# Double Auction Design on Networks

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## ABSTRACT

This paper studies a double auction market where a set of sellers sell homogeneous items to a set of buyers and each buyer (seller) buys (sells) one item. Moreover, each buyer is linked to a set of other potential buyers via a network. The goal is to design a double auction such that the buyers are incentivized to invite their neighbours to join the auction to improve the allocation efficiency and the sellers' revenue, which is not achievable under the existing double auction mechanisms. Moreover, it is well-known that budget balance is not compatible with other desirable properties at the same time. Hence, we design double auction mechanisms that both incentivize buyers to invite their neighbours and guarantee budget balance. We first show that an extension of McAfee's trade reduction [8] cannot achieve this. Therefore, we propose a new mechanism called Double Network Auction (DNA) to guarantee the properties.

## CCS CONCEPTS

• **Theory of computation → Algorithmic mechanism design**; *Social networks*; • **Computing methodologies → Multi-agent systems**.

## KEYWORDS

mechanism design, double auction, social network

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Double auctions (DAs) are important institutions in the modern economy. A double auction is a centralized market consisting of trading rules for the trading of commodities by traders. Double auctions can represent a broad class of trading institutions, and more details can be found in [4]. One purpose of studying DAs is to find a dominant strategy for sellers and buyers to report their valuations truthfully. Moreover, a well-known mechanism Vickrey-Clarke-Groves (VCG) [2, 5, 10] can be applied in this scenario. However, VCG will cause deficit to the market. Hence, another dominant strategy mechanism proposed by McAfee [1992] can avoid the deficit problem with some sacrifice to social welfare. Following that, many double auction studies (e.g. [11]) based on McAfee's work are directed at improving the social welfare while keeping no-deficit.

In this paper, we study double auctions from a new perspective considering sale promotions via buyers. Previous research on double auctions always has focused on designing a mechanism to prevent traders benefiting by lying about their valuations. This paper is interested in designing a mechanism to incentivize the traders to extend the market automatically. Specially, this new mechanism will incentivize traders in the primary market to invite their neighbours to join the double auction.

The challenge to design such a mechanism is to incentivize the traders to propagate the auction information to their neighbours voluntarily. In recent years, mechanism design on social networks has also been studied, and there already exist some closely related work. For instance, Borgatti's work [1] focused on building the basic social network theory assumptions, goals and explanatory mechanisms prevalent in the field of social network analysis. A more related work was proposed by Li *et al.* [2017]. They consider a single-item auction design problem on social networks where buyers have incentives to diffuse the auction information to their neighbours. They further generalized their work to sell multi-unit items [12]. The intuition behind their method is that information diffusion is rewarded if it leads to a more efficient allocation. While some other work [3, 6, 9] provided different views in terms of incentivising people to share information. Emek *et al.* [2011] presented a theoretical framework for multi-level marketing mechanisms. Kempe *et al.* [2003] focused on an optimization problem of selecting the most influential people to propagate information in a social network and Pickard *et al.* [2011] focused on how to design reward mechanisms to incentivize people to invite more people to accomplish a challenge together.

The goal of this paper is to design double auction mechanisms on social networks that can both incentivize all buyers to propagate the auction information to all their neighbours further and also keep the market from suffering deficit, which has not been studied in the existing double auction literature. This paper makes the following contributions to state of the art:

- We propose a model for a double auction on social networks where each buyer can only communicate with her neighbours.
- We introduce our mechanism called Double Network Auction (DNA). We prove that DNA can not only incentivize all buyers to invite all their neighbours to join the auction but also guarantee that there is no deficit.

The remainder of the paper is organized as follows. In Section 1, we introduce the basic model of our work by extending the traditional double auction model into the social network setting.


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Section 2 shows an extension of the well-known McAfee's trade reduction mechanism on social networks and demonstrates that it cannot guarantee buyers to propagate the auction information to all their neighbours. Section 3 proposes our new mechanism called Double Network Auction (DNA) and discusses the performance of it. Finally, we conclude our work in Section 4.

