Supplementary Materials for
"Efficient Point-in-Polygon Tests by Grids without the Trouble of Tuning the Grid Resolutions"

Here, we present many more experimental results for the submitted paper. There are 6 polygons tested, as listed in the following. The experimental results were collected on a PC with an Intel(R) Core (TM) i7-8700 CPU, 16 GB RAM and an NVIDIA GeForce GTX 1080Ti GPU with 11 GB memory, where 1,000,000 query points are randomly and evenly sampled in the bounding box of the tested polygon.

The statistics are listed in the following tables, from which it is known that our R-GP can considerably reduce the required multiplication, addition and comparison operations for inclusion tests, in comparison with the existing methods. This provides the solid foundation for our acceleration. As a result, we effectively reduce the sensitivity of the inclusion test efficiency to the grid resolution, and so able to handle the trade-off between the preprocessing and the inclusion test, a challenging problem for existing methods. We can use our R-GP with a lower grid resolution to save preprocessing time and storage requirement while speed up inclusion tests, when compared with the state-of-the-art method GCP with a higher grid resolution.

We have the following tables in the supplementary materials.

- Table 1. The tested 6 polygons.
- Table 2. Comparison between R-GP, GCP, Quad-tree, Kd-tree and Tri-tree on the CPU.
- Table 3. Statistics about the required calculations for the tests in Table 2.
- Table 4. Comparison between GCP and R-GP on preprocessing on the CPU.
- Table 5. Comparison between GCP and R-GP on storage requirements for precomputed results.
- Table 6. Comparison between GCP and R-GP on inclusion tests on the CPU.
- Table 7. Comparison between GCP and R-GP on inclusion tests on the GPU (Graphical Processing unit).
- Table 8. Statistics on handling singular cases.

✶ *R-GP*: our proposed method.
✶ *GCP*: the method in the reference [12] of the paper.
✶ *Quad-tree, Kd-tree*: the methods from the reference [6] of the paper.
✶ *Tri-tree*: the method in the reference [7] of the paper.

