Double Auction on Social Network

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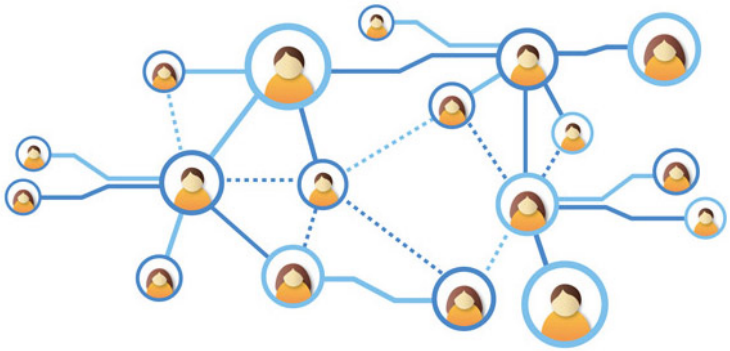
Outline

Mechanism Design on Social Network

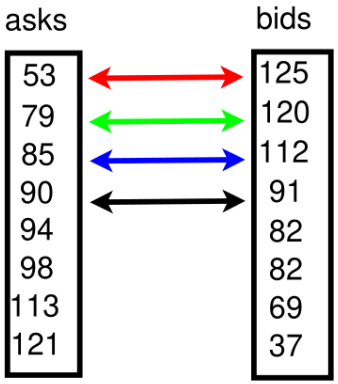


implication of the connectivity of social networks

opportunity social level welfare expansion and boost in individual utility

challenge possibility of manipulation in diffusion and bidding

Double Auction



Participants

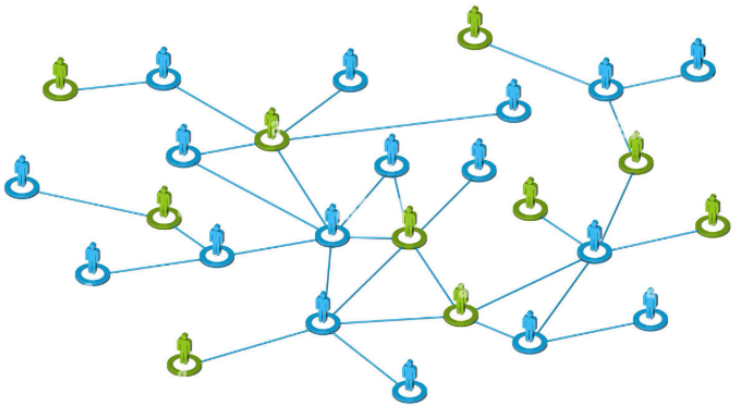
- two groups of disjoint players, buyers and sellers.
- each buyer b_i is willing to sell an item at price higher than v_i^b
- each seller s_j aims to sell an item at price lower than v_j^s

Mechanism

input reported valuation from buyers and sellers $(\hat{v}^b, \hat{v}^s)$

output allocation (π^b, π^s) and payment (p^b, p^s), determining the trade pairs and prices

Bilateral Trades on a Social Network



New Restrictions

- Trade only happens on neighbouring players
- Only a small proportion of the social network is aware of the trade.

Complicated Strategies

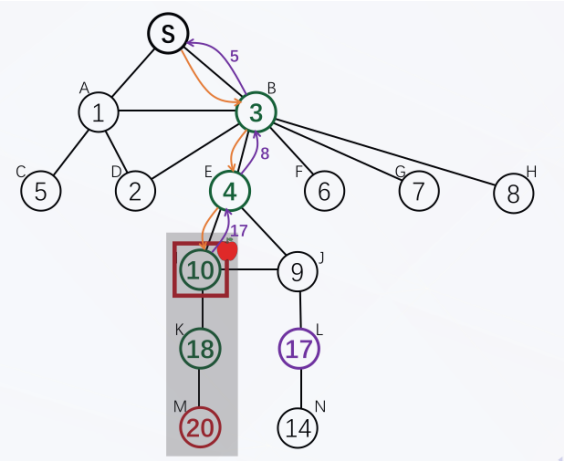
diffuse Inviting neighbours for potential rewards

block Blocking competitors from entering the auction

Existing Social Network Auction Mechanisms

players	supply	restriction	mechanism
one-sided	single item	arbitrary network	IDM
one-sided	multiple items	arbitrary network	MUDAN
two-sided	single item	disjoint buyer groups	DNA

IDM: one sided, single item



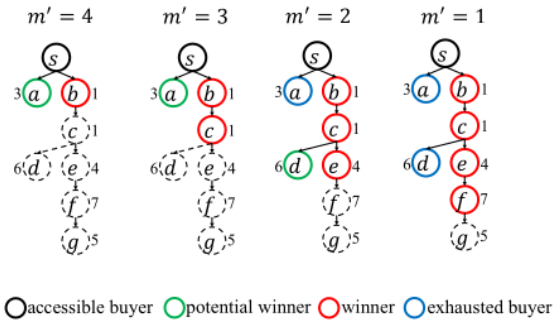
Brief of IDM

1. Determine the critical path to agent with highest valuation
2. Payment: Outside subtree highest price
3. Resaling if possible

Properties

IC, IR, WBB; Efficiency better than local auction

MUDAN: one sided, multiple items



Brief of MUDAN

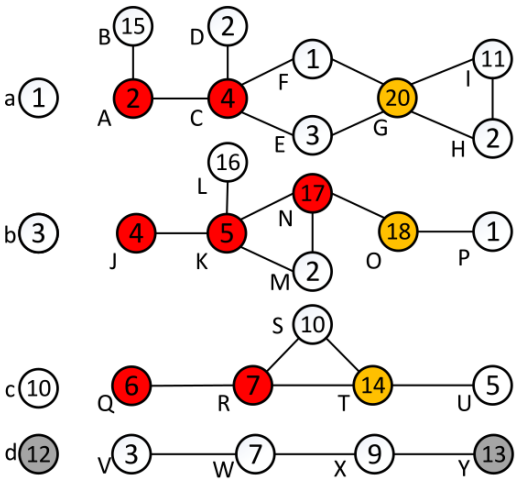
1. The priority of a buyer is the number of neighbours invited by him.
2. Iterative exploration of the network based on the priority.
3. Sell an item to the player with highest priority. Price is the $m' + 1$-th highest[a].

