering

$$\begin{array}{c}
 \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array} \\
 H
 \end{array}
 *
 \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ \hline 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ \hline 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ \hline 0 & 0 & 0 & 90 & 0 & 90 & 90 & 90 & 0 & 0 \\ \hline 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 90 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}
 =
 \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline & & & & & & & & & \\ \hline & 0 & 10 & 20 & 30 & 30 & 30 & 20 & 10 & \\ \hline & 0 & 20 & 40 & 60 & 60 & 60 & 40 & 20 & \\ \hline & 0 & 30 & 60 & 90 & 90 & 90 & 60 & 30 & \\ \hline & 0 & 30 & 50 & 80 & 80 & 90 & 60 & 30 & \\ \hline & 0 & 30 & 50 & 80 & 80 & 90 & 60 & 30 & \\ \hline & 0 & 20 & 30 & 50 & 50 & 60 & 40 & 20 & \\ \hline & 10 & 20 & 30 & 30 & 30 & 30 & 20 & 10 & \\ \hline & 10 & 10 & 10 & 0 & 0 & 0 & 0 & 0 & \\ \hline & & & & & & & & & \\ \hline \end{array}$$

F G

$$\frac{1}{9}
 \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}
 *
 \begin{array}{|c|} \hline \text{Original} \\ \hline \end{array}
 =
 \begin{array}{|c|} \hline \text{Blur (with a mean filter)} \\ \hline \end{array}$$

Linear filters: examples



Original



0	0	0
0	1	0
0	0	0



Identical image

Linear filters: examples



Original





0	0	0
1	0	0
0	0	0



Shifted left
By 1 pixel

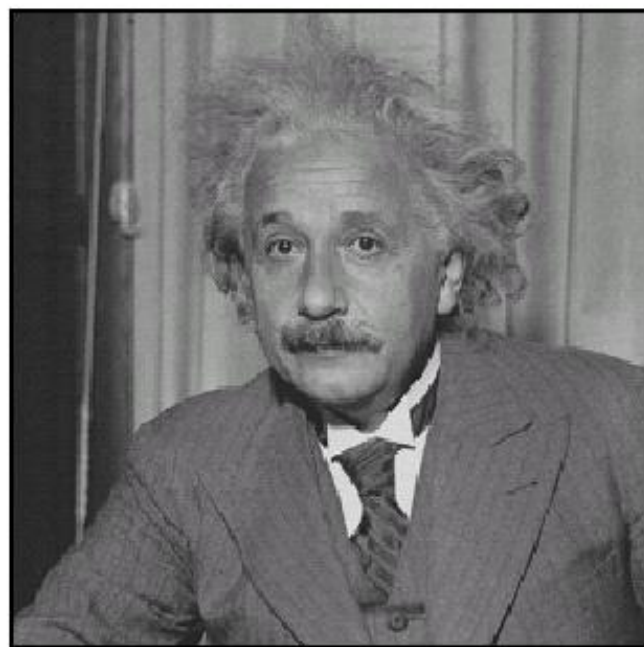
Linear filters: examples


$$* \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) =$$


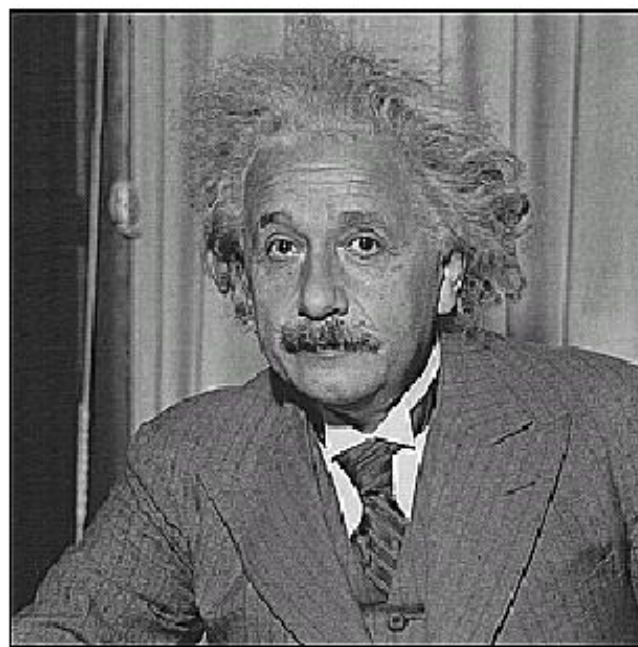
Original

Sharpening filter
(accentuates edges)

