



## STAT 485/685 Assignment Cover Page

Student Name

Heewon Oh

SFU Student Number

301268860

SFU email address

heewono@sfu.ca

Assignment Number

7

Due Date

Dec 16, 2021

Provide references for any data sets used in this assignment

Available on Crowdmark:

<https://app.crowdmark.com/student/assessments/project-7>

<https://www.rdocumentation.org/packages/forecast/versions/8.15/topics/forecast>

List software used in this assignment.

R, Rstudio

List ALL resources used to complete this assignment, including books, internet sources and people.

Tutorial code, my own previous assignments in this course (specifically projects 4 and 6) and lecture notes/videos.

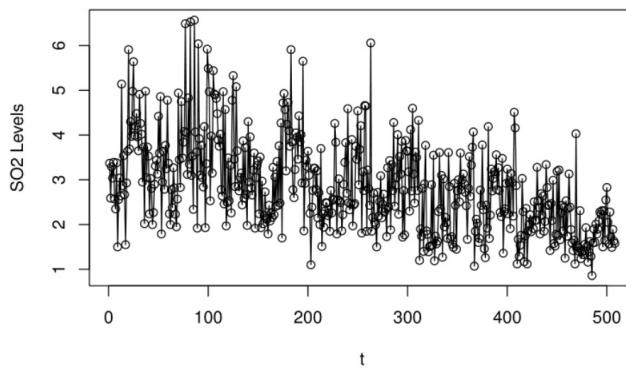
- I personally completed the computations and wrote the solutions submitted in this document.

Department of Statistics and Actuarial Science

**SFU** SIMON FRASER  
UNIVERSITY

2)

A)

**SO<sub>2</sub> Levels Over Time**

B) There appears to be a potential negative trend. It is difficult to conclude the type of the trend but it appears that it could be linear, polynomial, or even negative logarithmic; we do not have enough data to be sure from a quick inspection.



C) When we attempt a cubic polynomial OLS regression, the coefficients are:

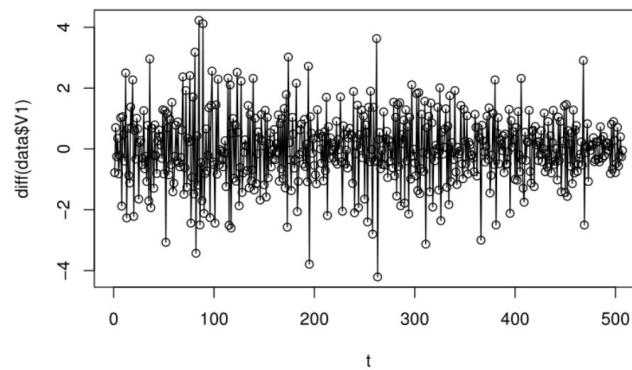
(Intercept) 3.516e+00  
time 2.206e-04  
time2 -1.329e-05  
time3 1.274e-08

D) We use d=1 because the time squared coefficient is roughly 0.06 the magnitude of time. We have a bias against using large d so we attempt this first. When we plot the 1st difference of the



data, we also see visually that the data now appears to have no trend. When we attempt to use a diagnostic tool by regressing the 1st-difference data against order-5 polynomial of time, we find coefficients we notice that the coefficients for other variables outside of time, are comparatively much smaller. We also notice that the residuals are very small(to the power of e-4). When attempt this again with the same diagnostic tool with 2nd-difference data, we get larger residual coefficients and again, only time is relatively important by size.

E)

**First Differenced Data**

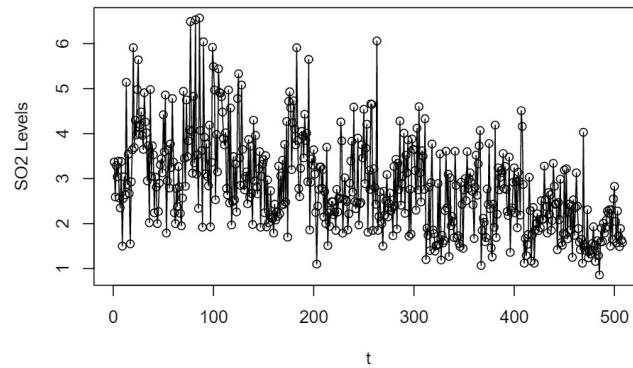
## P7 Q2 Code

Heewon Oh

14/12/2021

```
data <- read.table("so2.txt")
data1 <- diff(data$V1)

plot(data$V1,type="o", xlab = "t", ylab = "SO2 Levels", main = "SO2 Levels Over Time")
```

**SO2 Levels Over Time**

```
time = 1:508
time2 = time^2
time3 = time^3
cubicfit = lm(data$V1~time+time2+time3)
summary(cubicfit)
```

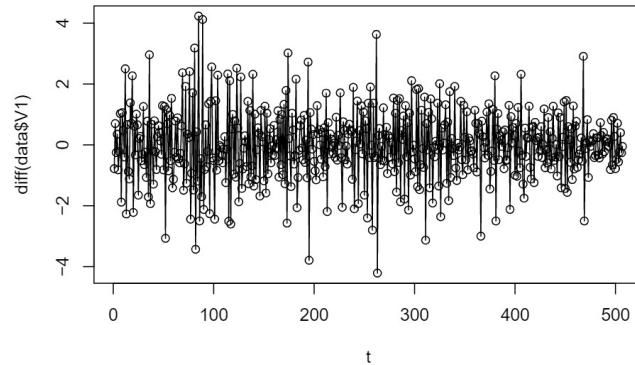
```
##
```

```
1
```

```
## Call:  
## lm(formula = data$V1 ~ time + time2 + time3)  
##  
## Residuals:  
##    Min     1Q Median     3Q    Max  
## -2.0196 -0.6509 -0.0949  0.5125  3.1737  
##  
## Coefficients:  
##                               Estimate Std. Error t value Pr(>|t|)  
## (Intercept) 3.516e+00  1.626e-01 21.625 <2e-16 ***  
## time        2.206e-04  2.764e-03  0.080   0.936  
## time2       -1.329e-05 1.261e-05 -1.054   0.292  
## time3       1.274e-08  1.629e-08  0.782   0.435  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 0.9094 on 504 degrees of freedom  
## Multiple R-squared:  0.2551, Adjusted R-squared:  0.2507  
## F-statistic: 57.54 on 3 and 504 DF,  p-value: < 2.2e-16  
  
quadfit = lm(data$V1~time+time2)  
summary(quadfit)  
  
##  
## Call:  
## lm(formula = data$V1 ~ time + time2)  
##  
## Residuals:  
##    Min     1Q Median     3Q    Max  
## -2.0845 -0.6360 -0.1004  0.5210  3.1695  
##  
## Coefficients:  
##                               Estimate Std. Error t value Pr(>|t|)  
## (Intercept) 3.601e+00  1.215e-01 29.639 <2e-16 ***  
## time        -1.761e-03 1.102e-03 -1.598  0.1106  
## time2       -3.569e-06  2.097e-06 -1.702  0.0894 .  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 0.9091 on 505 degrees of freedom  
## Multiple R-squared:  0.2542, Adjusted R-squared:  0.2513  
## F-statistic: 86.07 on 2 and 505 DF,  p-value: < 2.2e-16  
  
linfit = lm(data$V1~time)  
summary(linfit)
```

```
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 3.7550664  0.0809401   46.39 <2e-16 ***
## time        -0.0035781  0.0002756   -12.98 <2e-16 ***
## ---        
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.9108 on 506 degrees of freedom
## Multiple R-squared:  0.2499, Adjusted R-squared:  0.2485 
## F-statistic: 168.6 on 1 and 506 DF,  p-value: < 2.2e-16
```

plot(diff(data\$V1), type="o", xlab = "t", main="First Differenced Data")

**First Differenced Data**

```
time = 1:508
time2 = time^2
time3 = time^3
time4 = time^4
time5 = time^5
pentafit = lm(data$V1~time+time2+time3+time4+time5)
summary(pentafit)
```

```
## 
## Call:
## lm(formula = data$V1 ~ time + time2 + time3 + time4 + time5)
```

