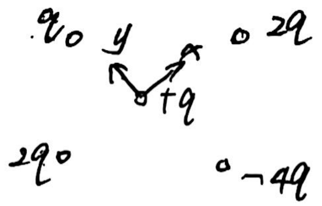
1. $\vec{F}_{x1} = \dfrac{2q\cdot e}{4\pi\varepsilon_0(\frac{\sqrt{2}}{2}a)^2} = \dfrac{eq}{\pi\varepsilon_0 a^2}$



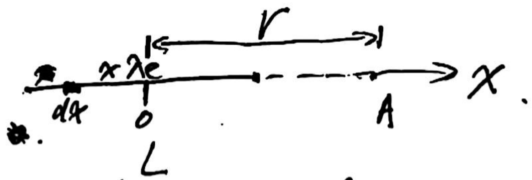
$\vec{F}_{x2} = \dfrac{-2q\cdot e}{4\pi\varepsilon_0(\frac{\sqrt{2}}{2}a)^2} = \dfrac{-eq}{\pi\varepsilon_0 a^2}$

$\vec{F}_{y1} = \dfrac{(-4q)\cdot e}{4\pi\varepsilon_0(\frac{\sqrt{2}}{2}a)^2} = \dfrac{-2eq}{\pi\varepsilon_0 a^2}$

$\vec{F}_{y2} = \dfrac{-q\cdot e}{4\pi\varepsilon_0(\frac{\sqrt{2}}{2}a)^2} = \dfrac{-eq}{2\pi\varepsilon_0 a^2}$

$\therefore \vec{F}_x = 0 \quad \vec{F}_y = -\dfrac{5eq}{2\pi\varepsilon_0 a^2}$ 即大小 $\dfrac{5eq}{2\pi\varepsilon_0 a^2}$. 指向 $-4q$ 电荷

3. $dE = \dfrac{\lambda_e dx}{4\pi\varepsilon_0(r-x)^2}$ 指向该点

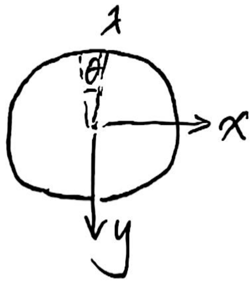


$$E = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\lambda_e}{4\pi\varepsilon_0}\cdot\frac{dx}{(r-x)^2} = \frac{\lambda_e}{4\pi\varepsilon_0}\cdot\frac{1}{r-x}\Big|_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{\lambda_e}{4\pi\varepsilon_0}\cdot\left(\frac{1}{r-\frac{L}{2}} - \frac{1}{r+\frac{L}{2}}\right)$$

$$= \frac{\lambda_e L}{4\pi\varepsilon_0(r^2-\frac{L^2}{4})}$$

$\lambda_e>0$ 时 方向指向延长线上该点. $\lambda_e<0$ 则相反

8. $\lambda = \dfrac{Q}{l} \approx \cancel{6.5\times10^{-9}}\ 1\times10^{-9}\ C/m$.



$\because$ 均匀分布. 由对称性. $E_x = 0$.

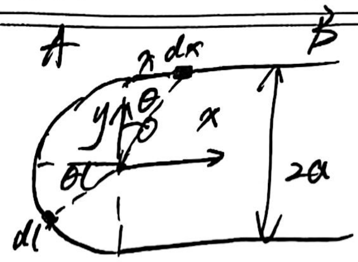
$\therefore$ 仅计算缺口圆对称处的场强.

设点到圆心与 y 负半轴夹角为 θ

$\therefore dE_y = \dfrac{\lambda dl}{4\pi\varepsilon_0 r^2}\cdot\cos\theta \quad \because dl = r d\theta$

$\therefore dE_y = \dfrac{\lambda d\theta}{4\pi\varepsilon_0 r}\cdot\cos\theta$

$\therefore E_y = 2\displaystyle\int_0^{\frac{\pi}{25}} \frac{\lambda}{4\pi\varepsilon_0 r}\cdot\cos\theta d\theta = \frac{\lambda}{2\pi\varepsilon_0 r}\sin\frac{\pi}{25}$



9、由对称性 $\vec{E_y}=0$

仅关注 $\vec{E_x}$、设线电荷密度为 λ.