[a] m' is the number of remained items to sell

Properties

IC, IR, WBB, $1/m$-efficiency

DNA: two sided, single item per seller



Brief of DNA

1. Partition graph into buyer groups
2. Sort buyer groups by the highest price within groups.
3. McAfee's mechanism determine the allocation and payment for sellers.
4. Sell items to the buyer with highest bid in each group.
5. VCG-alike payment/reward for buyers.

Properties

IC, IR, WBB

Limitation of Existing Mechanisms

players	supply	restriction	mechanism
one-sided	single item	arbitrary network	IDM
one-sided	multiple items	arbitrary network	MUDAN
two-sided	single item	disjoint buyer groups	DNA
two-sided	single item	connected buyers groups	(unknown)

Our Goal

Design an double auction mechanism applicable to realistic social networks

Outline

Model of Double Auction on Social Network

Components of the double auction

players Buyers B and Sellers S, which are disjoint $B \cap S = \varnothing$.

valuation Afforable price of buyers v_i^b and reserved price of sellers v_j^s

social network Each seller $s_i \in S$ can only interact with $N_s(s_i) \subseteq S$.
Each buyer $b_j \in B$ can only interact with $N_b(b_j) \subseteq B$.

initial/heads $S_0 \subseteq S$, every $s_i \in S_0$ knows $NB(s_i) \subseteq B$.
$B_0 \subseteq B$, every $b_j \in B_0$ knows $NS(b_j) \subseteq S$.

utility u_i^b for buyer and u_i^s for sellers, where

$$u_i^b = \begin{cases} v_i^b - p_i^b & \text{buy at price } p_i^b \\ 0 & \text{otherwise} \end{cases} \qquad u_i^s = \begin{cases} p_i^s - v_i^s & \text{sold at price } p_i^s \\ 0 & \text{otherwise} \end{cases}$$

Model of Diffusion Double Auction Mechanism

Strategies of agents

diffusion action Every agent $a \in S \cup B$ invite $\hat{N}(a) \subseteq N(a)$ into the market.

bid action Buyers and sellers report valuation $\hat{v}^b, \hat{v}^s$

Execution of the Mechanism

Payment Determine the price and rewards. $p_i^s : S \to \mathbb{R}, p_j^s : B \to \mathbb{R}$.

Allocation Determine the trade pairs. $\pi_i^s : S \to \{0, 1\}, \pi_j^s : B \to \{0, 1\}$.

Execution Computing feasible trades.

Goals

Desirable Properties of a Double Auction Mechanism

Incentive Compatbility Or truthfulness.

Indivdual Rationality Or participation.

Weakly Budget Balance Or non-deficit.

Economics Efficient Or social welfare maximizing

these properties may not be achieved in by one mechanism[1]

[1] *Efficiency in truthful auctions via a social network*. arXiv:1904.12422

Outline

Attempts 1: Adoption of IDM

Insight: We want people who have higher valuation hold the item, so resale mechanism is preferred.

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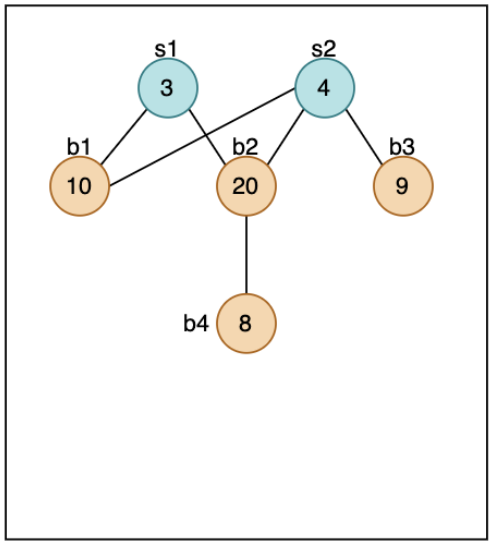
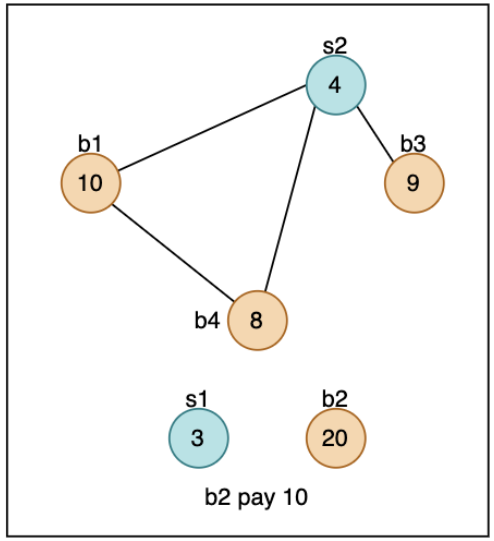
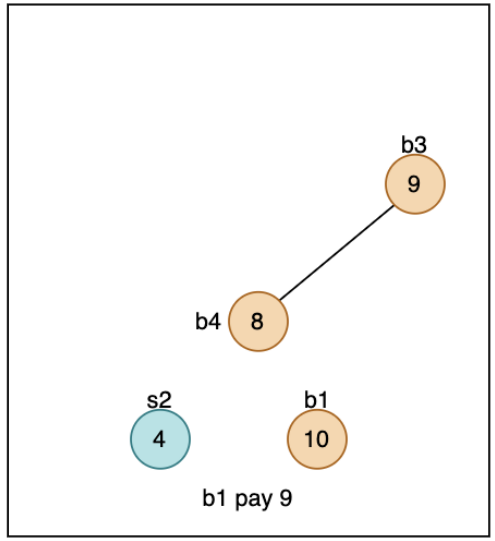
Initial attempts:

1. Sort the seller in ascending order and the buyer in descending order.
2. Run IDM.
3. If the payment is higher than the first seller's expectation, the seller is matched. Otherwise, stop.
4. The winner leave the network and share its connection.
5. Go to 2.

