

- $\begin{array}{l} \text{1) Time complexity \& Space Complexity} \\ \text{2) Asymptotic analysis (Big O)} \\ \text{3) Big O \rightarrow meaning} \\ \text{4) Time limit exceed (\underline{TLE})} \end{array} \quad \left. \right\} \text{TC II}$

Maths Concepts

1. $\log_2 N \rightarrow$ No of times we need to divide N by 2 to reduce it to 1.

Ques 2 $[3, 10] \rightarrow 3, 4, 5, 6, 7, 8, 9, 10 \Rightarrow \underline{8}$

$$[a, b] \Rightarrow b - a + 1$$

$$[1, 7] \Rightarrow 7 - 1 + 1 \Rightarrow 7$$

$$[12, 23] \Rightarrow 23 - 12 + 1 \Rightarrow 12$$

3. Arithmetic Progression

$$4, \underbrace{7,}_{3} \underbrace{10,}_{3} \underbrace{13,}_{3} \underbrace{16,}_{3} \dots$$

$$a, \underbrace{a+d,}_{d} \underbrace{a+2d,}_{d} \underbrace{a+3d,}_{d} \underbrace{a+4d,}_{d} \dots$$

$a \rightarrow$ first term

$d \rightarrow$ common diff

$N \rightarrow$ no. of terms

$$\boxed{\text{Sum of first } N \text{ terms} = \frac{N}{2} [2a + (N-1)d]}$$

$$\text{Sum of first } N \text{ natural no.} = \frac{N(N+1)}{2}$$

4. Geometric Progression (G.P.)

$$5, \underbrace{10,}_{10/5} \underbrace{20,}_{20/10} \underbrace{40,}_{40/2} \underbrace{80,}_{80/40} \underbrace{160, \dots}_{160/80}$$

$$a, ar, ar^2, ar^3, ar^4, \dots$$

$a \rightarrow$ first term

$r \rightarrow$ common ratio

$N \rightarrow$ no. of term.

$$\text{Sum of first } N \text{ terms of GP} = \frac{a(r^N - 1)}{r - 1} \quad [r > 1]$$

Log Basic

$$\log_a a^n = x \quad \left| \quad \log_2 2^{10} = 10 \quad \left| \quad \log_{10} 10^{12} = 12 \right. \right.$$

$\mathbb{Q} \quad \text{int } f_n(N) \{$

$i : [1, N]$

$$S = \emptyset;$$

for (*i*=1 ; *i*<=N ; *i*++) {

No of iterations = N

$$S+ = \lambda;$$

TC : O(N)

3

set S_i

۱

Q

Void fn(int N, int M){

`for (j=1; j<=N; j++) { ⇒ j : [1, N]`

```

    if ( i % 2 == 0 ) {
        print( i ),,
    }
}

```

$$\text{No of iterations} = N$$

for (j = 1 ; j <= M ; j++) { j : [1 , M]

```
if (j % 2 == 0) {  
    print(j);  
}
```

No. of iterations = M

2

Total No of iterations

$$= N + M$$

$T_C : O(N+M)$

Q3 $\text{int } f_n(\text{int } N) \{$

$$s = 0;$$

$\text{for } (i=1; i \leq N, i = i+2) \{$

$$s = s + i;$$

}

net s;

}

i

1	$\rightarrow i = i+2$
3	$\rightarrow i = i+2$
5	$\rightarrow i = i+2$
7	
9	
11	
13	
;	

iteration = No. of odd no.

in the range [1, N]

How many odd no. from 1 to N

• $N = 7$: $\{1, 2, 3, 4, 5, 6, 7\} \Rightarrow 4$

$N = 6$: $\{1, 2, 3, 4, 5, 6\} \Rightarrow 3$

No. of odd no. from 1 to N = $\frac{(N+1)}{2}$

No. of iterations = $\frac{(N+1)}{2} = \frac{N}{2} + \frac{1}{2}$

TC : $O(N)$

Q

$\text{int } f_n(N) \{$

$$s = 0;$$

$\text{for } (i=0; i \leq 100, i+1) \{$

$$s = s + i + i^2;$$

}

net s;

}

$i : [0, 100]$

$100 - 0 + 1 \Rightarrow 101$

No. of iteration = 101

$101 \times N^0 \Rightarrow 1$

TC : $O(1)$

Constant TC

Q void fn(N) {

```

    S = 0;           i <= √N
    for (i=1; i * i <= N; i++) {
        S = S + i * i;
    }
    net S;

