MATHS

CONTENTS

- LINEAR ALGEBRA
- CALCULUS
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CE, ME, CH

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LINEAR ALGEBRA

- ALGEBRA OF MATRICES
- RANK OF MATRICES
- SYSTEM OF EQUATIONS
- EIGEN VALUES & VECTORS

MATRIX

A set of mn objects/elements (real/complex) arranged in rectangular array of

$$\begin{array}{cc}
 & \begin{array}{ccccc} 1 & 2 & 3 & \cdots & n \\ \downarrow & \downarrow & \downarrow & & \end{array} \\
\begin{array}{c} 1 \rightarrow \\ 2 \rightarrow \\ 3 \rightarrow \\ \vdots \\ m \end{array} &
\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{2} & a_{22} & a_{23} & \\ a_{31} & a_{32} & a_{33} & \\ \vdots & & & \end{bmatrix}_{\underset{\text{Rows}}{m} \times \underset{\text{Columns}}{n}}
\end{array}$$

Element of Matrix $= [a_{ij}]$

$i \rightarrow$ Row $\{1 \text{ to } m\}$

$j \rightarrow$ Column $\{1 \text{ to } n\}$

TYPES OF MATRICES

1) ROW MATRIX :- A matrix having one row & any number of columns. Ex :- $\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \end{bmatrix}_{1\times n}$

$m \to 1$; n

2) COLUMN MATRIX :- A matrix having one column & any numbers of rows. Ex :- $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \end{bmatrix}_{m\times 1}$

m ; $n \to 1$

3) NULL MATRIX / ZERO MATRIX :- Any matrix in which all elements are 0. Ex :- $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$; $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

TYPES OF MATRICES

4) SQUARE MATRIX :- A matrix in which no. of rows & no. of columns are equal.

Ex :- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$ $\quad \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 5 \\ 6 & 7 & 1 \end{bmatrix}_{3 \times 3}$ Trace = 2

TRACE OF MATRIX :- Sum of principal diagonal elements / main diagonal / leading diagonal elements

5) DIAGONAL MATRIX :- A sq. matrix is called diagonal matrix if all its non-diagonal elements are zero.

TYPES OF MATRICES

Ex:- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3 \times 3}$ Diagonal $\neq 0$
Non-diagonal $= 0$

This is a diagonal matrix of order 3.

6) SCALAR MATRIX :- If all elements in a square diagonal matrix are equal.

Ex:- $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 5 I_3$

TYPES OF MATRICES

7) UNIT MATRIX :- A sq. matrix in which diagonal elements are 1.

→ IDENTITY MATRIX

Ex :- $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8) UPPER TRIANGULAR MATRIX :- A sq. matrix is K/a U.T.M. if all elements below principal diagonal are zero.

Ex :- $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$ → Above diag. $\neq 0$ → Below diag $= 0$

TYPES OF MATRICES

9) LOWER TRIANGULAR MATRIX :- A sq. matrix is K/a L.T.M. if all elements above principal diagonal is zero.

Ex :- $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 1 & -3 & 3 \end{bmatrix}$ → Above diag. = 0
→ Below diag. ≠ 0

10) SUB MATRIX :- A matrix obtained from given matrix by deleting some rows or columns.

$$A = \begin{bmatrix} 0 & 1 & 2 & 4 \\ 2 & -1 & 3 & 5 \\ -1 & 9 & 7 & 6 \\ -5 & 2 & 8 & 0 \end{bmatrix}_{4\times4} \Rightarrow A_1 = \begin{bmatrix} 2 & -1 & 5 \\ -1 & 9 & 6 \\ -5 & 2 & 0 \end{bmatrix}_{3\times3} \quad A_2 = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}_{2\times2}$$

TYPES OF MATRICES

11) HORIZONTAL MATRIX :- $m < n$

Ex :- $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -1 & 5 \\ 6 & 7 & 8 & -1 \end{bmatrix}_{3 \times 4}$

12) VERTICAL MATRIX :- $m > n$

Ex :- $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 1 \\ -1 & 0 & 5 \\ 5 & 6 & 8 \end{bmatrix}_{4 \times 3}$

TYPES OF MATRICES

Properties of Trace :-

1) $Tr(A+B) = Tr(A) + Tr(B)$

2) $Tr(A) = Tr(A^T)$

3) $Tr(\{A+B\}^T) = Tr(A^T) + Tr(B^T)$

PRODUCT BY A SCALAR

If a matrix is multiplied by scalar, then it is multiplied with each element.

