STAT 445/645 Assignment Cover Page

Student Name	Heewon Oh
SFU Student Number	301268860
SFU email address	heewono@sfu.ca
Assignment Number	6
Due Date	April 2,2022

Provide references for any data sets used in this assignment

Hand, D., Daly, F., Lunn, A., McConway, K. and Ostrowski, E. (eds), A Handbook of Small Data Sets, Chapman & Hall, London, 1994.

List software used in this assignment.

R, Rstudio, Excel

List **ALL** resources used to complete this assignment, including books, internet sources and people.

Tutorial Code, Notes from Class

https://stat.ethz.ch/R-manual/R-devel/library/Matrix/html/norm.html

https://www.rdocumentation.org/packages/psych/versions/2.2.3/topics/fa

☑ I personally completed the computations and wrote the solutions submitted in this document.

Department of Statistics and Actuarial Science

SFU SIMON FRASER UNIVERSITY

1)A)

i)

Variables	load 1	load 2	load 3	load 4	commun.	spec. var.
Red meat	0.61	-0.07	-0.32	0.63	0.87	0.129
White meat	0.62	-0.3	0.66	-0.04	0.92	0.082
Eggs	0.85	-0.05	0.19	0.31	0.86	0.138
Milk	0.76	-0.24	-0.41	0	0.8	0.205
Fish	0.27	0.83	-0.34	-0.21	0.92	0.081
Cereals	-0.88	-0.3	0.1	-0.01	0.87	0.133
Starchy foods	0.59	0.45	0.26	-0.33	0.73	0.268
Nuts	-0.84	0.18	-0.06	0.32	0.85	0.151
Fruits/Veg	-0.22	0.69	0.43	0.45	0.91	0.09
Var. Acc. For	4.01	1.63	1.13	0.95		
Prop. Tot. Var	0.45	0.18	0.13	0.11		

ii) Estimate

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.9999     0.1661  0.6529  0.6023  0.0795 -0.5453        0.0386
## White meat      0.1661     1.0005  0.6613  0.2702 -0.3002 -0.3865        0.4161
## Eggs            0.6529     0.6613  1.0002  0.5764  0.0642 -0.7170        0.4368
## Milk            0.6023     0.2702  0.5764  1.0001  0.1505 -0.6337        0.2387
## Fish            0.0795    -0.3002  0.0642  0.1505  0.9997 -0.5183        0.5160
## Cereals        -0.5453    -0.3865 -0.7170 -0.6337 -0.5183  1.0003       -0.6277
## Starchy foods   0.0386     0.4161  0.4368  0.2387  0.5160 -0.6277        1.0003
## Nuts           -0.3007    -0.6284 -0.6392 -0.6567 -0.1253  0.6746       -0.5390
## Fruits/Veg     -0.0348    -0.0745  0.0018 -0.5077  0.2641  0.0303        0.1410
```

```
##                  Nuts Fruits/Veg
## Red meat      -0.3007    -0.0348
## White meat    -0.6284    -0.0745
## Eggs          -0.6392     0.0018
## Milk          -0.6567    -0.5077
## Fish          -0.1253     0.2641
## Cereals        0.6746     0.0303
## Starchy foods -0.5390     0.1410
## Nuts           0.9999     0.4323
## Fruits/Veg     0.4323     1.0001
```

iii) Error Matrix

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.0001    -0.0131 -0.0673 -0.0994 -0.0186  0.0454        0.0968
## White meat     -0.0131    -0.0005 -0.0408  0.0113  0.0662 -0.0273       -0.1023
## Eggs           -0.0673    -0.0408 -0.0002 -0.0009  0.0014  0.0046        0.0154
## Milk           -0.0994     0.0113 -0.0009 -0.0001 -0.0126  0.0410       -0.0163
## Fish           -0.0186     0.0662  0.0014 -0.0126  0.0003 -0.0059       -0.1122
## Cereals         0.0454    -0.0273  0.0046  0.0410 -0.0059 -0.0003        0.0945
## Starchy foods   0.0968    -0.1023  0.0154 -0.0163 -0.1122  0.0945       -0.0003
## Nuts           -0.0488    -0.0065  0.0794  0.0357 -0.0219 -0.0236        0.0647
## Fruits/Veg     -0.0394     0.0132 -0.0474  0.0993  0.0020  0.0162       -0.0566
##                   Nuts Fruits/Veg
## Red meat      -0.0488    -0.0394
## White meat    -0.0065     0.0132
## Eggs           0.0794    -0.0474
## Milk           0.0357     0.0993
## Fish          -0.0219     0.0020
## Cereals       -0.0236     0.0162
## Starchy foods  0.0647    -0.0566
## Nuts           0.0001    -0.0573
## Fruits/Veg    -0.0573    -0.0001
```

Frobenius norm of error:

```
## [1] 0.4549
```

B)

i)

Variables	load 1	load 2	load 3	load 4	commun.	spec. var.
Red meat	-0.04	0.01	0.05	0.93	0.87	0.129
White meat	-0.05	-0.1	0.94	0.13	0.92	0.082
Eggs	-0.02	0.16	0.63	0.66	0.86	0.138
Milk	-0.58	0.22	0.2	0.61	0.8	0.205
Fish	0.1	0.92	-0.23	0.09	0.92	0.081
Cereals	0.15	-0.62	-0.39	-0.55	0.87	0.133
Starchy foods	0.03	0.68	0.51	0	0.73	0.268
Nuts	0.52	-0.33	-0.64	-0.26	0.85	0.151
Fruits/Veg	0.94	0.18	-0.01	0	0.91	0.09
Var. Acc. For	1.52	1.93	2.2	2.08		
Prop. Tot. Var	0.17	0.21	0.24	0.23		