Sharpening



before



after

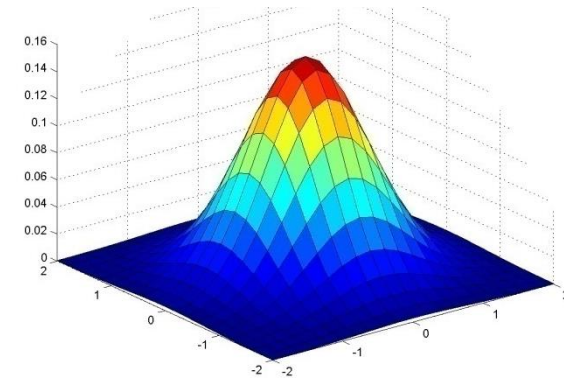
Gaussian filters

- A Gaussian kernel gives less weight to pixels further from the center of the window

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

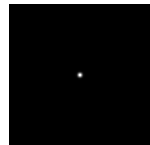
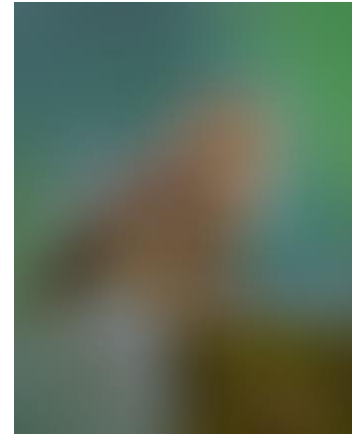
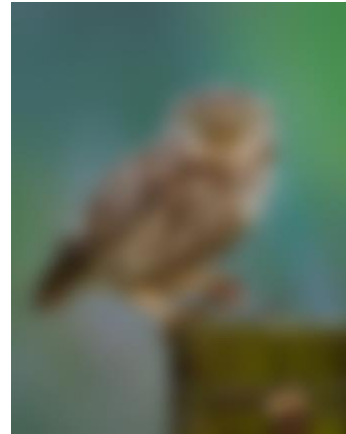
$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



