The Use of Optimization Algorithms for Solving Knapsack Problems

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***Abstract*—The main objective of this study is to review the various optimization algorithms used for solving the knapsack problem over the span of four decades. This study introduces the key aspects of the knapsack problem and its different formulations in the domain of dynamic programming. A critical review of four different peer reviewed research papers is presented discussing the approaches used by them and their merits and future directions. The first paper published in the year 1984 introduces the linear multiple choice knapsack problem having polynomial time complexity. The second paper published in the year 1998 discusses the application of genetic algorithms in the multidimensional knapsack problem. the third study published in the year 2008 introduces the ACO(ant colony optimization) algorithm for the same multidimensional knapsack problem. The final paper published in the year 2020 discusses a novel "swarm optimization search algorithm" for the multidimensional knapsack problem. For each of the above papers, a comprehensive review of their findings is presented along with any possible directions for the future work in the domain.**

I. Introduction

Dynamic Programming is an approach used for solving optimization problems. In this technique, a complex problem is broken down into simpler sub-problems, considering that the optimal solution of the general problem is dependent on the optimal solution of the sub-problems. One of the best characteristics of the technique is the multistage nature of the optimization procedure. In this approach, the solution to each sub-problem is solved only once and the results are stored, so that the system never gets to re-compute the same problem again. This method of practice, of the technique, guarantees correctness and efficiency which is rarely found in any other algorithms. Overall, the technique provides a general framework where several other optimization techniques can be employed to solve all the particular aspects of a generalized formulation. In this study, we discuss one of the most intensively studied problems in dynamic programming - the Knapsack Problem.

The Knapsack problem, one of the most famous dynamic programming problems can be understood correctly by the following scenario. Imagine a thief robbing a store, full of tantalizing jewellery, diamonds, and rare gems. But, the problem arises that he is new to this and has brought a single bag pack only. The ultimate goal at the end is to fill up the bag pack to the fullest with the most valuable and expensive items and objects, in such a way that the bag doesn't become too heavy and difficult to carry. This is where he needs to choose and decide the objects that maximize the loot. To be more precise, he needs to enlist all the items and their weights, considering that more is the number of objects more is the taxing for calculation. This fictional dilemma is termed as the *"Knapsack Problem"* which is stated as follows *"Given a set of items, each with a weight and a value, determine the number of each item include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible."* The problem has several real-time applications, many researchers have also taken advantage of the problem's complexity to create secure computer systems. There are several knapsack problems, one of the most complex is the *Multidimensional Knapsack Problem*, in which multiple resource constraints are considered.

Genetic Algorithm is one of the best evolutionary algorithms, in which the complex and large spaced problems are solved using biological processes such as selection, crossover, mutation and natural selection, etc. In this natural selection process, the most fitted individuals are selected for the process of reproduction for producing the off springs of the next generation. They encode their initial values or variables as integers or bit strings, depending on the type of variables i.e whether they are continuous or discrete. These variables play the role of chromosomes in all of the processes.

Ant Colony Optimization is a population based optimization meta heuristic technique to find optimal paths.It is an extension of Max Min Ant System (NMAS) in which we apply lower and upper trait limits to avoid any stagnation. This process was inspired by the foraging behaviour of ants. When applied to solve any problem, the artificial ants are built that construct the solution to the problem using heuristic information and pheromone trails. The algorithm has proved success in many other problems, when applied.

Swarm optimization is a class of optimization algorithms inspired from the movement patterns of flocks of birds and fishes. The swarm optimization algorithms share many similarities with the evolutionary algorithms in a way that both of them approach an optimization problem by iteratively improving the candidate solutions based on a given criteria. in the evolutionary algorithms, it is the fitness of the solution or solution set and in swarm optimization it is the stability analysis of the candidate solutions. It solves the given optimization problem by moving a swarm of particles (candidate solutions) in a search space by defining their positions and velocity. The movement of a particle is affected by best known local position and is guided towards the best position in the search space by other particles position updates.

In this study, we are going to review how these optimization algorithms can be applied to the Multidimensional Knapsack Problem. The state objective of the paper is to review the Modelling, Simulation, and Optimization studies from the 1980s, 1990's, 2000's and 2010's and review how they are correlated with each other.

II. Related Works

A. Paper-1: Introducing a multiple choice knapsack problem

"An O(n) algorithm for the linear multiple choice knapsack problem and related problems" [1]

In this introductory paper, the author introduces the linear knapsack problem with multiple choice and proposes an optimization algorithm with a time complexity of O(n) for solving it. The authors show that a linear problem having a fixed number of variables can be solved in linear time. The author also proposed a method for generalizing the algorithm for n-dimensions. He also considered a convex programming problem for the generalization of the proposed algorithm.

The author begins by introducing the linear one-dimensional knapsack problem with suitable constraints. He then introduces the multiple-choice knapsack problem as a special case for the 1-D knapsack problem with all coefficients equal to one. The algorithm for solving the multiple-choice knapsack problem is also presented briefly with general constraints. The author then explains the complexity constraints on the multiple-choice knapsack problems(MCKP). The author built on the previous work of Megiddo who showed how a linear programming problem containing a fixed no. of variables can be solved in linear time in O(n) steps.

