Numerical nonabelian Hodge correspondence

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Project status

Ready to start in April 2021.

Mentors

PD Dr. Jan Swoboda, Menelaos Zikidis.

Student(s) participating

Open!

Project description

[Prerequisites: A first course in differential geometry and partial differential equations. Basic Python programming skills.] This project is placed at the crossroads of harmonic maps, surface group representations and Higgs bundles. The link between these subjects is the nonabelian Hodge correspondence (NAH): every sufficiently "nice" surface group representation $\rho\colon \pi_1(S) \to G$ gives rise to a Higgs bundle, i.e. a holomorphic vector bundle with the extra datum of a Higgs field, and vice versa. While the case of an abelian Lie group G reduces to the classical Hodge theory of S, the situation becomes more intricate (and more interesting) for nonabelian Lie groups such as $G = \mathrm{SL}(n, \mathbb{C})$. NAH then involves the solution of the (nonlinear) harmonic map equation into hyperbolic space or more general noncompact symmetric spaces. Even in the case $n = 2$, many aspects of NAH are not entirely understood, and in recent years a number of new research directions and open questions have emerged. In addition to traditional analytic methods, in order to better understand the underlying harmonic maps, Dumas and Neitzke recently developed numerical tools to experimentally support some open conjectures related to NAH. The aim of this project is to extend this numerical study to more general cases and to use it to further explore the less understood aspects of NAH.