```

$i : [1, \sqrt{N}]$

No of iterations = \sqrt{N} TC: $O(\sqrt{N})$

Q 6

Void fn(N) {

int i = N;

while (i > 1) {

$i = i/2;$

}

}

$\log_2 N - 1$

$i :$

$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \frac{N}{8} \rightarrow \frac{N}{16} \dots 1$

K iterations

<u>i Before</u>	<u>Iteration</u>	<u>i after</u>	<u>i = i/2</u>
N	1	$N/2$	$N/2^1$
$N/2$	2	$N/4$	$N/2^2$
$N/4$	3	$N/8$	$N/2^3$
$N/8$	4	$N/16$	$N/2^4$
			\vdots
		1	$\rightarrow \frac{N}{2^K}$

$$1 = \frac{N}{2^K}$$

$$\Rightarrow 2^K = N$$

No of iterations = $\log_2 N$

TC: $O(\log N)$

$$\Rightarrow \log_2 2^K = \log_2 N$$

$$\Rightarrow \boxed{K = \log_2 N}$$

Q void fn(N) {

$S = 0;$

for ($i=0; i < N; i = i \times 2$) {

$S = S + i;$

}

}

i_b	iteration	i_a
0	1	0
0	2	0
0	3	0
0	4	0
⋮	⋮	⋮

No of iterations = ∞

∞

Q

void fn(N) {

$S = 0;$

for ($i=1; i < N; i = i \times 2$) {

$S = S + i;$

}

}

$i = 1, 2, 3, 4, \dots, N$

$\xrightarrow{x_2}$

$\xleftarrow{1/2}$

i_b	iteration	i_a
1	1	$2 \rightarrow 2^1$
2	2	$4 \rightarrow 2^2$
4	3	$8 \rightarrow 2^3$
8	4	$16 \rightarrow 2^4$
⋮	⋮	⋮

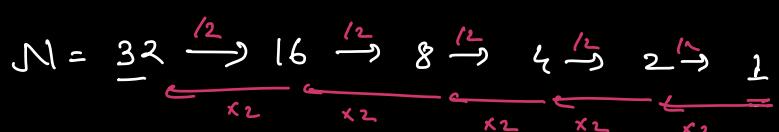
$N \rightarrow 2^K$

$\log_2 N$

$2^K = N$

$\Rightarrow K = \underline{\log_2 N}$

Tc: $O(\log N)$



Q void fm (N) {

 for (i=1; i<=10; i++) {

 for (j=1; j<=N; j++) {
 Print (i,j),
 }

 }

}

i	j	Inner Loop status
1	[1, N]	N
2	[1, N]	+ N
3	[1, N]	+ N
:	:	+
10	[1, N]	N
11	x	0

$$\text{No of iteration} = \frac{10N}{T C : O(N)}$$

Q void fm (N) {

 for (i=1; i<=N; i++) {

 for (j=1; j<=N; j++) {
 Print (i,j),

 }

}

}

i	j	Inner Loop status
1	[1, N]	N
2	[1, N]	+ N
3	[1, N]	+ N
:	:	+
N	[1, N]	N
N+1	x	x

$$\text{No of iteration} = N^2$$

$$TC: O(N^2)$$

Q void fn(N) {
 for (i=1; i <= N; i++) { j <= N
 for (j=1; j < N; j=j*2) {
 print(i*j);
 } } }

i	j	iterations
1	1 → N	$\log_2 N$
2	1 → N	$\log_2 N$
3	1 → N	$\log_2 N$
⋮	⋮	⋮
N	1 → N	$\log_2 N$
N+1	0	0

No of iterations = $N \log_2 N$
 $T.C : O(N \log_2 N)$

Q void fn(N) {
 for (i=1; i <= 2^n; i++) {
 } } }

$i : [1, 2^n]$
 # of iterations = 2^n
 $T.C : O(2^n)$

Q void fn(N) {
 for (i=1; i <= N; i++) {
 for (j=1; j <= 2^i; j++) {
 print(i+j);
 } } }

i	j : [1, 2^i]	I-terations
1	[1, 2]	<u>2</u> 2^1
2	[1, 2^2]	<u>4</u> 2^2
3	[1, 2^3]	<u>8</u> 2^3
4	[1, 2^4]	<u>16</u> 2^4
5	[1, 2^5]	<u>32</u> 2^5
⋮	⋮	⋮
N	[1, 2^N]	<u>2^N</u> 2^N

$$\# \text{ of iterations} = 2 + 4 + 8 + 16 + 32 + \dots + 2^N$$

$\underbrace{\hspace{10em}}_G P$

$$a = 2$$

$$r = 2$$

$$\text{No of iter} = N$$

$$\text{Sum} = a \times \left(\frac{r^N - 1}{r - 1} \right)$$

$$= \frac{2 (2^N - 1)}{2 - 1}$$

$$= 2 (2^N - 1) \quad T.C : O(2^N)$$

$$\# \text{ of iterations} = 2 (2^N - 1)$$

$$\underbrace{2^1, 2^2, 2^3, 2^4, \dots, 2^N}_{\substack{N \text{ iter} \\ \uparrow \\ N^{th} \text{ iter}}} \quad //$$