The proposed algorithm is generalized for the MCKP having n constraints in n dimensions. While explaining the proposed algorithm, the author showed how the O(n) time complexity is achieved by eliminating certain variables(roughly 25%) in each iteration of the algorithm.

B. Paper-2: Evolutionary algorithms approach for solving Knapsack problem

"A Genetic Algorithm for the Multidimensional Knapsack Problem" [2]

This paper comes with the idea of a heuristic-based genetic algorithm (GA'S) for a well-known Knapsack Problem. The problem considered for the study is *Multidimensional knapsack (MKP)*. Multidimensional knapsack problems are the knapsack problem with multiple source constraints, also called NP-hard. The multidimensional knapsack problem is formulated using multiple constraints, called as *knapsack constraint* to deal with all kinds of formations in the Multidimensional Knapsack (MKP). The authors have mentioned and referred to the previous studies that incorporate other practical uses of the problem in real-time, such as capital budgeting, allocation of resources like processors and databases in the distributed computer systems, cutting the stock problems, cargo loading, and project selection, etc.

The genetic algorithms work on the exercise where the biological processes of mutation, crossover, etc. are emulated in producing the solutions to all the big and large spaced problems. For this study, the components used for the genetic algorithm are comparable to the ones that are used in standard Generalised algorithms. The authors have incorporated a heuristic operator into the standard and generalized genetic algorithm approach. The heuristic operator included, utilizes the knowledge specific to the problem, which is directly included in the generalized algorithm, which differs our approach from all the previous techniques as this technique guarantees that the child solutions are feasible in the future.

The results prove that this is one of the most desirable approaches that can be applied to multi-dimensional knapsack problems for tacking constraints and feasibility issues. When given some modest amount of computational efforts on a large set of problems, high-quality results can be obtained for various characteristics from the genetic algorithm heuristic. Also, high-quality solutions could be provided to several heuristics, using the genetic algorithm heuristic.

C. Paper-3: ACO algorithm for tackling multi-dimensional knapsack problem

"An ant colony optimization approach for the multidimensional knapsack problem" [3]

This study proposes to introduce the *Ant colony optimization (ACO)* approach for solving the *Multidimensional Knapsack problem (MKP)*, called DMMAS. The algorithm extends the Max-Min Ant System (NMAS) in the procedure that is applied, which is termed as one of the best ACO algorithms.

For this optimization technique, they have proposed a novel method to choose the lower trail limit, which takes into consideration the influence of the heuristic information. The proposed algorithm, DMMAS, follows the standard procedure of the Ant Colony optimizations, along with several characteristics that are dependent on the components of MKP. DMMAS differs from the other ACO algorithms, which is applied to any other scenarios, because of the MKP characteristics added in the algorithm. DMMAS has shown commendable results when compared with the other ACO-based algorithms. The final statistics are showing that DMMAS is superior to the other algorithms.

Further, they have proposed a local search procedure to improve the solution constructed by ants. This is a combination of DMMAS and a local search procedure. They showed that when the hybrid algorithm is applied to the benchmark problems and compared with the other two hybrid algorithms, their hybrid algorithm turned out to be quite competitive. Therefore, it is summarised by providing evidence that shows that the proposed algorithm is well competitive with the other promising algorithms of the problem.

Comparing the current study with the second paper, we can say that although being slow in convergence, the ant colony optimization algorithms perform better than the evolutionary algorithms while solving the multidimensional knapsack problem. Evolutionary algorithms, on the other hand, require more computational resources and memory to achieve the optimal candidate solutions. The local search procedure introduced in the ACO algorithm certainly improves the algorithm's performance.

D. Paper-4: Swarm optimization search approach for solving knapsack problem

"A swarm optimization-based search algorithm for the quadratic knapsack problem with conflict Graphs" [4]

In this study, the authors have dealt with a special type of Knapsack problem, that is called *quadratic knapsack problem*, with conflict graphs (QKPCG). They have used a population-based search algorithm, inspired by the binary particle swarn optimization technique. They combined their search algorithm with a quick and efficient local search. This process of optimization works in two steps, first, that particle swarn optimization creates the population of all the particles while the local search works on all the infeasibilities of the solutions. The local search focuses on the repair of the feasibility of the solution created or either improve its quality.

They have used a simple population-based algorithm. They worked on the hybrid approach but unlike the usual practice of hybrid procedure in which continuous optimization is applied, they have introduced a method in which they substitute it with a discrete optimization method. The results produced showed that the model proposed performed better than the GLPX, Cplex, and many other recent algorithms, that were considered in the literature. Overall, the proposed methodology produced high-quality solutions for the problem. The authors also tested the algorithm for the special knapsack problems and saw that outputs were encouraged. They also mentioned that the proposed methodology reduced the average run time, achieved all the bounds. The method has turned out to be successful due to its simplicity.

III. Conclusions

In this study, we explored several different optimization algorithms that have been used over the years for solving the knapsack problem. We studied how the linear one dimensional knapsack problem can be generalized to the multiple knapsack problem with additional constraints. We also compared and contrasted the results and findings of different optimization algorithms used over the time period of four decades for tackling the knapsack problem. We also discussed the newest swarm optimization search based approach for solving the knapsack problem. In the study, we compared several different algorithms on the basis of their accuracy, convergence and time and space complexities for solving the multidimensional knapsack problem.

References

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