# 1 THE MODEL

We consider a double auction market where there exist $n$ sellers denoted by a set $N = \{1, 2, \cdots, n\}$ and $m$ buyers denoted by a set $M = \{1, 2, \cdots, m\}$. Each seller $i \in N$ wants to sell one-unit item with a price no less than $s_i$. Each buyer $j \in M$ wants to buy the item with price no more than $b_j$. Particularly, all the buyers are connected via social networks and each buyer $j \in M$ has a set of neighbours $r_j \subseteq M \setminus \{j\}$ with whom $j$ can communicate directly. In most cases, only a few buyers join the auction initially. We regard these buyers who are in the auction initially as head buyers and denote the set of head buyers as $A$ and $M_{-A} = M \setminus A$.

Intuitively, sellers in the auction want to expand the auction to attract more buyers to enter the auction to obtain higher transaction prices. However, buyers are not willing to tell other buyers in the networks since it will increase the risk of losing the opportunity to get an item. Hence, the goal of this paper is to design mechanisms to encourage all buyers to inform their neighbours who are not aware of this market to participant in. It is quite difficult and counter-intuitive and has not been studied in other double auction literature.

In this paper, the mechanisms demand each seller $i \in N$ in the market reports her valuation on the item, denoted by $s'_i \in S_i$ ($s'_i \geq 0$) where $S_i$ is the seller $i$'s action space and $s' = (s'_1, \cdots, s'_n) \in S$ is all sellers' action profile where $S$ is the action profile space of all sellers. Let $s'_{-i}$ be the action profile of all sellers except $i$ and $s' = (s'_i, s'_{-i})$. Let $s_i$ be seller $i$'s truthful valuation. Each buyer $j \in M$ has an action $\theta'_j = (b'_j, r'_j)$ where $b'_j \geq 0$ denotes $j$'s valuation she reports and $r'_j$ denotes the set of neighbours $j$ invites. Specially, when a buyer is not in the auction (i.e. she doesn't know the auction information), her action is set to *null*. Let $\Theta_j$ be buyer $j$'s action space. The joint vector $\theta' = (\theta'_1, \cdots, \theta'_m)$ is assumed to be the action profile of all buyers. Let $\theta'_{-j}$ be the action profile of all buyers except $j$ and we have $\theta' = (\theta'_j, \theta'_{-j})$. Let $\theta_j = (b_j, r_j)$ be the action that buyer $j$ truthfully report her valuation and invite all her neighbours $r_j$.

Definition 1. *We say an action profile of all buyers $\theta'$ is feasible if for all $i \in M_{-A}$, $\theta'_i \neq null$, $i$ must receive the auction information following the actions of the others.*

Each buyer except the head buyers must have at least one inviter who have participated in the auction to informs her of the auction.

Note that we only consider the feasible action profiles and let $f(\theta')$ be a feasible action profile which is transformed from $\theta'$ by removing uninvited buyers in order. Let $\Theta$ be the space of all feasible action profiles $f(\theta')$.

In what we have discussed so far, we allow each head buyer to invite her neighbours to join the auction. Each head buyer will invite a group of her neighbours to take part in the auction, and her neighbours will invite the neighbours' neighbours into the auction. Finally as the phase of information diffusion ends, we will extend the auction from $|A|$ buyers to $|A|$ buyer groups. Due to the networks complexity and practical implementation issue, we only consider the situation where the intersection of these $|A|$ buyer groups are empty in the rest of this paper.

According to the description above, there only exists a few buyers initially. The revenue of both sellers and the market owner would be limited. That is to say all sellers and the market owner have the incentive to extend the market. However, buyers do not want to do so because it will increase the risk of losing in this auction. To combat this challenge, we can find a solution to deal with this dilemma in the view of mechanism design. Before that, we need to formulate the form of mechanisms in double auctions with social networks.