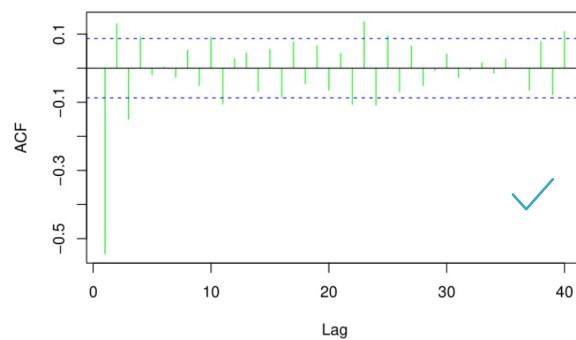
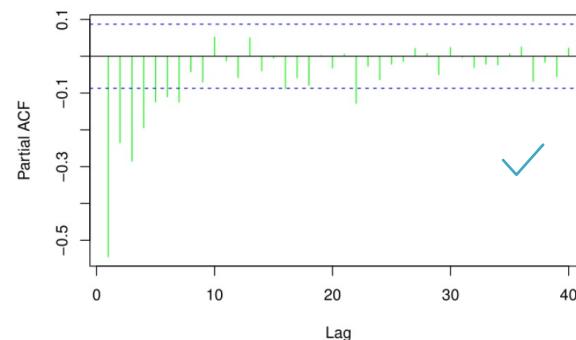
```
##  
## Residuals:  
##   Min     1Q Median     3Q    Max  
## -1.8925 -0.6346 -0.1145  0.4990  3.2763  
##  
## Coefficients:  
##             Estimate Std. Error t value Pr(>|t|)  
## (Intercept) 3.013e+00 2.448e-01 12.310 <2e-16 ***  
## time        2.344e-02 9.684e-03 2.421  0.0158 *  
## time2       -2.718e-04 1.176e-04 -2.311  0.0212 *  
## time3        1.130e-06 5.852e-07  1.931  0.0540 .  
## time4       -2.065e-09 1.267e-09 -1.630  0.1038  
## time5        1.368e-12 9.906e-13  1.381  0.1678  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 0.9037 on 502 degrees of freedom  
## Multiple R-squared:  0.2675, Adjusted R-squared:  0.2602  
## F-statistic: 36.66 on 5 and 502 DF,  p-value: < 2.2e-16  
  
time = 1:507  
time2 = time^2  
time3 = time^3  
time4 = time^4  
time5 = time^5  
pentafit = lm(data1~time+time2+time3+time4+time5)  
summary(pentafit)  
  
##  
## Call:  
## lm(formula = data1 ~ time + time2 + time3 + time4 + time5)  
##  
## Residuals:  
##   Min     1Q Median     3Q    Max  
## -4.2058 -0.6479 -0.0182  0.7017  4.2299  
##  
## Coefficients:  
##             Estimate Std. Error t value Pr(>|t|)  
## (Intercept) 7.196e-04 3.204e-01  0.002  0.998  
## time        1.650e-04 1.270e-02  0.013  0.990  
## time2       -3.308e-06 1.546e-04 -0.021  0.983  
## time3        1.820e-08 7.705e-07  0.024  0.981  
## time4       -3.896e-11 1.671e-09 -0.023  0.981  
## time5        2.855e-14 1.310e-12  0.022  0.983  
##  
## Residual standard error: 1.182 on 501 degrees of freedom  
## Multiple R-squared:  9.916e-06, Adjusted R-squared: -0.00997  
## F-statistic: 0.0009936 on 5 and 501 DF,  p-value: 1  
  
time = 1:506  
time2 = time^2  
time3 = time^3  
time4 = time^4
```

4

```
time5 = time^5
pentafit2 = lm(diff(data1)-time+time2+time3+time4+time5)
summary(pentafit2)

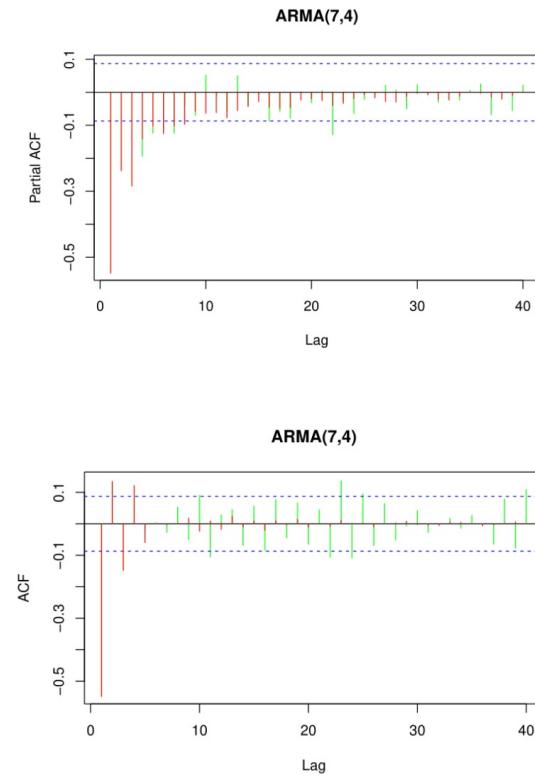
##
## Call:
## lm(formula = diff(data1) ~ time + time2 + time3 + time4 + time5)
##
## Residuals:
##    Min      1Q  Median      3Q     Max 
## -7.8421 -1.2174  0.0223  1.3130  5.8285 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 5.816e-02 5.642e-01  0.103   0.918    
## time        -2.026e-03 2.241e-02 -0.090   0.928    
## time2       2.149e-05 2.732e-04  0.079   0.937    
## time3      -9.611e-08 1.365e-06 -0.070   0.944    
## time4      1.910e-10 2.967e-09  0.064   0.949    
## time5      -1.392e-13 2.329e-12 -0.060   0.952    
## 
## Residual standard error: 2.079 on 500 degrees of freedom
## Multiple R-squared:  2.081e-05, Adjusted R-squared:  -0.009979 
## F-statistic: 0.002081 on 5 and 500 DF, p-value: 1
```

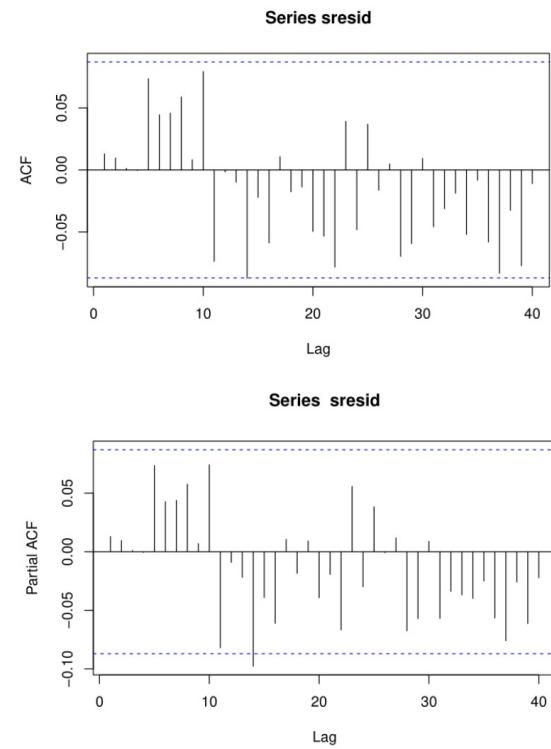
3) A)

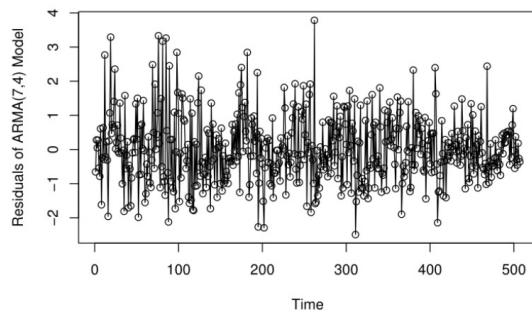
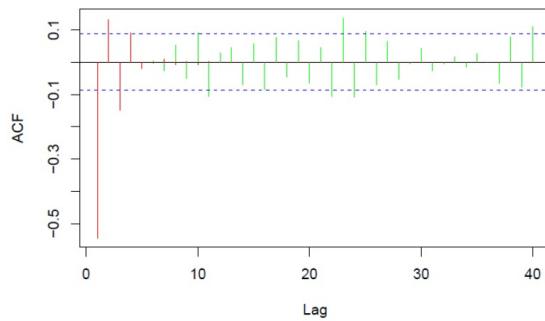
**Series data1****Series data1**

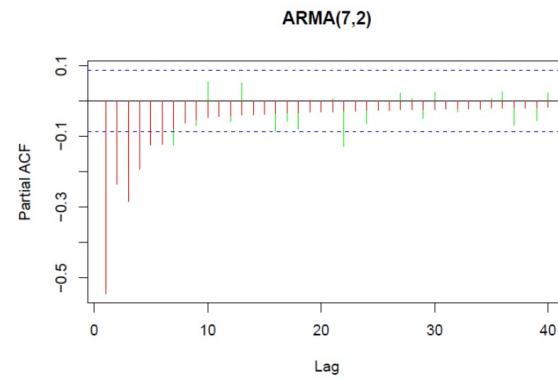
B) We see that there are 4 significant leading lag values in the acf and 7 in the pacf. A possible order of dependency could be  $p=(1,2,3,4,5,6,7)$ ,  $q=(1,2,3,4)$ , for example ARMA(7,4).