$$A = [a_{ij}]$$

$$KA = [Ka_{ij}]$$

$$A = \begin{bmatrix} 1 & 0 & 3 \\ -5 & 2 & 4 \end{bmatrix} \qquad 4A = \begin{bmatrix} 4 & 0 & 12 \\ -20 & 8 & 16 \end{bmatrix}$$

ADDITION & SUBTRACTION OF MATRICES

The order of two matrices should be same.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} 0 & -1 \\ -2 & 1 \end{bmatrix}_{2\times 2}$$

$$A + B = \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$$

$$A + (-B)$$

ADDITION & SUBTRACTION OF MATRICES

Properties of Matrix Addition :-

1) Matrix addition is commutative.

$$A+B = B+A$$

2) Matrix addition is associative.

$$(A+B)+C = A+(B+C)$$

3) Cancellation law for matrix addition :-

$$\cancel{A}+B = \cancel{A}+C$$ holds only if $B=C$.

TYPES OF MATRICES

MULTIPLICATION OF MATRICES

The product AB of any two matrices is possible if & only if matrices are conformable.

Matrices are conformable :- $A_{m \times n} \times B_{n \times p}$

No. of columns (A) = No. of rows (B)

Ex :- $\underset{(A)}{\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 5 \end{bmatrix}_{2\times 3}} \underset{(B)}{\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 3 & 4 \end{bmatrix}_{3\times 2}} = \begin{bmatrix} 1+0+9 & 0+0+12 \\ -1-2+15 & 0+4+20 \end{bmatrix} = \begin{bmatrix} 10 & 12 \\ 12 & 24 \end{bmatrix}_{2\times 2}$

Total multiplications $= 2 \times 3 \times 2 = 12$

Total additions $= 2 \times (3-1) \times 2 = 8$

MULTIPLICATION OF MATRICES

- Order of $(AB) = m \times p$
- No. of multiplications to obtain each element in AB $= (n)$
- No. of additions to obtain each element in AB $= (n-1)$
- Total no. of multiplications $= mnp$
- Total no. of additions $= m(n-1)p$

MULTIPLICATION OF MATRICES

Properties of Matrix Multiplication :-

1) Matrix multiplication may or may not be commutative.

$$AB \neq BA$$

Ex :- $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}_{3\times 3}$ $B = \begin{bmatrix} 2 & 1 \\ 3 & 0 \\ 4 & 1 \end{bmatrix}_{3\times 2}$. Will BA exist ?

Matrices multiplication do not commute.

2) Matrix multiplication is associative.

$$(AB)C = A(BC)$$

MULTIPLICATION OF MATRICES

3) Matrix multiplication is distributive w.r.t. addition.

$$A(B+C) = AB + AC$$

" " is not distributive w.r.t. multiplication.

$$A(B \times C) \neq AB \times AC$$

4) Positive integral power of square matrix :-

$$A^m \cdot A^n = A^{m+n} \qquad A \rightarrow \text{square matrix.}$$

5) Zero divisor :- $AB = 0$

- If either $A = 0$ or $B = 0$; then $AB = 0$
- If both $A \neq 0$ & $B \neq 0$; then $AB = 0$ is possible

MULTIPLICATION OF MATRICES

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A \neq O \quad B \neq O \quad AB = O$

6) Multiplication with Identity matrix :-

$A \rightarrow$ Sq. matrix or order n.

$$\boxed{A I_n = I_n A = A}$$

MULTIPLICATION OF MATRICES

Minimum number of multiplications & additions :-

$A_{3\times 2}$ $B_{2\times 5}$ $C_{5\times 3}$

I) $(AB)C \rightarrow (AB)_{3\times 5}\, C_{5\times 3} \rightarrow (ABC)_{3\times 3}$

→ No. of multiplications $\underbrace{(3\times 2\times 5)}_{AB} + \underbrace{(3\times 5\times 3)}_{(AB)C} = 75$

→ No. of additions $\underbrace{\{3\times(2-1)\times 5\}}_{AB} + \underbrace{\{3\times(5-1)\times 3\}}_{(AB)C} = 51$

II) $(BC)A \rightarrow (BC)_{2\times 3}\, A_{3\times 2} \rightarrow (BCA)_{2\times 2}$

No. of multiplication $2\times 5\times 3 + 2\times 3\times 2 = 42$ ✓

No. of additions ; $2\times(5-1)\times 3 + 2\times(3-1)\times 2 = 32$ ✓

MINORS $[M_{ij}]$

The determinant value of square matrix obtained from original matrix of any order by deleting its row & column.