Table 1. The tested 6 polygons

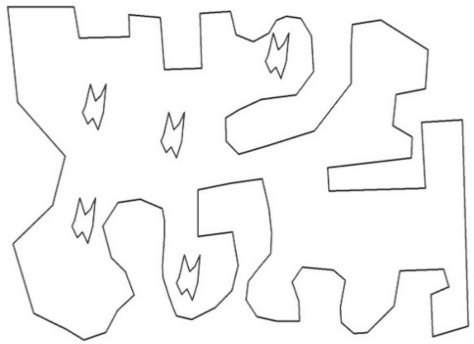
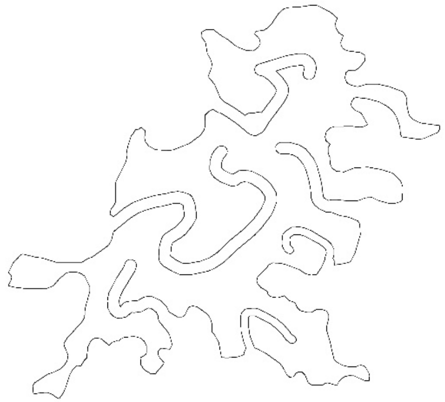
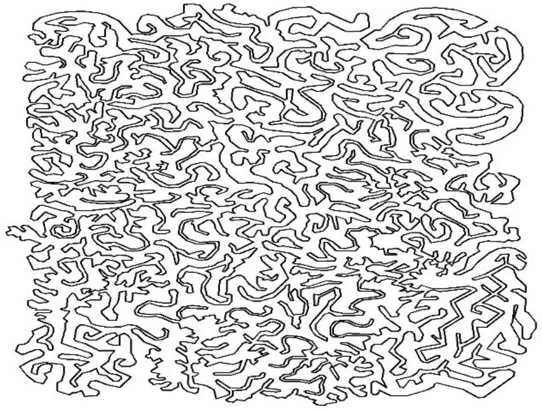
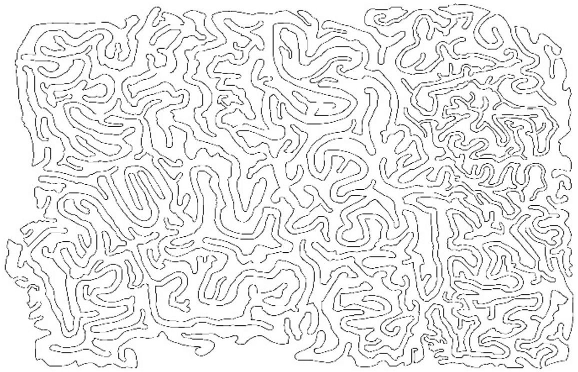
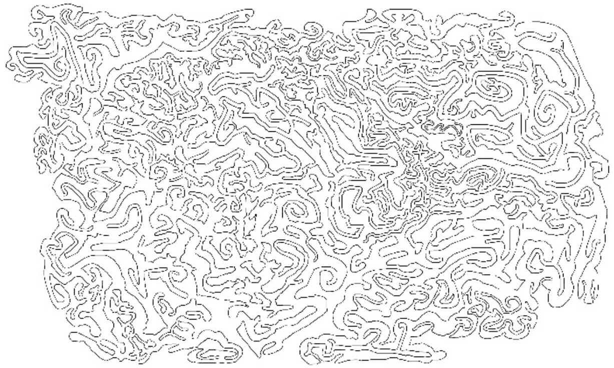
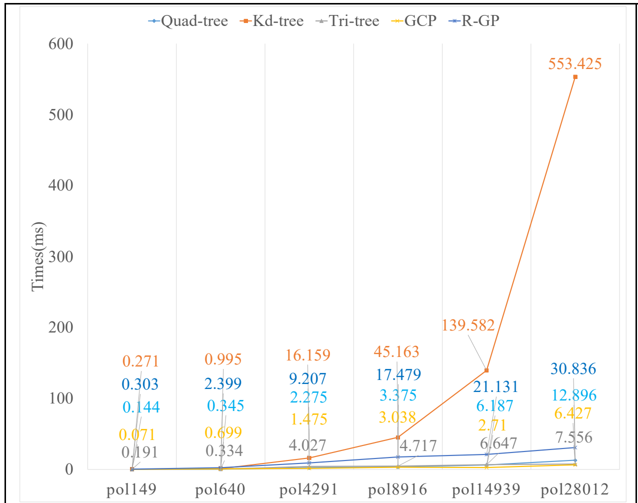
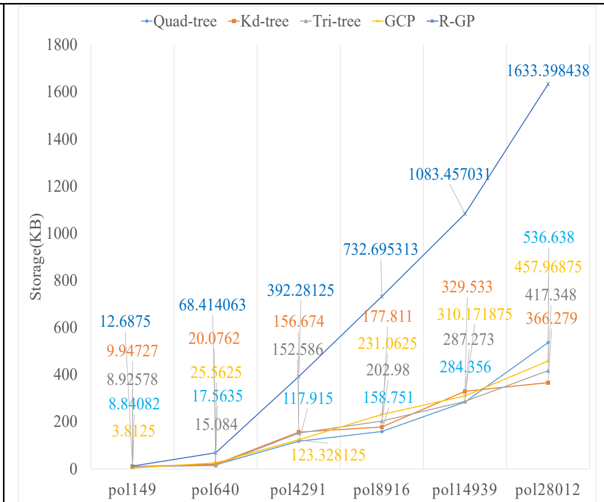
		
pol149	**pol640**	**pol4291**
		
pol8916	**pol14939**	**pol28012**

Table 2. Comparison between R-GP, GCP, Quad-tree, Kd-tree and Tri-tree on the CPU