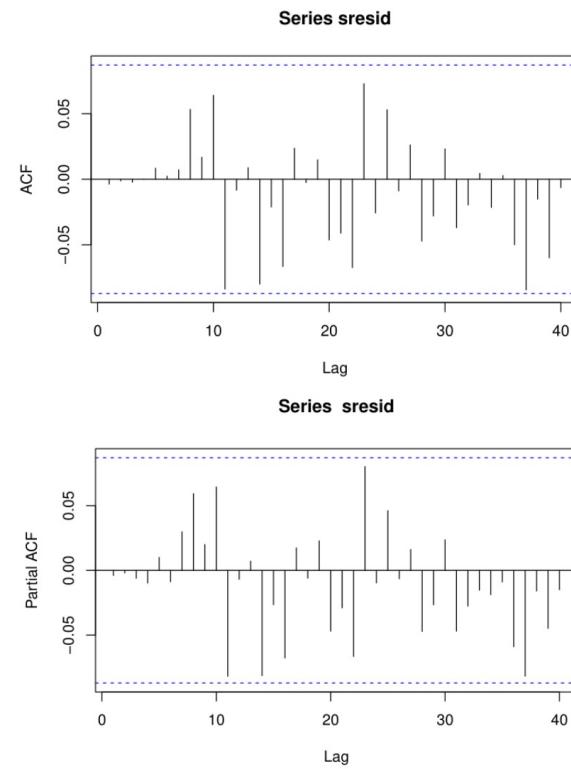
C)

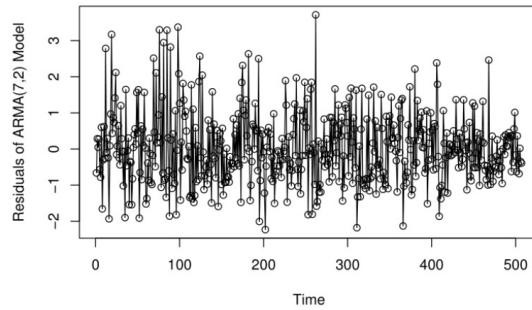
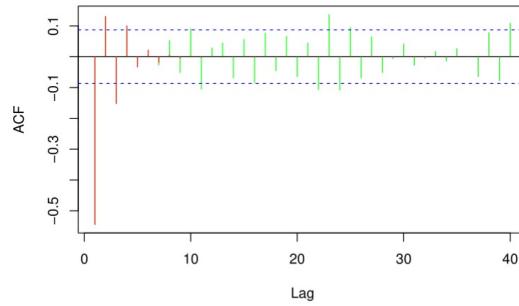


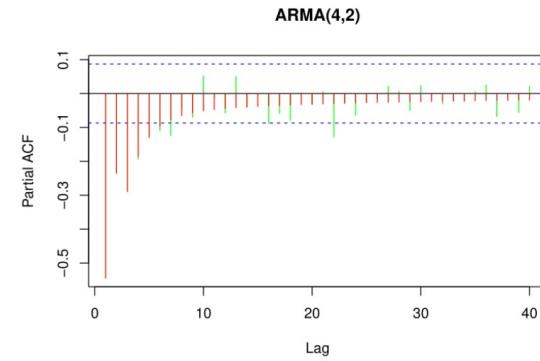


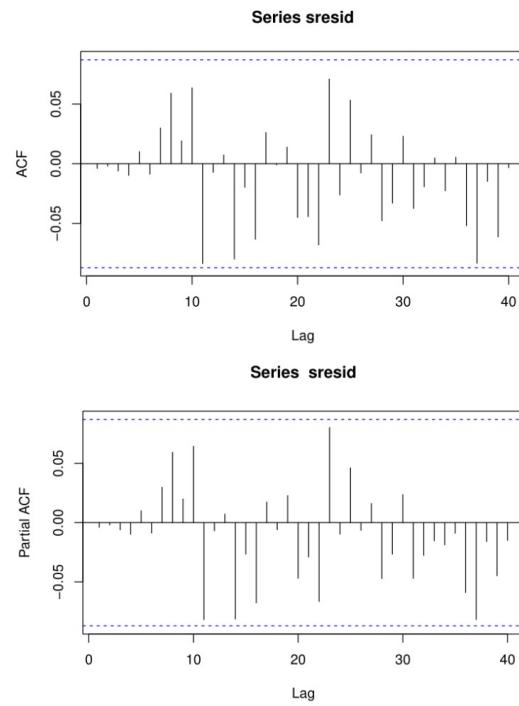
**Plot of Standardized Residuals of ARMA(7,4) Model****ARMA(7,2)**

