Project Milestone for Team#6

Double Auction Mechanism on Social Network

JINTIAN HU (2020533167), HUIZHE SU (2020533009), CHENG PENG (2020533068), LIYU YANG (2020533162), WEIMING LUO (2020533168)

1 INTRODUCTION

1.1 background

The establishment of huge social network such as Tiktok, Wechat and Weibo has bring both challenges and opportunity. Leveraging the connectivity of social network to promote social welfare has become a fruitful area of study in algorithmic game theory research , particularly since the publication of [4] in 2017.

Our project aims to develop a mechanism for facilitating general trading activity within social networks.

1.2 prior work

Initially, our research focused on distributed mechanisms that could be executed without prior knowledge of the network structure and without requiring a central authority to enforce the allocation. In this regard, [6] proposed the Sequential Resale Auction (SRA) mechanism for selling a single item with Ex-post Incentive Compatibility (EPIC) property. Although SRA provides a sufficient solution for single-source single-item auction problems in distributed network settings, realistic trading scenarios are often more complex than the settings of SRA. For example, the network may contains multiple sellers who are willing to trade an item with potential buyers. This requires a distributed double auction mechanism to be properly addressed.

However, after a brief literature review, we found that developing a double auction mechanism for social networks is a challenging task. Even a centralized truthful mechanism for double auctions on general networks has yet to be developed. As a result, we changed the direction of our research to focus on inventing a centralized mechanism.

1.3 related work

[8] is the only paper that covers double auctions on a network. However, the mechanism proposed in that paper relies on unrealistic assumptions. Specifically, it assumes that the graph of buyers can be decomposed into disjoint groups, which is unlikely to hold in modern social networks that are characterized by strong connectivity. Therefore, we need to derive a mechanism that works for more general network structures.

1.4 overall plan for research

The overall plan for approaching the problem will involve conducting a literature review to establish connections between single seller mechanisms and our current multiple seller model, and using this information to develop a new mechanism that is suitable for our needs. The mechanism will then be subject to theoretical analysis using small test cases to verify key properties such as individual rationality (IR), incentive compatibility (IC), and weak budget balance (WB). Empirical evaluation will be conducted under different problem backgrounds in real life, using appropriate evaluation metrics to measure the mechanism's performance. Insights gained from the theoretical and empirical evaluations will be used to fine-tune and optimize the mechanism. Finally, the findings will be summarized and conclusions drawn about the mechanism's effectiveness, along with recommendations for future research in the field.

2 THE MODEL

Consider a double auction market of $n + m$ participants where n out of them are sellers and the rest m participants are potential buyers. Let $S = \{s_1, s_2, \ldots, s_n\}$ and $B = \{b_1, b_2, \ldots b_m\}$ be the set of all sellers and buyers. Each seller $s_i \in S$ is holding a one-unit item and is willing to sell it at price no lower than v_i^s. Each buyer $b_j \in B$ wants to purchase one item with price no more than v_j^b.

Every participant of the market is connected to some other participants forming a social network. More precisely, every seller or buyer $x \in S \cup B$ can only communicate directly with its neighbors $r_x \subseteq (S \cup B \setminus \{x\})$. Initially, only part of the buyers (called head buyers) are aware of the incoming auction. Head buyers are those who is directly linked to a seller. $H = \bigcup_{s_i \in S}(r_x \cap B)$ denotes all the head buyers in the market.

To sell the item at higher price, the sellers want to invite potential buyers other than the initial ones i.e, the head buyers, to expand the auction. However, once neighboring potential buyers are invited to the auction, they will compete with the initial buyers for the items. Intuitively, a buyer has incentive to keep his neighboring buyers from entering the market. Therefore, the market owner must adopt a mechanism encouraging the buyers in the market to inform the neighbors not being aware of the auction.

A mechanism for double auction on social network requires the buyers and the sellers to report their private information and decide the allocation of items and payment for each agent. For the sellers, the mechanism asks them to reveal their expected price $\hat{v}^s = (\hat{v}_1^s, \hat{v}_2^s, \ldots \hat{v}_n^s)$. The mechanism also requires the buyers to report their type $\hat{\theta}_i = (\hat{v}_i^b, \hat{r}_{s_i})$ and use $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_m)$ to denote a reported type profile.

After receiving the reported information $\hat{\theta}$ and $\hat{v}^s$, the mechanism determines the allocation π^s, π^b and the payment p^s, p^b.