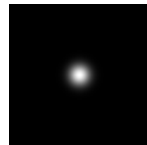
1	2	1
2	4	2
1	2	1

$H[u, v]$

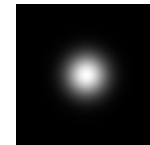
Gaussian filters



$\sigma = 1$ pixel



$\sigma = 5$ pixels



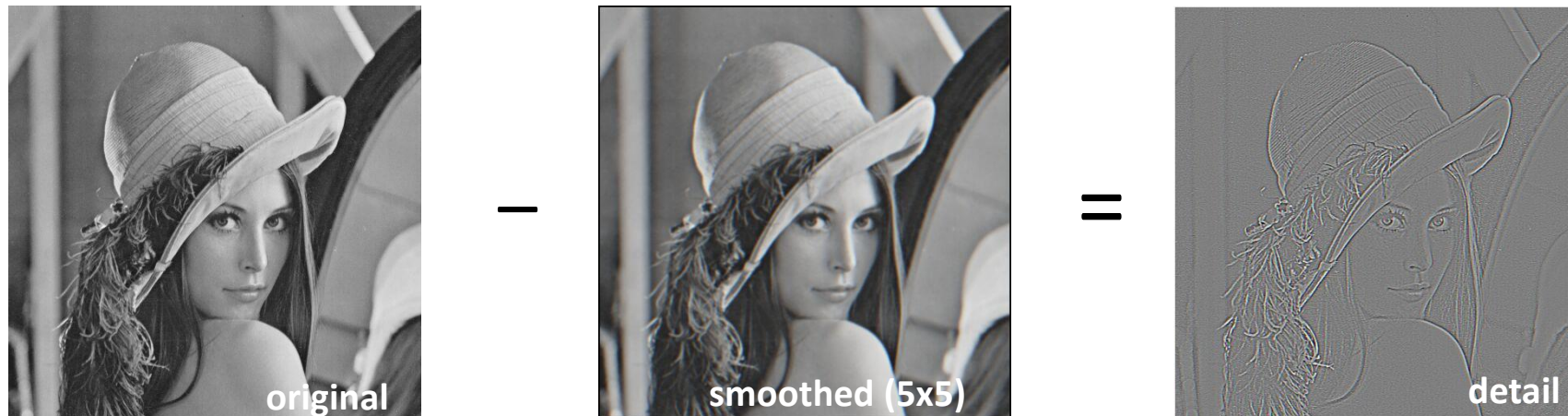
$\sigma = 10$ pixels



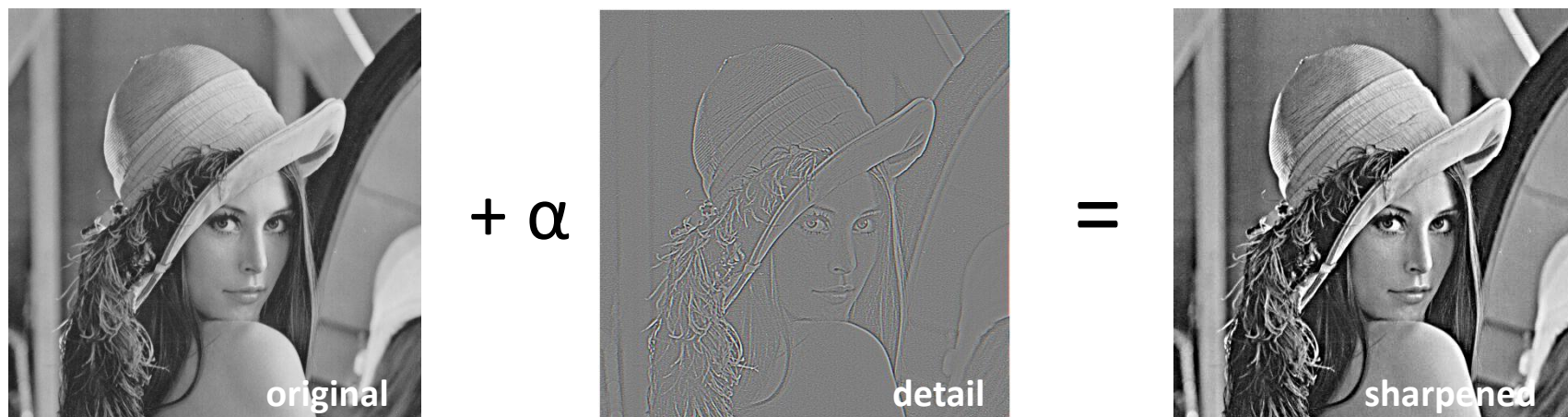
$\sigma = 30$ pixels

Sharpening revisited

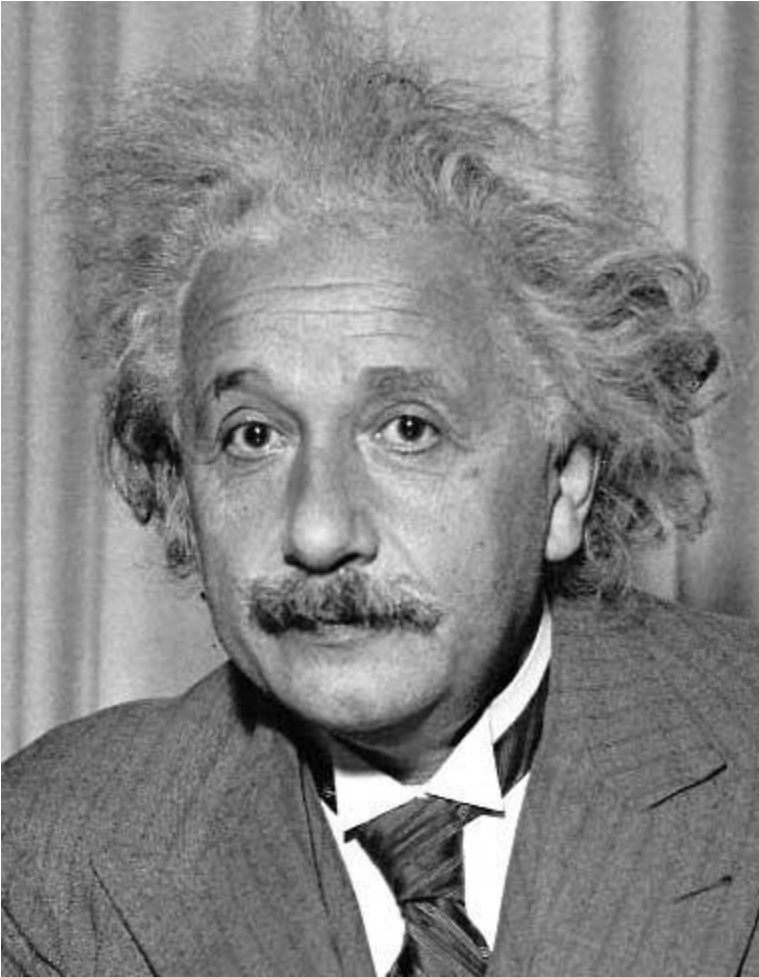
- What does blurring take away?