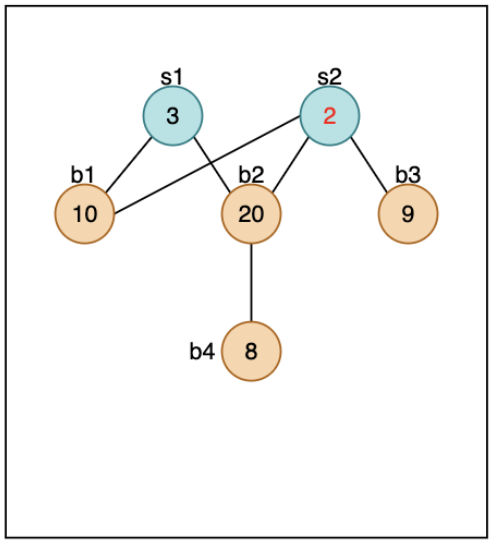
Seller not IC: Counterexample

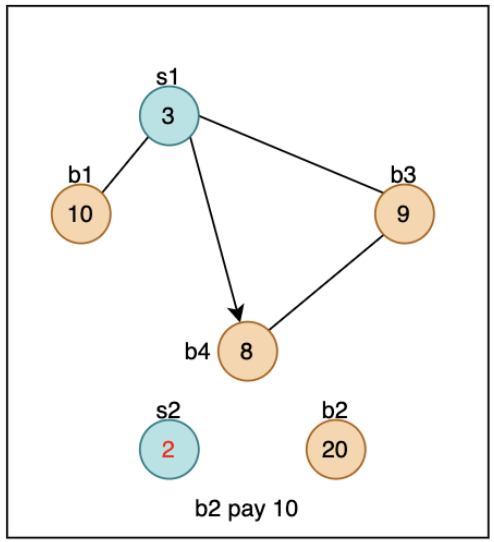


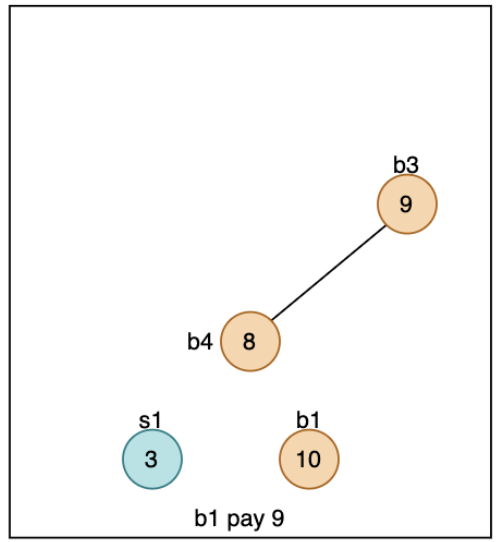




If play truthfully, the utility of s2 is 5







s2 misreport 2, the utility of s2 is 6

Fix of the mechanism

1. Sort the seller in ascending order and the buyer in descending order.
2. Let m = the number of the remaining seller
3. Run IDM. When calculating the payment, omit the first m higher bids.
4. If the payment is higher than the first seller's expectation, the seller is matched. Otherwise, stop.
5. The winner leave the network and share its connection.
6. Go to 2.

Now sellers will play truthfully...

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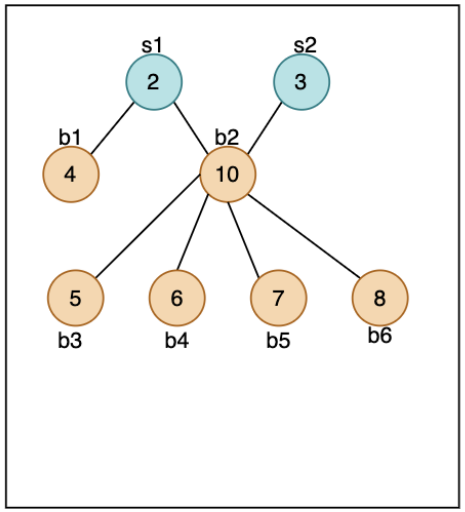
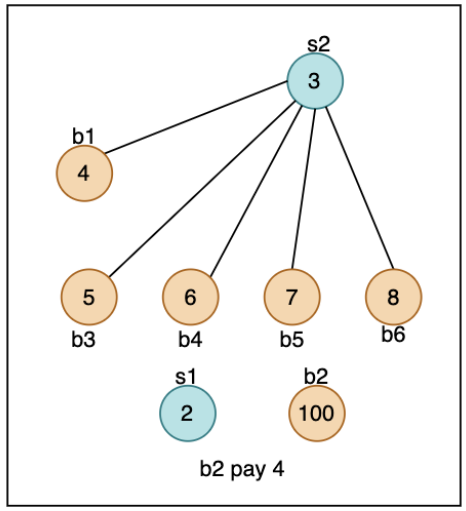
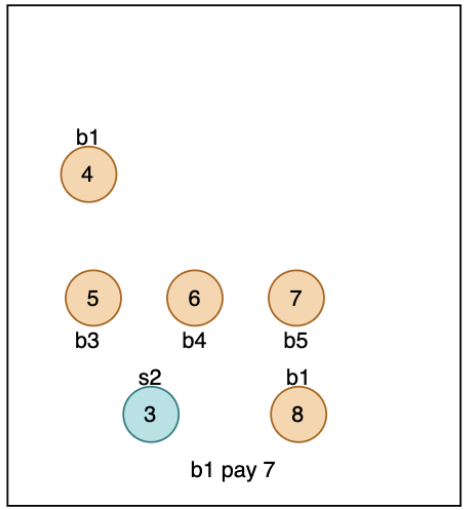
Now sellers will play truthfully...
However, the buyer have a chance to misreport.

