KEY CONCEPTS
MATRICES

1. Definition : Rectangular array of mn numbers . Unlike determinants it has no value.

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ : & : & : & : \\ a_{m1} & a_{m2} & \ldots\ldots & a_{mn} \end{bmatrix} \quad \text{or} \quad \begin{pmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ : & : & : & : \\ a_{m1} & a_{m2} & \ldots\ldots & a_{mn} \end{pmatrix}$$

Abbreviated as : $A = \left[a_{ij}\right]$ $1 \le i \le m$; $1 \le j \le n$, i denotes the row and j denotes the column is called a matrix of order $m \times n$.

2. Special Type Of Matrices :

(a) Row Matrix : $A = [\, a_{11}, a_{12}, \ldots\ldots a_{1n} \,]$ having one row . $(1 \times n)$ matrix.
(or row vectors)

(b) Column Matrix : $A = \begin{bmatrix} a_{11} \\ a_{21} \\ : \\ a_{m1} \end{bmatrix}$ having one column. $(m \times 1)$ matrix
(or column vectors)

(c) Zero or Null Matrix : $(A = O_{m \times n})$
An $m \times n$ matrix all whose entries are zero .

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ is a 3×2 null matrix & $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is 3×3 null matrix

(d) Horizontal Matrix : A matrix of order $m \times n$ is a horizontal matrix if $n > m$.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 1 & 1 \end{bmatrix}$$

(e) Verical Matrix : A matrix of order $m \times n$ is a vertical matrix if $m > n$. $\begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}$

(f) Square Matrix : (Order n)

If number of row = number of column $\Rightarrow$ a square matrix.

Note (i) In a square matrix the pair of elements a_{ij} & a_{ji} are called **Conjugate Elements** .

e.g. $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

(ii) The elements $a_{11}, a_{22}, a_{33}, \ldots\ldots a_{nn}$ are called **Diagonal Elements** . The line along which the diagonal elements lie is called **"Principal or Leading"** diagonal.
The qty $\Sigma\, a_{ii}$ = trace of the matrice written as , i.e. $t_r A$

Square Matrix

Triangular Matrix

$A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{pmatrix}$; $B = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -3 & 0 \\ 4 & 3 & 3 \end{pmatrix}$

Upper Triangular $a_{ij} = 0 \ \forall \ i > j$ — Lower Triangular $a_{ij} = 0 \ \forall \ i < j$

Note that : Minimum number of zeros in a triangular matrix of order $n = n(n-1)/2$

Diagonal Matrix denote as $d_{dia}(d_1, d_2, \ldots.., d_n)$ all elements except the leading diagonal are zero

diagonal Matrix

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

Unit or Identity Matrix

$$a_{ij} = \begin{bmatrix} 1 & \text{if} & i = j \\ 0 & \text{if} & i \neq j \end{bmatrix}$$

Note: (1) If $d_1 = d_2 = d_3 = a$ Scalar Matrix

(2) If $d_1 = d_2 = d_3 = 1$ Unit Matrix

Note: Min. number of zeros in a diagonal matrix of order $n = n(n-1)$

"It is to be noted that with square matrix there is a corresponding determinant formed by the elements of A in the same order."

3. Equality Of Matrices :

Let $A = [a_{ij}]$ & $B = [b_{ij}]$ are equal if,

(i) both have the same order . **(ii)** $a_{ij} = b_{ij}$ for each pair of i & j.

4. Algebra Of Matrices :

Addition : $A + B = \left[a_{ij} + b_{ij}\right]$ where A & B are of the same type. (same order)

(a) Addition of matrices is commutative.

i.e. $A + B = B + A$ $A = m \times n$; $B = m \times n$

(b) Matrix addition is associative .

$(A + B) + C = A + (B + C)$ **Note :** A, B & C are of the same type.

(c) Additive inverse.