Let's add it back:



Other filters



1	0	-1
2	0	-2
1	0	-1

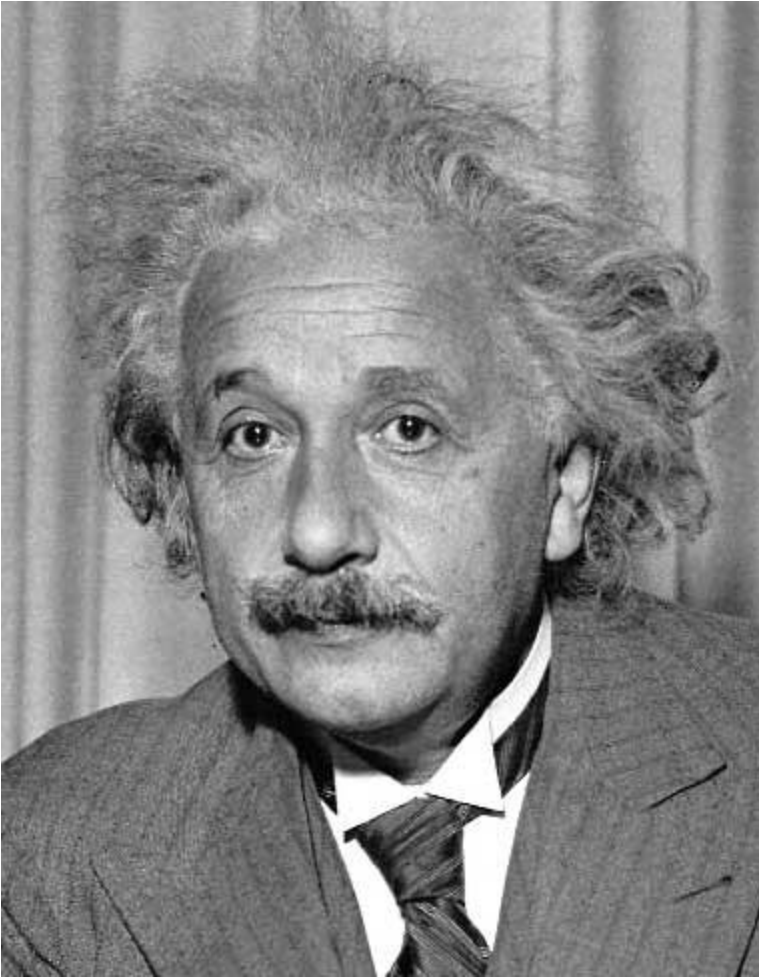
Sobel



Vertical Edge
(absolute value)

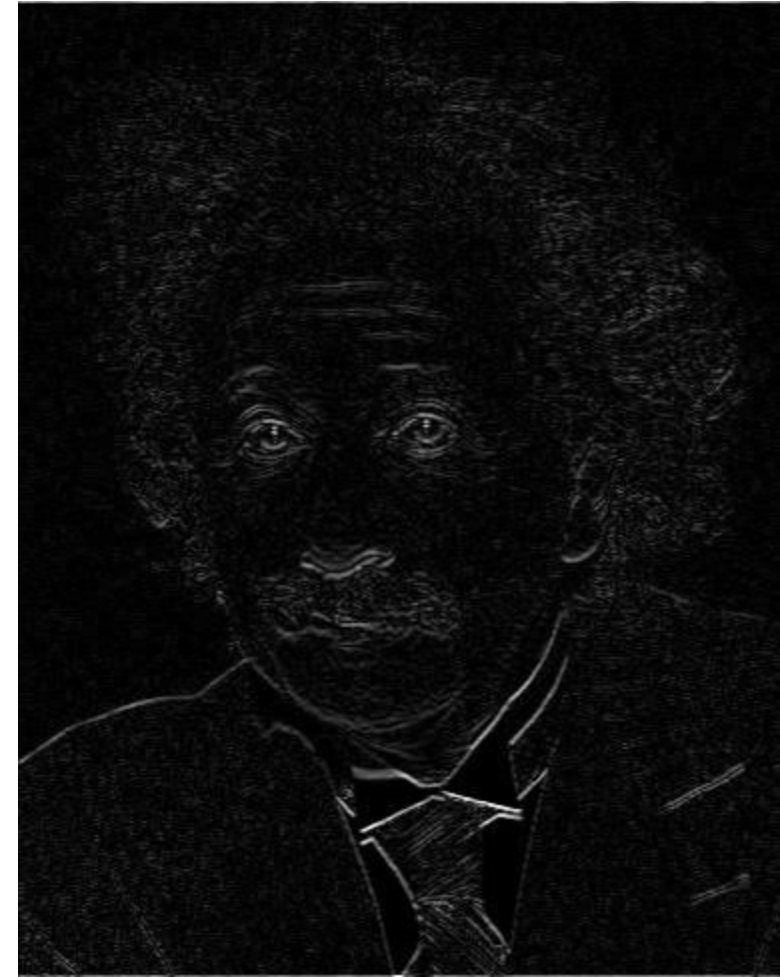
Source: D. Lowe

Other filters



1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

Source: D. Lowe

Linear vs. Non-linear

- “Convolution”/Linear Filters
 - Linear operation
 - Have corresponding frequency domain filter
- Non-linear Filters
 - Mask used to determine the proper substitution of a “good” pixel value
 - Examine neighbors using various orderings
 - Often use Rank or Order Statistics
 - Harder to interpret effect in frequency domain