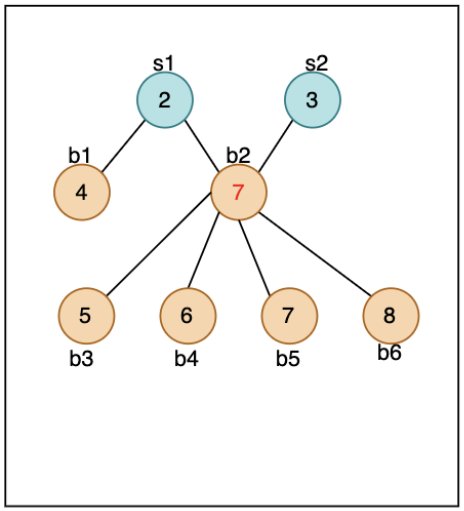
Extra reward from resale

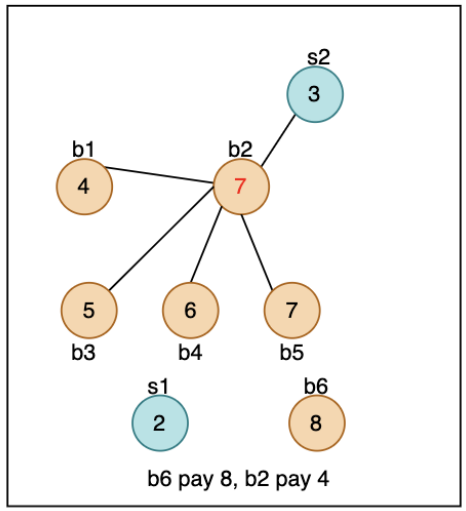


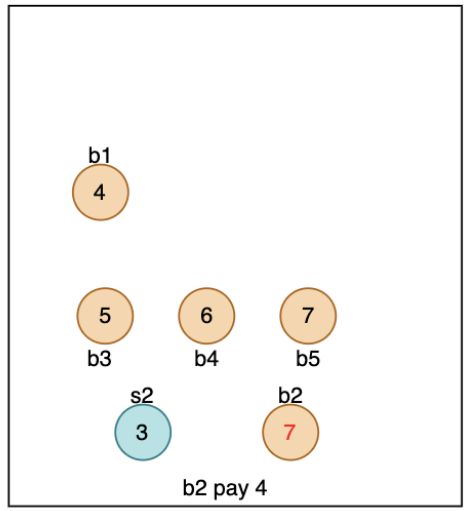




If play truthfully, the utility of b2 is 6

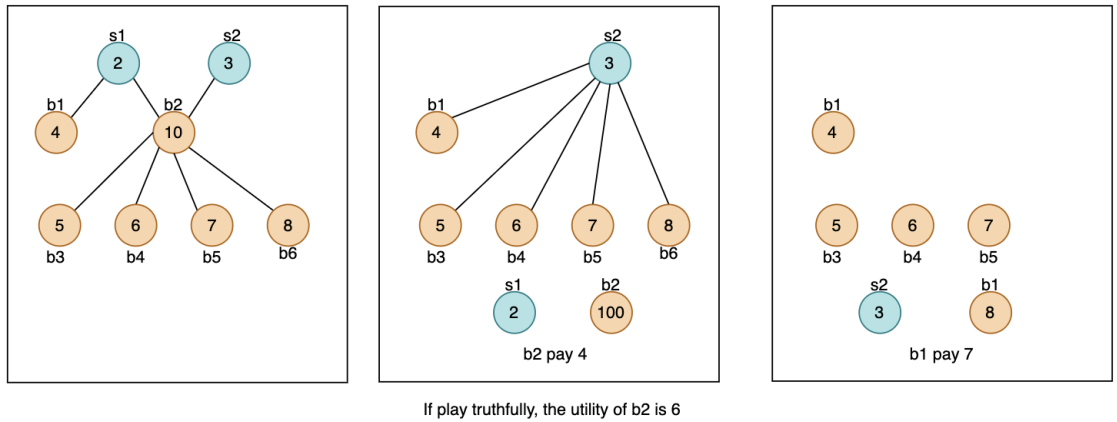






b2 misreport 7, the utility of b2 is 4 + 6 = 10

Extra reward from resale



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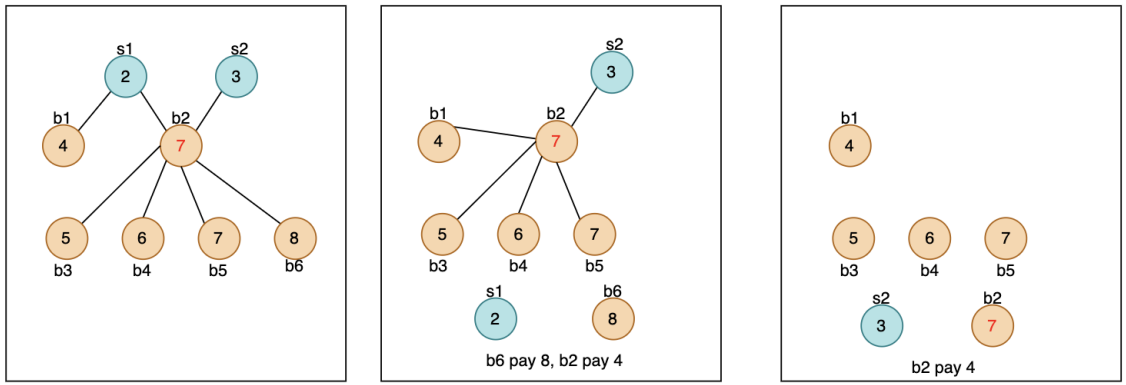


b2 misreport 7, the utility of b2 is 4 + 6 = 10

This problem come with any resale mechanism that rewards player who plays as a reseller. Therefore, we have to abandon this kind of mechanism.

Attempts 2: Connected buyer group

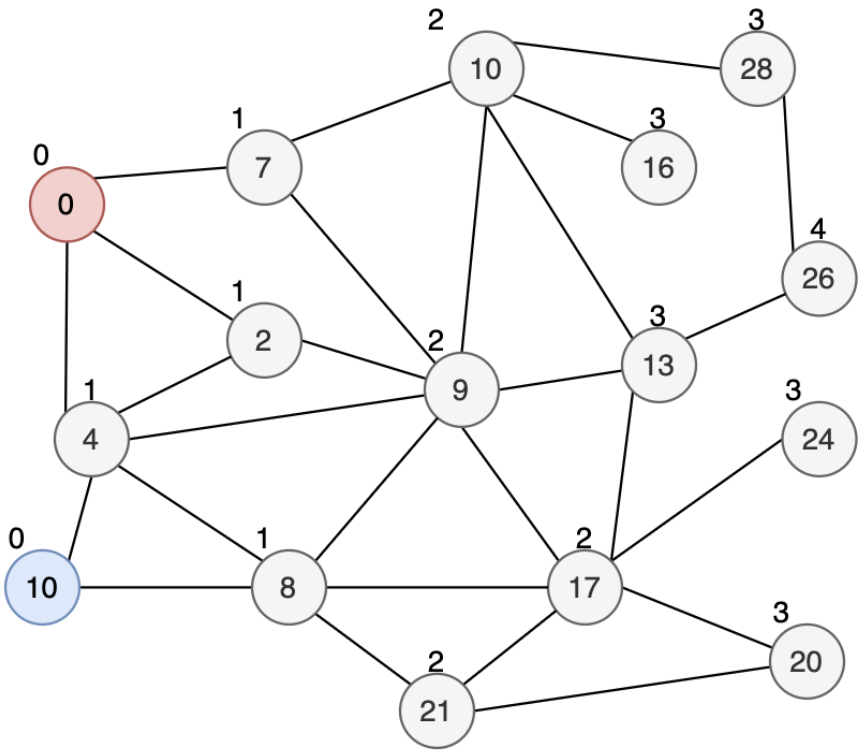
- Recall that: DNA runs on several isolated buyer groups with one head buyers in each group connected with seller.
- Intuition: If the graph has the property that the buyer conncets to at most one seller, the the buyer can be viewed as head buyer.
- If we have a method to partition the graph into different buyer group, we can then run DNA on that graph.

Graph Partition

Partition Algorithm:

1. Calculate each buyer's depth by finding the minimal distance to the nearest head buyer.
2. Remove the edge between nodes with the same level.
3. Let each head buyer be a buyer group and keep recording each buyer group's highest valuation.
4. Start from the deepest layer and the node with highest valuation.
5. Find critical paths to all the buyer groups where the node has the highest value among all the nodes on the path.
6. Omit the nodes have no critical path.
7. If there's only one critical path, add all the nodes on that path to the corresponding buyer group.
8. Otherwise, choose the path to the buyer group with lowest highest valuation.
9. Repeat untill no new buyers can be added to a group.
10. Randomly add the omitted buyers.

