Hamming Code

An Error Correction Method

Yusuf Hegazy – 2018005

Under the Supervision of Dr. Mohammed Safi

May 5, 2021

Historical Background

- It was invented by Richard Hamming
- He worked at Bell Labs in the 1940s on The Bell Model V computer
- It was an electromechanical relay machine which produced a lot of errors
- Whenever it detected an error it stopped, and needed to be restarted manually
- He said to himself "If the machine can detect an error, why can't it locate the position of the error and correct it?"

Error Correction

- A simple way to get error-free data, is to send the message 3 times in a row.

Example:

1 1 0 1

1 0 0 1

1 1 0 1

Parity

- However the previous method is very ineffecient and waste a lot of potential space
- A better way to spot errors that is widely used to this dat is **Parity Checks**

Parity: Method

Types of Parity

1. Even Parity:
   We add a parity bit to our message which is set if the sum of the 1's are an **odd** number.
2. Odd Parity:
   We add a parity bit to our message which is set if the sum of the 1's are an **even** number.

Parity: Example

Let's use even parity, since It's used the most.

Message

1 1 0 1

Message (With Parity Bit)

1 1 0 1 1

Parity In Hardware

XOR Gates are sometimes called parity gates since they compute the even parity of their inputs.

i.e

the inputs and outputs will always sum to an even number

Input A	Input B	Output Y
0	0	0
0	1	1
1	0	1
1	1	0

Figure 1: XOR Truth Table

- Richard Hamming thought of a clever way to use multiple parity bits on a single message.
- He arranged the parity bits in a way so that each parity bit covers certain bits of the message that is to be sent.
- This made the bit's that are covered overlap with one another.
- This will let us locate and even correct 1-bit errors very effeciently with so little space wasted.

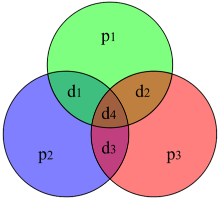

Figure 2: Parity Distribution

Hamming(7,4): A Practical Example

Let's Say we have the following message:

D4	D3	D2	D1
1	1	0	1

We will start by placing the parity bits according to the relation '2^n' (where n is the bit position)

at $n = 0 : 2^0 = \mathbf{1}$

at $n = 1 : 2^1 = \mathbf{2}$

at $n = 2 : 2^2 = \mathbf{4}$

at $n = 3 : 2^3 = 8$ (Out of range so we will stop at $n = 2$)

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1

7   6   5   4   3   2   1
D4  D3  D2  P3  D1  P2  P1
```

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1

7   6   5   4   3   2   1
1   D3  D2  P3  D1  P2  P1
```

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1

7    6    5    4    3    2    1
1    1    D2   P3   D1   P2   P1
```

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1

7   6   5   4   3   2   1
1   1   0   P3  D1  P2  P1
```

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1

7    6    5    4    3    2    1
1    1    0    P3   1    P2   P1
```

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1
7   6   5   4   3   2   1
                P3      P2  P1
```

$P1 = ?$
$P2 = ?$
$P3 = ?$

Hamming(7,4): A Practical Example

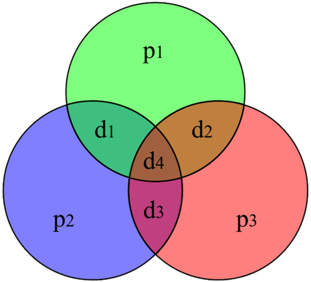

Figure 3: Parity Distribution

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1
7   6   5   4   3   2   1
D4      D2      D1      P1
```

$P1 = ?$
$P2 = ?$
$P3 = ?$

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1
7    6    5    4    3    2    1
1         0         1         P1
```

$P1 = ?$
$P2 = ?$
$P3 = ?$

Hamming(7,4): A Practical Example

`Data: 1 1 0 1`

7	6	5	4	3	2	1
1		0		1		0

$P1 = 0$
$P2 = ?$
$P3 = ?$

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1
7   6   5   4   3   2   1
D1  D2          D4  P2
```

$$\mathrm{P1} = 0$$
$$\mathrm{P2} = ?$$
$$\mathrm{P3} = ?$$

Hamming(7,4): A Practical Example

`Data: 1 1 0 1`

7	6	5	4	3	2	1
1	1			1	P2	

$$P1 = 0$$
$$P2 = ?$$
$$P3 = ?$$

Hamming(7,4): A Practical Example

`Data: 1 1 0 1`

7	6	5	4	3	2	1
1	1			1	1	

$$P1 = 0$$
$$P2 = 1$$
$$P3 = ?$$

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1
7    6    5    4    3    2    1
D4   D3   D2   P3
```

$P1 = 0$
$P2 = 1$
$P3 = ?$

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1
7    6    5    4    3    2    1
1    1    0    P3
```

$P1 = 0$
$P2 = 1$
$P3 = ?$

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1
7   6   5   4   3   2   1
1   1   0   0
```

$$P1 = 0$$
$$P2 = 1$$
$$P3 = 0$$

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1
7    6    5    4    3    2    1
1    1    0    0    1    1    0
```

$$P1 = 0$$
$$P2 = 1$$
$$P3 = 0$$

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1
```

7	6	5	4	3	2	1
1	0	0	0	1	1	0

Hamming(7,4): A Practical Example

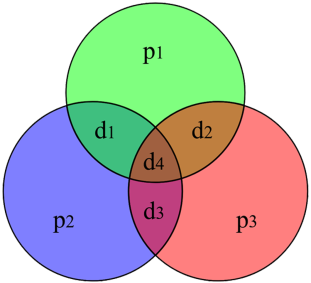

Figure 4: Parity Distribution

Hamming(7,4): A Practical Example

```
Data: 1 1 0 1
```

Hamming Code with Error Inserted

7	6	5	4	3	2	1
D4	D3	D2	P3	D1	P2	P1
1	0	0	0	1	1	0

Parity Groups Table

D4	D3	D2	P3	D1	P2	P1
X		X		X		X
X	X			X	X	
X	X	X	X			

Simulation

The End

Thanks for your attention.