If $A + B = \mathbf{O} = B + A$ $A = m \times n$

5. Multiplication Of A Matrix By A Scalar :

If $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$; $kA = \begin{bmatrix} ka & kb & kc \\ kb & kc & ka \\ kc & ka & kb \end{bmatrix}$

6. Multiplication Of Matrices : (Row by Column)

AB exists if, $A = m \times n$ & $B = n \times p$

2×3 3×3

AB exists, but BA does not $\Rightarrow AB \neq BA$

Note : In the product AB, $\begin{cases} A = \text{pre factor} \\ B = \text{post factor} \end{cases}$

$A = (a_1, a_2, \ldots\ldots a_n)$ & $B = \begin{bmatrix} b_1 \\ b_2 \\ : \\ b_n \end{bmatrix}$

$1 \times n$ $n \times 1$

$AB = [a_1 b_1 + a_2 b_2 + \ldots\ldots + a_n b_n]$

If $A = \left[a_{ij}\right]$ $m \times n$ & $B = \left[b_{ij}\right]$ $n \times p$ matrix, then $\boxed{(AB)_{ij} = \sum_{r=1}^{n} a_{ir} \cdot b_{rj}}$

Properties Of Matrix Multiplication :

1. Matrix multiplication is not commutative .

$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$; $AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$; $BA = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow$ $AB \neq BA$ (in general)

2. $AB = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\Rightarrow$ $AB = \mathbf{O}$ $\not\Rightarrow$ $A = \mathbf{O}$ or $B = \mathbf{O}$

Note: If A and B are two non- zero matrices such that $AB = \mathbf{O}$ then A and B are called the divisors of zero. Also if $[AB] = \mathbf{O} \Rightarrow |AB| \Rightarrow |A|\,|B| = 0 \Rightarrow |A| = 0$ or $|B| = 0$ but not the converse.

If A and B are two matrices such that

(i) $AB = BA$ $\Rightarrow$ A and B commute each other

(ii) $AB = -BA$ $\Rightarrow$ A and B anti commute each other

3. **Matrix Multiplication Is Associative :**

If A, B & C are conformable for the product AB & BC, then

$$(A\,.\,B)\,.\,C = A\,.\,(B\,.\,C)$$

4. **Distributivity :**

$$\left.\begin{aligned} A\,(B + C) &= AB + AC \\ (A + B)\,C &= AC + BC \end{aligned}\right] \text{ Provided A, B \& C are conformable for respective products}$$

5. **Positive Integral Powers Of A Square Matrix :**

For a square matrix A, $A^2 A = (AA)\,A = A\,(AA) = A^3$.

Note that for a unit matrix I of any order , $I^m = I$ for all $m \in N$.

6. **MATRIX POLYNOMIAL :**

If $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots\ldots + a_nx^0$ then we define a matrix polynomial

$$f(A) = a_0A^n + a_1A^{n-1} + a_2A^{n-2} + \ldots\ldots + a_nI^n$$

where A is the given square matrix. If f (A) is the null matrix then A is called the zero or root of the polynomial f (x).

DEFINITIONS :

(a) **Idempotent Matrix :** A square matrix is idempotent provided $A^2 = A$.

Note that $A^n = A\ \forall\ n \geq 2$, $n \in N$.

(b) **Nilpotent Matrix:** A square matrix is said to be nilpotent matrix of order m, $m \in N$, if $A^m = \mathbf{O}$, $A^{m-1} \neq \mathbf{O}$.

(c) **Periodic Matrix :** A square matrix is which satisfies the relation $A^{K+1} = A$, for some positive integer K, is a periodic matrix. The period of the matrix is the least value of K for which this holds true.

Note that period of an idempotent matrix is 1.

(d) **Involutary Matrix :** If $A^2 = I$, the matrix is said to be an involutary matrix.

Note that $A = A^{-1}$ for an involutary matrix.