Ordered Statistic Filters

- Also called “rank” filters

2	3	8
3	4	10
4	2	9

Consider a neighborhood about a pixel.
Rank (sort) the pixels.
{2, 2, 3, 3, 4, 4, 8, 9, 10}

Rank Filters: Median Filter

- One of the most popular non-linear filter
- Find the median of the window
- Preserves edges
- Removes impulse noise, avoids excessive smoothing

2	3	8
3	4	10
4	2	9

pixel values about (x,y) window 3x3

neighbor sort = {2,2,3,3,4,4,8,9,10}

$f(x,y) = \text{median}$

Rank Filters: Min/Max Filter

- Find the min or max of the neighborhood
- Not as “mainstream” as median filter
- Has various uses, will talk about these more later.

2	3	8
3	4	10
4	2	9

pixel values about (x,y) window 3x3

neighbor sort = {2,2,3,3,4,4,8,9,10}

$f(x,y) = \min$

$f(x,y) = \max$

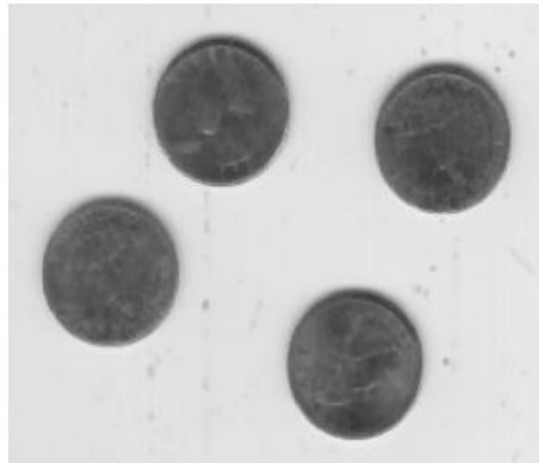
Examples



Original Image



Median



Min



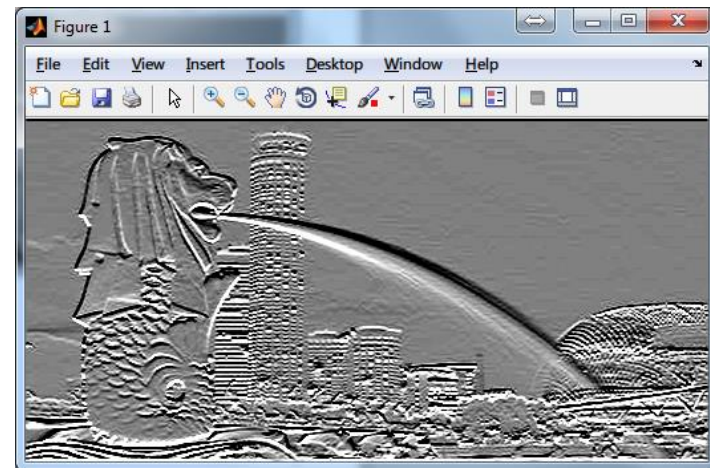
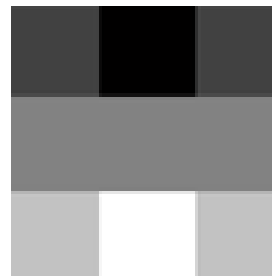
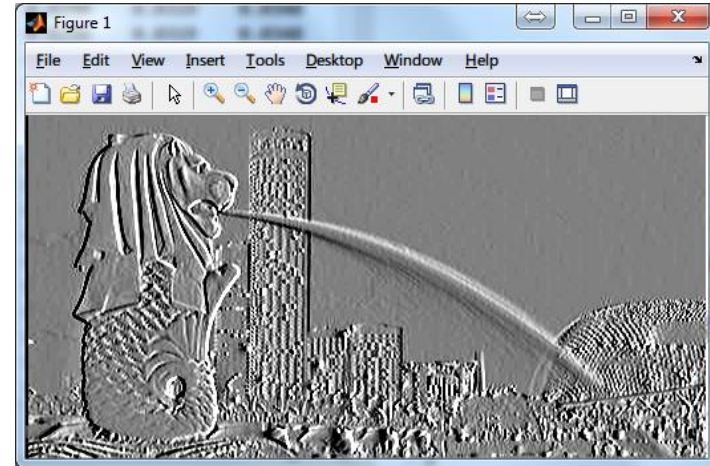
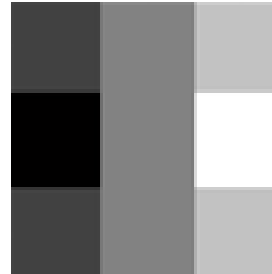
Max

Sobel Filter

Kernel = [-1 0 1;
 -2 0 2;
 -1 0 1];

