

2-18. ∴ 铜球上下表面等势

$$\therefore \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\therefore C_1 = 4\pi\epsilon_{r1}\epsilon_0 R \quad C_2 = 4\pi\epsilon_{r2}\epsilon_0 R$$

$$\therefore Q_1 + Q_2 = Q$$

$$\therefore Q_1 = \frac{\epsilon_{r1}}{\epsilon_{r1} + \epsilon_{r2}} Q \quad Q_2 = \frac{\epsilon_{r2}}{\epsilon_{r1} + \epsilon_{r2}} Q$$

$$\therefore E_1 = \frac{\sigma_1}{\epsilon_{r1}\epsilon_0} = \frac{Q}{2\pi R^2(\epsilon_{r1} + \epsilon_{r2})\epsilon_0} \quad E_2 = \frac{Q}{2\pi R^2\epsilon_{r2}} = E_1$$

$$\therefore \sigma_1' = \frac{F_1}{A} \quad P_1 = \frac{F_1}{A\epsilon_0} = \chi\epsilon_0 E = \frac{(\epsilon_{r1} - 1)Q}{2\pi R^2(\epsilon_{r1} + \epsilon_{r2})}$$

$$\sigma_1' = \vec{P}_1 \cdot \vec{n}_2 = \frac{(\epsilon_{r1} - 1)Q}{2\pi R^2(\epsilon_{r1} + \epsilon_{r2})}$$

$$\sigma_2' = - \frac{(\epsilon_{r2} - 1)Q}{2\pi R^2(\epsilon_{r1} + \epsilon_{r2})}$$

2-21. 取圆柱形高斯面

$$\oint_S D \cdot dS = Q_{enc} = \lambda L$$

$$D_1 = \frac{\lambda}{2\pi r}$$

$$\therefore E_1 = \frac{\lambda}{2\pi r\epsilon_1}$$

$$\therefore E_{1max} = \frac{\lambda}{2\pi R_1\epsilon_1}$$

$$\text{同理 } E_{2max} = \frac{\lambda}{2\pi R_2\epsilon_2}$$

$$\text{即有 } R_1\epsilon_1 = R_2\epsilon_2$$

$$\therefore U = \int_{R_1}^{R_2} E_1 dr + \int_{R_2}^{R_3} E_2 dr = \frac{\lambda}{2\pi\epsilon_1} \ln \frac{R_2}{R_1} + \int_{R_2}^{R_3} \frac{\lambda}{2\pi\epsilon_2} \ln \frac{R_3}{R_2}$$

$$\therefore C = \frac{Q}{U} = \frac{\lambda L}{\frac{\lambda}{2\pi\epsilon_1} \ln \frac{R_2}{R_1} + \frac{\lambda}{2\pi\epsilon_2} \ln \frac{R_3}{R_2}} = \frac{2\pi\epsilon_1\epsilon_2 L}{\epsilon_2 \ln \frac{R_2}{R_1} + \epsilon_1 \ln \frac{R_3}{R_2}}$$

$$C_{\text{单位}} = \frac{2\pi\epsilon_1\epsilon_2}{\epsilon_2 \ln \frac{R_2}{R_1} + \epsilon_1 \ln \frac{R_3}{R_2}} \quad \triangle$$



2-23. 设 A、B 带  $Q, -Q$ .

$$\therefore E_{AK} = E_{BK} = \frac{Q}{\epsilon_0 S}$$

$$\therefore Q_{AK} = Q_{BK} = E_{AK} d_1 = \frac{Q d_1}{\epsilon_0 S} \quad \therefore C_{AK} = C_{BK} = \frac{\epsilon_0 S}{d_1}$$

$$\therefore \frac{1}{C_1} = \frac{1}{C_{AK}} + \frac{1}{C_{BK}} = \frac{2\epsilon_0 S}{Q d_1} \quad C_1 = \frac{C_{AK} C_{BK}}{C_{AK} + C_{BK}} = \frac{Q d_1}{2\epsilon_0 S} \frac{\epsilon_0 S}{2 d_1}$$

$$\therefore E_{AB} = \frac{2Q}{\epsilon_0 S}$$

$$\therefore \frac{Q_2}{U} = \frac{2Q d_2}{\epsilon_0 S} \quad C_2 = \frac{\epsilon_0 S}{2 d_2}$$

$$\therefore C = C_1 + C_2 = \frac{Q d_1 + 4 Q d_2}{2 \epsilon_0 S}$$

$$\therefore C = C_1 + C_2 = \frac{\epsilon_0 S (d_1 + d_2)}{2 d_1 d_2} = 7.12 \times 10^{-10} \text{ F}$$

$$C' = C_{AK} + C_2 = 1.07 \times 10^{-9} \text{ F}$$

2-24. ~~将电容器看作两个电容器串联~~ 平板电容.