Ex1:-

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & -1 \\ 7 & 8 & 1 & 2 \end{bmatrix}_{3\times 4}$$

Minors of A

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 1 \end{vmatrix}_{3\times 3} \quad \begin{vmatrix} 5 & 6 \\ 8 & 1 \end{vmatrix}_{2\times 2} \quad \begin{vmatrix} 2 & 0 \\ 8 & 2 \end{vmatrix}_{2\times 2}$$

Ex2:-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 0 & 5 \end{bmatrix}$$

$$M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 5 \end{vmatrix} = 5 \quad M_{21} = \begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = 10 \quad M_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix}$$

$$M_{12} = \begin{vmatrix} 0 & 0 \\ 2 & 5 \end{vmatrix} = 0 \quad M_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} = -1 \quad M_{32} = \begin{vmatrix} 1 & 3 \\ 0 & 0 \end{vmatrix}$$

$$M_{13} = \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = -2 \quad M_{23} = \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = -4 \quad M_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$$

COFACTORS

$$A_{ij} = (-1)^{i+j} |M_{ij}|$$

$$3 \times 3 \rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$A_{11} = (-1)^{1+1} M_{11}$$
$$A_{12} = (-1)^{1+2} M_{12}$$
$$A_{13} = (-1)^{1+3} M_{13}$$

$$2 \times 2 \rightarrow \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} + & - \\ - & + \end{vmatrix}$$

DETERMINANT

- only defined for square matrix.
- Determinant (Δ) = Product of any row/column elements $\times$ their Cofactors

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 0 & 1 \\ 0 & -1 & 2 \end{vmatrix} = +2\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} - 4\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + 6\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

$$= 2(1) - 4(2) + 6(-1)$$

$$= 2 - 8 - 6 = -12$$

$$\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

$$\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$$

$$\Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$$

DETERMINANT & ITS PROPERTIES

TRANSPOSE OF MATRIX :- $A' = A^T$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

Interchanging Rows $\rightleftarrows$ Columns

- $(A^T)^T = A$
- $(A+B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$

1) $|A| = |A^T|$

2) If we interchange any two rows & columns then the sign of determinant will change.

DETERMINANT & ITS PROPERTIES

$$\begin{vmatrix} 1 & -1 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{vmatrix} 0 & 0 & 3 \\ 0 & 2 & 0 \\ 1 & -1 & 4 \end{vmatrix} \xrightarrow{C_1 \leftrightarrow C_2} \begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 0 \\ -1 & 1 & 4 \end{vmatrix}$$

$\Delta = 6$ $\qquad$ $\Delta = -6$ $\qquad$ $\Delta = 6$

NOTE :- Δ of U.T.M. / L.T.M. / Diagonal / Scalar matrix

Δ = Product of diagonal elements

3. If any two rows or columns are identical (or proportional) ; then the value of $\Delta = 0$

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 5 & 6 \end{vmatrix} \qquad \begin{vmatrix} 1 & 2 & 3 \\ 3 & 6 & 9 \\ 5 & 10 & 15 \end{vmatrix}$$

Identical $\qquad$ Proportional

DETERMINANT & ITS PROPERTIES

4. Multiplying a determinant by scalar K; then it means multiplying K in any row or column.

$$\begin{vmatrix} 1 & 2 \\ 3 & 8 \end{vmatrix} \xrightarrow{\times 5} 5\begin{vmatrix} 1 & 2 \\ 3 & 8 \end{vmatrix} = \begin{vmatrix} 5 & 10 \\ 3 & 8 \end{vmatrix}$$

$\Delta = 2$

$\Delta = 5 \times 2 = 10 \Leftrightarrow \Delta = 10$

$$= \begin{vmatrix} 1 & 2 \\ 15 & 40 \end{vmatrix} = \begin{vmatrix} 5 & 2 \\ 15 & 8 \end{vmatrix}$$

5. Elements of a row/column can be expressed as a sum of two or more elements then the given Δ can be expressed as a sum 2 or more determinants.