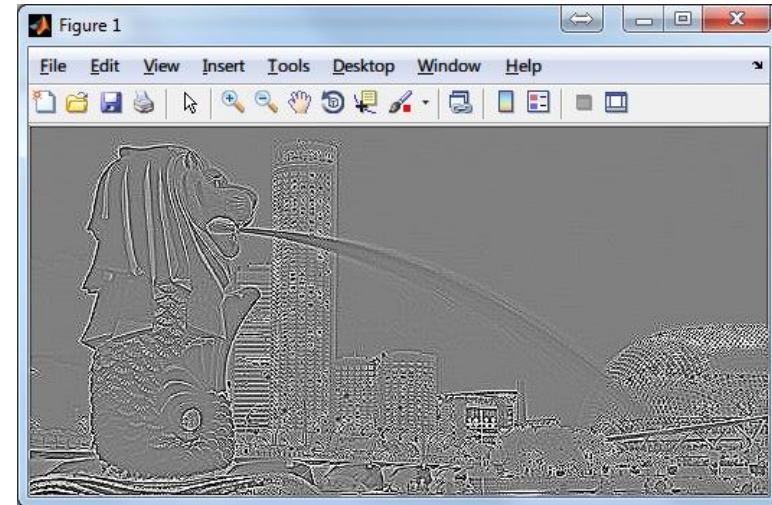
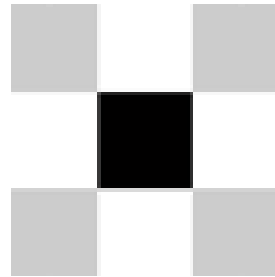
J1 = imfilter(I, kernel, 'replicate');

J2 = imfilter(I, kernel', 'replicate');



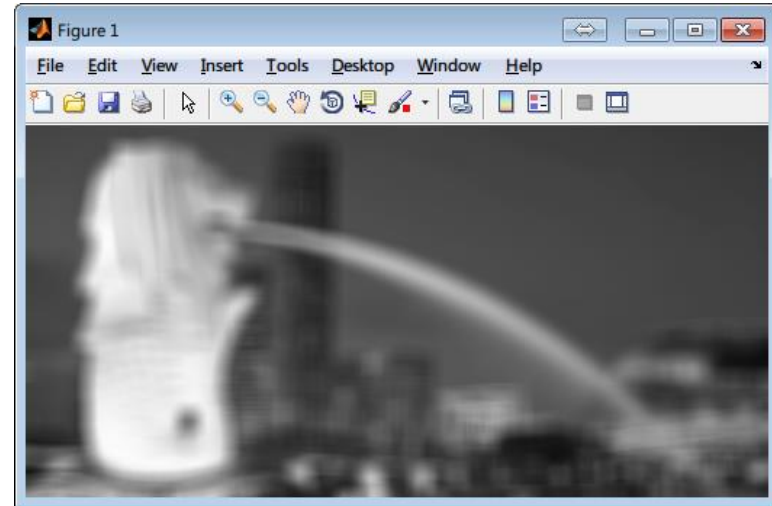
Laplacian Filter

Kernel = $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$;



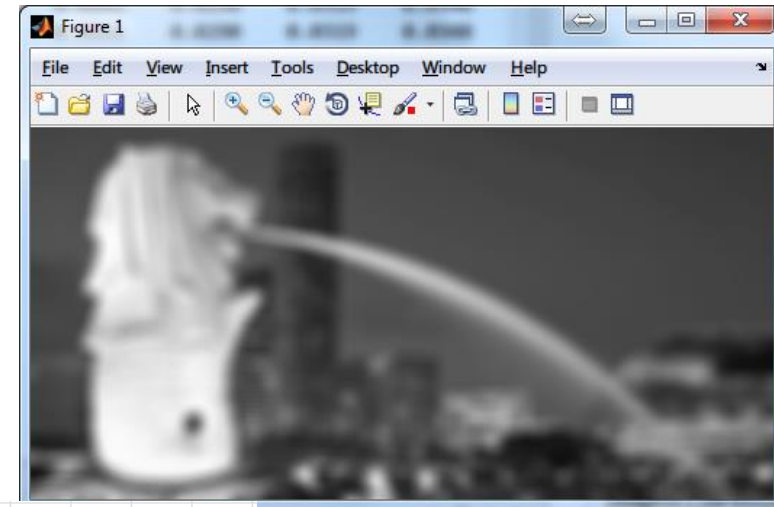
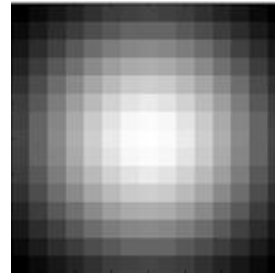
Mean Filter

Kernel = ones(15,15) / 15^2;



Gaussian Filter

Kernel = fspecial('gaussian', [15 15], 5);