- For each seller s_i: The allocation $\pi_i^s = 1$ means that he can trade the item with a buyer, otherwise $\pi_i^s = 0$. The payment $p_i^s \leq 0$ gives the money s_i can get from the market owner.
- For each buyer b_i: The allocation $\pi_i^b = 1$ indicates that he get one item from a seller while $\pi_i^b = 0$ if he get no item. The payment $p_i^b > 0$ if b_i is asked to pay money to the market owner and $p_i^b < 0$ when b_i can get get some rewards from the market owner.

We make a few definition of properties for further discussion.

Definition 2.1 (active buyers). *Given a profile $\hat{\theta}$, we say buyer b_i is active if there exists $q_1, q_2, \ldots q_k$ such that $b_{q_1} \in H$ is a head buyer, $b_{q_k} = b_i$ is the active buyer and $b_{q_{i+1}} \in \hat{r}_{b_{q_i}}$ for every $i = 1, 2 \ldots k-1$. That is a buyer is active if he is a head buyer or an active buyer invites him.*
Let $Q \subseteq B$ be the set of all active buyers.

Next we definition the desired properties of the mechanism.

Definition 2.2 (valid). *A mechanism is valid if for all possible input of $(\hat{\theta}, \hat{v}^s)$ the following conditions are satisfied.*

- *If $b_i \notin Q$ then $\pi_i^b = p_i^b = 0$.*
- $\sum_{b_i \in B} \pi_i^b = \sum_{s_i \in S} \pi^s$.

Definition 2.3 (utility). *The utility is the welfare an agent gains from participanting the double auction mechanism trading.*

- *For $b_i \in B$, his utility is $u_i^b = \pi_i^b v_i^b - p_i^b$.*
- *For $s_i \in S$, his utility is $u_i^s = \pi_i^s v_i^s - p_i^s$.*

Definition 2.4 (individual rational (IR)). *A mechanism is IR if for all possible input of $(\hat{\theta}, \hat{v}^s)$,*

- *For all* $b_i \in B$, $u_i^b((\theta_i, \hat{\theta}_{-i}), \hat{v}^s) \geq 0$.
- *For all* $s_i \in S$, $u_i^s(\hat{\theta}, (v_i^s, \hat{v}_{-i}^s) \geq 0$.

That is, no agent is punished for participating truthfully.

Definition 2.5 (incentive compatible (IC)). *A mechanism is IC if*

- *For all* $b_i \in B$, $u_i^b((\theta_i, \hat{\theta}_{-i}), \hat{v}^s) \geq u_i^b(\hat{\theta}, \hat{v}^s)$.
- *For all* $s_i \in S$, $u_i^s(\hat{\theta}, (v_i^s, \hat{v}_{-i}^s)) \geq u_i^s(\hat{\theta}, \hat{v}^s)$

That is, an agent's utility is maximized when he participate truthfully.

Definition 2.6 (weak budget balance (WBB)). *A mechanism is WBB if* $\sum_{s_i \in S} p_i^s(\hat{\theta}, \hat{v}^s) + \sum_{b_i \in B} p_i^b(\hat{\theta}, \hat{v}^s) \geq 0$ *for all* $(\hat{\theta}, \hat{v}^s)$. *That is, the market owner never pay extra money for running the mechanism.*

3 TECHNICAL APPROACH

Given the current model and prior works, we would like to design a novel double-auction mechanism on social network that satisfies Individual Rational(IR), Incentive Compatible(IC) and Budget Balance(BB). However, by Myerson-Satterthwaite theorem[7], it's impossible to also guranteen the Economic Efficiency(EE). Therefore, our mechanism will be slightly inefficient because it does not guarantee that the buyer with a higher evaluation will always get the item.

The mechanism will then be subject to theoretical analysis of IR, IC and WB. Firstly, we will evaluate whether such properties does not hold by some small test cases. If such test cases are not found, we will try to give an exhaustive analysis of our mechanism with thorough proof. For EE, we will try to give a bound analysis to see how worse the social welfare can be compared to the most efficient one. In addition to theoretical analysis, we will evaluate the performance of the proposed mechanism in simulated settings based on empirical observations. These experiments will serve as a supplement to the theoretical analysis and will assist in validating the analytical properties.

4 CURRENT RESULTS

We have made several attempts to adapt existing mechanisms into our model.