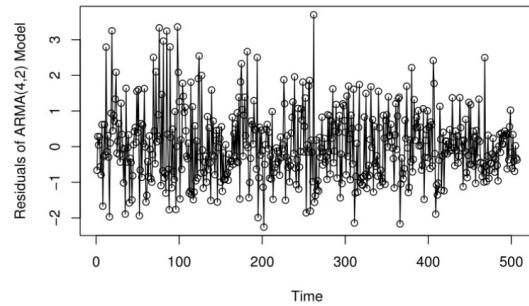


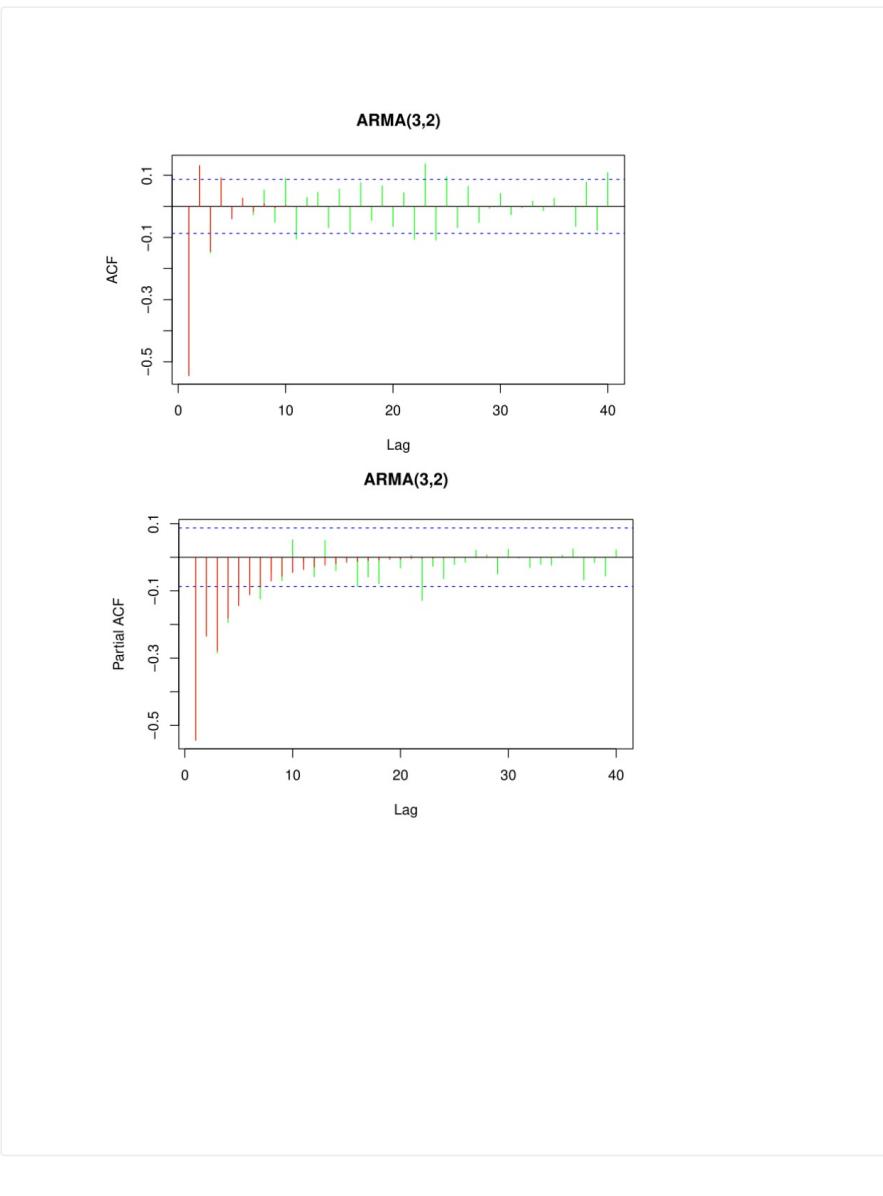


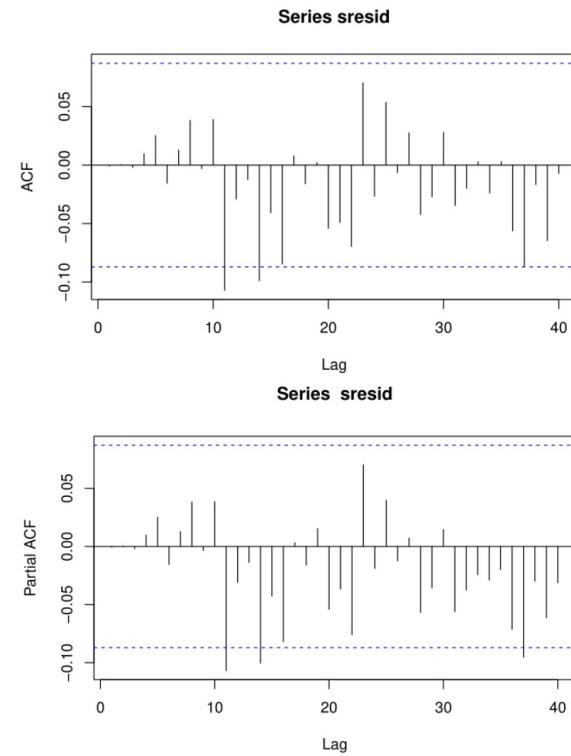
**Plot of Standardized Residuals of ARMA(7,2) Model****ARMA(4,2)**

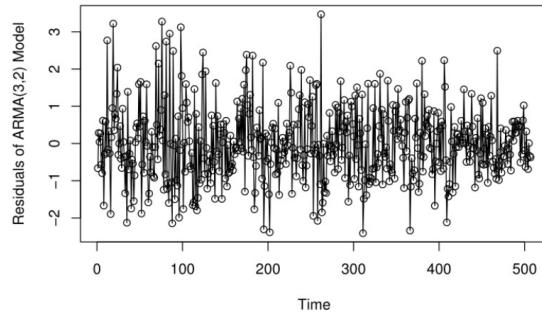




**Plot of Standardized Residuals of ARMA(4,2) Model**





**Plot of Standardized Residuals of ARMA(3,2) Model**

D) AIC for:

ARMA(7,4):1326.94

ARMA(7,2):1310.51 ✓

ARMA(4,2):1305.18

ARMA(3,2):1321.24

E) I choose ARMA(4,2) model with (4,2) p and q values because it is the overall best fit based on the model vs sample acf/pacf and has the lowest AIC. While (3,2) is superior in the respect that it has the correct number of leading significant lag values(7) on pacf compared to 6 in the ARMA(4,2) model, there is larger inaccuracy in the (3,2) model and the model lag values decrease too quickly compared to the sample in the (3,2) model. Thus, the ARMA(4,2) model with (4,2) (p,q) values is the best.

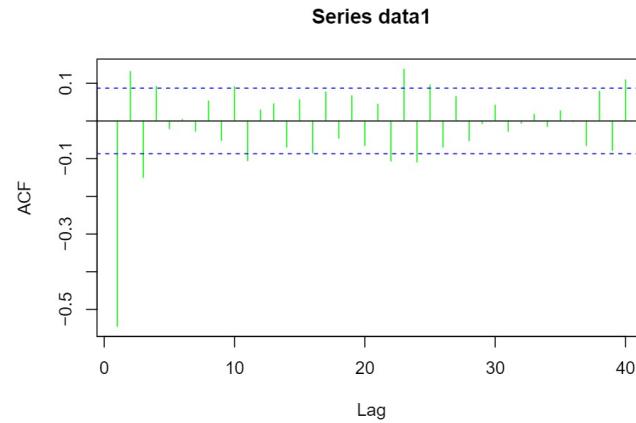
## P7 Q3 Code

Heewon Oh

15/12/2021

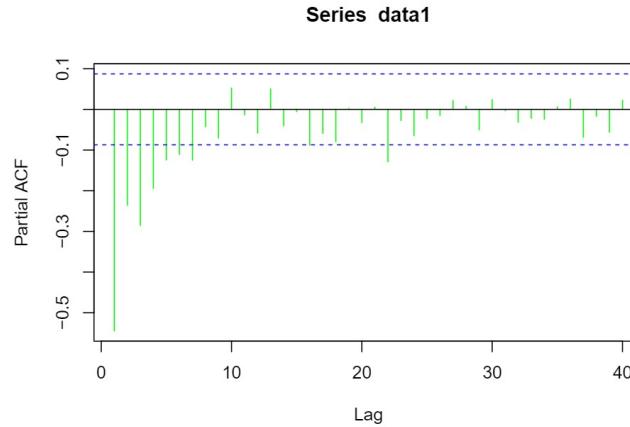
```
data <- read.table("so2.txt")
data1 <- diff(data$V1)

library(TSA)
acf(data1,col="green",lag.max=40)
```



1

```
pacf(data1,col="green",lag.max=40)
```



```
library(itsmnr)
arma74=arima(data1,order=c(7, 0, 4),method="ML")

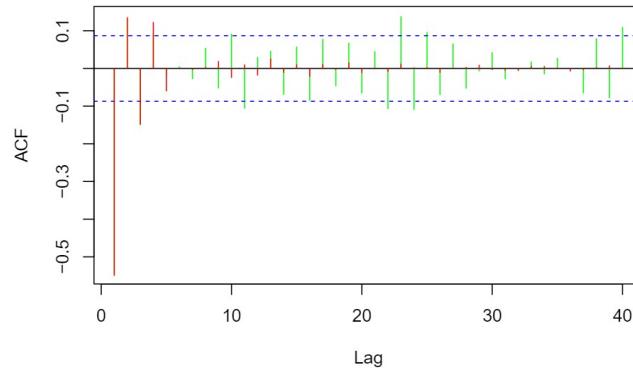
arma74

##
## Call:
## arima(x = data1, order = c(7, 0, 4), method = "ML")
##
## Coefficients:
##
## Warning in sqrt(diag(x$var.coef)): NaNs produced

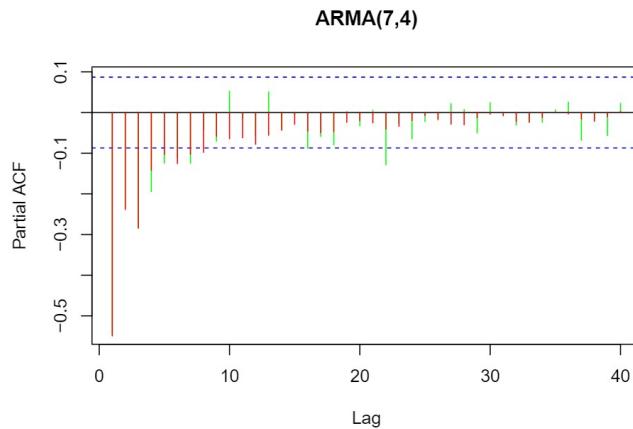
##          ar1      ar2      ar3      ar4      ar5      ar6      ar7      mai
##     -0.1566 -0.2220 -0.8557  0.1985  0.1333 -0.0096  0.0944 -0.7261
## s.e.    0.2558  0.1706  0.2444   NaN   0.0854  0.0454  0.0523  0.2527
##          ma2      ma3      ma4  intercept
##     0.1873  0.5324 -0.8361   -0.0035
## s.e.   0.3951  0.4289  0.2645   0.0035
##          ## sigma^2 estimated as 0.7593: log likelihood = -651.47, aic = 1326.94
```

```
acf(data1,lag.max=40,col="green", main="ARMA(7,4)")
lines(x=1:40,ARMAacf(ar=c(-0.1566,-0.2220,-0.8557,0.1985,0.1333,-0.0096,0.0944),ma=c(-0.7261,0.1873,0.5
```

ARMA(7,4)