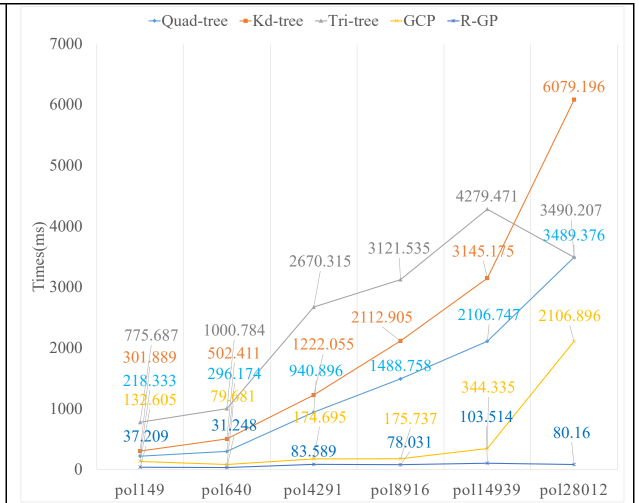
polygons	pol149	pol640	pol4291	pol8916	pol14939	pol28012
	Tree-levels					
quad-tree	6	6	7	7	8	9
Kd-tree	8	8	11	11	12	14
Tri-tree	5	5	8	8	8	8
	grid resolutions					
GCP	9×4	25×26	29×37	55×40	43×30	20×15
R-GP	9×4	25×26	29×37	55×40	43×30	20×15



preprocessing time



storage

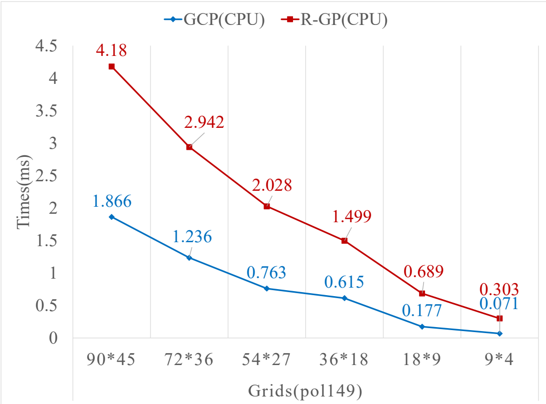


test time

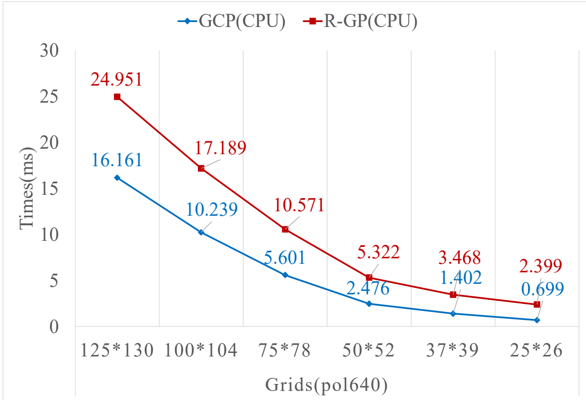
Table 3. Statistics about the required calculations for the tests in Table 2.

polygons	methods	Additions	Multiplications	Comparisons	polygons	methods	Additions	Multiplications	Comparisons
pol149	quad-tree	63219421	19948160	130821535	pol8916	quad-tree	190847060	52180006	389228188
	Kd-tree	82325047	25354556	161948878		Kd-tree	231799286	64518034	451099834
	Tri-tree	132250617	66975497	166813081		Tri-tree	884799544	442460638	965437432
	GCP	249235197	114655772	115758556		GCP	259555775	119402485	120244009
	R-GP	15782552	5571815	36592443		R-GP	14401402	5159280	31787803
pol640	quad-tree	85153108	24002676	183038300	pol14939	quad-tree	165119524	45548076	343877289
	Kd-tree	140698281	40913150	275806219		Kd-tree	211735925	59439564	417584560
	Tri-tree	220023736	109098592	270376116		Tri-tree	1342446480	671984217	1438236676
	GCP	77200694	35857003	39779670		GCP	639112449	293350788	287624256
	R-GP	9288599	3987445	16018407		R-GP	19060940	5362575	47093087
pol4291	quad-tree	127943911	37275614	276483172	pol28012	quad-tree	92079846	24991412	200961439
	Kd-tree	139884470	40907112	286664622		Kd-tree	362108310	97699414	649730050
	Tri-tree	711947131	359410893	768751896		Tri-tree	1095391080	546169300	1226919002
	GCP	289784037	133205755	133627544		GCP	4231593904	1947560873	1875142462
	R-GP	15944708	5523514	37596834		R-GP	45779154	4722137	126669700

