[5] K.Inoue, J. Lightwave Technol. Vol. 13, pp. 856-861, 1995
[6] E.Lichtman, J. Opt. Commun. Vol. 12, pp. 53-58, 1991
[7] D.Marcuse, J. Lightwave Technol. Vol. 9, pp. 356-361, 1991
[8] D.Marcuse, J. Lightwave Technol. Vol. 9, pp. 1330, 1991
[9] K.O.Hill, D.C.Johnson, B.S.Kawasaki, and R.I.MacDonald, J. Appl. Phys. Vol. 49, pp. 5098-5106, 1978
[10] N.Shibata, F.P.Braun, and R.G.Waarts, J. Quantum Electron. Vol. QE-23, pp. 1205-1210, 1987
[11] N.A.Olsson, J. Lightwave Technol. Vol. 7, 1071-1082, 1989

Symmtry-breaking Subharmonic Bifurcations of Periodically Parameter-excited Systems [1]

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Abstract: In this communication, the existence of the symmetry-breaking subharmonic branches in periodically parameter-exited systems is proved, which supports the previous methods for computing bifurcation structures. Following these results, we can obtain all the possible subharmonic branches only by classifying subgroups.
Key Words: Symmetry-breaking, Subharmonical bifurcation.

The dynamic systems of finite but very high dimensions, such as coupled map lattices and coupled oscillators, are suggested to simulate the spatiotemporal behaviors of the extended systems. Although there exist some differences between these two cases, such high-dimensional systems still exhibit the non-trivial collective behaviors of spatially extended systems[1]: their evolution depends on each unit, and morever, their statistical quantities are not stationary. In generic cases, this kind of system can no longer be reduced. Thus it is very difficult to predict their bifurcations in the determinate methods. However, for the symmetric systems, the problem seems to be reduced since symmetries may impose constraints on their bifurcations. This technique has been utilized in our investagations on symmetry-breaking bifurctions of the Mathieu-Duffing equation and the flutters of soft tubes [2] (also, see for the scale symmetry in turbulence [4]. In this communication, we present the rigorously mathematical results to support this technique. In fact, Golubitsky et al. [3] have obtained the same results on autonomous systems independently explicitly on time. Our results work for a kind of non-autonomous systems which depends periodically on time.

Let us consider a general periodically non-autonomous system of high dimensions instead of the coupled oscillators:

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$$\dot{u} + A(\lambda)u + g(u, \lambda, nt) = 0 \tag{1}$$

where $g(u, \lambda, nt) = (||u||^2)$ is an infinite smooth function from R^n to R^n. For simplicity, we only consier the resonance Hopf bifucation: $A(\lambda)$ has a simple pair of eigenvalues $\sigma(\lambda) \pm i\omega(\lambda)$, where $\sigma(0) = 0, \omega(0) = 1$, $d\sigma/d\lambda(0) \neq 0$.

(1) Equation (1) is defined as temporal symmetry if g is Z_n-equivariant with respect to the temporal argument t, where $Z_n = \{k\frac{2\pi}{n}, k = 0, 1, \ldots, n-1\}$, that is

$$g(u, \lambda, n(t + \frac{2\pi}{n}) = g(u, \lambda, nt)$$

(2) Equation (1) is defined as spatial symmtries if g is Γ-equivariant with respect to the argument u (where Γ is a compact Lie group), that is,

$$\gamma g(u, \lambda, nt) = g(\gamma u, \lambda, nt)$$

Theorem 1. If equation (1) satisfies the resonant Hopf condition, Its 2π subharmonic bifurcation diagrams can be determined only by the non-degenerated conditions.

where the non-degenetated conditions are refered to as the non-trivial relations for the coefficients of the algbraic bifucation equations equivalent to the original equation (1). We can compute these coefficients directly by the formal differentials of the equation (1) with respect to its argument and parameters. If these coefficients are substituted into some specified expressions and produce non-zero value, it is defined as non-degenerated conditions. Moreover, it is just by these values that we can obtain the bifurcation structures.

Theorem 1 confirms the existence of a periodic branch. The periodic solution may as well be considered as time-shift invariance, then it is temporally symmetric. However, this result seems to be a little trivial. We are more interested in the spatial and space-time symmetry as follows:

Theorem 2. Assume that equation (1) in the case of the Hopf resonance is Γ-equivariant with respect to the argument u. Σ **is an isotropic subgroup of** Γ so that the dimension of its fixed-point subspace $Fix(\Sigma)$ is equal to two. If the strong non-degenerated conditions are satisfied, there exists a unique 2π subharmonic branch with Σ **symmetry**.

Theorem 3. Assume that equation (1) with the Hopf resonance is Γ-equivariant with respect to the argument u. Σ' **is an isotropic subgroup of** $\Gamma \times Z_n$ so that the dimension of its fixed-point subspace $Fix(\Sigma)$ is equal to two. If strong non-degenerated conditions are satisfied, there exists a unique 2π subharmonic branch with $\Sigma' \times Z_n$ **symmetry** .

where strong non-degenerate conditions meants more constraints on the coefficients than the non-degenerate ones in theorem 1.

From theorem 2, there exists a unique periodic branch with spatial symmtry. Its symmtry is the maximal isotropic subgroup of Γ, which can be explained as a pure modal solution in physics. Theorem 3 indicates the existence of subharmonic branches with the spatiotemporal symmetry. Their symmetry is the maximal isotropic subgroup of $\Gamma \times Z_n$.

The main idea in proving these theorems is as follows: we reduce equation (1), by the Liapunov-Schmidt procedure, to the algbraic bifurcation equations, which fall just in the scope of singularity theory. The symmetries provide all possible forms of the algbraic bifurcation equations. The equivariant singularity theory is utilized to formulate the bifucation structure of each form.

The key task for obtainning bifurcation diagrams is to compute the coefficients of algebraic bifurcation equations. it calls for quite grand computations. Actually, we may predict all subharmonic branches by the maximal isotropic subgroup. Each isotropic subgroup corresponds to one branch. It provides an access to obtain all the possible symmetry-breaking branches without any complicated computation. Numerical simulation shows that symmetries of subharmonic solutions will decrease toward triviality with seccessive bifurcations. This procedure does not stop until a chaotic atrractor jumps out. When there exists a unique chaotic attractor, all symmetries will restore. Following this idea, we will investigate statistical symmetries on chaotic attractors, which leads to the interesting scaling laws.

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References

[1] H. Chate and J. Losson, Non-trivial collective behavior in couples map lattice: a transfer operator perspective, submmited to Physica D.
[2] G. He and T. Fang. Science in China, Vol.25, No.3 (1995).
[3] M. Golubistsky, I. Stewart and D.G. Schaeffer, Singularities and group in bifurcation theory, Vol.2. Springer-verlag, New York (1985).
[4] G. He and B. Dubrulle, Scale invariances and log-poisson distributions of passive scalars in fully developed turbulence, submitted to J. Physique.

Numerical Simulation of Line Puff Via RNG $K-\epsilon$ Model [1]

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Abstract: The time evolution of a line puff is studied using the renormalization group (RNG) $k-\epsilon$ model. The predicted puff flow and mixing rate are substantially similar to those obtained from the standard $k-\epsilon$ model, and are well-supported by experimental data. The computed scalar field reveals significant secondary concentration peaks trailing behind in the wake of the puff. The present results suggest that the overall mixing rate of a puff is primarily determined by the large scale motion, and that streamline curvature probably plays a minor role.
Key Words: Environmental Fluid Mechanics, Turbulence Model, Line Puff

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