Example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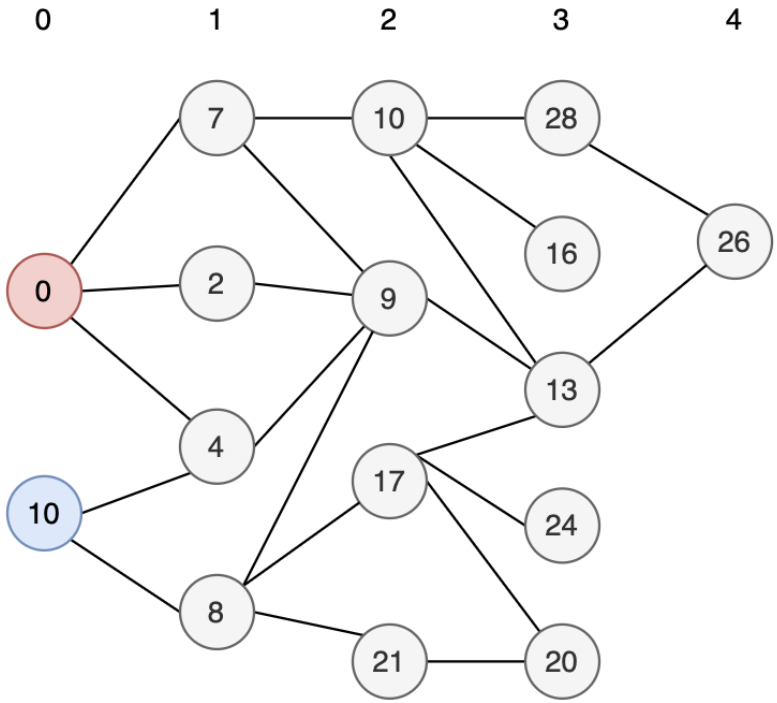
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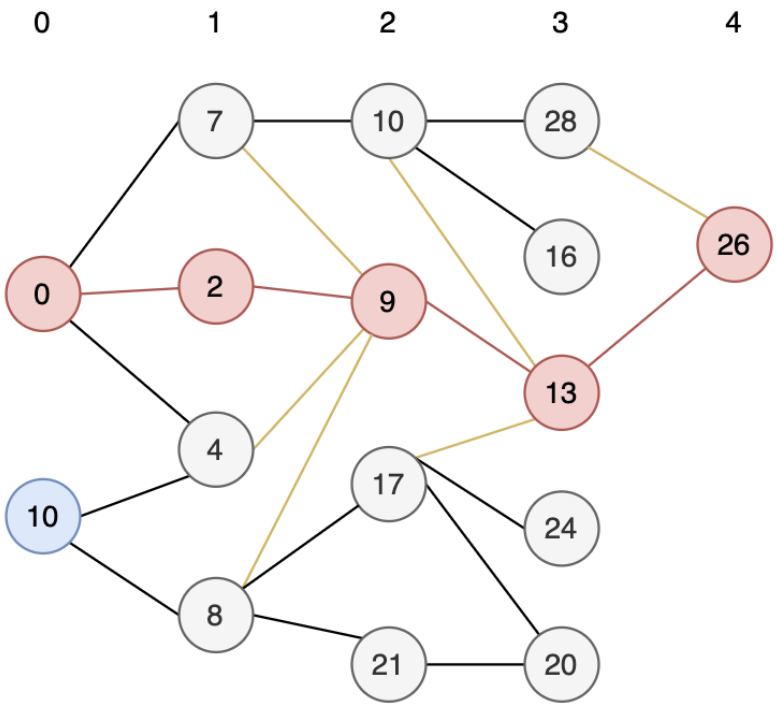
Example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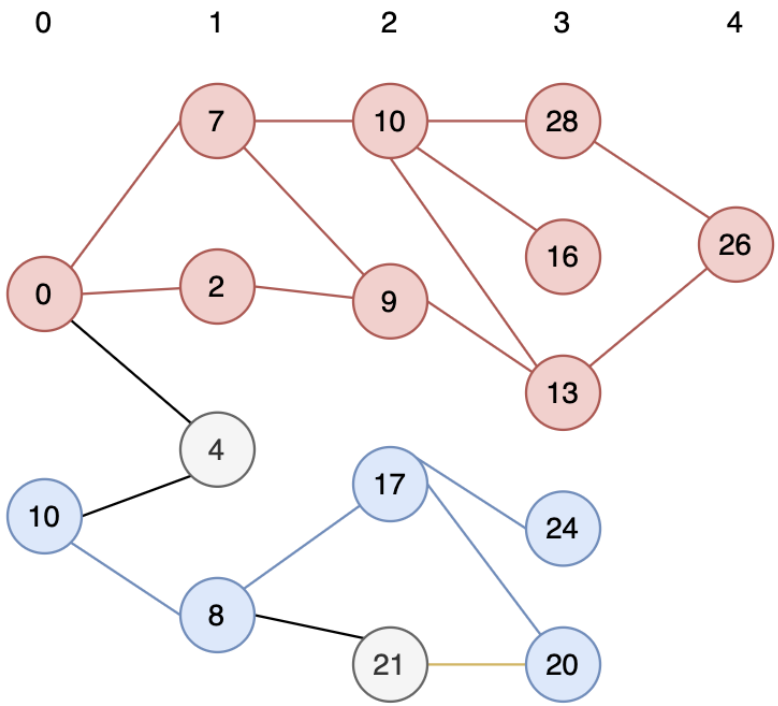