Definition 2. *A mechanism $\mathcal{M}$ in double auctions with social networks is defined by a pair $\{\pi, p\}$ where $\pi = \{\pi_i\}_{i \in M \cup N}$ and $p = \{p_i\}_{i \in M \cup N}$ are allocation and payment functions respectively. Specially, the allocation function is defined as $\pi_i : S \times \Theta \rightarrow \{0, 1\}$ for all $i \in M \cup N$ and the payment function is defined as $p_i : S \times \Theta \rightarrow R$ for all $i \in M \cup N$.*

For the simplicity of expression, we denote $\pi_i, p_i$ as $\pi^s_i, p^s_i$ for all sellers and $\pi^b_i, p^b_i$ for all buyers i.e. $\pi = \{\pi^s_i\}_{i \in N} \cup \{\pi^b_i\}_{i \in M}$, $p = \{p^s_i\}_{i \in N} \cup \{p^b_i\}_{i \in M}$. For $\pi^s_i = 0$, it means the seller sells the item and for $\pi^s_i = 1$, it means the seller reserves the item. Meanwhile, $\pi^b_i = 1$ means the buyer receives an item and $\pi^b_i = 0$ means the buyer does not receive an item. For all buyers and sellers $i \in M \cup N$, $p_i \geq 0$ means $i$ gives $p_i$ to the market and if $p_i < 0$, $i$ receives a payment of $|p_i|$ from the market.

Next, we define the related properties of the mechanism.

Definition 3. *We say an allocation $\pi$ is feasible if for all sellers' action profile $s' \in S$ and all feasible buyers' action profile $\theta' \in \Theta$, we have :*

- *for all $i \in M$, if $\theta'_i = null$, then $\pi^b_i(s', \theta') = 0$;*
- *$\sum_{i \in N} \pi^s_i(s', \theta') + \sum_{i \in M} \pi^b_i(s', \theta') = n$.*

A feasible allocation can only allocate the items to the buyers who participate in the auction. Besides, the number of items at the end of the auction must equal to $n$ wherever the items are allocated. We only consider feasible allocations in the rest of this paper. When an allocation is determined, social welfare can be defined as $SW(s', \theta', \pi) = \sum_{i \in N} \pi^s_i(s', \theta') s'_i + \sum_{i \in M} \pi^b_i(s', \theta') b'_i$ where $\pi$ is a feasible allocation.

Definition 4. *We say a mechanism $\mathcal{M}$ is efficient if for all sellers' action profile $s' \in S$ and all feasible buyers' action profile $\theta' \in \Theta$, $\pi \in argmax_{\pi' \in \Pi} SW(s', \theta', \pi')$ where $\Pi$ is the set of all feasible allocations.*

Given a buyer $i \in M$, a feasible buyers' action profile $\theta' \in \Theta$ and a sellers' action profile $s' \in S$, the utility of $i$ under a mechanism $\mathcal{M}$ is quasilinear and defined as:

$$u^b_i(\theta_i, s', \theta', \mathcal{M}) = \pi^b_i(s', \theta') b_i - p^b_i(s', \theta')$$

Accordingly, given a seller $i \in N$, a sellers' action profile $s' \in S$ and a feasible buyers' action profile $\theta' \in \Theta$, the utility of $i$ under a mechanism $\mathcal{M}$ is quasilinear and defined as:

$$u^s_i(s_i, s', \theta', \mathcal{M}) = (\pi^s_i(s', \theta') - 1) s_i - p^s_i(s', \theta')$$

A mechanism is individually rational if for all buyers and sellers $i \in M \cup N$, $i$'s utility is always non-negative if she reports valuation truthfully. That's to say the mechanism does not force buyers to inform their neighbours under the guarantee of individually rational property.