7. **The Transpose Of A Matrix : (Changing rows & columns)**

Let A be any matrix . Then, $A = a_{ij}$ of order $m \times n$

$\Rightarrow$ A^T or $A' = [\,a_{ji}\,]$ for $1 \leq i \leq n$ & $1 \leq j \leq m$ of order $n \times m$

Properties of Transpose : If A^T & B^T denote the transpose of A and B ,

(a) $(A \pm B)^T = A^T \pm B^T$; note that A & B have the same order.

IMP. (b) $(AB)^T = B^T A^T$ A & B are conformable for matrix product AB.

(c) $(A^T)^T = A$

(d) $(kA)^T = kA^T$ k is a scalar .

General : $(A_1, A_2, \ldots\ldots A_n)^T = A_n^T , \ldots\ldots. , A_2^T , A_1^T$ (reversal law for transpose)

8. Symmetric & Skew Symmetric Matrix :

A square matrix $A = \left[a_{ij}\right]$ is said to be ,

symmetric if,

$a_{ij} = a_{ji} \quad \forall \quad i \ \& \ j$ (conjugate elements are equal) **(Note $A = A^T$)**

Note: Max. number of distinct entries in a symmetric matrix of order n is $\frac{n(n+1)}{2}$.

and skew symmetric if,

$a_{ij} = -a_{ji} \quad \forall \quad i \ \& \ j$ (the pair of conjugate elements are additive inverse of each other) **(Note $A = -A^T$)**

Hence If A is skew symmetric, then

$$a_{ii} = -a_{ii} \quad \Rightarrow \quad a_{ii} = 0 \quad \forall \quad i$$

Thus the digaonal elements of a skew symmetric matrix are all zero , but not the converse .

Properties Of Symmetric & Skew Matrix :

P – 1 A is symmetric if $A^T = A$

A is skew symmetric if $A^T = -A$

P – 2 $A + A^T$ is a symmetric matrix

$A - A^T$ is a skew symmetric matrix .

Consider $(A+A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T$

$A + A^T$ is symmetric .

Similarly we can prove that $A - A^T$ is skew symmetric .

P – 3 The sum of two symmetric matrix is a symmetric matrix and the sum of two skew symmetric matrix is a skew symmetric matrix .

Let $A^T = A$; $B^T = B$ where A & B have the same order .

$\mathbf{(A+B)^T = A+B}$

Similarly we can prove the other

P – 4 If A & B are symmetric matrices then ,

(a) $AB + BA$ is a symmetric matrix

(b) $AB - BA$ is a skew symmetric matrix .

P – 5 Every square matrix can be uniquely expressed as a sum of a symmetric and a skew symmetric matrix.

$$A = \frac{1}{2}\underbrace{(A + A^T)}_{\substack{P \\ \text{Symmetric}}} + \frac{1}{2}\underbrace{(A - A^T)}_{\substack{Q \\ \text{Skew Symmetric}}}$$

9. Adjoint Of A Square Matrix :

Let $A = \left[a_{ij}\right] = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ be a square matrix and let the matrix formed by the cofactors of $[a_{ij}]$ in determinant $|A|$ is $= \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}$.

Then $(\text{adj } A) = \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$

V. Imp. Theorem : $A\,(\text{adj. } A) = (\text{adj. } A).A = |A|\ I_n$, If A be a square matrix of order n.

Note : If A and B are non singular square matrices of same order, then

(i) $|\operatorname{adj} A| = |A|^{n-1}$

(ii) $\operatorname{adj}(AB) = (\operatorname{adj} B)(\operatorname{adj} A)$

(iii) $\operatorname{adj}(KA) = K^{n-1}(\operatorname{adj} A)$, K is a scalar

Inverse Of A Matrix (Reciprocal Matrix) :

A square matrix A said to be invertible (non singular) if there exists a matrix B such that,

$$AB = I = BA$$

B is called the inverse (reciprocal) of A and is denoted by A^{-1}. Thus

$$A^{-1} = B \Leftrightarrow AB = I = BA.$$

We have,

$$A.(\operatorname{adj} A) = |A|\ I_n$$

$$A^{-1} A\ (\operatorname{adj} A) = A^{-1} I_n\ |A|$$

$$I_n\ (\operatorname{adj} A) = A^{-1}\ |A|\ I_n$$

$$\therefore \quad A^{-1} = \frac{(\operatorname{adj} A)}{|A|}$$

Note : The necessary and sufficient condition for a square matrix A to be invertible is that $|A| \neq 0$.