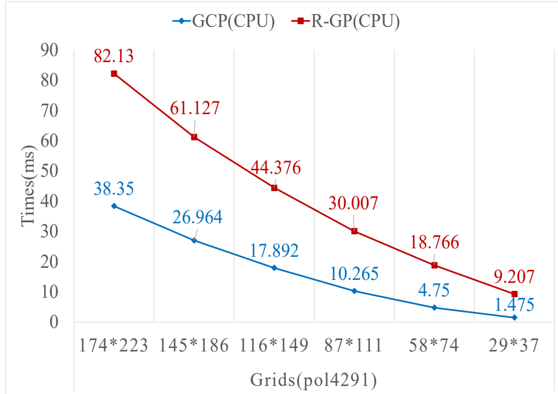
Table 4. Comparison between GCP and R-GP on preprocessing time (ms) on the CPU



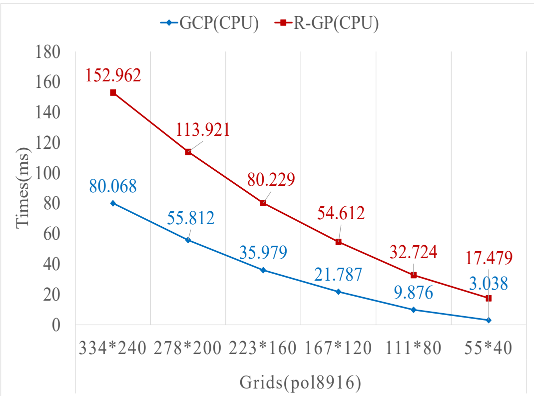
pol149



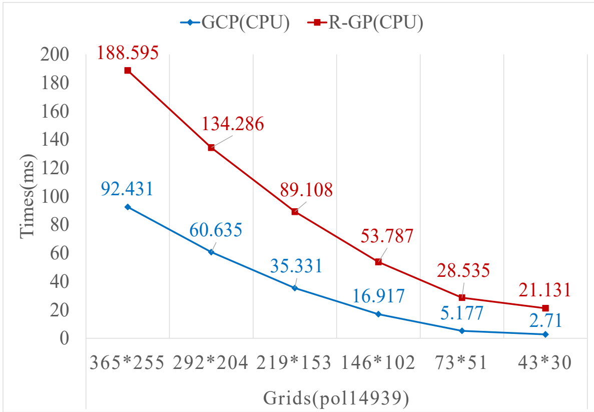
pol640



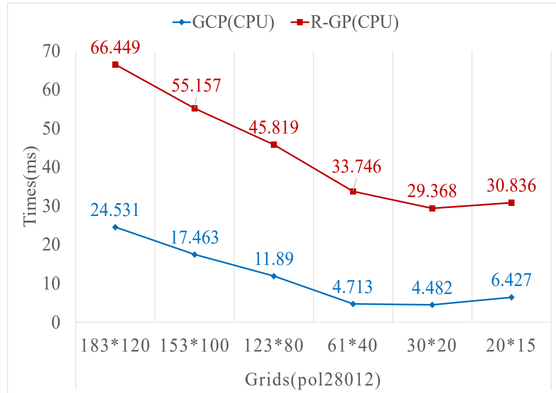
pol4291



pol8916

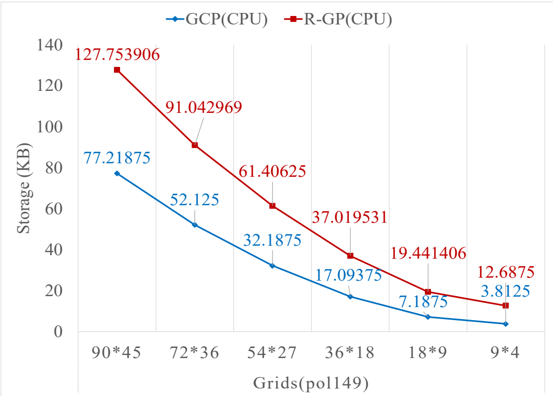


pol14939