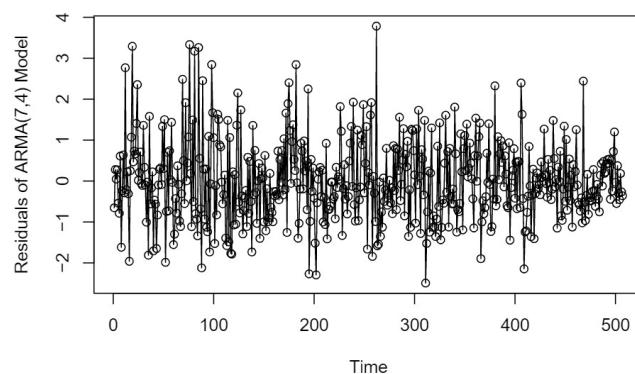


```
pacf(data1,col="green",lag.max=40, main="ARMA(7,4)")
lines(ARMAacf(ar=c(-0.1566,-0.2220,-0.8557,0.1985,0.1333,-0.0096,0.0944),ma=c(-0.7261,0.1873,0.5324,-0.
```

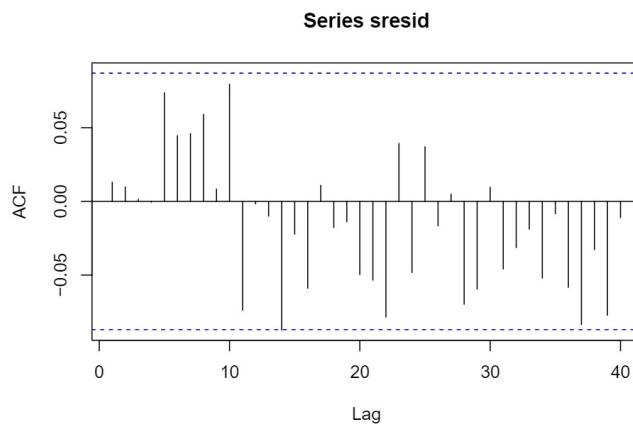


```
plot(rstandard(arma74), type="o", main="Plot of Standardized Residuals of ARMA(7,4) Model", ylab="Residu
```

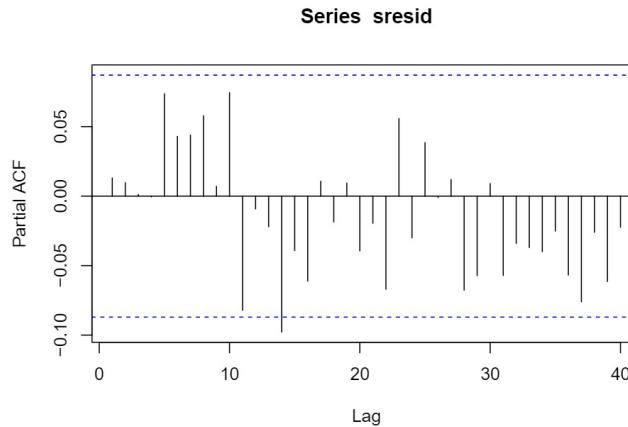
4

**Plot of Standardized Residuals of ARMA(7,4) Model**

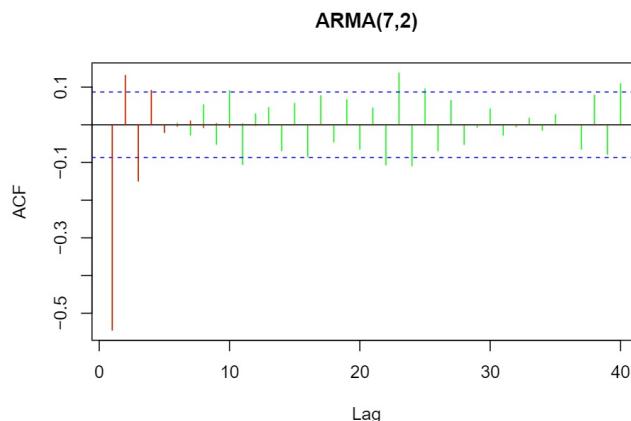
```
sresid=rstandard(arma74)
acf(sresid,lag.max=40)
```



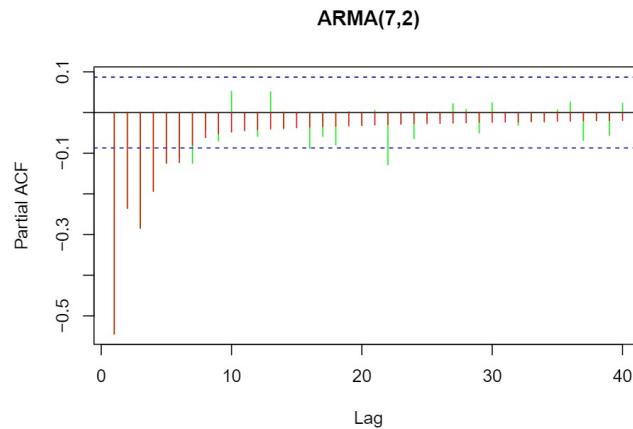
```
pacf(sresid,lag.max=40)
```



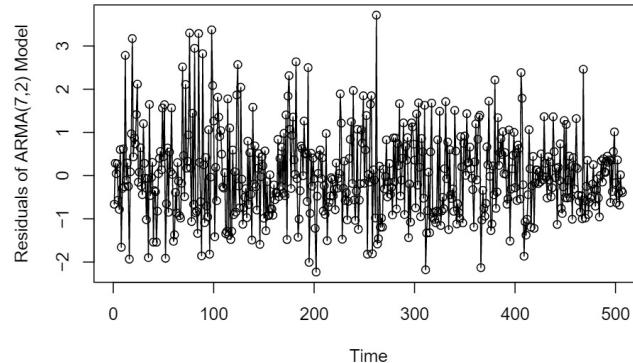
```
arma72=arima(data1,order=c(7, 0, 2),method="ML")  
arma72  
  
##  
## Call:  
## arima(x = data1, order = c(7, 0, 2), method = "ML")  
##  
## Coefficients:  
##             ar1      ar2      ar3      ar4      ar5      ar6      ar7      ma1      ma2  
##       0.5138  0.1316 -0.0824  0.1264  0.0289  0.0031  0.0386 -1.4142  0.4142  
##  s.e.   0.2817  0.0572  0.0701  0.0499  0.0603  0.0554  0.0486  0.2792  0.2791  
##  
##          intercept  
##              -0.0035  
##  s.e.      0.0006  
##  
## sigma^2 estimated as 0.7394:  log likelihood = -645.26,  aic = 1310.51  
  
acf(data1,lag.max=40,col="green",main="ARMA(7,2)")  
lines(x=1:40,ARMAacf(ar=c(0.5138,0.1316,-0.0824,0.1264,0.0289,0.0031,0.0386), ma = c(-1.4142,0.4142), l
```



```
pacf(data1,col="green",lag.max=40,main="ARMA(7,2)")
lines(ARMAacf(ar=c(0.5138,0.1316,-0.0824,0.1264,0.0289,0.0031,0.0386), ma = c(-1.4142,0.4142), lag.max=
```

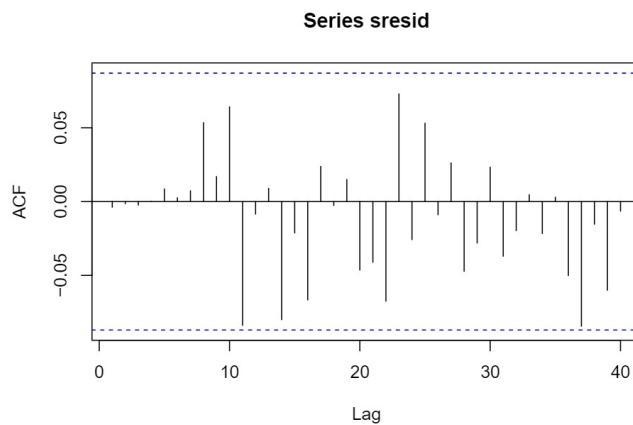


```
plot(rstandard(arma72), type="o", main="Plot of Standardized Residuals of ARMA(7,2) Model", ylab="Residu
```

**Plot of Standardized Residuals of ARMA(7,2) Model**

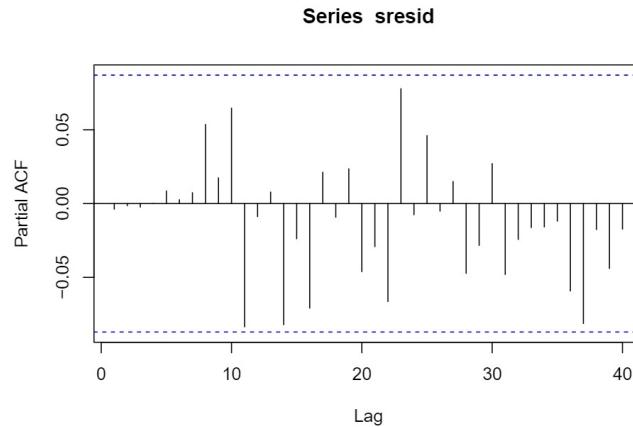
```
sresid=rstandard(arma72)
acf(sresid,lag.max=40)
```