ii) Estimate

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.9999     0.1661  0.6529  0.6023  0.0795 -0.5453        0.0386
## White meat      0.1661     1.0005  0.6613  0.2702 -0.3002 -0.3865        0.4161
## Eggs            0.6529     0.6613  1.0002  0.5764  0.0642 -0.7170        0.4368
## Milk            0.6023     0.2702  0.5764  1.0001  0.1505 -0.6337        0.2387
## Fish            0.0795    -0.3002  0.0642  0.1505  0.9997 -0.5183        0.5160
## Cereals        -0.5453    -0.3865 -0.7170 -0.6337 -0.5183  1.0003       -0.6277
## Starchy foods   0.0386     0.4161  0.4368  0.2387  0.5160 -0.6277        1.0003
## Nuts           -0.3007    -0.6284 -0.6392 -0.6567 -0.1253  0.6746       -0.5390
## Fruits/Veg     -0.0348    -0.0745  0.0018 -0.5077  0.2641  0.0303        0.1410
##                 Nuts Fruits/Veg
## Red meat      -0.3007    -0.0348
## White meat    -0.6284    -0.0745
## Eggs          -0.6392     0.0018
## Milk          -0.6567    -0.5077
## Fish          -0.1253     0.2641
## Cereals        0.6746     0.0303
## Starchy foods -0.5390     0.1410
## Nuts           0.9999     0.4323
## Fruits/Veg     0.4323     1.0001
```

Identical to that in A

iii)

Error Matrix

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.0001    -0.0131 -0.0673 -0.0994 -0.0186  0.0454        0.0968
## White meat     -0.0131    -0.0005 -0.0408  0.0113  0.0662 -0.0273       -0.1023
## Eggs           -0.0673    -0.0408 -0.0002 -0.0009  0.0014  0.0046        0.0154
## Milk           -0.0994     0.0113 -0.0009 -0.0001 -0.0126  0.0410       -0.0163
## Fish           -0.0186     0.0662  0.0014 -0.0126  0.0003 -0.0059       -0.1122
## Cereals         0.0454    -0.0273  0.0046  0.0410 -0.0059 -0.0003        0.0945
## Starchy foods   0.0968    -0.1023  0.0154 -0.0163 -0.1122  0.0945       -0.0003
## Nuts           -0.0488    -0.0065  0.0794  0.0357 -0.0219 -0.0236        0.0647
## Fruits/Veg     -0.0394     0.0132 -0.0474  0.0993  0.0020  0.0162       -0.0566
##                 Nuts Fruits/Veg
## Red meat      -0.0488    -0.0394
## White meat    -0.0065     0.0132
## Eggs           0.0794    -0.0474
## Milk           0.0357     0.0993
## Fish          -0.0219     0.0020
## Cereals       -0.0236     0.0162
## Starchy foods  0.0647    -0.0566
## Nuts           0.0001    -0.0573
## Fruits/Veg    -0.0573    -0.0001
```

Frobenius norm of error:

```
## [1] 0.4549
```

Both of these are identical to that in A.

C)
i)

Variables	load 1	load 2	load 3	load 4	commun.	spec. var.
Red meat	0.09	0.41	0.57	0.2	0.55	0.453
White meat	-0.25	0.89	-0.32	0.01	0.95	0.049
Eggs	0.08	0.83	0.31	0.33	0.9	0.101
Milk	0.16	0.56	0.54	-0.22	0.68	0.322
Fish	0.99	0	-0.04	0	0.99	0.014
Cereals	-0.54	-0.7	-0.24	-0.04	0.84	0.156
Starchy foods	0.41	0.47	-0.04	0.02	0.39	0.609
Nuts	-0.15	-0.8	-0.11	0.44	0.87	0.133
Fruits/Veg	0.25	-0.15	-0.4	0.58	0.58	0.417
Var. Acc. For	1.63	3.34	1.05	0.73		
Prop. Tot. Var	0.18	0.37	0.12	0.08		

ii) Estimate

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.9998     0.1596  0.5906  0.5110  0.0626 -0.4831        0.2097
## White meat      0.1596     0.9999  0.6197  0.2784 -0.2341 -0.4123        0.3313
## Eggs            0.5906     0.6197  1.0001  0.5686  0.0656 -0.7147        0.4188
## Milk            0.5110     0.2784  0.5686  1.0004  0.1365 -0.5984        0.2999
## Fish            0.0626    -0.2341  0.0656  0.1365  0.9999 -0.5239        0.4053
## Cereals        -0.4831    -0.4123 -0.7147 -0.5984 -0.5239  0.9998       -0.5418
## Starchy foods   0.2097     0.3313  0.4188  0.2999  0.4053 -0.5418        1.0000
## Nuts           -0.3188    -0.6346 -0.5666 -0.6255 -0.1475  0.6527       -0.4261
## Fruits/Veg     -0.1527    -0.0642 -0.0340 -0.3873  0.2658  0.0399        0.0620
##                   Nuts Fruits/Veg
## Red meat       -0.3188    -0.1527
## White meat     -0.6346    -0.0642
## Eggs           -0.5666    -0.0340
## Milk           -0.6255    -0.3873
## Fish           -0.1475     0.2658
## Cereals         0.6527     0.0399
## Starchy foods -0.4261     0.0620
## Nuts            0.9995     0.3782
## Fruits/Veg      0.3782     1.0004
```

iii)
Error Matrix

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.0002    -0.0066 -0.0050 -0.0081 -0.0017 -0.0168        -0.0743
## White meat     -0.0066     0.0001  0.0007  0.0030  0.0000 -0.0015        -0.0175
## Eggs           -0.0050     0.0007 -0.0001  0.0069  0.0000  0.0022         0.0334
## Milk           -0.0081     0.0030  0.0069 -0.0004  0.0014  0.0056        -0.0775
## Fish           -0.0017     0.0000  0.0000  0.0014  0.0001 -0.0003        -0.0014
## Cereals        -0.0168    -0.0015  0.0022  0.0056 -0.0003  0.0002         0.0086
## Starchy foods  -0.0743    -0.0175  0.0334 -0.0775 -0.0014  0.0086         0.0000
## Nuts           -0.0307    -0.0004  0.0068  0.0044  0.0004 -0.0017        -0.0482
## Fruits/Veg      0.0785     0.0029 -0.0115 -0.0210  0.0003  0.0066         0.0224
##                  Nuts Fruits/Veg
## Red meat      -0.0307     0.0785
## White meat    -0.0004     0.0029
## Eggs           0.0068    -0.0115
## Milk           0.0044    -0.0210
```

```
## Fish           0.0004     0.0003
## Cereals       -0.0017     0.0066
## Starchy foods -0.0482     0.0224
## Nuts           0.0005    -0.0032
## Fruits/Veg    -0.0032    -0.0004
```