Imp. Theorem : If A & B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1}A^{-1}$. This is reversal law for inverse.

Note :

(i) If A be an invertible matrix, then A^T is also invertible & $(A^T)^{-1} = (A^{-1})^T$.

(ii) If A is invertible, **(a)** $(A^{-1})^{-1} = A$; **(b)** $(A^k)^{-1} = (A^{-1})^k = A^{-k}$, $k \in N$

(iii) If A is an Orthogonal Matrix. $AA^T = I = A^TA$

(iv) A square matrix is said to be **orthogonal** if, $A^{-1} = A^T$.

(v) $|A^{-1}| = \dfrac{1}{|A|}$

1. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ = and B = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then B is equal to

(A) I cos θ + J sinθ
(B) I cosθ – Jsinθ
(C) I sinθ + Jcosθ
(D) –I cosθ + Jsinθ

2. The number of different orders of a matrix having 12elements is

(A) 3 (B) 1
(C) 6 (D) None of these

3. $\begin{bmatrix} x^2+x & x \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -x+1 & x \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$ then x is equal to

(A) –1 (B) 2
(C) 1 (D) No value of x

4. If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 4 & 0 \\ 0 & 2 & -1 \\ 1 & -3 & 2 \end{bmatrix}$, then

(A) $AB = \begin{bmatrix} -5 & 8 & 0 \\ 0 & 4 & -2 \\ 3 & -9 & 6 \end{bmatrix}$

(B) AB = [–2 –1 4]

(C) $AB = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

(D) AB does not exist

5. If $[1\ x\ 1] \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = [0]$ then x is

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$
(C) 1 (D) –1

6. If AB = A and BA = B, then B^2 is equal to

(A) B (B) A
(C) I (D) 0

7. If A = diag (2, –1, 3), B = diag (–1, 3, 2), then
A^2 B equal to

(A) diag (5, 4, 11)
(B) diag (–4, 3, 18)
(C) diag (3, 1, 8)
(D) B

8. If the matrix AB is a zero matrix, then

(A) A = O or B = O
(B) A = O and B = O
(C) It is not necessary that either A = O or B = O
(D) All the above statements are wrong

9. Which relation true for $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$

(A) $(A + B)^2 = A^2 + 2AB + B^2$
(B) $(A - B)^2 = A^2 - 2AB + B^2$
(C) AB = BA
(D) None of these

10. If A and B are square matrices of size n × n such
that $A^2 - B^2 = (A - B)(A + B)$, then which of the
following will be always true ?

(A) AB = BA
(B) either of A or B is a zero matrix
(C) either of A or B is an identity matrix
(D) A = B

11. $A = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$ then B^TA^T is

(A) a null matrix
(B) an identity matrix
(C) scalar, but not an identity matrix
(D) such that $T_r(B^T A^T) = 4$

12. If A is a non–singular matrix and A^T denotes the transporse of A, then

(A) $|A| \neq |A^T|$ (B) $|A.A|^T \neq |A|^2$

(C) $|A^T. A| \neq |A^T|^2$ (D) $|A| + |A^T| \neq 0$

13. If A is a skew – symmetric matrix and n is an even
positive integer, then A^n is

(A) a symmetric matrix

(B) a skew–symmetric matrix

(C) a diagonal matrix

(D) None of these

14. Which one of the following is wrong ?

(A) The elements on the main diagonal of a symmetric matrix are all zero

(B) The elements on the main diagonal of a skew – symmetric matrix are all zero

(C) For any square matrix A, 1/2 (A + A') is symmetric

(D) For any square matrix, 1/2 (A – A') is skew – symmetric

15. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, then adj equal to

(A) $\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -2 \\ -2 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$

16. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then value of A^{-1} is equal to

(A) A (B) A^2

(C) A^3 (D) A^4

17. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and X be a matrix such that A = BX is equal to

(A) $\frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (B) $\frac{1}{2}\begin{bmatrix} -2 & 4 \\ 3 & -5 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (D) None of these

18. Given $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}; I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. If $A - \lambda I$ is a singular matrix then

(A) $\lambda \in \phi$ (B) $\lambda^2 - 3\lambda - 4 = 0$

(C) $\lambda^2 + 3\lambda + 4 = 0$ (D) $\lambda^2 - 3\lambda - 6 = 0$

19. From the matrix equation AB = AC, we conclude
B = C provided

(A) A is singular

(B) A is non–singular

(C) A is symmetric

(D) A is a square

20. If $A^2 - A + I = 0$, then the inverse of A is

(A) I – A (B) A – I

(C) A (D) A + I

21. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. The only correct statement about the matrix A is

(A) A is a zero matrix

(B) A = (–1) I, where I is a unit matrix

(C) A^{-1} does not exist

(D) $A^2 = I$

22. A and B be 3 × 3 matrices . Then AB = 0 implies

(A) A = 0 and B = 0

(B) 0

(C) |A| = 0 and |B| = 0

(D) A = 0 or B = 0

23. Which of the following statements is incorrect for a square matrix A. ($|A| \neq 0$)

(A) If A is a diagonal matrix, A^{-1} will also be a diagonal matrix

(B) If A is symmetric matrix , A^{-1} will also be a symmetric matrix

(C) If $A^{-1} = A \Rightarrow A$ is an idempotent matrix
(D) If $A^{-1} = A \Rightarrow A$ an involutary matrix

24. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. I is the unit matrix of order 2 and a ,b are arbitrary constants,. then $(aI + bA)^2$ is equal to
(A) $a^2I + b^2A$ (B) $a^2I = abA$
(C) $a^2I + 2abA$ (D) None of these

25. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I_2 = 0$, then the value of k is
(A) 3 (B) 5
(C) 7 (D) –7

26. If a matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$, then A^{-1} is equal to-
(A) $-(3A^2 + 2A + 5)$
(B) $(3A^2 + 2A + 5)$
(C) $(3A^2 - 2A - 5)$
(D) none of these

27. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$, then
(A) a = 1, c = –1 (B) $a = 2, c = -\frac{1}{2}$
(C) a = –1, c = 1 (D) $a = \frac{1}{2}, c = \frac{1}{2}$

28. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & a \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of matrix A, then α is
(A) –2 (B) –1
(C) 2 (D) 5

29. If $A^2 - A + I = 0$, then the inverse of A is - **[AIEEE-2005]**
(A) A + I (B) A
(C) A – I (D) I – A

30. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction - **[AIEEE-2005]**
(A) $A^n = nA - (n-1)I$
(B) $A^n = 2^{n-1}A - (n-1)I$
(C) $A^n = nA + (n-1)I$
(D) $A^n = 2^{n-1}A + (n-1)I$

31. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true – **[AIEEE 2006]**
(A) AB = BA
(B) Either of A or B is a zero matrix
(C) Either of A or B is an identity matrix
(D) A = B

32. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ If $|A^2| = 25$, then $|\alpha|$ equals- **[AIEEE 2007]**
(A) 5^2 (B) 1
(C) $\frac{1}{5}$ (D) 5

33. If $w \neq 1$ is the complex cube root of unity and matrix H = 0 $\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to **[AIEEE 2011]**
(A) H (B) 0
(C) –H (D) H^2

34. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, then $u_1 + u_2$ is equal to **[AIEEE 2012]**

(A) $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ (B) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

35. Let P and Q be 3×3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to : **[AIEEE 2012]**

(A) 0 (B) –1

(C) –2 (D) 1

36. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to : **[AIEEE 2013]**

(A) 5 (B) 0

(C) 4 (D) 11

37. If A is a 3×3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T is equal to **[AIEEE 2014]**

(A) I + B (B) I

(C) B–1 (D) (B–1)T

38. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to **[AIEEE 2015]**

(A) (2, 1) (B) (– 2, – 1)

(C) (2, – 1) (D) (– 2, 1)

39. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \operatorname{adj} A = AA^T$, then 5a + b is equal to : **[AIEEE 2016]**

(A) 5 (B) 4

(C) 13 (D) –1

40. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then $\operatorname{adj}(3A^2 + 12A)$ is equal to : **[AIEEE 2017]**

(A) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (B) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

(C) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (D) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

41. If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $x = P^TQ^{2005}$ then x is equal to **[JEE 2005 (Scr.)]**

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-200\sqrt{3} \end{bmatrix}$

(C) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$

(D) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

42. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. IF $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals **[JEEAdv.2016]**

(A) 52 (B) 103

(C) 201 (D) 205

43. If $A - 2B = \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}$ and $2A - 3B = \begin{bmatrix} -2 & 5 \\ 0 & 7 \end{bmatrix}$, then matrix B is equal to -

(A) $\begin{bmatrix} -4 & -5 \\ -6 & -7 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 6 \\ -3 & 7 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 6 & -1 \\ 0 & 1 \end{bmatrix}$

44. If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then $A_\alpha A_\beta$ is equal to -

(A) $A_{\alpha+\beta}$ (B) $A_{\alpha\beta}$

(C) $A_{\alpha-\beta}$ (D) none of these

45. If number of elements in a matrix is 60 then how many different order of matrix are possible -

(A) 12 (B) 6

(C) 24 (D) none of these

46. Matrix A has x rows and x + 5 columns. Matrix B has y rows and 11 – y columns. Both AB and BA exist, then -

(A) x = 3, y = 4 (B) x = 4, y = 3

(C) x = 3, y = 8 (D) x = 8, y = 3

47. If $A^2 = A$, then $(I + A)^4$ is equal to -

(A) I + A (B) I + 4A

(C) I + 15A (D) none of these

48. If the product of n matrices $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \ldots\ldots \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is equal to the matrix $\begin{bmatrix} 1 & 378 \\ 0 & 1 \end{bmatrix}$ then the value of n is equal to -

(A) 26 (B) 27

(C) 377 (D) 378

49. If A is a skew symmetric matrix such that $A^TA = I$, then A^{4n-1} $(n \in N)$ is equal to -

(A) $-A^T$ (B) I

(C) – I (D) A^T

50. If $AA^T = I$ and det(A) = 1, then -

(A) Every element of A is equal to it's co-factor.

(B) Every element of A and it's co-factor are additive inverse of each other.

(C) Every element of A and it's co-factor are multiplicative inverse of each other.

(D) None of these

51. Which of the following is an orthogonal matrix -

(A) $\begin{bmatrix} 6/7 & 2/7 & -3/7 \\ 2/7 & 3/7 & 6/7 \\ 3/7 & -6/7 & 2/7 \end{bmatrix}$

(B) $\begin{bmatrix} 6/7 & 2/7 & 3/7 \\ 2/7 & -3/7 & 6/7 \\ 3/7 & 6/7 & -2/7 \end{bmatrix}$

(C) $\begin{bmatrix} -6/7 & -2/7 & -3/7 \\ 2/7 & 3/7 & 6/7 \\ -3/7 & 6/7 & 2/7 \end{bmatrix}$