10

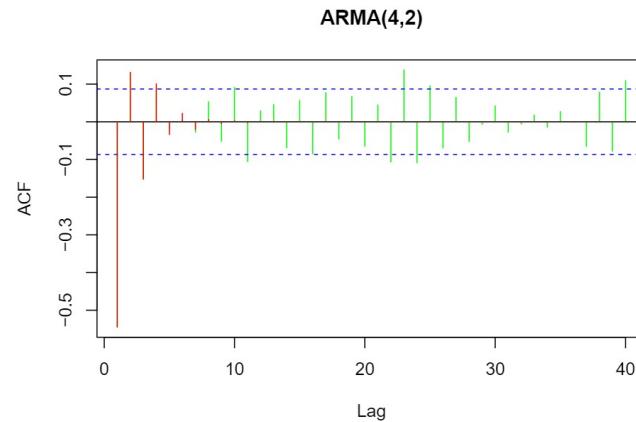


```
pacf(sresid,lag.max=40)
```

11

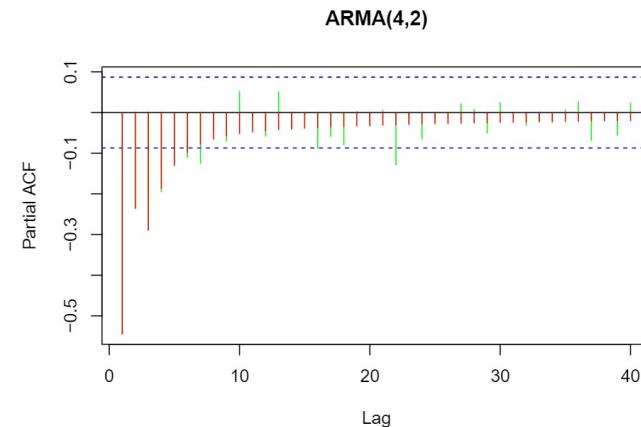


```
arma42=arima(data1,order=c(4,0,2),method="ML")  
arma42  
  
##  
## Call:  
## arima(x = data1, order = c(4, 0, 2), method = "ML")  
##  
## Coefficients:  
##             ar1      ar2      ar3      ar4      ma1      ma2  intercept  
##       0.6953  0.1130 -0.1096  0.1320 -1.5939  0.5939   -0.0035  
##  s.e.  0.1235  0.0545  0.0582  0.0481  0.1192  0.1191    0.0006  
##  
## sigma^2 estimated as 0.7403:  log likelihood = -645.59,  aic = 1305.18  
  
acf(data1,lag.max=40,col="green",main="ARMA(4,2)")  
lines(x=1:40,ARMAacf(ar=c(0.6953,0.1130,-0.1096,0.1320), ma = c(-1.5939,0.5939), lag.max=40)[2:41],type
```



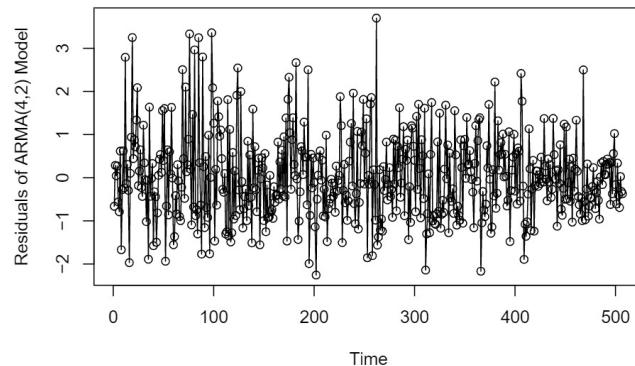
```
pacf(data1,col="green",lag.max=40,main="ARMA(4,2)")
lines(ARMAacf(ar=c(0.6953,0.1130,-0.1096,0.1320), ma = c(-1.5939,0.5939), lag.max=40,pacf=TRUE),type="h")
```

13

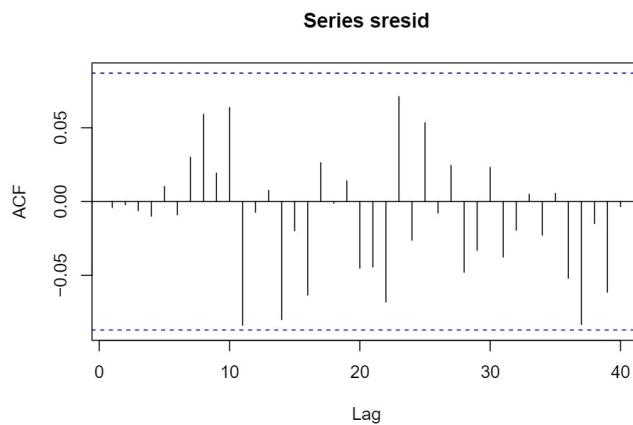


```
plot(rstandard(arma42), type="o", main="Plot of Standardized Residuals of ARMA(4,2) Model", ylab="Residu
```

14

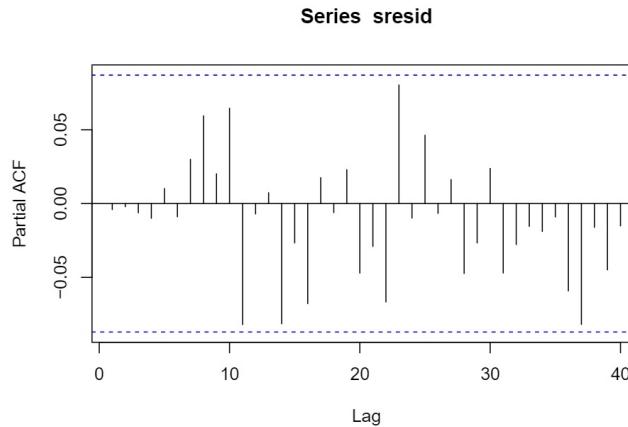
**Plot of Standardized Residuals of ARMA(4,2) Model**

```
sresid=rstandard(arma42)
acf(sresid,lag.max=40)
```

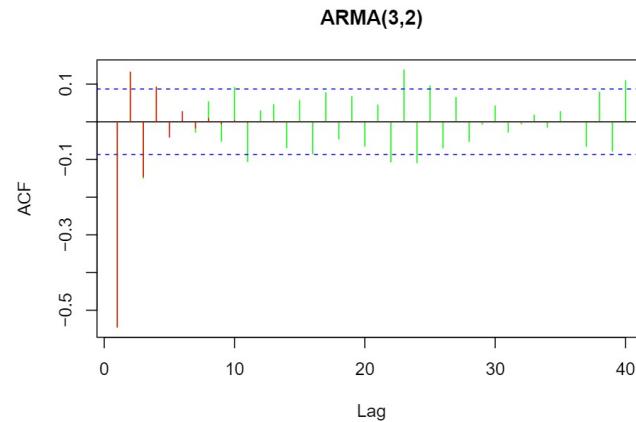


```
pacf(sresid,lag.max=40)
```

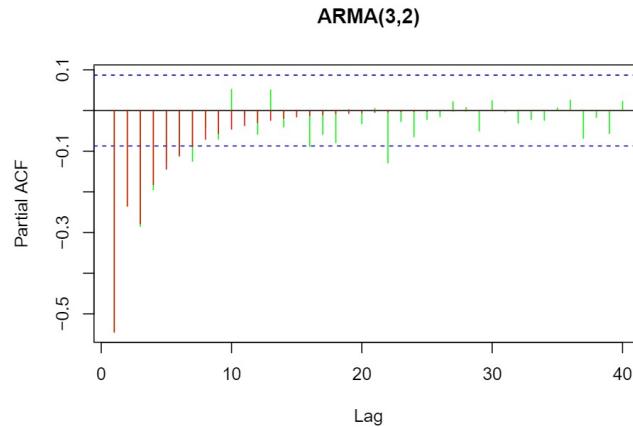
16



```
arma32=arima(data1,order=c(3,0,2),method="ML")  
arma32  
  
##  
## Call:  
## arima(x = data1, order = c(3, 0, 2), method = "ML")  
##  
## Coefficients:  
##             ar1      ar2      ar3      ma1      ma2  intercept  
##       -0.2510  0.0358 -0.0930 -0.6073 -0.1634   -0.0024  
##  s.e.    0.3765  0.0776  0.0809  0.3761  0.3418    0.0069  
##  
## sigma^2 estimated as 0.7726:  log likelihood = -654.62,  aic = 1321.24  
  
acf(data1,lag.max=40,col="green",main="ARMA(3,2)")  
lines(x=1:40,ARMAacf(ar=c(-0.2510,0.0358,-0.0930), ma = c(-0.6073,-0.1634), lag.max=40)[2:41],type="h",
```