4.1 DNA on Connected Network

One of the outstanding works on double auctions on social networks is Double Network Auction (DNA)[8]. The method is an extension of McAfee's trade reduction mechanism that runs on several disjoint buyer networks. Therefore, we may reduce our model to DNA's model by dividing the connected buyer networks into several disjoint smaller buyer networks according to the allocation given by McAfee's method. However, we can find a situation where the network is individable. Fig 1 shows one of the situations when several sellers together are connected to a chain of buyers. Therefore, our problem setting cannot be reduced.

4.2 Multi-item Mechanism

In a double auction scenario, we have multiple sellers, which means we also have multiple items. Therefore, the multi-item mechanism may provide some valuable insights.

First, we try to adapt Generalized IDM(GIDM)[9] and Distance-based Network Auction mechanism for Multi-unit, Unit-demand buyers(DNA-MU)[2]. However, these two methods are not IC. The counterexample is given in 2. The seller has $m = 4$ items. The example is from [1].

- **GIDM** If all player report truthfully. The winner will be b, c, d, e(Table 1). However, if f misreport her connection, then the allocation result will be a, b, c, f (Table 2).

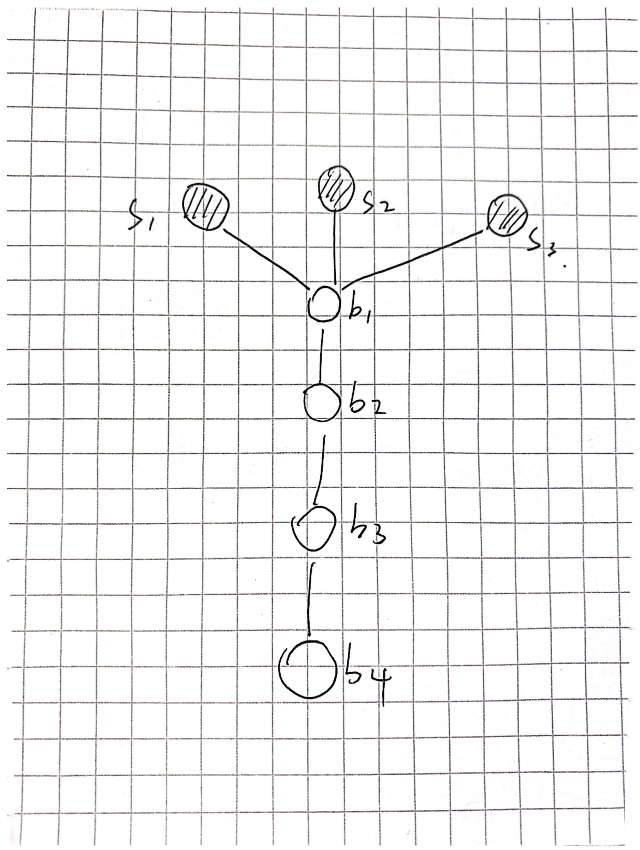

Fig. 1. Network that can not be divided

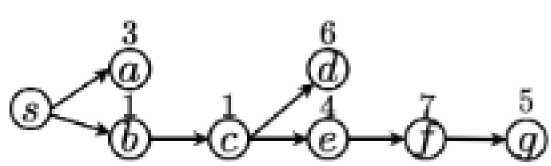

Fig. 2. Counter example for both GIDM and DNA-MU

round	left items	winners	π, p
1	4	$\varnothing$	$\pi_b = 1, p_b = 0$
2	3	$\{b\}$	$\pi_c = 1, p_c = 0$
3	2	$\{b, c\}$	$\pi_d = 1, p_d = 5$
4	1	$\{b, c, d\}$	$\pi_e = 1, p_e = 3$

Table 1. GIDM with $m = 4$. All buyer report truthfully[1]

round	left items	winners	π, p
1	4	$\varnothing$	$\pi_a = 1, p_b = 0$
2	3	$\{a\}$	$\pi_b = 1, p_c = 0$
3	2	$\{a, b\}$	$\pi_c = 1, p_d = 0$
4	1	$\{a, b, c\}$	$\pi_f = 1, p_e = 6$

Table 2. GIDM with $m = 4$. f misreport connection $r'_f = \varnothing$[1]

.