(D) $\begin{bmatrix} 6/7 & -2/7 & 3/7 \\ 2/7 & 2/7 & -3/7 \\ -6/7 & 2/7 & 3/7 \end{bmatrix}$

52. Let the matrix A and B be defined as $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}$ then the value of Det.$(2A^9B^{-1})$, is -

(A) 2 (B) 1

(C) –1 (D) –2

53. If $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then matrix A equals -

(A) $\begin{bmatrix} 7 & 5 \\ -11 & -8 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 7 & 1 \\ 34 & 5 \end{bmatrix}$ (D) $\begin{bmatrix} 5 & 3 \\ 13 & 8 \end{bmatrix}$

54. If $A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$ and $f(x) = 1 + x + x^2 + ... + x^{16}$, then $f(A) =$

(A) 0 (B) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 5 \\ 1 & 1 \end{bmatrix}$

55. If $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - \lambda M - I_2 = O$, then λ equals -

(A) –2 (B) 2

(C) –4 (D) 4

56. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then $|A|\,|\text{Adj } A|$ is equal to -

(A) a^{25} (B) a^{27}

(C) a^{81} (D) none of these

57. If A and B are square matrices of same order and $AA^T = I$ then $(A^TBA)^{10}$ is equal to -

(A) $AB^{10}A^T$ (B) $A^TB^{10}A$

(C) $A^{10}B^{10}(A^T)^{10}$ (D) $10A^TBA$

58. If A is a invertible idempotent matrix of order n, then adj A is equal to -

(A) $(\text{adj } A)^2$ (B) I

(C) A^{-1} (D) none of these

59. Matrix $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if xyz = 60 and 8x + 4y + 3z = 20, then A (adj A) is equal to -

(A) $\begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$ (B) $\begin{bmatrix} 88 & 0 & 0 \\ 0 & 88 & 0 \\ 0 & 0 & 88 \end{bmatrix}$

(C) $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$ (D) $\begin{bmatrix} 34 & 0 & 0 \\ 0 & 34 & 0 \\ 0 & 0 & 34 \end{bmatrix}$

60. Let three matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$ then

$$t_r(A) + t_r\left(\frac{ABC}{2}\right) + t_r\left(\frac{A(BC)^2}{4}\right) + t_r\left(\frac{A(BC)^3}{8}\right) + ... + \infty =$$

(A) 6 (B) 9

(C) 12 (D) none of these

61. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$, then -

(A) a = 1, c = –1 (B) a = 2, $c = -\frac{1}{2}$

(C) a = –1, c = 1 (D) $a = \frac{1}{2}$, $c = \frac{1}{2}$

62. Let $\Delta_0 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ (where $\Delta_0 \neq 0$) and let Δ_1 denote the determinant formed by the cofactors of elements of Δ_0 and Δ_2 denote the determinant formed by the cofactor at Δ_1 and so on Δ_n denotes the determinant formed by the cofactors at Δ_{n-1} then the determinant value of Δ_n is -

(A) Δ_0^{2n} (B) $\Delta_0^{2^n}$

(C) $\Delta_0^{n^2}$ (D) Δ_0^{2}

ANSWER KEY

1. A **2.** C **3**. A **4**. D **5**. B **6**. A **7**. A **8**. C **9**. D **10.** A **11**. B **12**. B **13**. A
14. A **15.** A **16**. C **17.** A **18.** B **19.** B **20**. A **21**. D **22.** C **23.** D **24.** C **25.** B **26.** C
27. A **28.** D **29.** D **30.** A **31.** A **32.** C **33.** A **34.** D **35.** A **36.** D **37.** B **38.** B **39.** A
40. B **41.** A **42.** B **43.** A **44.** A **45.** A **46.** C **47.** C **48.** B **49.** D **50.** A **51.** A **52.** D
53. A **54.** B **55.** D **56.** D **57.** B **58.** ABC**59.**C **60.** A **61.** A **62.** B