Definition 5. *A mechanism $\mathcal{M}$ is individually rational (IR) if*

- *for all $i \in M$, all feasible buyers' action profile $((b_i, r_i'), \theta_{-i}') \in \Theta$ and all sellers' action profile $s' \in S$, we have $u_i^b(\theta_i, s', ((b_i, r_i'), \theta_{-i}'), \mathcal{M}) \geq 0$*
- *for all $j \in N$, all sellers' action profile $(s_j, s_{-j}') \in S$ and all feasible buyers' action profile $\theta' \in \Theta$, we have $u_j^s(s_j, (s_j, s_{-j}'), \theta', \mathcal{M}) \geq 0$*

Next, we introduce another property called incentive compatible. The incentive compatible mechanisms in double auctions with social networks are not only required to let all sellers and buyers report their valuations truthfully, but also to let all the buyers have the incentive to invite all their neighbours to join the auction.

Definition 6. *A mechanism $\mathcal{M}$ is incentive compatible (IC) if*

- *for all $i \in M$, all feasible buyers' action profile $(\theta_i, \theta_{-i}') \in \Theta$, all sellers' action profile $s' \in S$ and all $\theta_i' \in \Theta_i$, we have*
$$u_i^b(\theta_i, s', (\theta_i, \theta_{-i}'), \mathcal{M}) \geq u_i^b(\theta_i, s', f((\theta_i', \theta_{-i}')), \mathcal{M})$$
- *for all $j \in N$, all sellers' action profile $(s_j, s_{-j}') \in S$, all feasible buyers' action profile $\theta' \in \Theta$ and all $s_j' \in S_j$, we have $u_j^s(s_j, (s_j, s_{-j}'), \theta', \mathcal{M}) \geq u_j^s(s_j, (s_j', s_{-j}'), \theta', \mathcal{M})$*

Note that when a buyer $i \in M$ changes her action from $\theta_i$ to $\theta_i'$, which means that $i$ may not invite all her neighbours to the auction, $(\theta_i', \theta_{-i}')$ may not be feasible any more. That's why we need $f((\theta_i', \theta_{-i}'))$ to transform it to a feasible action profile. Particularly, according to the definition of IC, truthfully reporting valuation and inviting all neighbours at the same time is a dominant strategy for all buyers.

Given a sellers' action profile $s' \in S$, a feasible buyers' action profile $\theta' \in \Theta$ and a mechanism $\mathcal{M}$, the revenue of the market owner is defined by the sum of all buyers' and sellers' payments, which is denoted by $R^{\mathcal{M}}(s', \theta') = \sum_{i \in N} p_i^s(s', \theta') + \sum_{i \in M} p_i^b(s', \theta')$.

Definition 7. *A mechanism $\mathcal{M}$ is weakly budget balanced if for all sellers' action profile $s' \in S$ and all feasible buyers' action profile $\theta' \in \Theta$, $R^{\mathcal{M}}(s', \theta') \geq 0$.*

With all the definitions well defined, we can introduce our new mechanisms. Our goal is to design a mechanism that is IR, IC and weakly budget balanced. In the next section, we first introduce a simple extension of the well-known McAfee's trade reduction mechanism on social networks and show it cannot guarantee incentive compatible and therefore, we need to design new solutions for this model.

## 2 MCAFEE'S MECHANISM IN SOCIAL NETWORKS

McAfee proposed a trade reduction mechanism for the setting where a trader can either sell or buy. The idea is to remove the pair with the lowest buying and selling valuations from the efficient allocation to set up the payments to the other buyers and sellers. In our situation, since buyers are connected via social networks, we take every single group as a whole single buyer and take the maximum valuation of one group as the whole single buyer's bid. McAfee's trade reduction mechanism can be extended to this model as follows:

**McAfee's Mechanism in Social Networks**

1. Given a sellers' action profile $s' \in S$, a feasible buyers action profile $\theta' \in \Theta$, we can obtain $|A|$ social network groups denoted by $G$, and each group in $G$ starts with a head buyer in $A$.
2. Since the intersection of each head buyers' group is empty, we can find $|A|$ connected groups denoted by $G = \{G_1, \cdots, G_{|A|}\}$ and each group takes one of the head buyers as a root.
3. In each group $G_i \in G$, let $b_{G_i} = max_{k \in G_i}(b_k')$ be the highest valuation in group $G_i$.
4. Sort all the groups in descending order according to $b_{G_i}$. To simplify representation, we let $b_{G_1} \geq b_{G_2} \geq \cdots \geq b_{G_{|A|}}$. Meanwhile, sort all the sellers in ascending order according to their valuations such as $s_1' \leq s_2' \leq \cdots \leq s_n'$.
5. Find the efficient trade quantity $q$ (satisfying $b_{G_q} \geq s_q$ and $b_{G_{q+1}} < s_{q+1}'$). Let the $q-1$ highest groups trade with $q-1$ lowest sellers, with sellers being paid $s_q'$ (i.e. the payment for each traded seller is $-s_q'$).
6. For all buyers $i \in G_j$, the allocation is defined as:
$$\pi_i^b(s', \theta') = \begin{cases} 1 & \text{if } b_i' = b_{G_j}, j < q\ , \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$
That is, give the item to the buyer with the highest bid in $G_j$. If there exist multiple buyers in the same $G_i$ with $\pi^b(s', \theta') = 1$, allocate the item randomly among them to break the tie.
Let $\mathcal{W} = \{w_1, w_2, \cdots, w_{q-1}\}$ be the winners who receive the item. The payment of $i$ is defined as:
$$p_i^b(s', \theta') = \begin{cases} b_{G_q} & \text{if } i \in \mathcal{W}\ , \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Now, let us consider an example shown in Figure 1. Left column is the sellers $a, b, c, d$. The right graph is the networks generated by head buyers $A, J, Q, V$. Numbers in each circle are the valuations they report, and lines between buyers represent the information diffusion. Note that if head buyers do not diffuse the information, there is only one pair of seller and buyer can trade with each other in McAfee's trade reduction mechanism, since $s_a' < b_Q'$, $s_b' < b_J'$ and $s_c' > b_V'$, $s_d' > b_A'$. In McAfee's Mechanism with social networks, we can compute that $q = 4$, and sellers $a, b, c$ sell the item to buyers $G, O, T$. Sellers $a, b, c$ receive $s_d' = 12$. As for buyers, $G, O, T$ should pay $b_Y' = 13$.

In the traditional exchange market, McAfee's trade reduction mechanism is incentive compatible, individually rational and weakly

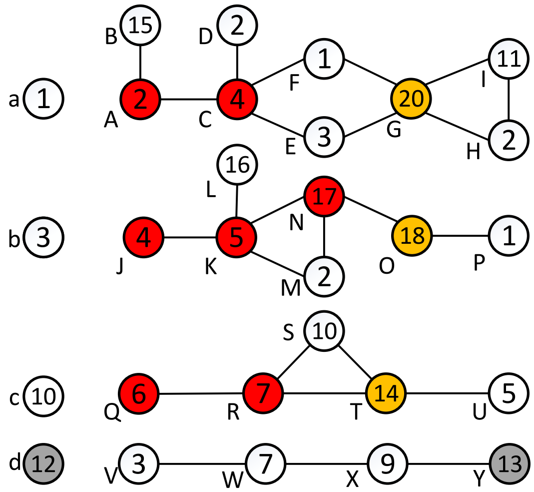


**Figure 1: An example of double auction mechanism with social networks, where yellow nodes receive the items in McAfee's Mechanism and gray nodes are the points to stop trading.**

budget balanced. However, McAfee's Mechanism on social networks is not incentive compatible since $N$ can get the item if she does not diffuse the auction information to $O$. Therefore, there is no motivation for buyers to invite their neighbours. Hence, in the next section, we propose a new mechanism which will incentive buyers to spread the information.

## 3 DOUBLE NETWORK AUCTION MECHANISM

McAfee's mechanism on social networks mentioned in Section 2 cannot run in our model directly to incentivize buyers to diffuse the information to their neighbours. In this section, we propose a new mechanism which is individually rational, incentive compatible and weakly budget balanced. In particular, the revenue of the market owner is equal to McAfee's trade reduction mechanism in the social networks described in Section 2.