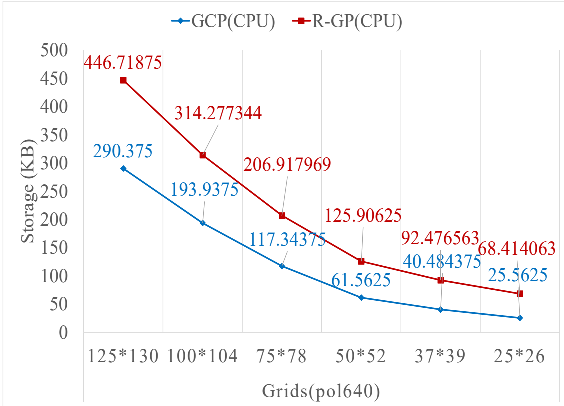


pol28012

Table 5. Comparison between GCP and R-GP on storage requirements (KB) for precomputed results



pol149



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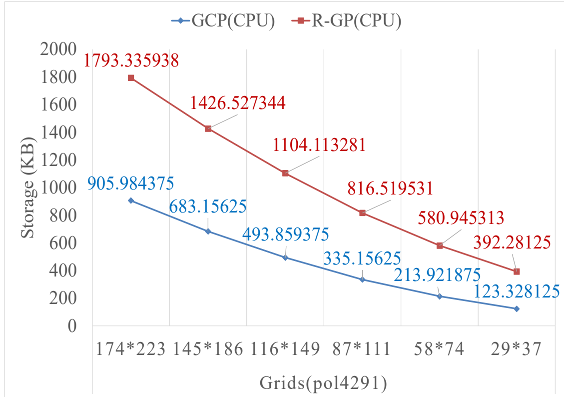
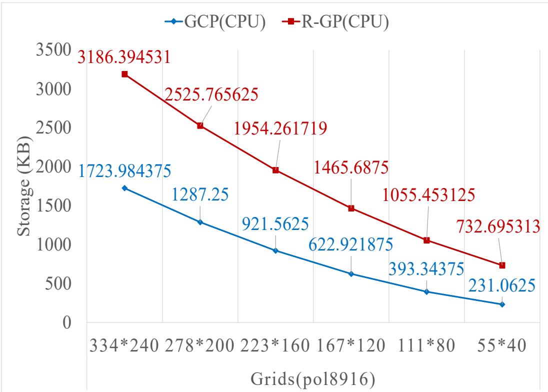
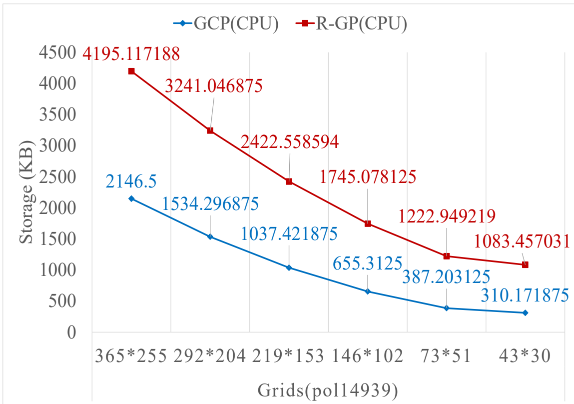