- **DAN-MU** If all player report truthfully. The winner will be b, c, d, e (Table 3). However, if f misreport her connection, then the allocation result will be a, b, c, f(Table **??**).

round	left items	agent	π, p
1	4	b	$\pi_b = 1, p_b = 0$
2	3	c	$\pi_c = 1, p_c = 0$
3	2	d	$\pi_d = 1, p_d = 5$
4	1	e	$\pi_e = 1, p_e = 3$

Table 3. DAN-MU with $m = 4$. All buyer report truthfully[1]

round	left items	agent	π, p
1	4	a	$\pi_a = 1, p_b = 1$
2	3	b	$\pi_b = 1, p_c = 0$
3	2	c	$\pi_c = 1, p_d = 0$
4	1	f	$\pi_f = 1, p_e = 6$

Table 4. DAN-MU with $m = 4$. f misreport connection $r'_f = \varnothing$[1]

.

We can reduce our problem to a multi-item problem if we connect several sellers to the same buyer with the same prices. An example is shown in Fig 3. It can be expected that the adapted method will not be IC either.

We also look into two other IC mechanisms: MUDAN and LDM. These two methods are both layer-based mechanisms. In our setting, items are held by different sellers. If the sellers are scattered around the network, the network will degenerate into one layer since the children cannot get them, even if they may have higher evaluations. An example is shown in Fig 4.

Resale mechanism with leave and share

In double-auction, we want people with high valuations to hold the item. Therefore, resale mechanisms are better than layer-based mechanisms. Buyers with higher evaluations have more chances to get the item, while buyers with lower evaluations are rewarded if they invite someone who gets it. From this insight, we designed a naive resale mechanism that combined IDM with leave and share. The mechanism is described as follows,

(1) Sort the sellers in order of their price from lowest to highest.
(2) Let the seller with the lowest price sell the item using IDM. Other sellers are viewed as buyers with 0 valuations.
(3) The seller and the winner leave the network and share their connection.
(4) Repeat step 2, 3 until no seller can sell the item.

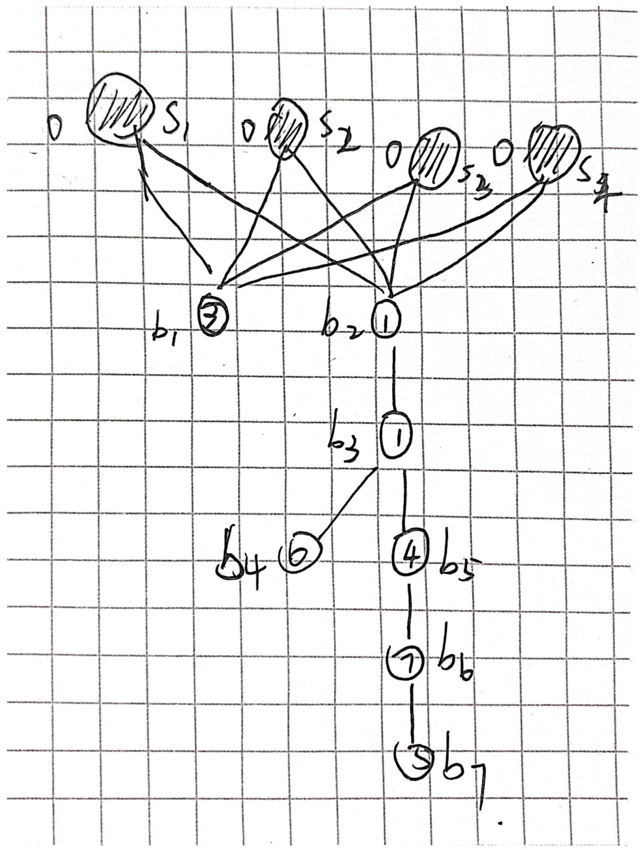

Fig. 3. A reduction from multi-seller to multi-item

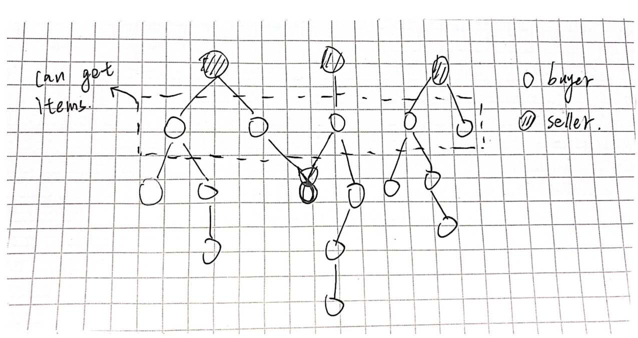

Fig. 4. An example of the degeneration of a network to a single layer.