$$C_0 = \frac{\epsilon_0 S}{d} = \frac{\epsilon_0 a b}{d}$$

$$\therefore dC = \frac{r \tan \theta}{(d + r \tan \theta)} \frac{Q}{\epsilon_0 a b} dr = \frac{\epsilon_0 b}{d + r \cdot \frac{L}{a}} dr = \frac{\epsilon_0 a b}{r L} dr$$

$$\therefore \frac{1}{C'} = \int_0^L \frac{\epsilon_0 a b}{r L} dr =$$

$$\frac{1}{C'} = \int_0^L \frac{\epsilon_0 b}{d + \frac{L}{a} r} dr = \frac{\epsilon_0 b a}{L} \ln \left( d + \frac{L}{a} r \right) \Big|_0^L$$

$$= \frac{\epsilon_0 b a}{L} \ln \left( \frac{d+L}{d} \right)$$



2-27. 设上板带+Q. 则下板带-Q

$$\text{则 } E_1 = \frac{\sigma}{\epsilon_1 \epsilon_0} = \frac{Q}{S \epsilon_1 \epsilon_0}$$

$$E_2 = \frac{Q}{S \epsilon_2 \epsilon_0}$$

$$\therefore U = E_1 \frac{d}{2} + E_2 \frac{d}{2} = \frac{Qd}{2S\epsilon_0} \left( \frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} \right) = \frac{Qd(\epsilon_1 + \epsilon_2)}{2S\epsilon_0 \epsilon_1 \epsilon_2}$$

$$\therefore C = \frac{Q}{U} = \frac{2S\epsilon_0 \epsilon_1 \epsilon_2}{d(\epsilon_1 + \epsilon_2)}$$

2-29. 设上半部分带电  $Q_1$ . 下半带  $Q_2$ . 则球总电荷  $Q_1 + Q_2$

$$\text{则 } D_1 = \frac{Q_1}{2\pi r^2} \quad E_1 = \frac{Q_1}{2\pi r^2 \epsilon_0}$$

$$E_2 = \frac{Q_2}{2\pi r^2 \epsilon_0 \epsilon_r}$$

$$\therefore U = E_1 d = E_2 d \quad \text{即 } Q_1 = \frac{Q_2}{\epsilon_r}$$

$$\therefore Q_1 + Q_2 = Q \quad U = \int_{R_1}^{R_2} E_1 dr = \frac{R_2}{2\pi(R_1 + R_2)\epsilon_0} \frac{(R_2 - R_1)Q}{2\pi R_1 R_2 \epsilon_0}$$

$$\therefore C = \frac{Q}{U} = \frac{Q_1 + Q_2}{\frac{Q_1(R_2 - R_1)}{2\pi\epsilon_0(R_1 + R_2)}} = \frac{(Q_1 + Q_2)Q_1}{Q_1(R_2 - R_1)} = \frac{(\epsilon_r + 1)Q_1}{Q_1(R_2 - R_1)} = \frac{2\pi\epsilon_0(\epsilon_r + 1)R_1 R_2}{R_2 - R_1}$$

2-31. 设第二个电容  $C'$

$$\therefore C = C_1 + C'$$

$$\therefore Q_0 = C_1 U_0 = (C_1 + C') U'$$

$$\text{即 } C' = \frac{C_1 U_0}{U'} - C_1 = \frac{700}{3} \text{ pF} \approx 233 \text{ pF}$$

$$\Delta W = \frac{1}{2} C_1 U_0^2 - \frac{1}{2} C U'^2 = 3.5 \times 10^{-7} \text{ J}$$

电能转化为电荷从原电容转移至另一个电容时产生的热能



$$2-35. (1) \because E = \frac{Q}{S} \cdot \frac{1}{\epsilon_0}$$

$$U_0 = Ed = \frac{Qd}{\epsilon_0 S} \quad U' = E(d-b) = \frac{Q(d-b)}{\epsilon_0 S}$$

$$\therefore \Delta A = \frac{1}{2} Q(U' - U_0) = -\frac{Qb}{2\epsilon_0 S}$$

(2)  $\because \Delta A < 0$ . 故系统对外做功

$$\therefore \text{导体板能量增大} \quad W = -\Delta A = \frac{Qb}{2\epsilon_0 S}$$

被吸入

$$(3) \because \frac{Qd}{\epsilon_0 S} = \frac{Q'(d-b)}{\epsilon_0 S}$$

$$Q' = \frac{d}{d-b} Q_0$$

$$\therefore \Delta A = \frac{1}{2} U(Q' - Q_0) = \frac{1}{2} U \cdot \frac{b}{d-b} Q_0 = \frac{1}{2} U \cdot \frac{b}{d-b} \cdot \frac{U\epsilon_0 S}{d} = \frac{b\epsilon_0 S U^2}{2d(d-b)}$$

$\therefore$  导体板进入时, 电源做功  $W_{\text{源}} = U\Delta Q = 2\Delta A$

$$\therefore \text{外力做功 } \Delta A' = \Delta A - W_{\text{源}} = -\frac{b\epsilon_0 S U^2}{2d(d-b)}$$

$$W = -\Delta A' = \frac{b\epsilon_0 S U^2}{2d(d-b)} \quad \text{故需推入}$$

$$W = 2\Delta A' = \frac{b\epsilon_0 S U^2}{2d(d-b)} \quad \text{被吸入}$$