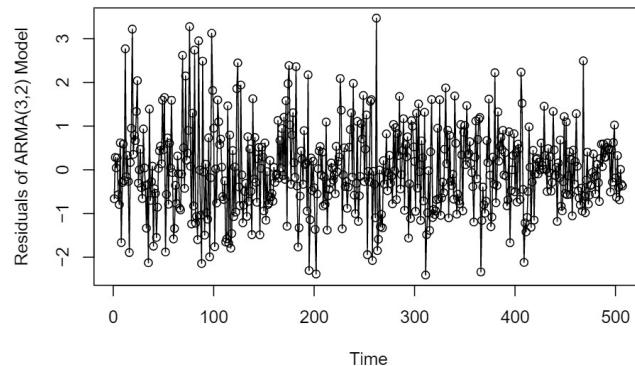


```
pacf(data1,col="green",lag.max=40,main="ARMA(3,2)")
lines(ARMAacf(ar=c(-0.2510,0.0358,-0.0930), ma = c(-0.6073,-0.1634), lag.max=40,pacf=TRUE),type="h",col="green")
```

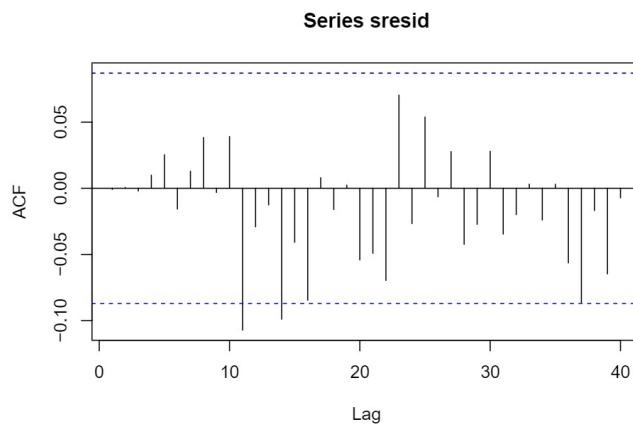


```
plot(rstandard(arma32), type="o", main="Plot of Standardized Residuals of ARMA(3,2) Model", ylab="Residu
```

19

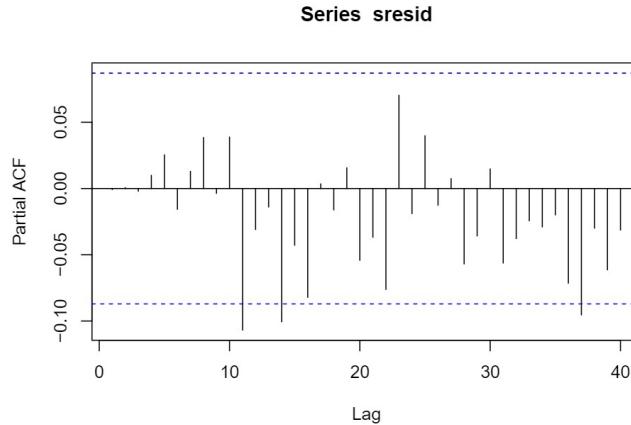
**Plot of Standardized Residuals of ARMA(3,2) Model**

```
sresid=rstandard(arma32)
acf(sresid,lag.max=40)
```



```
pacf(sresid,lag.max=40)
```

21



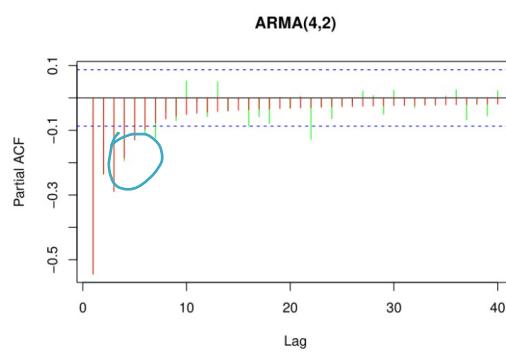
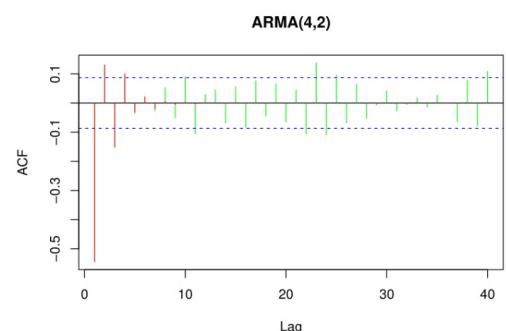
22

4) A)  $p=4$ ,  $d=1$ ,  $q=2$

B) ar1      ar2      ar3      ar4      ma1      ma2      intercept  
0.6953    0.1130   -0.1096   0.1320   -1.5939   0.5939   -0.0035

C) aic = 1305.18

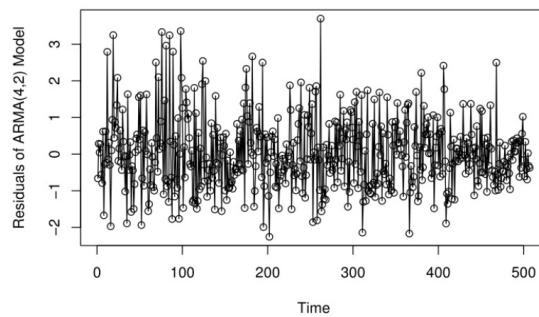
D)



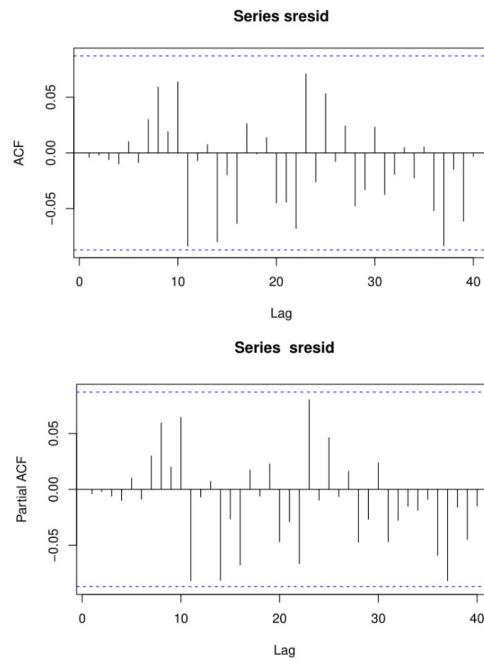
E) Overall, the quality of the model fit is relatively fair (somewhat good). The leading significant model ACF values are very close to the sample ACF ones. The model ACF non-significant lag values however decrease faster than we'd like to see compared to the sample ACF ones. The model PACF 1-3 lag values (significant) are very accurate to the sample PACF ones. The 4<sup>th</sup> and 6<sup>th</sup> lag value (significant) is just smaller in the model than sample PACF and the 5<sup>th</sup> (significant) larger. Critically however, the 7<sup>th</sup> leading lag value in the model PACF is significantly smaller than that in the sample PACF and is non-significant in the model while it is significant in the sample. Overall, the rate of decrease of the non-significant values in the model PACF is reasonably consistent to that in the sample PACF.

F)

**Plot of Standardized Residuals of ARMA(4,2) Model**



G)



H) The standardized model residuals, ACF residuals and PACF residuals plots are consistent with and thus support the hypothesis that the model residuals behave like iid noise. Visually, the standardized model residuals appears to be stationary. The ACF and PACF residuals plots have no significant values.

I)

```
[1] 7.452796e-05
Box-Ljung test
data: sresid
X-squared = 15.645, df = 20, p-value = 0.7384
```

The McLeod-Li test returned a p-value of 7.45e-05, which rejects the iid noise model residuals hypothesis. In Contrast, the Ljung-Box test returned a P-value of 0.7384, which supports the iid noise model residuals hypothesis.

J) Overall, the results support the validity of the hypothesis that the model residuals behave like iid noise. The McLeod-Li test does fail; however, the very strong results for the Ljung-Box test, acf and pacf residuals test, and standardized model residuals inspection, support the hypothesis. Thus, we uphold the validity of the hypothesis.