Frobenius norm of error:

```
## [1] 0.2834
```

D)

i)

Variables	load 1	load 2	load 3	load 4	commun.	spec. var.
Red meat	0.07	0.04	0.72	-0.15	0.55	0.453
White meat	-0.07	0.96	0.17	-0.05	0.95	0.049
Eggs	0.14	0.52	0.78	0.01	0.9	0.101
Milk	0.23	0.18	0.57	-0.52	0.68	0.322
Fish	0.97	-0.18	0.03	0.14	0.99	0.014
Cereals	-0.61	-0.37	-0.56	0.14	0.84	0.156
Starchy foods	0.47	0.34	0.23	0	0.39	0.609
Nuts	-0.33	-0.61	-0.26	0.56	0.87	0.133
Fruits/Veg	0.17	0	-0.08	0.74	0.58	0.417
Var. Acc. For	1.76	1.87	1.93	1.19		
Prop. Tot. Var	0.2	0.21	0.21	0.13		

ii) Estimate:

```
##              Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat       0.9998     0.1596  0.5906  0.5110  0.0626 -0.4831        0.2097
## White meat     0.1596     0.9999  0.6197  0.2784 -0.2341 -0.4123        0.3313
## Eggs           0.5906     0.6197  1.0001  0.5686  0.0656 -0.7147        0.4188
## Milk           0.5110     0.2784  0.5686  1.0004  0.1365 -0.5984        0.2999
## Fish           0.0626    -0.2341  0.0656  0.1365  0.9999 -0.5239        0.4053
## Cereals       -0.4831    -0.4123 -0.7147 -0.5984 -0.5239  0.9998       -0.5418
## Starchy foods  0.2097     0.3313  0.4188  0.2999  0.4053 -0.5418        1.0000
## Nuts          -0.3188    -0.6346 -0.5666 -0.6255 -0.1475  0.6527       -0.4261
## Fruits/Veg    -0.1527    -0.0642 -0.0340 -0.3873  0.2658  0.0399        0.0620
##                  Nuts Fruits/Veg
## Red meat      -0.3188    -0.1527
## White meat    -0.6346    -0.0642
## Eggs          -0.5666    -0.0340
## Milk          -0.6255    -0.3873
## Fish          -0.1475     0.2658
## Cereals        0.6527     0.0399
## Starchy foods -0.4261     0.0620
## Nuts           0.9995     0.3782
## Fruits/Veg     0.3782     1.0004
```

Identical to that in C).

iii) Error Matrix

```
##              Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat       0.0002    -0.0066 -0.0050 -0.0081 -0.0017 -0.0168       -0.0743
```

```
## White meat     -0.0066     0.0001  0.0007  0.0030  0.0000 -0.0015       -0.0175
## Eggs           -0.0050     0.0007 -0.0001  0.0069  0.0000  0.0022        0.0334
## Milk           -0.0081     0.0030  0.0069 -0.0004  0.0014  0.0056       -0.0775
## Fish           -0.0017     0.0000  0.0000  0.0014  0.0001 -0.0003       -0.0014
## Cereals        -0.0168    -0.0015  0.0022  0.0056 -0.0003  0.0002        0.0086
## Starchy foods  -0.0743    -0.0175  0.0334 -0.0775 -0.0014  0.0086        0.0000
## Nuts           -0.0307    -0.0004  0.0068  0.0044  0.0004 -0.0017       -0.0482
## Fruits/Veg      0.0785     0.0029 -0.0115 -0.0210  0.0003  0.0066        0.0224
##                   Nuts Fruits/Veg
## Red meat       -0.0307     0.0785
## White meat     -0.0004     0.0029
## Eggs            0.0068    -0.0115
## Milk            0.0044    -0.0210
## Fish            0.0004     0.0003
## Cereals        -0.0017     0.0066
## Starchy foods -0.0482     0.0224
## Nuts            0.0005    -0.0032
## Fruits/Veg     -0.0032    -0.0004
```

Frobenius norm of error:

```
## [1] 0.2834
```

Both are identical to those in C).

1)E)

i) Based on the values of the error matrices, both methods (principal components and maximum likelihood), with and without rotations produce reasonably good approximations, as seen from the reasonably small off-diagonal entries.

ii) Based on the value of the error matrices, maximum likelihood produces the best approximation (regardless of rotation), as the Frobenius norm of the error is much smaller, meaning that the eigenvalues we drop are smaller. The varimax rotation here does not affect the error matrices. However, rotations produce better approximations in both methods, each variable

good job! 64

is a potential dependency to at most, 1 load, while as before, this was not as apparent. Thus, the rotations were successful, with the varimax rotation on the MLE method more distinctly so.

