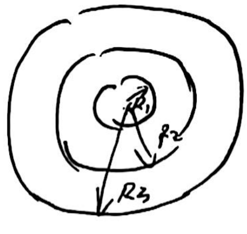
2-2. 设A带电量为Q_1



$$\int_{R_3}^{\infty}\frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2}dr+\int_{R_1}^{R_2}\frac{Q_1}{4\pi\varepsilon_0 r^2}dr$$

$$=\frac{Q_1+Q_2}{4\pi\varepsilon_0 R_3}+\frac{Q_1}{4\pi\varepsilon_0 R_1}-\frac{Q_1}{4\pi\varepsilon_0 R_2}=\varphi_1$$

$$\therefore Q_1=4\pi\varepsilon_0\frac{R_1R_2R_3\varphi_1-R_1R_2Q}{R_2R_3-R_1R_3+R_1R_2}$$

$r<R_1$:　$\varphi=\varphi_1$　$E=0$

$R_1<r<R_2$:　$\varphi=\int_{R_3}^{\infty}\frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2}dr+\int_{r}^{R_2}\frac{Q_1}{4\pi\varepsilon_0 r^2}dr=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r}-\frac{Q_1}{R_2}+\frac{Q+Q_1}{R_2}\right)$

$E=\frac{Q_1}{4\pi\varepsilon_0 r^2}$

$R_2<r<R_3$:　$\varphi=\int_{R_3}^{\infty}\frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2}dr=\frac{Q_1+Q_2}{4\pi\varepsilon_0 R_3}$　$E=0$

$r>R_3$:　$\varphi=\frac{Q_1+Q_2}{4\pi\varepsilon_0 r}$　$E=\frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2}$

2-3. $r<R_1$:　$\varphi=\int_{R_3}^{\infty}\frac{q_1+q_2}{4\pi\varepsilon_0 r^2}dr+\int_{R_1}^{R_2}\frac{q_1}{4\pi\varepsilon_0 r^2}dr=\frac{q_1+q_2}{4\pi\varepsilon_0 R_3}+\frac{q_1}{4\pi\varepsilon_0 R_1}-\frac{q_1}{4\pi\varepsilon_0 R_2}$

$E=0$

$R_1<r<R_2$:　$\varphi=\frac{q_1+q_2}{4\pi\varepsilon_0 R_3}+\frac{q_1}{4\pi\varepsilon_0 r}-\frac{q_1}{4\pi\varepsilon_0 R_2}$　$E=\frac{q_1}{4\pi\varepsilon_0 r^2}$

$R_2<r<R_3$:　$\varphi=\frac{q_1+q_2}{4\pi\varepsilon_0 R_3}$　$E=0$

$r>R_3$:　$\varphi=\frac{q_1+q_2}{4\pi\varepsilon_0 r}$　$E=\frac{q_1+q_2}{4\pi\varepsilon_0 r^2}$

2-5. 由上至下，设6个表面电荷面密度为$\sigma_1, \sigma_2, \dots \sigma_6$

$\therefore \sigma_3+\sigma_4=1.3\times10^{-5}C/m^2$　　$\sigma_1+\sigma_2+\sigma_5+\sigma_6=0$ ③　　$\sigma_2+\sigma_5+\sigma_3+\sigma_4=0$ ④

对两个导体板取高斯面，有 $\sigma_2+\sigma_3=0$　$\sigma_4+\sigma_5=0$　~~$\sigma_2+\sigma_3=0$~~

$\because$ A、C相连，电势相同．$\therefore\left(\frac{\sigma_1}{2\varepsilon_0}+\frac{\sigma_2}{2\varepsilon_0}+\frac{\sigma_3}{2\varepsilon_0}+\frac{\sigma_4}{2\varepsilon_0}-\frac{\sigma_5}{2\varepsilon_0}-\frac{\sigma_6}{2\varepsilon_0}\right)d_{12}=$

$\left(-\frac{\sigma_1}{2\varepsilon_0}-\frac{\sigma_2}{2\varepsilon_0}+\frac{\sigma_3}{2\varepsilon_0}+\frac{\sigma_4}{2\varepsilon_0}+\frac{\sigma_5}{2\varepsilon_0}+\frac{\sigma_6}{2\varepsilon_0}\right)d_{23}$

对A、B、C板内任一点P．$E_P=0$．即有 $\sigma_1-\sigma_2-\sigma_3-\sigma_4-\sigma_5-\sigma_6=0$

$\sigma_1+\sigma_2+\sigma_3-\sigma_4-\sigma_5-\sigma_6=0$

$\sigma_1+\sigma_2+\sigma_3+\sigma_4+\sigma_5-\sigma_6=0$

联立得 $\sigma_1=\sigma_6=6.5\times10^{-6}C/m^2$　　$\sigma_2=-4.9\times10^{-6}C/m^2$　$\sigma_3=4.9\times10^{-6}C/m^2$