A侧：$dE_y=\frac{\lambda dl}{4\pi\varepsilon_0 a^2}\cos\theta$

$=\frac{\lambda d\theta}{4\pi\varepsilon_0 a}\cos\theta$

$\therefore E_{yA}=\cancel{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}\ 2\int_0^{\frac{\pi}{2}}\frac{\lambda}{4\pi\varepsilon_0 a}\cos\theta\, d\theta=\frac{\lambda}{2\pi\varepsilon_0 a}$

B侧：$dE_y=\frac{-\lambda dx}{4\pi\varepsilon_0(x^2+a^2)}\sin\theta$

$\cancel{\therefore E_y=-2\int_0^{+\infty}\frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{dx}{a^2+x^2}=-2\int\frac{\lambda}{4\pi\varepsilon_0 a}\arctan\frac{x}{a}\Big|_0^{+\infty}=-\frac{\lambda}{4\varepsilon_0 a}}$

$\because r=\frac{a}{\cos\theta}\qquad \tan\theta=\frac{x}{a}\qquad x=a\tan\theta\quad dx=a\cdot\frac{1}{\cos^2\theta}$

$\therefore dE_y=-\frac{\lambda d\theta}{4\pi\varepsilon_0 a}\sin\theta$

$E_{yB}=-2\int_0^{\pi}\frac{\lambda}{4\pi\varepsilon_0 a}\sin\theta\, d\theta=\frac{\lambda}{4\pi\varepsilon_0 a}(\cos\frac{\pi}{2}-\cos 0)=-\frac{\lambda}{2\pi\varepsilon_0 a}$

综上 $E_{yA}+E_{yB}=0$ 即O处场强为0.

10、 ~~由高斯定律. $\vec{E}_{环} = \vec{E}\oint d\vec{S}$~~

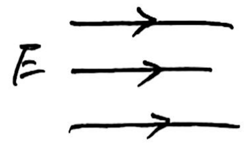
由对称性、~~球形表面 $\vec{E}_{球}=0$~~.

$\vec{E}_{环} = 0$.

$$\vec{E}_{线} = \cancel{\int_R^{4R}\frac{\lambda dx}{4\pi\varepsilon_0(R+x)^2}} \int_R^{4R}\frac{\lambda dx}{4\pi\varepsilon_0 x^2} = \frac{\frac{Q}{3R}}{4\pi\varepsilon_0}\left(-\frac{1}{x}\right)\Big|_R^{4R} = \frac{Q}{12\pi R\varepsilon_0}\cdot\frac{3}{4R}$$

$$= \frac{Q}{16\pi R^2\varepsilon_0}$$

方向沿线指向中心.

11. 

(1) $\Phi_{e1} = \vec{E}\cdot\int_S d\vec{S} = 7.5\times10^{-3}\,N\cdot m^2\cdot C^{-1}$

(2) $\Phi_{e2} = E\int_S dS\cos\theta_2 = \Phi_{e1}\cos\theta_2 = 3.75\sqrt{3}\times10^{-3}\,N\cdot m^2\cdot C^{-1}$

(3) $\Phi_{e3} = 0$

(4) $\Phi_{e4} = \Phi_{e1}\cos\theta_4 = -3.75\times10^{-3}\,N\cdot m^2\cdot C^{-1}$

(5) $\Phi_{e5} = -7.5\times10^{-3}\,N\cdot m^2\cdot C^{-1}$

13、令 $\vec{E}_1 = E_0\vec{i}$ $\vec{E}_2 = \cancel{E_0}\, E_0\left(\frac{z}{a}\right)(\vec{i}+\vec{j})$ 即 $\vec{E} = \vec{E}_1 + \vec{E}_2$

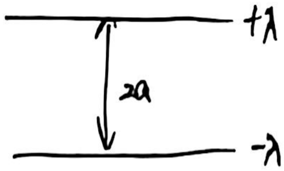
由 $\because \oint\vec{E}d\vec{S} = \frac{1}{\varepsilon_0}q$

$$\oint\vec{E}d\vec{S} = \oint\vec{E}_1 d\vec{S} + \oint\vec{E}_2 d\vec{S} = E_0a^2 + \int_0^a E_0\frac{z}{a}\cdot\sqrt{2}a^2 dz$$

$$= E_0a^2 + \frac{\sqrt{2}}{2}E_0a^2 = \left(1+\frac{\sqrt{2}}{2}\right)E_0a^2$$

$$\therefore q = \varepsilon_0\left(1+\frac{\sqrt{2}}{2}\right)E_0a^2 = \varepsilon_0\left(1+\frac{\sqrt{2}}{2}\right)$$

14、



$$\vec{F} = \vec{E}q = \frac{\lambda}{2\pi\varepsilon_0 \cdot 2a} \cdot (-\lambda) = -\frac{\lambda^2}{4\pi\varepsilon_0 a}$$

∴大小 $\frac{\lambda^2}{4\pi\varepsilon_0 a}$　方向指向另一根直线