Q1

1)F)

```
library(readxl)
data <- read_excel("europe_protein_data.xls",col_names=TRUE)
```

```
## New names:
## * `` -> ...1
```

```
data<- data[,-1]
```

Code for 1A)i)

```
R<-cor(data)
out<-eigen(R,symmetric=T,only.values=F)
library(psych)
```

```
## Warning: package 'psych' was built under R version 4.1.3
```

```
fa_pc1<-principal(R,nfactors=4,n.obs=25,rotate="none")
L<-fa_pc1$loadings
com<-fa_pc1$communality
fa_pc1
```

```
## Principal Components Analysis
## Call: principal(r = R, nfactors = 4, rotate = "none", n.obs = 25)
## Standardized loadings (pattern matrix) based upon correlation matrix
##                  PC1   PC2   PC3   PC4   h2    u2 com
## Red meat        0.61 -0.07 -0.32  0.63 0.87 0.129 2.5
## White meat      0.62 -0.30  0.66 -0.04 0.92 0.082 2.4
## Eggs            0.85 -0.05  0.19  0.31 0.86 0.138 1.4
## Milk            0.76 -0.24 -0.41  0.00 0.80 0.205 1.8
## Fish            0.27  0.83 -0.34 -0.21 0.92 0.081 1.7
## Cereals        -0.88 -0.30  0.10 -0.01 0.87 0.133 1.3
## Starchy foods  0.59  0.45  0.26 -0.33 0.73 0.268 2.9
## Nuts          -0.84  0.18 -0.06  0.32 0.85 0.151 1.4
## Fruits/Veg    -0.22  0.69  0.43  0.45 0.91 0.090 2.8
##
##                         PC1  PC2  PC3  PC4
## SS loadings            4.01 1.63 1.13 0.95
## Proportion Var         0.45 0.18 0.13 0.11
## Cumulative Var         0.45 0.63 0.75 0.86
## Proportion Explained   0.52 0.21 0.15 0.12
## Cumulative Proportion 0.52 0.73 0.88 1.00
```

```
##
## Mean item complexity =  2
## Test of the hypothesis that 4 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.05
##  with the empirical chi square  5.17  with prob <  0.52
##
## Fit based upon off diagonal values = 0.98
```

```
L
```

```
##
## Loadings:
##               PC1    PC2    PC3    PC4
## Red meat       0.606         -0.316  0.632
## White meat     0.622 -0.303  0.663
## Eggs           0.854          0.193  0.306
## Milk           0.756 -0.236 -0.410
## Fish           0.272  0.827 -0.341 -0.211
## Cereals       -0.876 -0.299  0.102
## Starchy foods  0.595  0.451  0.258 -0.329
## Nuts          -0.841  0.183         0.323
## Fruits/Veg    -0.221  0.686  0.433  0.451
##
##                  PC1   PC2   PC3   PC4
## SS loadings    4.006 1.635 1.128 0.955
## Proportion Var 0.445 0.182 0.125 0.106
## Cumulative Var 0.445 0.627 0.752 0.858
```

```
com
```

```
##      Red meat    White meat          Eggs          Milk          Fish
##     0.8709187     0.9184707     0.8622249     0.7951075     0.9187090
##       Cereals Starchy foods          Nuts    Fruits/Veg
##     0.8672571     0.7323342     0.8489215     0.9100768
```

Code for A)ii)

```
Est1 <- (L%*% t(L) + diag(c(0.129,0.082,0.138,0.205,0.081,0.133,
                            0.268,0.151,0.090)))
round(Est1,4)
```

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.9999     0.1661  0.6529  0.6023  0.0795 -0.5453         0.0386
## White meat      0.1661     1.0005  0.6613  0.2702 -0.3002 -0.3865         0.4161
## Eggs            0.6529     0.6613  1.0002  0.5764  0.0642 -0.7170         0.4368
## Milk            0.6023     0.2702  0.5764  1.0001  0.1505 -0.6337         0.2387
## Fish            0.0795    -0.3002  0.0642  0.1505  0.9997 -0.5183         0.5160
## Cereals        -0.5453    -0.3865 -0.7170 -0.6337 -0.5183  1.0003        -0.6277
## Starchy foods   0.0386     0.4161  0.4368  0.2387  0.5160 -0.6277         1.0003
## Nuts           -0.3007    -0.6284 -0.6392 -0.6567 -0.1253  0.6746        -0.5390
## Fruits/Veg     -0.0348    -0.0745  0.0018 -0.5077  0.2641  0.0303         0.1410
```

```
##                  Nuts Fruits/Veg
## Red meat      -0.3007    -0.0348
## White meat    -0.6284    -0.0745
## Eggs          -0.6392     0.0018
## Milk          -0.6567    -0.5077
## Fish          -0.1253     0.2641
## Cereals        0.6746     0.0303
## Starchy foods -0.5390     0.1410
## Nuts           0.9999     0.4323
## Fruits/Veg     0.4323     1.0001
```

Code for A)iii)

```
#R-L*L^T + Phi
#calculate error matrix E = R - (L*L^T + Phi)

E1 <- R - (L%*% t(L) + diag(c(0.129,0.082,0.138,0.205,0.081,0.133,
                              0.268,0.151,0.090)))
round(E1,4)
```

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.0001    -0.0131 -0.0673 -0.0994 -0.0186  0.0454        0.0968
## White meat     -0.0131    -0.0005 -0.0408  0.0113  0.0662 -0.0273       -0.1023
## Eggs           -0.0673    -0.0408 -0.0002 -0.0009  0.0014  0.0046        0.0154
## Milk           -0.0994     0.0113 -0.0009 -0.0001 -0.0126  0.0410       -0.0163
## Fish           -0.0186     0.0662  0.0014 -0.0126  0.0003 -0.0059       -0.1122
## Cereals         0.0454    -0.0273  0.0046  0.0410 -0.0059 -0.0003        0.0945
## Starchy foods   0.0968    -0.1023  0.0154 -0.0163 -0.1122  0.0945       -0.0003
## Nuts           -0.0488    -0.0065  0.0794  0.0357 -0.0219 -0.0236        0.0647
## Fruits/Veg     -0.0394     0.0132 -0.0474  0.0993  0.0020  0.0162       -0.0566
##                  Nuts Fruits/Veg
## Red meat      -0.0488    -0.0394
## White meat    -0.0065     0.0132
## Eggs           0.0794    -0.0474
## Milk           0.0357     0.0993
## Fish          -0.0219     0.0020
## Cereals       -0.0236     0.0162
## Starchy foods  0.0647    -0.0566
## Nuts           0.0001    -0.0573
## Fruits/Veg    -0.0573    -0.0001
```

```
E1norm <- norm(R - (L%*% t(L) + diag(c(0.129,0.082,0.138,0.205,0.081,0.133,
                              0.268,0.151,0.090))),type="F")
round(E1norm,4)
```

```
## [1] 0.4549
```