How to calculate Big O Notation from no. of iterations

- 1) Neglect all lower order terms
- 2) Neglect all constant coefficient term

$$\# \text{ Iterations} = 4N^2 + 3N + 1$$

$\underbrace{\hspace{2em}}_{\substack{\hookrightarrow \text{Neglect} \\ 4N^2 \rightarrow N^2}} \Rightarrow O(N^2)$

$$Q \quad 4N^3 + 3N + 10^6$$

$\xrightarrow{N^3}$

$$Q \quad 3N\sqrt{N} + 4\log N + 31N\log N \Rightarrow O(N\sqrt{N})$$

$$N = 2^{32}$$

$\frac{N}{\log N}$ $\underline{\underline{=}}$	$N \log N$
$2^{32} \times 2^{16}$	$2^{32} \log_2 2^{32} \Rightarrow 2^{32} \times 2^5$
2^{48}	$= 2^{37}$

$$O(1) < \log_2 N < \sqrt{N} < N < N \log N < N\sqrt{N} < N^2 < \underline{\underline{2^N}} < \underline{\underline{N!}} < \underline{\underline{N^N}}$$

$$N = 32$$

$$\begin{array}{c|c} \log_2 32 & \sqrt{32} \\ 5 & 5.65 \\ \hline & 32 \\ & 32 \end{array} \quad \begin{array}{c|c} 32 \log_2 32 & 32 \times \sqrt{32} \\ 160 & 181 \\ \hline 1024 & 4 \times 10^3 \end{array} \quad \begin{array}{c|c} 32^2 & 2^{32} \\ 1024 & 4 \times 10^3 \\ \hline & \gg \\ & 4 \times 10^3 \end{array}$$

DOUBTS

Void $f(n)$ {

int $i = N; \leftarrow$

while ($i >= 1$)

$i = i/2;$

}

}

Void $f(n)$ {

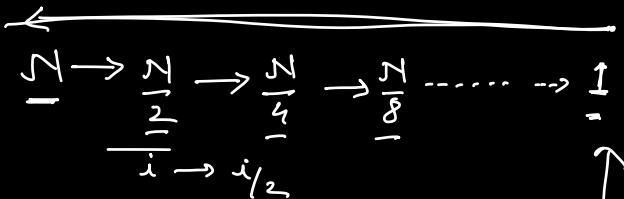
$s = 0;$

for ($\underbrace{i=1}_{s_1}; \underbrace{i \leq n}_{s_2}; \underbrace{i = i*2}_{s_3})$ {

$s = s + i;$

}

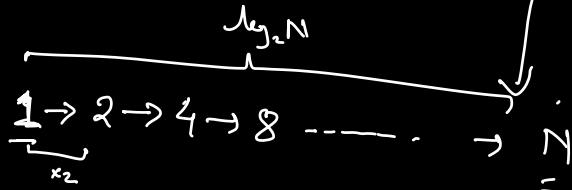
}



$\log_2 N$

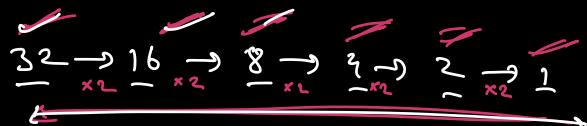
$i \geq 1$

$i_{\max} \Rightarrow 1$



$i \leq n$

$$N = 32$$



$$\times [a, b] \Rightarrow [1, 10] \Rightarrow 10$$

$$1 \rightarrow 2^0$$

$$2 \rightarrow 2^1$$

$$4 \rightarrow 2^2$$

$$8 \rightarrow 2^3$$

⋮

$$N \rightarrow 2^K$$

K times

$$\frac{a}{n}$$

$$\underline{\sqrt{n}}$$

$$\underline{O(\log N)}$$

$$\Rightarrow N = 2^K$$

$$\Rightarrow \underline{\log_2 N} = K$$