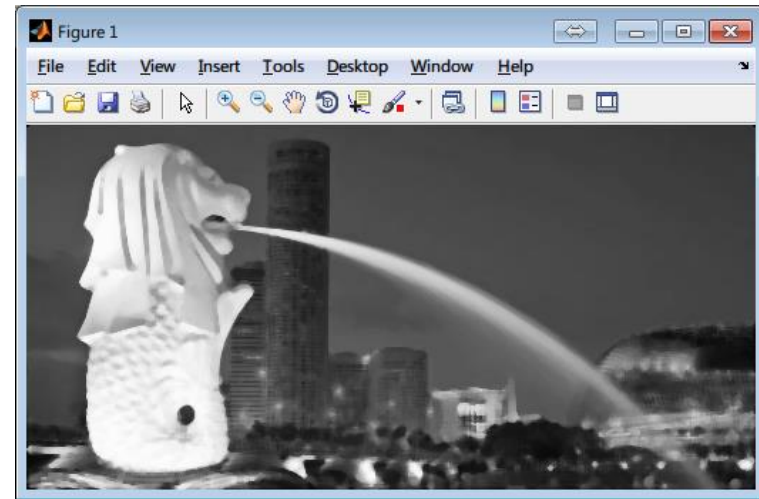


0.0012	0.0015	0.0019	0.0023	0.0027	0.0029	0.0031	0.0032	0.0031	0.0029	0.0027	0.0023	0.0019	0.0015	0.0012
0.0015	0.0020	0.0025	0.0030	0.0034	0.0038	0.0040	0.0041	0.0040	0.0038	0.0034	0.0030	0.0025	0.0020	0.0015
0.0019	0.0025	0.0031	0.0037	0.0043	0.0047	0.0050	0.0051	0.0050	0.0047	0.0043	0.0037	0.0031	0.0025	0.0019
0.0023	0.0030	0.0037	0.0045	0.0051	0.0057	0.0060	0.0061	0.0060	0.0057	0.0051	0.0045	0.0037	0.0030	0.0023
0.0027	0.0034	0.0043	0.0051	0.0059	0.0065	0.0069	0.0071	0.0069	0.0065	0.0059	0.0051	0.0043	0.0034	0.0027
0.0029	0.0038	0.0047	0.0057	0.0065	0.0072	0.0077	0.0078	0.0077	0.0072	0.0065	0.0057	0.0047	0.0038	0.0029
0.0031	0.0040	0.0050	0.0060	0.0069	0.0077	0.0081	0.0083	0.0081	0.0077	0.0069	0.0060	0.0050	0.0040	0.0031
0.0032	0.0041	0.0051	0.0061	0.0071	0.0078	0.0083	0.0085	0.0083	0.0078	0.0071	0.0061	0.0051	0.0041	0.0032
0.0031	0.0040	0.0050	0.0060	0.0069	0.0077	0.0081	0.0083	0.0081	0.0077	0.0069	0.0060	0.0050	0.0040	0.0031
0.0029	0.0038	0.0047	0.0057	0.0065	0.0072	0.0077	0.0078	0.0077	0.0072	0.0065	0.0057	0.0047	0.0038	0.0029
0.0027	0.0034	0.0043	0.0051	0.0059	0.0065	0.0069	0.0071	0.0069	0.0065	0.0059	0.0051	0.0043	0.0034	0.0027
0.0023	0.0030	0.0037	0.0045	0.0051	0.0057	0.0060	0.0061	0.0060	0.0057	0.0051	0.0045	0.0037	0.0030	0.0023
0.0019	0.0025	0.0031	0.0037	0.0043	0.0047	0.0050	0.0051	0.0050	0.0047	0.0043	0.0037	0.0031	0.0025	0.0019
0.0015	0.0020	0.0025	0.0030	0.0034	0.0038	0.0040	0.0041	0.0040	0.0038	0.0034	0.0030	0.0025	0.0020	0.0015
0.0012	0.0015	0.0019	0.0023	0.0027	0.0029	0.0031	0.0032	0.0031	0.0029	0.0027	0.0023	0.0019	0.0015	0.0012