Code for B) i)

```
fa_rotation1<-principal(R,nfactors=4,n.obs=25,rotate="varimax")
L_rotation1<-fa_rotation1$loadings
fa_rotation1
```

```
## Principal Components Analysis
## Call: principal(r = R, nfactors = 4, rotate = "varimax", n.obs = 25)
## Standardized loadings (pattern matrix) based upon correlation matrix
##                 RC3   RC4   RC2   RC1   h2    u2 com
## Red meat       0.05  0.93  0.01 -0.04 0.87 0.129 1.0
## White meat     0.94  0.13 -0.10 -0.05 0.92 0.082 1.1
## Eggs           0.63  0.66  0.16 -0.02 0.86 0.138 2.1
## Milk           0.20  0.61  0.22 -0.58 0.80 0.205 2.5
## Fish          -0.23  0.09  0.92  0.10 0.92 0.081 1.2
## Cereals       -0.39 -0.55 -0.62  0.15 0.87 0.133 2.8
## Starchy foods  0.51  0.00  0.68  0.03 0.73 0.268 1.9
## Nuts          -0.64 -0.26 -0.33  0.52 0.85 0.151 2.9
## Fruits/Veg    -0.01  0.00  0.18  0.94 0.91 0.090 1.1
##
##                        RC3  RC4  RC2  RC1
## SS loadings           2.20 2.08 1.93 1.52
## Proportion Var        0.24 0.23 0.21 0.17
## Cumulative Var        0.24 0.48 0.69 0.86
## Proportion Explained  0.29 0.27 0.25 0.20
## Cumulative Proportion 0.29 0.55 0.80 1.00
##
## Mean item complexity =  1.8
## Test of the hypothesis that 4 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.05
##  with the empirical chi square  5.17  with prob <  0.52
##
## Fit based upon off diagonal values = 0.98
```

```
L_rotation1
```

```
##
## Loadings:
##               RC3    RC4    RC2    RC1
## Red meat             0.931
## White meat     0.943  0.127
## Eggs           0.628  0.664  0.164
## Milk           0.197  0.610  0.219 -0.579
## Fish          -0.228         0.921  0.105
## Cereals       -0.394 -0.549 -0.624  0.145
## Starchy foods  0.513         0.684
## Nuts          -0.638 -0.263 -0.327  0.516
## Fruits/Veg                   0.178  0.937
##
##                  RC3   RC4   RC2   RC1
## SS loadings    2.203 2.075 1.929 1.517
## Proportion Var 0.245 0.231 0.214 0.169
## Cumulative Var 0.245 0.475 0.690 0.858
```

Code for B)ii)

```
Est2<-(L_rotation1%*% t(L_rotation1) + diag(c(0.129,0.082,0.138,0.205,0.081,0.133,0.268,0.151,0.090)))
round(Est2,4)
```

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.9999     0.1661  0.6529  0.6023  0.0795 -0.5453        0.0386
## White meat      0.1661     1.0005  0.6613  0.2702 -0.3002 -0.3865        0.4161
## Eggs            0.6529     0.6613  1.0002  0.5764  0.0642 -0.7170        0.4368
## Milk            0.6023     0.2702  0.5764  1.0001  0.1505 -0.6337        0.2387
## Fish            0.0795    -0.3002  0.0642  0.1505  0.9997 -0.5183        0.5160
## Cereals        -0.5453    -0.3865 -0.7170 -0.6337 -0.5183  1.0003       -0.6277
## Starchy foods   0.0386     0.4161  0.4368  0.2387  0.5160 -0.6277        1.0003
## Nuts           -0.3007    -0.6284 -0.6392 -0.6567 -0.1253  0.6746       -0.5390
## Fruits/Veg     -0.0348    -0.0745  0.0018 -0.5077  0.2641  0.0303        0.1410
##                  Nuts Fruits/Veg
## Red meat      -0.3007    -0.0348
## White meat    -0.6284    -0.0745
## Eggs          -0.6392     0.0018
## Milk          -0.6567    -0.5077
## Fish          -0.1253     0.2641
## Cereals        0.6746     0.0303
## Starchy foods -0.5390     0.1410
## Nuts           0.9999     0.4323
## Fruits/Veg     0.4323     1.0001
```

Code for B)iii)