$\sigma_4=8.1\times10^{-6}C/m^2$　$\sigma_5=-8.1\times10^{-6}C/m^2$

2-7 (1) $E_o = 0$

$$\varphi = \int_R^{\infty} \frac{Q}{4\pi\varepsilon_0 r^2} dr + \int_{\cancel{R+}x}^{\infty} \frac{q}{4\pi\varepsilon_0 \cancel{(R+x)} r^2} dr = \frac{Q}{4\pi\varepsilon_0 R} + \frac{q}{\cancel{4\pi\varepsilon_0 (R+x)}\ 4\pi\varepsilon_0 x}$$

(2) $\because E_P = E_Q + E_q = 0$

$$E_q = \frac{q}{4\pi\varepsilon_0 (x-\frac{R}{2})^2}$$

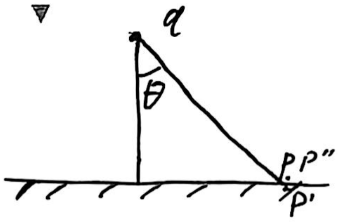
$$E_Q = -\cancel{Q} - \frac{q}{4\pi\varepsilon_0 (x-\frac{R}{2})^2}$$ 即大小 $\frac{q}{4\pi\varepsilon_0 (x-\frac{R}{2})^2}$ 方向沿径矢向外
指向q

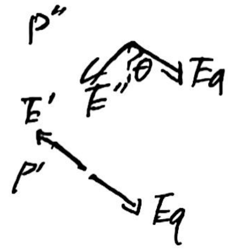
$$\because \varphi_P = \varphi = \frac{Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 \cancel{(R+x)}\, x} = \varphi_Q + \varphi_q$$

$$\because \varphi_q = \int_{x-\frac{R}{2}}^{\infty} \frac{q}{4\pi\varepsilon_0 r^2} dr = \frac{q}{4\pi\varepsilon_0 (x-\frac{R}{2})}$$

$$\therefore \varphi_Q = \frac{Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 x} - \frac{q}{4\pi\varepsilon_0 (x-\frac{R}{2})}$$

2-9. 取P点延长线紧邻P的点P'





∵ P'位于板内部 $E_p=0$.

∴ $E_q+E'=0$

$E'=-E_q=-\frac{q}{4\pi\varepsilon_0 R^2}$

取与P'关于板面对称且与P'紧邻的点P''

则平板其他点对P''点产生的电场强度E''与E'关于平面对称

∴ $E=-2E'\cos\theta=-2E'\cdot\frac{h}{R}=\frac{qh}{2\pi\varepsilon_0 R^3}$

∴ $\sigma=\cancel{2}\varepsilon_0 E=\frac{qh}{2\pi R^3}$

2-11:(1) 当 $r<R_1$: $\cancel{\text{由}}$ $\oint_S \vec{D}d\vec{S}=0$

$\vec{D}=0$. $\vec{E}=0$

当 $R_1<r<R$ $\oint_S \vec{D}d\vec{S}=Q$

$D\cdot 4\pi r^2=Q$

$D=\frac{Q}{4\pi r^2}$

方向均沿半径向外

$E=\frac{D}{\varepsilon}=\frac{D}{\varepsilon_{r1}\varepsilon_0}=\frac{Q}{4\pi\varepsilon_0\varepsilon_{r1}r^2}$

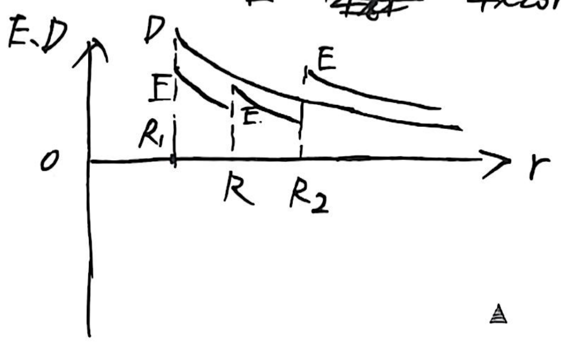
当 $R<r<R_2$ $\oint_S \vec{D}d\vec{S}=Q$

$D=\frac{Q}{4\pi r^2}$

$E=\frac{Q}{4\pi\varepsilon_0\varepsilon_{r2}r^2}$

当 $r>R_2$ $D=\frac{Q}{4\pi r^2}$

$E=\cancel{\frac{Q}{4\pi r^2}}\ \frac{Q}{4\pi\varepsilon_0 r^2}$



(2) $U_{12}=\int_{R_1}^{R}\frac{Q\,dr}{4\pi\varepsilon_0\varepsilon_{r1}r^2}+\int_{R}^{R_2}\frac{Q\,dr}{4\pi\varepsilon_0\varepsilon_{r2}r^2}=\frac{Q}{4\pi\varepsilon_0\varepsilon_{r1}R_1^2}-\frac{Q}{4\pi\varepsilon_0\varepsilon_{r1}R^2}+\frac{Q}{4\pi\varepsilon_0\varepsilon_{r2}R^2}-\frac{Q}{4\pi\varepsilon_0\varepsilon_{r2}R_2^2}$