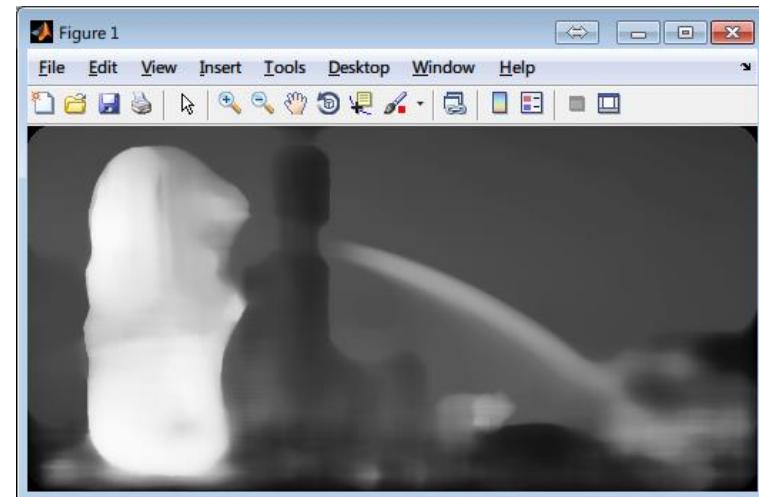
Median Filter



5x5



25x25



Sharpening



$\times 2 -$



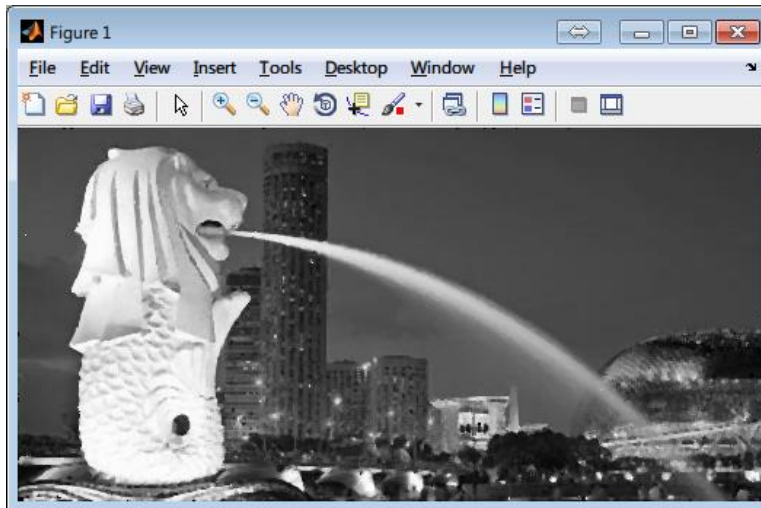
$=$



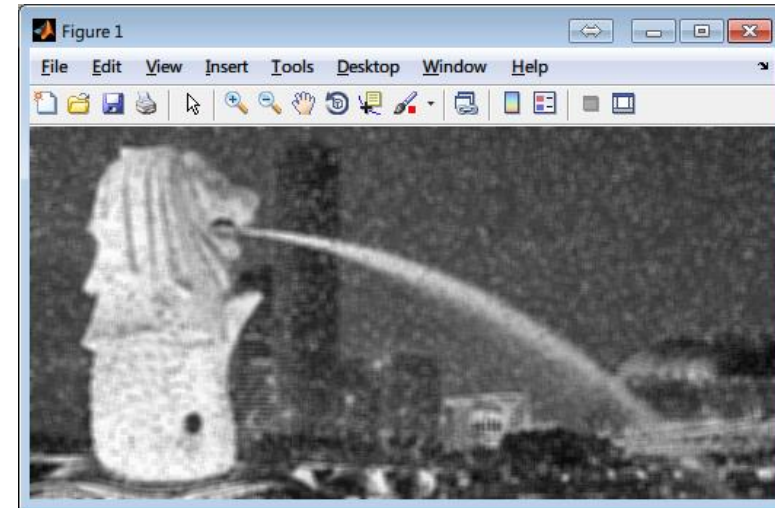
Denoising



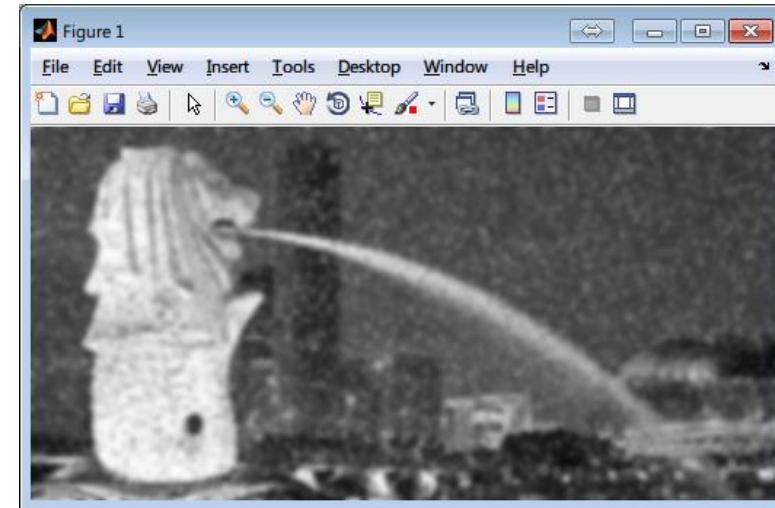
Median



Mean



Gaussian



Q&A