```
E1b <- R-(L_rotation1%*% t(L_rotation1) + diag(c(0.129,0.082,0.138,0.205,0.081,0.133,0.268,0.151,0.090)
round(E1b,4)
```

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.0001    -0.0131 -0.0673 -0.0994 -0.0186  0.0454        0.0968
## White meat     -0.0131    -0.0005 -0.0408  0.0113  0.0662 -0.0273       -0.1023
## Eggs           -0.0673    -0.0408 -0.0002 -0.0009  0.0014  0.0046        0.0154
## Milk           -0.0994     0.0113 -0.0009 -0.0001 -0.0126  0.0410       -0.0163
## Fish           -0.0186     0.0662  0.0014 -0.0126  0.0003 -0.0059       -0.1122
## Cereals         0.0454    -0.0273  0.0046  0.0410 -0.0059 -0.0003        0.0945
## Starchy foods   0.0968    -0.1023  0.0154 -0.0163 -0.1122  0.0945       -0.0003
## Nuts           -0.0488    -0.0065  0.0794  0.0357 -0.0219 -0.0236        0.0647
## Fruits/Veg     -0.0394     0.0132 -0.0474  0.0993  0.0020  0.0162       -0.0566
##                  Nuts Fruits/Veg
## Red meat      -0.0488    -0.0394
## White meat    -0.0065     0.0132
## Eggs           0.0794    -0.0474
## Milk           0.0357     0.0993
## Fish          -0.0219     0.0020
## Cereals       -0.0236     0.0162
## Starchy foods  0.0647    -0.0566
## Nuts           0.0001    -0.0573
## Fruits/Veg    -0.0573    -0.0001
```

```
E1bnorm <- norm(R - (L_rotation1%*% t(L_rotation1) + diag(c(0.129,0.082,0.138,0.205,0.081,0.133,0.268,0
              type="F")
round(E1bnorm,4)
```

```
## [1] 0.4549
```

Code for 1C)i)

```
fa(cor(data),nfactors=4,n.obs=25,rotate="none",fm='ml')
```

```
## Factor Analysis using method =  ml
## Call: fa(r = cor(data), nfactors = 4, n.obs = 25, rotate = "none",
##     fm = "ml")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                 ML2   ML1   ML3   ML4   h2    u2 com
## Red meat       0.41  0.09  0.57  0.20 0.55 0.453 2.1
## White meat     0.89 -0.25 -0.32  0.01 0.95 0.049 1.4
## Eggs           0.83  0.08  0.31  0.33 0.90 0.101 1.6
## Milk           0.56  0.16  0.54 -0.22 0.68 0.322 2.5
## Fish           0.00  0.99 -0.04  0.00 0.99 0.014 1.0
## Cereals       -0.70 -0.54 -0.24 -0.04 0.84 0.156 2.1
## Starchy foods  0.47  0.41 -0.04  0.02 0.39 0.609 2.0
## Nuts          -0.80 -0.15 -0.11  0.44 0.87 0.133 1.7
## Fruits/Veg    -0.15  0.25 -0.40  0.58 0.58 0.417 2.4
##
##                        ML2  ML1  ML3  ML4
## SS loadings           3.34 1.63 1.05 0.73
## Proportion Var        0.37 0.18 0.12 0.08
## Cumulative Var        0.37 0.55 0.67 0.75
## Proportion Explained  0.49 0.24 0.16 0.11
## Cumulative Proportion 0.49 0.74 0.89 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 factors are sufficient.
##
## The degrees of freedom for the null model are  36  and the objective function was  5.7 with Chi Squa
## The degrees of freedom for the model are 6  and the objective function was  0.21
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.06
##
## The harmonic number of observations is  25 with the empirical chi square  1.21  with prob <  0.98
## The total number of observations was  25  with Likelihood Chi Square =  3.61  with prob <  0.73
##
## Tucker Lewis Index of factoring reliability =  1.224
## RMSEA index =  0  and the 90 % confidence intervals are  0 0.194
## BIC =  -15.7
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                 ML2  ML1  ML3  ML4
## Correlation of (regression) scores with factors   0.98 0.99 0.92 0.88
## Multiple R square of scores with factors          0.97 0.99 0.85 0.78
## Minimum correlation of possible factor scores     0.94 0.97 0.70 0.56
```

```
mle1<- factanal(data,factors=4,n.obs=25,rotation="none")
mle1
```

```
##
## Call:
```

```
## factanal(x = data, factors = 4, n.obs = 25, rotation = "none")
##
## Uniquenesses:
##      Red meat    White meat          Eggs          Milk          Fish
##         0.453         0.049         0.101         0.322         0.014
##       Cereals Starchy foods          Nuts    Fruits/Veg
##         0.156         0.609         0.133         0.417
##
## Loadings:
##               Factor1 Factor2 Factor3 Factor4
## Red meat       0.412           0.575   0.197
## White meat     0.886  -0.248  -0.322
## Eggs           0.831           0.307   0.329
## Milk           0.557   0.161   0.541  -0.222
## Fish                   0.992
## Cereals       -0.703  -0.540  -0.239
## Starchy foods  0.472   0.408
## Nuts          -0.801  -0.154  -0.106   0.436
## Fruits/Veg    -0.150   0.252  -0.397   0.583
##
##                Factor1 Factor2 Factor3 Factor4
## SS loadings      3.336   1.630   1.050   0.729
## Proportion Var   0.371   0.181   0.117   0.081
## Cumulative Var   0.371   0.552   0.669   0.750
##
## Test of the hypothesis that 4 factors are sufficient.
## The chi square statistic is 3.61 on 6 degrees of freedom.
## The p-value is 0.729
```

```
Lmle1<- mle1$loadings
Lmle1
```

```
##
## Loadings:
##               Factor1 Factor2 Factor3 Factor4
## Red meat       0.412           0.575   0.197
## White meat     0.886  -0.248  -0.322
## Eggs           0.831           0.307   0.329
## Milk           0.557   0.161   0.541  -0.222
## Fish                   0.992
## Cereals       -0.703  -0.540  -0.239
## Starchy foods  0.472   0.408
## Nuts          -0.801  -0.154  -0.106   0.436
## Fruits/Veg    -0.150   0.252  -0.397   0.583
##
##                Factor1 Factor2 Factor3 Factor4
## SS loadings      3.336   1.630   1.050   0.729
## Proportion Var   0.371   0.181   0.117   0.081
## Cumulative Var   0.371   0.552   0.669   0.750
```

Code for 1C)ii)

```
Est3<-(Lmle1%*% t(Lmle1) + diag(c(0.453,0.049,0.101,0.322,0.014,
                                  0.156,0.609,0.133,0.417)))
round(Est3,4)
```