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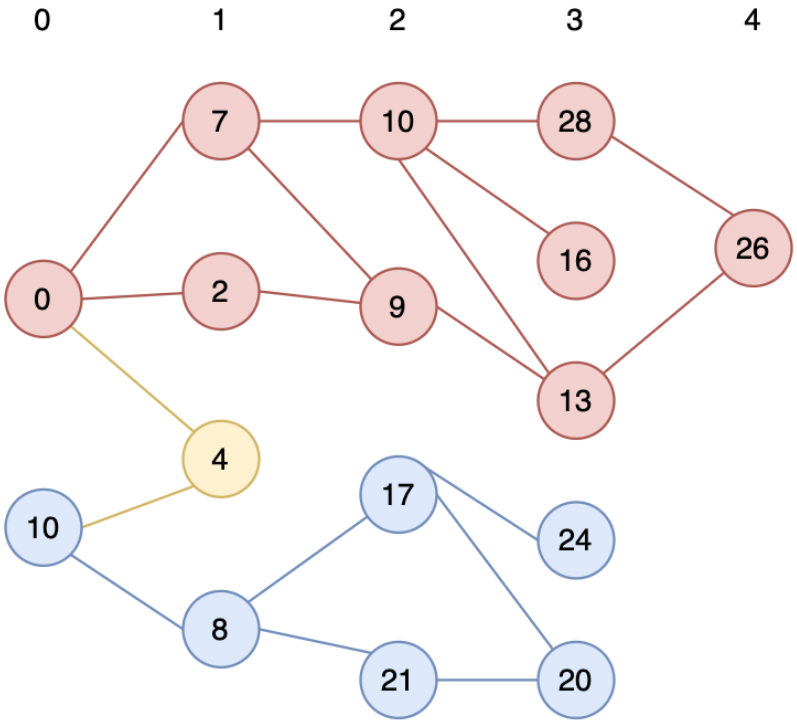
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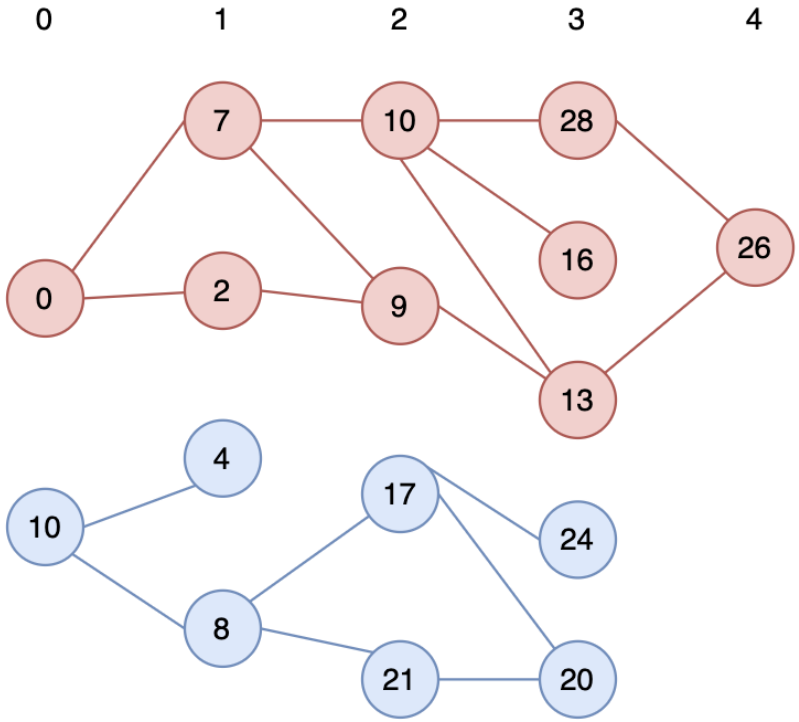
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Properties