First, we introduce some important concepts which are needed to describe the mechanism.

Definition 8. *In a buyer group $G_i \in G$, given a feasible action profile $\theta' \in \Theta$, for all $j, k \in G_i$ and for a head buyer $a \in A \cap G_i$, we say $k$ is a cut point of $j$ in the networks generated by $\theta'$ if $a$ and $j$ are disconnected when $k$ is removed from the networks.*

Intuitively, $k$ is a cut point of $j$ if all simple paths from $a$ to $j$ in the networks pass through $k$. Especially, $a$ and $j$ themselves are also cut points of $j$. Informally, definition 8 is proposed to select out some key buyers of buyer $j$. Without these key buyers, $j$ would never be aware of the market. Hence, more attention should be paid to these key buyers. For instance, in Figure 1, $A$ is the head buyer of $G$ and $A$, $C$ are cut points of $G$, which means if $A$ and $C$ choose not to diffuse the information, $G$ will not participate in the auction. $F$ is not a cut point of $G$ since $G$ can still be invited by $E$ without $F$'s participation.

Moreover, in a group $G_i \in G$, for all $j, k \in G_i$, $j \neq k$, if $k$ is $j$'s cut point, we say that $j$ is in $k$'s successor buyers set which is denoted by $d_k$. The set $d_k$ contains all buyers who can only get auction information through $k$. Therefore, when $k$ does not invite her neighbours to join the market, buyers in $d_k$ would never take part in the auction. Besides, let $l_k = d_k \cup \{k\}$.

Definition 9. *In a buyer group $G_i \in G$, given a feasible action profile $\theta' \in \Theta$, for all buyers $j \in G_i$ and for a head buyer $a \in A \cap G_i$, let $k_1, k_2, \cdots$ be the all cut points of $j$, we define $CP_j = \{a, k_1, k_2, \cdots, j\}$ as the cut path of $j$ which includes all cut points of $j$.*

To simplify the expression, let $CP_j = \{a, a+1, \cdots, j-1, j\}$ be the cut path of $j$ and obviously we have $CP_a \subseteq CP_{a+1} \cdots \subseteq CP_j$.

The set $CP_j$ includes all the key buyers for buyer $j$ taking part in the market. Meanwhile, $CP_j$ indicates how $a$ diffuses the information to $j$. That is to say, if buyer $k \in CP_j$ decides not to invite her neighbours, $k+1, k+2, \cdots, j$ would never take part in the auction.

In a buyer group $G_i \in G$, given a feasible action profile $\theta' \in \Theta$, for all buyer $k \in G_i$, let $p = argmax_{i \in d_k} b'_i$ and we can verify that $k \in CP_p$ since $k$ is $p$'s cut point. Hence, we have $CP_p = \{a, a+1, \cdots, k, k+1, \cdots, p\}$ and $p$ may be one of the potential winner candidates of this group. Meanwhile, let $c_k$ be the closest successor $(k+1)$ whom $k$ can inform of in $CP_p$.

Let $G^{-l_i} = \{G_1^{-l_i}, G_2^{-l_i}, \cdots, G_{|A|}^{-l_i}\}$ denotes the new buyer groups set when buyers in set $l_i$ are removed from the market, i.e. buyer $i$ didn't participate in the auction. Note that $G_j$ and $G_j^{-l_i}$ may not refer to the same group that starts with the same head buyer since when $l_i$ is removed from the market, all groups will resort accordingly.

Now we can start to define the new mechanism called Double Network Auction Mechanism.

**Double Network Auction Mechanism (DNA)**

As defined in last mechanism, find the efficient trade quantity $q$ (satisfying $b_{G_q} \geq s'_q$ and $b_{G_{q+1}} < s'_{q+1}$). Let the $q-1$ highest groups trade with $q-1$ lowest sellers, with sellers being paid $s'_q$ (i.e. the payment for each traded seller is $-s'_q$).