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.9998     0.1596  0.5906  0.5110  0.0626 -0.4831        0.2097
## White meat      0.1596     0.9999  0.6197  0.2784 -0.2341 -0.4123        0.3313
## Eggs            0.5906     0.6197  1.0001  0.5686  0.0656 -0.7147        0.4188
## Milk            0.5110     0.2784  0.5686  1.0004  0.1365 -0.5984        0.2999
## Fish            0.0626    -0.2341  0.0656  0.1365  0.9999 -0.5239        0.4053
## Cereals        -0.4831    -0.4123 -0.7147 -0.5984 -0.5239  0.9998       -0.5418
## Starchy foods   0.2097     0.3313  0.4188  0.2999  0.4053 -0.5418        1.0000
## Nuts           -0.3188    -0.6346 -0.5666 -0.6255 -0.1475  0.6527       -0.4261
## Fruits/Veg     -0.1527    -0.0642 -0.0340 -0.3873  0.2658  0.0399        0.0620
##                  Nuts Fruits/Veg
## Red meat      -0.3188    -0.1527
## White meat    -0.6346    -0.0642
## Eggs          -0.5666    -0.0340
## Milk          -0.6255    -0.3873
## Fish          -0.1475     0.2658
## Cereals        0.6527     0.0399
## Starchy foods -0.4261     0.0620
## Nuts           0.9995     0.3782
## Fruits/Veg     0.3782     1.0004
```

Code for 1)C)iii)

```
E1c <- R- (Lmle1%*% t(Lmle1) + diag(c(0.453,0.049,0.101,0.322,0.014,
                                  0.156,0.609,0.133,0.417)))
round(E1c,4)
```

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.0002    -0.0066 -0.0050 -0.0081 -0.0017 -0.0168       -0.0743
## White meat     -0.0066     0.0001  0.0007  0.0030  0.0000 -0.0015       -0.0175
## Eggs           -0.0050     0.0007 -0.0001  0.0069  0.0000  0.0022        0.0334
## Milk           -0.0081     0.0030  0.0069 -0.0004  0.0014  0.0056       -0.0775
## Fish           -0.0017     0.0000  0.0000  0.0014  0.0001 -0.0003       -0.0014
## Cereals        -0.0168    -0.0015  0.0022  0.0056 -0.0003  0.0002        0.0086
## Starchy foods  -0.0743    -0.0175  0.0334 -0.0775 -0.0014  0.0086        0.0000
## Nuts           -0.0307    -0.0004  0.0068  0.0044  0.0004 -0.0017       -0.0482
## Fruits/Veg      0.0785     0.0029 -0.0115 -0.0210  0.0003  0.0066        0.0224
##                  Nuts Fruits/Veg
## Red meat      -0.0307     0.0785
## White meat    -0.0004     0.0029
## Eggs           0.0068    -0.0115
## Milk           0.0044    -0.0210
## Fish           0.0004     0.0003
## Cereals       -0.0017     0.0066
## Starchy foods -0.0482     0.0224
## Nuts           0.0005    -0.0032
## Fruits/Veg    -0.0032    -0.0004
```

```
E1cnorm <- norm(R-(Lmle1%*% t(Lmle1) + diag(c(0.453,0.049,0.101,0.322,0.014,
                                0.156,0.609,0.133,0.417))))
round(E1cnorm,4)
```

```
## [1] 0.2834
```

Code for 1D) i)

```
fa(cor(data),nfactors=4,n.obs=25,rotate="varimax",fm='ml')
```

```
## Factor Analysis using method =  ml
## Call: fa(r = cor(data), nfactors = 4, n.obs = 25, rotate = "varimax",
##     fm = "ml")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                 ML3   ML2   ML1   ML4   h2    u2 com
## Red meat       0.72  0.04  0.07 -0.15 0.55 0.453 1.1
## White meat     0.17  0.96 -0.07 -0.05 0.95 0.049 1.1
## Eggs           0.78  0.52  0.14  0.01 0.90 0.101 1.8
## Milk           0.57  0.18  0.23 -0.52 0.68 0.322 2.5
## Fish           0.03 -0.18  0.97  0.14 0.99 0.014 1.1
## Cereals       -0.56 -0.37 -0.61  0.14 0.84 0.156 2.8
## Starchy foods  0.23  0.34  0.47  0.00 0.39 0.609 2.3
## Nuts          -0.26 -0.61 -0.33  0.56 0.87 0.133 3.0
## Fruits/Veg    -0.08  0.00  0.17  0.74 0.58 0.417 1.1
##
##                        ML3  ML2  ML1  ML4
## SS loadings           1.93 1.87 1.76 1.19
## Proportion Var        0.21 0.21 0.20 0.13
## Cumulative Var        0.21 0.42 0.62 0.75
## Proportion Explained  0.29 0.28 0.26 0.18
## Cumulative Proportion 0.29 0.56 0.82 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 factors are sufficient.
##
## The degrees of freedom for the null model are  36  and the objective function was  5.7 with Chi Squa
## The degrees of freedom for the model are 6  and the objective function was  0.21
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.06
##
## The harmonic number of observations is  25 with the empirical chi square  1.21  with prob <  0.98
## The total number of observations was  25  with Likelihood Chi Square =  3.61  with prob <  0.73
##
## Tucker Lewis Index of factoring reliability =  1.224
## RMSEA index =  0  and the 90 % confidence intervals are  0 0.194
## BIC =  -15.7
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                ML3  ML2  ML1  ML4
## Correlation of (regression) scores with factors   0.93 0.97 0.99 0.90
## Multiple R square of scores with factors          0.86 0.94 0.98 0.80
## Minimum correlation of possible factor scores     0.72 0.88 0.96 0.61
```

```
mle2<- factanal(data,factors=4,n.obs=25,rotation="varimax")
mle2
```

```
##
## Call:
## factanal(x = data, factors = 4, n.obs = 25, rotation = "varimax")
##
## Uniquenesses:
##      Red meat    White meat          Eggs          Milk          Fish
##         0.453         0.049         0.101         0.322         0.014
##       Cereals Starchy foods          Nuts    Fruits/Veg
##         0.156         0.609         0.133         0.417
##
## Loadings:
##               Factor1 Factor2 Factor3 Factor4
## Red meat       0.720                   -0.147
## White meat     0.168   0.957
## Eggs           0.780   0.520   0.138
## Milk           0.572   0.179   0.228  -0.516
## Fish                  -0.177   0.967   0.136
## Cereals       -0.564  -0.366  -0.610   0.137
## Starchy foods  0.227   0.339   0.474
## Nuts          -0.264  -0.610  -0.334   0.560
## Fruits/Veg                    0.173   0.740
##
##                Factor1 Factor2 Factor3 Factor4
## SS loadings      1.930   1.872   1.755   1.189
## Proportion Var   0.214   0.208   0.195   0.132
## Cumulative Var   0.214   0.422   0.617   0.750
##
## Test of the hypothesis that 4 factors are sufficient.
## The chi square statistic is 3.61 on 6 degrees of freedom.
## The p-value is 0.729
```

```
Lmle2<- mle2$loadings
Lmle2
```

```
##
## Loadings:
##               Factor1 Factor2 Factor3 Factor4
## Red meat       0.720                   -0.147
## White meat     0.168   0.957
## Eggs           0.780   0.520   0.138
## Milk           0.572   0.179   0.228  -0.516
## Fish                  -0.177   0.967   0.136
## Cereals       -0.564  -0.366  -0.610   0.137
## Starchy foods  0.227   0.339   0.474
## Nuts          -0.264  -0.610  -0.334   0.560
## Fruits/Veg                    0.173   0.740
##
##                Factor1 Factor2 Factor3 Factor4
## SS loadings      1.930   1.872   1.755   1.189
## Proportion Var   0.214   0.208   0.195   0.132
```

```
## Cumulative Var   0.214   0.422   0.617   0.750
```