However, we are not sure if this mechanism is IC. The proof is beyond our capacity, but we have some vague directions that may be helpful:

- If the buyer misreport more and successfully get the item, his utility will be negative (DNA's IC).
- If the buyer misreport less, he will risk the chance to lose the item.
- If the buyer misreport less to get into another buyer group, he will enter a group with more competitive buyers. This may decrease his utility.
- If the buyer misreport more to get into another group, either he will lose the item or he will get a negative utility.

Attempts 3: Reduced seller network

There exists some situation that the network cannot be partitioned: When multiple sellers are connected to one buyer.

Attempts 3: Reduced seller network

There exists some situation that the network cannot be partitioned: When multiple sellers are connected to one buyer.

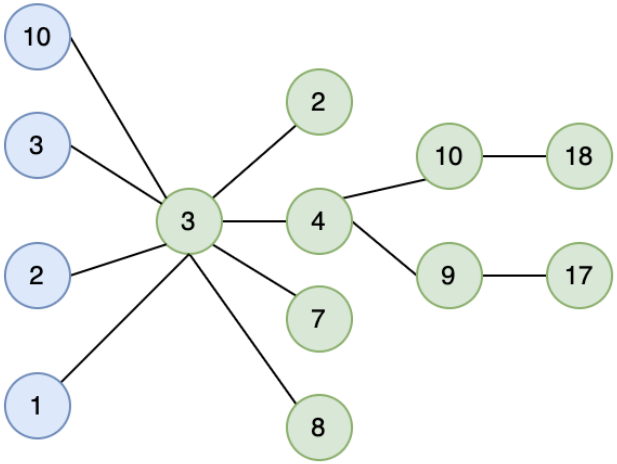
- Insight: This is very similar to multi-unit auction on social network with reserved prices.
- Therefore, we can try to adopt the currently IC mechanism on this setting: MUDAN.
- Since all the seller are connected together, the sequence of selling is not important.

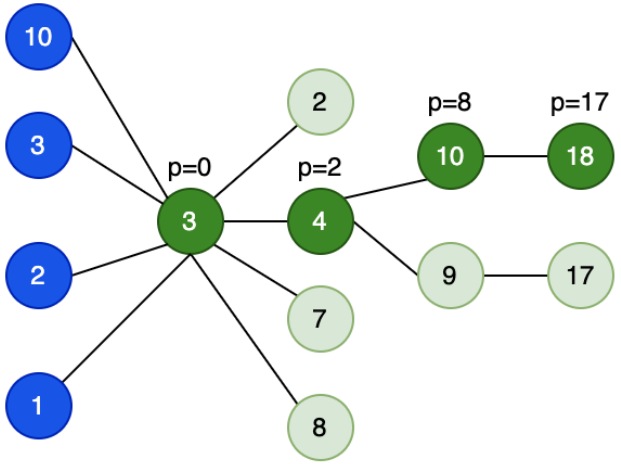
Club Auction

Algorithm:

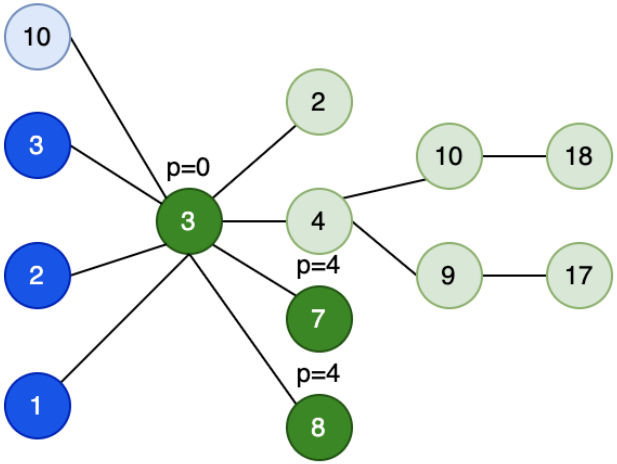
1. Let $m =$ the number of sellers. For convenience, suppose that: $v_1^s < v_2^s < \ldots < v_m^s$.
2. Run MUDAN with m items.
3. Let $p =$ the sum of the buyers' payments given by MUDAN.
4. If for all $v_i^s \geq \frac{p}{m}$, then let $p^s = -\frac{p}{m}$ be the payment of the seller.
5. Otherwise, let $m = m - 1$ and go to 2.