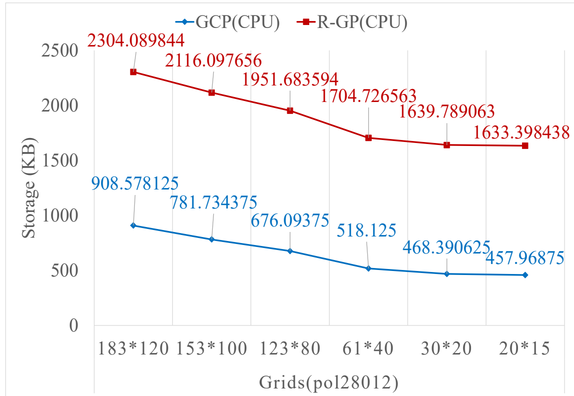
pol4291



pol8916

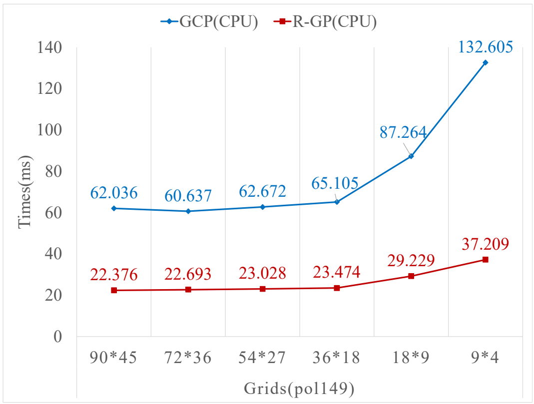


pol14939

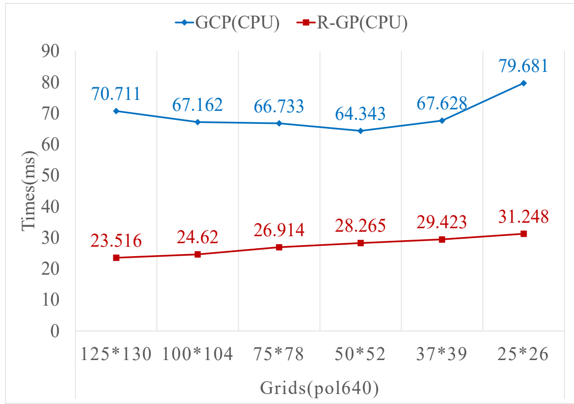


pol28012

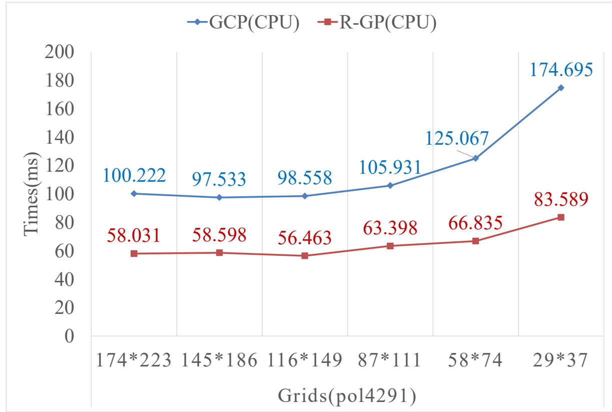
Table 6. Comparison between GCP and R-GP on inclusion tests on the CPU