Code for 1)D)ii)

```
Est4<-(Lmle2%*% t(Lmle2) + diag(c(0.453,0.049,0.101,0.322,0.014,
                                  0.156,0.609,0.133,0.417)))
round(Est4,4)
```

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.9998     0.1596  0.5906  0.5110  0.0626 -0.4831        0.2097
## White meat      0.1596     0.9999  0.6197  0.2784 -0.2341 -0.4123        0.3313
## Eggs            0.5906     0.6197  1.0001  0.5686  0.0656 -0.7147        0.4188
## Milk            0.5110     0.2784  0.5686  1.0004  0.1365 -0.5984        0.2999
## Fish            0.0626    -0.2341  0.0656  0.1365  0.9999 -0.5239        0.4053
## Cereals        -0.4831    -0.4123 -0.7147 -0.5984 -0.5239  0.9998       -0.5418
## Starchy foods   0.2097     0.3313  0.4188  0.2999  0.4053 -0.5418        1.0000
## Nuts           -0.3188    -0.6346 -0.5666 -0.6255 -0.1475  0.6527       -0.4261
## Fruits/Veg     -0.1527    -0.0642 -0.0340 -0.3873  0.2658  0.0399        0.0620
##                  Nuts Fruits/Veg
## Red meat      -0.3188    -0.1527
## White meat    -0.6346    -0.0642
## Eggs          -0.5666    -0.0340
## Milk          -0.6255    -0.3873
## Fish          -0.1475     0.2658
## Cereals        0.6527     0.0399
## Starchy foods -0.4261     0.0620
## Nuts           0.9995     0.3782
## Fruits/Veg     0.3782     1.0004
```

Code for 1D)iii)

```
E1d <- R- (Lmle2%*% t(Lmle2) + diag(c(0.453,0.049,0.101,0.322,0.014,
                                  0.156,0.609,0.133,0.417)))
round(E1d,4)
```

```
##               Red meat White meat    Eggs    Milk    Fish Cereals Starchy foods
## Red meat        0.0002    -0.0066 -0.0050 -0.0081 -0.0017 -0.0168       -0.0743
## White meat     -0.0066     0.0001  0.0007  0.0030  0.0000 -0.0015       -0.0175
## Eggs           -0.0050     0.0007 -0.0001  0.0069  0.0000  0.0022        0.0334
## Milk           -0.0081     0.0030  0.0069 -0.0004  0.0014  0.0056       -0.0775
## Fish           -0.0017     0.0000  0.0000  0.0014  0.0001 -0.0003       -0.0014
## Cereals        -0.0168    -0.0015  0.0022  0.0056 -0.0003  0.0002        0.0086
## Starchy foods  -0.0743    -0.0175  0.0334 -0.0775 -0.0014  0.0086        0.0000
## Nuts           -0.0307    -0.0004  0.0068  0.0044  0.0004 -0.0017       -0.0482
## Fruits/Veg      0.0785     0.0029 -0.0115 -0.0210  0.0003  0.0066        0.0224
##                  Nuts Fruits/Veg
## Red meat      -0.0307     0.0785
## White meat    -0.0004     0.0029
## Eggs           0.0068    -0.0115
## Milk           0.0044    -0.0210
## Fish           0.0004     0.0003
## Cereals       -0.0017     0.0066
```

```
## Starchy foods -0.0482     0.0224
## Nuts           0.0005    -0.0032
## Fruits/Veg    -0.0032    -0.0004
```

```
E1dnorm <- norm(R-(Lmle2%*% t(Lmle2) + diag(c(0.453,0.049,0.101,0.322,0.014,
                                  0.156,0.609,0.133,0.417))))
round(E1dnorm,4)
```

```
## [1] 0.2834
```