$=-3.8\times10^3V$

(3) ∵ $\vec{P}=\cancel{\frac{E}{(\varepsilon_{r1}-1)\varepsilon_0}}=(\varepsilon_{r1}-1)\varepsilon_0E=\frac{(\varepsilon_{r1}-1)Q}{4\pi\varepsilon_{r1}R_1^2}$

∴ $\sigma'=P\cos\pi=-\frac{(\varepsilon_{r1}-1)Q}{4\pi\varepsilon_{r1}R_1^2}=9.9\times10^{-6}C/m^2$

2-14. 当 $R_1<r<r_0$, $\oint_S\vec{D}\,d\vec{S}=q_内$

$$D=\frac{q_内}{4\pi r^2}$$

$$E_1=\frac{D}{\varepsilon_0\varepsilon_{r1}}=\frac{q_内}{4\pi\varepsilon_0\varepsilon_{r1}r^2}$$

同理 $E_2=\frac{2D}{\varepsilon_0\varepsilon_{r1}}=\frac{q_内}{2\pi\varepsilon_0\varepsilon_{r1}r^2}$

∴ $E_{1max}=\frac{q_内}{4\pi\varepsilon_0\varepsilon_{r1}R_1^2}$ $E_{2max}=\frac{q_内}{2\pi\varepsilon_0\varepsilon_{r1}r_0^2}$

设内部电荷线密度为λ

$U=\int_{R_1}^{r_0}E_1dr+\int_{r_0}^{R_2}E_2dr=\int_{R_1}^{r_0}\frac{\lambda}{2\varepsilon_0\varepsilon_{r1}r}dr+\int_{r_0}^{R_2}\frac{\lambda}{\varepsilon_0\varepsilon_{r1}r}dr$

$=\frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r1}}\ln\frac{r_0}{R_1}+\frac{\lambda}{\pi\varepsilon_0\varepsilon_{r1}}\ln\frac{R_2}{r_0}=\frac{\lambda}{2\pi\varepsilon_0\varepsilon_d}\ln\frac{R_2^2}{R_1r_0}$

∵ $E_{1max}=\frac{\lambda}{2\varepsilon_0\varepsilon_{r1}R_1^2}=\frac{U}{R_1\ln\frac{R_2^2}{R_1r_0}}$

$E_{2max}=\frac{2U}{r_0\ln\frac{R_2^2}{R_1r_0}}$

∴ $\frac{E_{1max}}{E_{2max}}=\frac{r_0}{2R_1}<1$ ∴ $E_{1max}<E_{2max}$

∴ 外层先被击穿

∴ $U_{max}=\frac{E_{max}r_0\ln\frac{R_2^2}{R_1r_0}}{2}$

2-16. $\cancel{D_1 = \oint_S \vec{D_1} d\vec{S_1} = \frac{Q}{S_1+S_2} \Delta S}$

$$\cancel{D_1 = \frac{Q}{S_1+S_2}}$$

$$\oint_S \vec{D_1} d\vec{S} = \sigma_1 \Delta S = D_1 \Delta S$$

$$D_1 = \sigma_1$$

$$\therefore E_1 = \frac{\sigma_1}{\varepsilon_1}$$

同理 $D_2 = \sigma_2$ $E_2 = \frac{\sigma_2}{\varepsilon_2}$

∵ 上下板电势差一定

$\therefore E_1 = E_2$ $\because \sigma_1 S_1 + \sigma_2 S_2 = Q$ $\frac{\sigma_1}{\varepsilon_1} = \frac{\sigma_2}{\varepsilon_2}$

$$\begin{cases} \sigma_1 = \dfrac{\varepsilon_1 Q}{\varepsilon_1 S_1 + \varepsilon_2 S_2} \\ \sigma_2 = \dfrac{\varepsilon_2 Q}{\varepsilon_1 S_1 + \varepsilon_2 S_2} \end{cases}$$

$$\therefore E_1 = \frac{Q}{(\varepsilon_1 S_1 + \varepsilon_2 S_2)}$$

$$P_1 = \cancel{\frac{(\varepsilon_1)Q}{\varepsilon_1 S_1 + \varepsilon_2 S_2}} \frac{(\varepsilon_1 - \varepsilon_0)Q}{\varepsilon_1 S_1 + \varepsilon_2 S_2} = \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_1}\sigma_1$$

$$\therefore \sigma_1' = P_1 = \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_1}\sigma_1 = \frac{(\varepsilon_1 - \varepsilon_0)Q}{\varepsilon_1 S_1 + \varepsilon_2 S_2}$$

同理 $\sigma_2' = P_2 = \frac{\varepsilon_2 - \varepsilon_0}{\varepsilon_2}\sigma_2 = \frac{(\varepsilon_2 - \varepsilon_0)Q}{\varepsilon_1 S_1 + \varepsilon_2 S_2}$