DETERMINANT & ITS PROPERTIES

$$\begin{vmatrix} a+b & c+d \\ e & f \end{vmatrix} = \begin{vmatrix} a & c \\ e & f \end{vmatrix} + \begin{vmatrix} b & d \\ e & f \end{vmatrix}$$

$$\begin{vmatrix} a+b+c & d+e+f \\ g & h \end{vmatrix} = \begin{vmatrix} a+b & d+e \\ g & h \end{vmatrix} + \begin{vmatrix} c & f \\ g & h \end{vmatrix}$$

$$= \begin{vmatrix} a & d \\ g & h \end{vmatrix} + \begin{vmatrix} b & e \\ g & h \end{vmatrix} + \begin{vmatrix} c & f \\ g & h \end{vmatrix}$$

6. $\boxed{\sum \text{Any row/column elements} \times \text{Corresponding factors} = \Delta}$

DETERMINANT & ITS PROPERTIES

7. $\boxed{\sum \text{Any row/column elements} \times \text{Non-corresponding cofactors} = 0}$

Ex:- $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

- $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = \Delta$
- $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$
- $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$

*** 8. Operations like $\left\{\begin{matrix} R_i \to R_i + KR_j \\ C_i \to C_i + KC_j \end{matrix}\right\}$, then the value of determinant will not change.

$K \to +, -, 0,$ fraction

DETERMINANT & ITS PROPERTIES

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 0 \\ 3 & 2 & 1 \end{vmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & -6 \\ 3 & 2 & 1 \end{vmatrix}$$

$$\Delta \qquad\qquad \Delta$$

$$\xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & -6 \\ 0 & -4 & -8 \end{vmatrix} \xrightarrow{R_3 \to R_3 + 4R_2} \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & -6 \\ 0 & 0 & -32 \end{vmatrix}$$

$$\Delta = -32$$

- $R_1 \to R_1 + 5R_2$
  $\Delta \qquad \Delta$
- $C_2 \to C_2 - 2.5C_3$
  $\Delta \qquad \Delta$
- $R_1 \to 5R_1 + 6R_2$
  $\Delta \qquad 5\Delta$
- $C_2 \to 3C_2 - 7C_1$
  $\Delta \qquad 3\Delta$

$$\begin{vmatrix} 5 & 0 & 1 \\ 3 & 0 & 2 \\ 2 & 1 & 5 \end{vmatrix} \xrightarrow[R_3 \to R_3 - \frac{2R_1}{5}]{R_2 \to R_2 - \frac{3R_1}{5}} \begin{vmatrix} 5 & 0 & 1 \\ 0 & 0 & 7/5 \\ 0 & 1 & 23/5 \end{vmatrix} \longrightarrow \begin{vmatrix} 5 & 0 & 1 \\ 0 & 1 & 23/5 \\ 0 & 0 & 7/5 \end{vmatrix}$$

$$\Delta = -7 \qquad\qquad \Delta = 7$$

DETERMINANT & ITS PROPERTIES

9. $|ABCD| = |A||B||C||D|$

$$|A^n| = |A|.|A|\ldots n = |A|^n$$

Ex:-

$$\begin{vmatrix} 0 & 0 & 0 & -5 \\ 1 & 0 & 2 & -3 \\ 0 & 0 & 3 & -2 \\ 0 & 1 & 0 & 3 \end{vmatrix} \qquad \begin{vmatrix} 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 3 & -2 \\ 0 & 1 & 0 & 3 \end{vmatrix} \qquad \begin{vmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & -5 \end{vmatrix}$$

$\boxed{\Delta = -15}$ $\qquad \Delta = 15 \qquad \Delta = -15$

DETERMINANT & ITS PROPERTIES

ADJOINT OF MATRIX :-

$$\boxed{A.(Adj.A) = (Adj\ A)A = |A|I_n}$$

$$Adj\ A = (\text{Cofactor matrix})^T$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$Adj\ A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}}_{adj.\ A} = \begin{bmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{bmatrix} = \Delta \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

INVERSE OF A MATRIX :-

$$\boxed{A^{-1} = \frac{(\mathrm{adj}.A)}{|A|}}$$

$$A.(\mathrm{Adj}\,A) = |A|\,I_n$$

$$A^{-1}A(\mathrm{Adj}\,A) = A^{-1}|A|\,I_n$$

$$I(\mathrm{Adj}\,A)$$

$$\mathrm{Adj}\,A = |A|\,A^{-1}$$

$$A^{-1} = \frac{\mathrm{Adj}\,A}{|A|}$$