K) Overall, the quality of the fitted model is good. The iid noise model residuals hypothesis is upheld and the quality of the acf/pacf model fits is relatively fair. We do have some concerns, such as the McLeod-Li test failing and the 7<sup>th</sup> leading significant lag value on the model pacf not being significant while it is in the sample pacf. However, these concerns do not detract from the matter that the fitted model is overall well representative of the data.

## P7 Q4 Code

Heewon Oh

15/12/2021

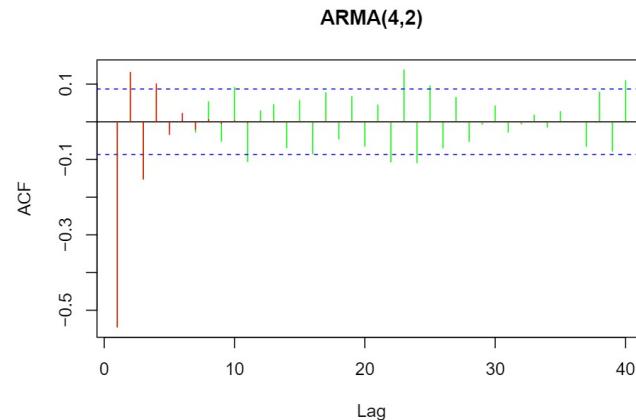
```
data <- read.table("so2.txt")
data1 <- diff(data$V1)
library(itsmr)
library(TSA)

arma42=arima(data1,order=c(4,0,2),method="ML")
arma42

##
## Call:
## arima(x = data1, order = c(4, 0, 2), method = "ML")
##
## Coefficients:
##             ar1      ar2      ar3      ar4      ma1      ma2  intercept
##             0.6953  0.1130 -0.1096  0.1320 -1.5939  0.5939   -0.0035
## s.e.    0.1235  0.0545  0.0582  0.0481  0.1192  0.1191    0.0006
##
## sigma^2 estimated as 0.7403:  log likelihood = -645.59,  aic = 1305.18

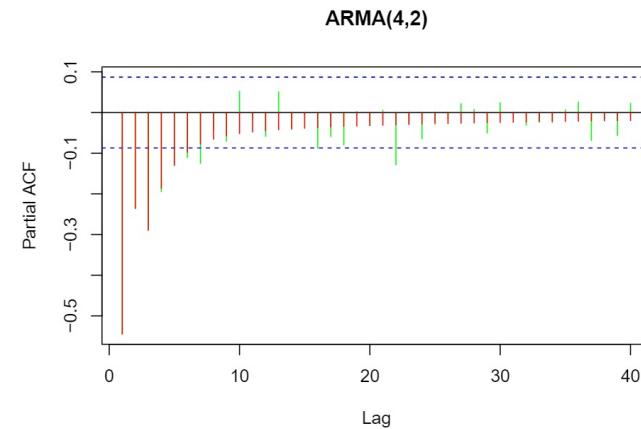
acf(data1,lag.max=40,col="green",main="ARMA(4,2)")
lines(x=1:40,ARMAacf(ar=c(0.6953,0.1130,-0.1096,0.1320), ma = c(-1.5939,0.5939), lag.max=40)[2:41],type
```

1



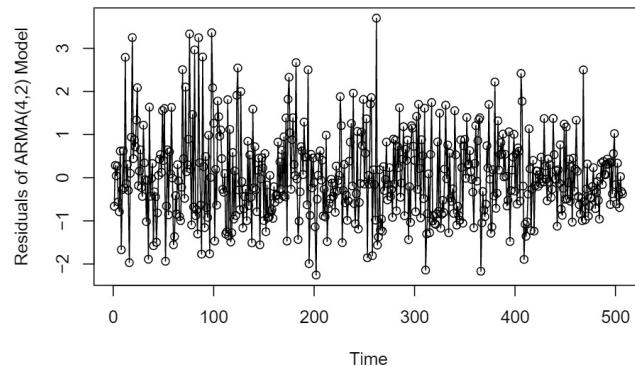
```
pacf(data1,col="green",lag.max=40,main="ARMA(4,2)")
lines(ARMAacf(ar=c(0.6953,0.1130,-0.1096,0.1320), ma = c(-1.5939,0.5939), lag.max=40,pacf=TRUE),type="h")
```

2

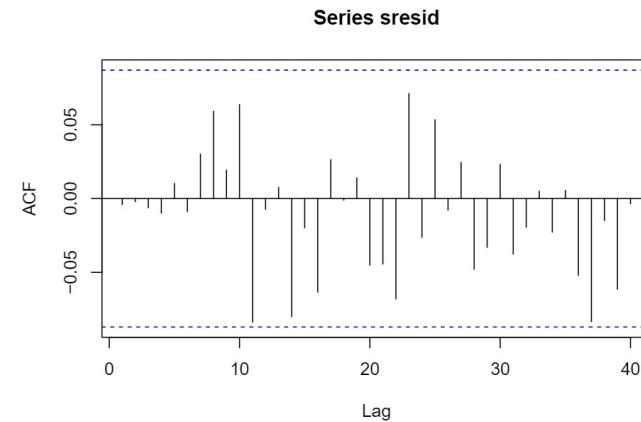


```
plot(rstandard(arma42), type="o", main="Plot of Standardized Residuals of ARMA(4,2) Model", ylab="Residu
```

3

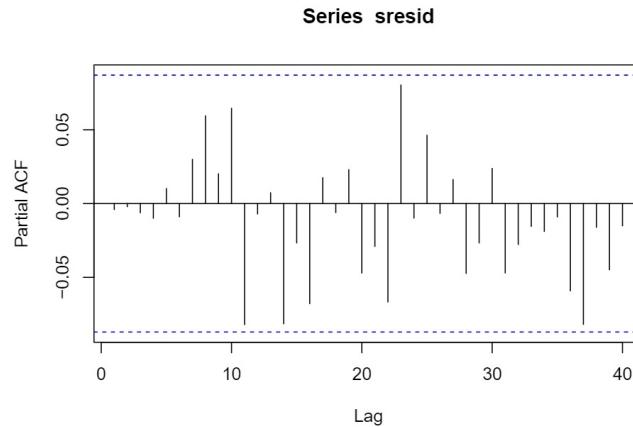
**Plot of Standardized Residuals of ARMA(4,2) Model**

```
sresid=rstandard(arma42)
acf(sresid,lag.max=40)
```



```
pacf(sresid,lag.max=40)
```

5



```
m1 = McLeod.Li.test(y=sresid,gof.lag=20,plot=FALSE)
m1$p.values[20]

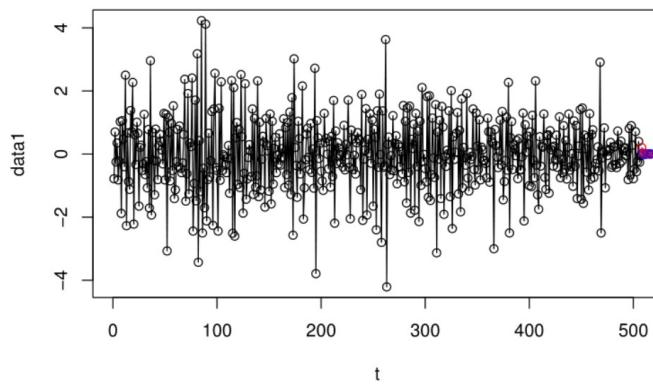
## [1] 7.452796e-05

Box.test(sresid,type="Ljung-Box",lag=20)

##
## Box-Ljung test
##
## data: sresid
## X-squared = 15.645, df = 20, p-value = 0.7394
```

Q5)  
A)

**First-Differenced Data with Forecast**



## P7 Q5 Code

Heewon Oh

15/12/2021

```
data <- read.table("so2.txt")
data1 <- diff(data$V1)
library(itsmr)
library(TSA)
set.seed(123)
library(forecast)
arma42=arima(data1,order=c(4,0,2),method="ML")
arma42

##
## Call:
## arima(x = data1, order = c(4, 0, 2), method = "ML")
##
## Coefficients:
##             ar1      ar2      ar3      ar4      ma1      ma2  intercept 
##             0.6953  0.1130 -0.1096  0.1320 -1.5939  0.5939   -0.0035 
## s.e.    0.1235  0.0545  0.0582  0.0481  0.1192  0.1191    0.0006 
## 
## sigma^2 estimated as 0.7403:  log likelihood = -645.59,  aic = 1305.18

plot(data1,type="o", xlab = "t", main = "First-Differenced Data with Forecast")
fc = forecast(arma42,initial = "simple", h=10,level=c(80,95),model=arma42)
lines(fc$mean,type="o",col=c("red","blue"))
```

1