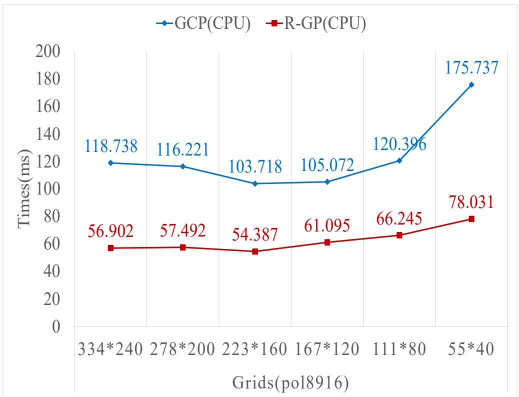
pol149



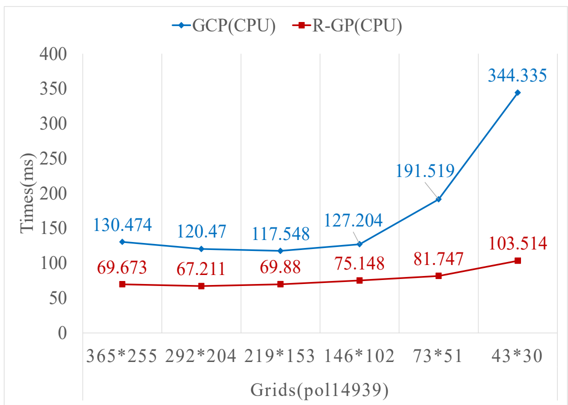
pol640



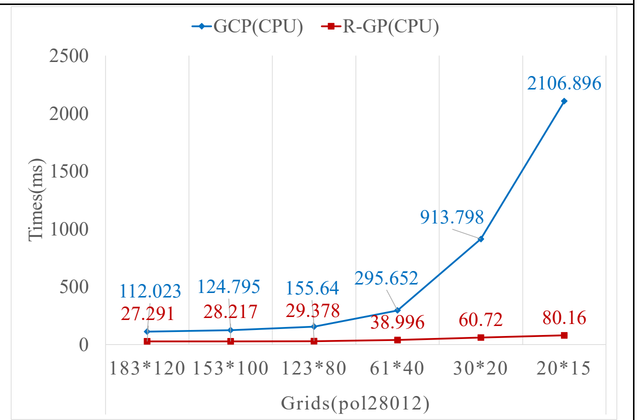
pol4291



pol8916

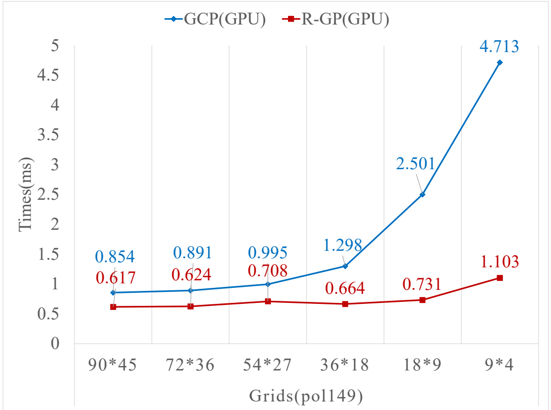


pol14939

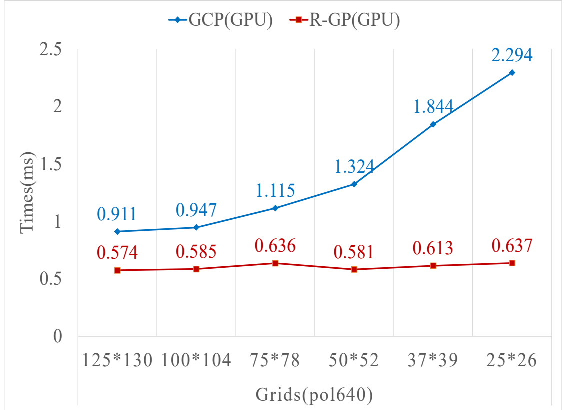


pol28012

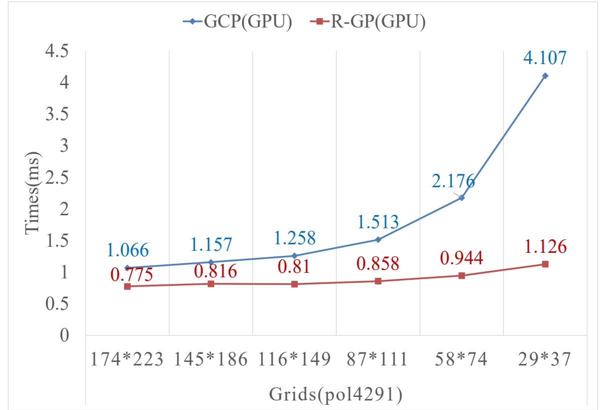
Table 7. Comparison between GCP and R-GP on inclusion tests on the GPU (Graphical Processing unit)