Example

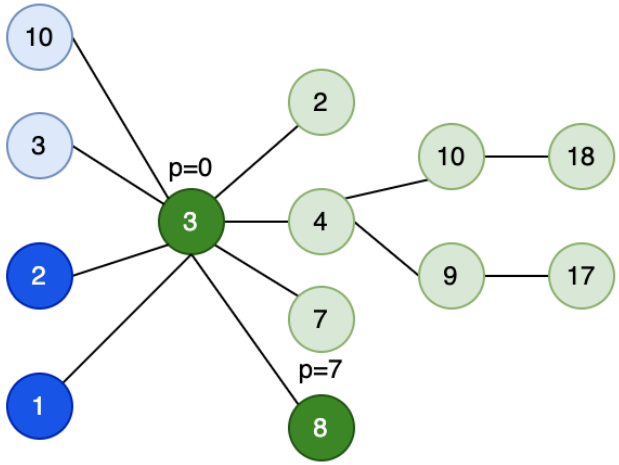




(0+2+8+17)/4 = 6.75 < 10



(4+4)/3 = 2.67 < 3



(0+7)/2 = 3.5

properties

The Mechanism is IR, IC and BB.

properties

The Mechanism is IR, IC and BB.
IR and BB are trivial.
The sketch of the proof for IC:

- The buyer will play truthfully since MUDAN is IC.
- If $v_i^s < p^s$ and the buyer misreport $v_i^{s*} > p^s$, his utility will become negative.
- If $v_i^s > p^s$ and the buyer misreport $v_i^{s*} < p^s$, his utility will decrease to zero.
- Under other circumstances, the utility of the buyer will not change.

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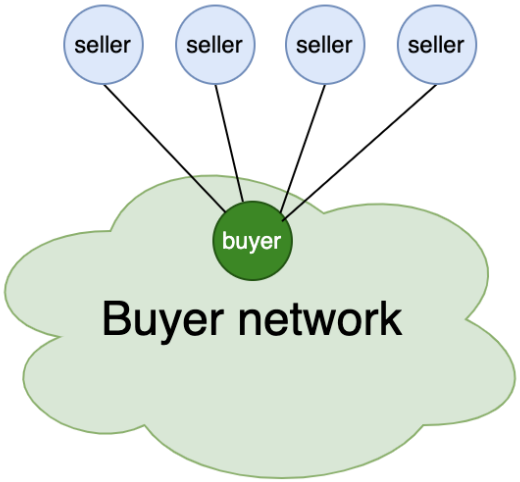
- Under other circumstances, the utility of the buyer will not change.

Any other IC mechanism besides MUDAN also work.

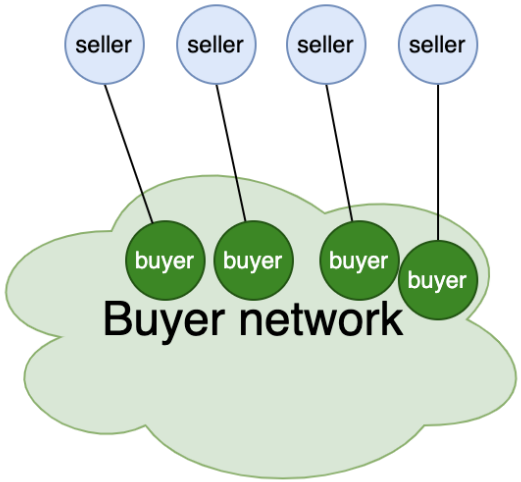
Outline

Conclusion

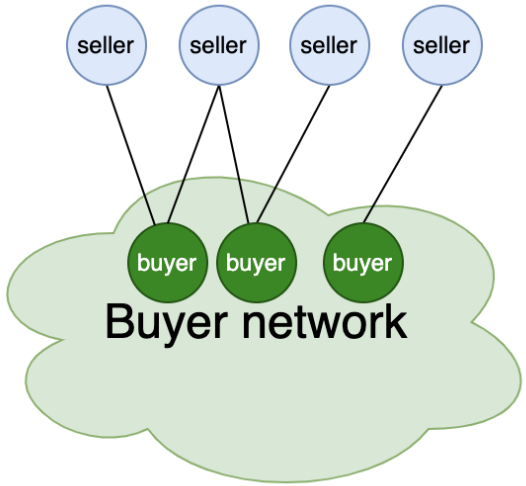
- This problem can be devide into three classes based on the complexity of the graph.



Club Auction.



DNA with graph partition.



Combination of the other two networks.

Future Work

- Club Auction depends on multi-unit Single Auction Mechanism.
  - Find other multi-unit Single Auction Mechanism.
  - Find metrices to evaluate Club Auction with different Mechanism.
- Graph partition can be a seperate direction.
  - Find other graph partition mechanism.
  - Analysis the properties of the graph partition mechanism.
- Using Graph Partition, a large network can be divide into smaller network with special properties.
- Find a method to combine different mechanisms on the seperated networks. (DNA is one of them)

Outline

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