However, this mechanism is not IC since the seller with a lower price has more chance to sell their item at a high price, and the seller has the incentive to misreport a lower price. In the following example, if s_1 s_2 both report truthfully (Fig 5), then s_2 will receive 9. But if he misreport 2, he will receive 10(Fig 6). Therefore, he has the incentive to misreport.

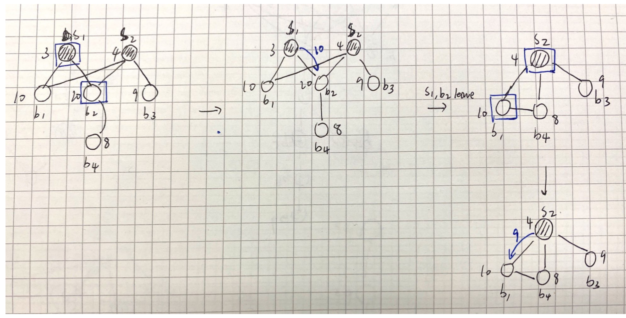

Fig. 5. A run of the naive algorithm. All player report truthfully

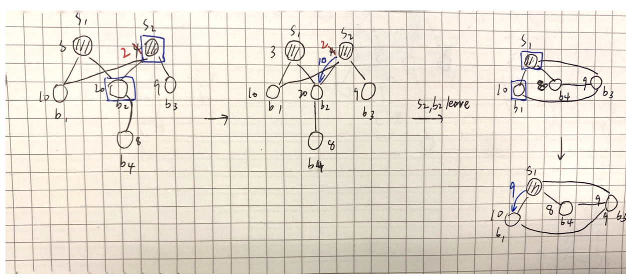

Fig. 6. A run of the naive algorithm. s_2 misreport 2

Resale with leave and share, eliminating highest m bid

Then we modified the original algorithm. The main idea is still the same, but when calculating the price using IDM, we first eliminate the highest m bids, where m is the number of unmatched sellers. As a result, the seller cannot profit more by misreporting, and neither can the buyer. However, we still have to do more work to show and prove that this mechanism is IC. Moreover, Since we have eliminated the highest bids, some deals that might have worked will fail. However, this will not be a problem if the network is big enough and the number of the buyer is much greater than the seller.

REFERENCES

[1] Yuan Fang et al. "Multi-unit Auction over a Social Network". In: *arXiv preprint arXiv:2302.08924* (2023).

[2] Takehiro Kawasaki et al. "Strategy-Proof and Non-Wasteful Multi-Unit Auction via Social Network". In: *CoRR* abs/1911.08809 (2019). arXiv: 1911.08809. URL: http://arxiv.org/abs/1911.08809.

[3] Bin Li et al. "Diffusion auction design". In: *Artificial Intelligence* 303 (2022), p. 103631. ISSN: 0004-3702. DOI: https://doi.org/10.1016/j.artint.2021.103631.

[4] Bin Li et al. "Mechanism Design in Social Networks". In: *Proceedings of the Thirty-First AAAI Conference on Artificial Intelligence*. AAAI'17. San Francisco, California, USA: AAAI Press, 2017, pp. 586–592.

[5] Haolin Liu, Xinyuan Lian, and Dengji Zhao. "Diffusion multi-unit auctions with diminishing marginal utility buyers". In: *arXiv preprint arXiv:2201.08616* (2022).

[6] Haoxin Liu, Yao Zhang, and Dengji Zhao. *Distributed Mechanism Design in Social Networks*. 2023. arXiv: 2303.03077 [cs.GT].

[7] Roger B Myerson and Mark A Satterthwaite. "Efficient mechanisms for bilateral trading". In: *Journal of Economic Theory* 29.2 (1983), pp. 265–281. ISSN: 0022-0531. DOI: https://doi.org/10.1016/0022-0531(83)90048-0. URL: https://www.sciencedirect.com/science/article/pii/0022053183900480.

[8] Junping Xu, Xin He, and Dengji Zhao. "Double Auction Design on Networks". In: *Proceedings of the First International Conference on Distributed Artificial Intelligence.* DAI '19. Beijing, China: Association for Computing Machinery, 2019. ISBN: 9781450376563. DOI: 10.1145/3356464.3357708.

[9] Dengji Zhao et al. "Selling Multiple Items via Social Networks". In: *CoRR* abs/1903.02703 (2019). arXiv: 1903.02703. URL: http://arxiv.org/abs/1903.02703.