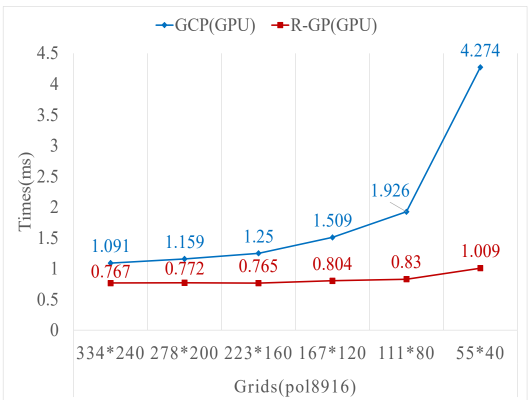
pol149



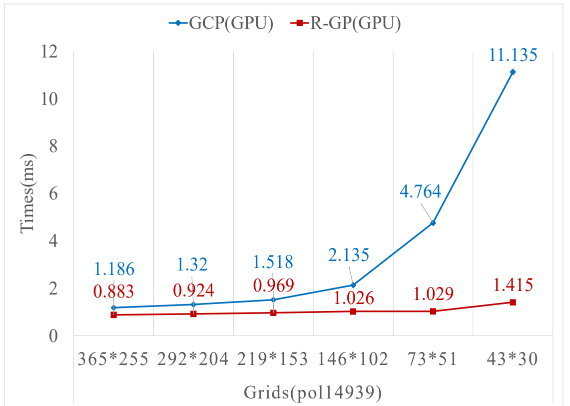
pol640



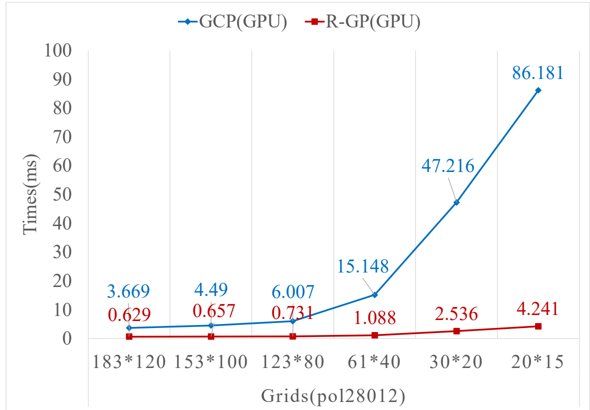
pol4291



pol8916



pol14939



pol28012

Table 8. Statistics on handling singular cases（time: ms）

Polygons		pol149		pol640		pol4291		pol8916		pol14939		pol28012	
Grid resolutions		36×18		50×52		116×149		223×160		292×204		123×80	
Query points		[#]inter	[$]lower	[#]inter	[$]lower	[#]inter	[$]lower	[#]inter	[$]lower	[#]inter	[$]lower	[#]inter	[$]lower
Number of points		138	149	330	640	4816	4291	6728	8916	10986	14939	1926	28012
CPU time	R-GP	0.012	0.014	0.025	0.041	0.567	0.581	0.947	1.211	1.574	2.017	0.345	1.796
	GCP	0.043	0.044	0.06	0.092	0.955	0.967	1.358	1.599	2.275	2.707	0.754	6.891
GPU time	R-GP	0.067	0.066	0.065	0.063	0.073	0.079	0.092	0.096	0.099	0.103	0.095	0.121
	GCP	0.067	0.067	0.07	0.075	0.111	0.109	0.116	0.121	0.128	0.136	0.158	0.301
Add.	R-GP	1859	2411	4335	9435	64414	67676	88811	130867	146420	219438	33071	483150
	GCP	12494	16978	32594	81245	402691	448187	544924	984026	932980	1673531	836578	15504885
Mult.	R-GP	635	835	1485	3221	22275	24108	30850	45889	50703	77133	9214	128965
	GCP	5805	7923	15141	37905	186641	208735	253262	459372	433331	779921	389654	7180717
Comp.	R-GP	3682	4678	8510	18820	127213	129199	175510	257974	290559	430942	70771	1021690
	GCP	6436	8583	16522	40713	208431	228854	288292	503726	488235	852082	393350	7114046

[#]*inter:* referring to the query points that are obtained by slightly decreasing the *y* coordinates of the intersection points between the grid row lines and polygons edges.

[$]*lower*: referring to the query points that are obtained by slightly decreasing the *y* coordinates of the vertices of the tested polygons.

Add., *Mult.*, and *Comp.*: referring to the numbers of the performed multiplication, addition and comparison operations in answering all the query points against a polygon at a grid resolution.