For buyers, we have:

- For each buyer $i \in G_j$, the allocation is defined as
  - If $j \leq q-1$,

$$\pi_i^b(s', \theta') = \begin{cases} 1 & \text{if } b'_i = b_{G_j^{-l_{c_i}}} \geq b_{G_q^{-l_{c_i}}}, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

  - If $j > q-1$, then $\pi_i^b(s', \theta') = 0$.

  That is, if there exist some buyers $k \in G_i$ on the cut path of the highest valuation buyer and $k$ becomes the biggest valuation buyer in $G_i$ when she does not diffuse the auction information to $c_k$, DNA will allocate the item to $k$ rather than current highest valuation buyer. If there exist multiple buyers $i$ in the same $G_j$ with $\pi_i^b(s', \theta') = 1$, allocate the item to the buyer $i$ with minimum size of $CP_i$.

- Let $\mathcal{W} = \{w_1, w_2, \cdots, w_{q-1}\}$ be the winners who receive the item. Let $D_{\mathcal{W}} = \bigcup_{j=1}^{q-1} d_{w_j}$ be the buyers who are any one of the winners' successor buyers.
- For each buyer $i \in G_j$, the payment is defined as:

$$p_i^b(s', \theta') = \begin{cases} 0 & \text{if } i \in D_{\mathcal{W}}, \\ W_{-l_i}^q - (W_{-l_{c_i}}^q - \pi_i^b b_i') & \text{otherwise}. \end{cases} \tag{4}$$

where $W_{-l_i}^q = \sum_{k=1}^{q-1} b_{G_k^{-l_i}}$, and $W_{-l_{c_i}}^q = \sum_{k=1}^{q-1} b_{G_k^{-l_{c_i}}}$.

That is, buyer $i$ should pay the difference between the sum of the $q-1$ highest groups' valuations without $c_i$'s participation and without $i$'s participation.

Note that the idea behind DNA is to combine the McAfee's trade reduction mechanism and Vickrey-Clarke-Groves (VCG) mechanism. Intuitively, we can treat each group as a whole and use McAfee's trade reduction mechanism first to get a reserved price for the single item that will be allocated in some groups later. Then, we allocate every single item in each group separately. Buyers can obtain some rewards for their diffusion by a VCG-like payment (i.e. $i$ pays an amount equal to the total damage that she causes the first $q-1$ buyer groups).

The payment of winner ($p_w$) in each group may be higher than what each group pay to the market ($b_{G_q}$). We take the rest money ($p_w - b_{G_q}$) as the rewards of all cut points of winners. That is why the revenue of the market owner of DNA is equal to McAfee's mechanism described in Section 2.

Now, let's consider the same example shown in Figure 1 where yellow nodes have the biggest valuation report but that doesn't mean they are the final winners, red nodes are their cut points. In DNA, we can compute that $q = 4$, and sellers $a, b, c$ sell the items to buyers $G, N, T$. Buyer $N$ is a winner because if she doesn't tell $O$, then she will have the highest valuation report in her group. Sellers $a, b, c$ receive $s_d' = 12$. As for buyers, $G$ should pay $(15+18+14)-(20+18+14-20) = 15$; $N$ should pay $(20+16+14)-(20+17+14-17) = 16$, $T$ should pay $(20+18+13)-(20+18+14-14) = 13$. Specially, $A$ would get a payment $(18+14+13)-(15+18+14) = -2$, $K$ would get a payment $(20+14+13)-(20+16+14) = -3$.

Then we prove that DNA is individually rational, incentive compatible and weakly budget balanced.

Theorem 1. *The double network auction mechanism is individually rational.*

Proof. we prove that the double network auction mechanism is individually rational for sellers and buyers respectively.

**Sellers:** For sellers' part, since sellers' allocation policy and payment policy are the same as the McAfee's mechanism. It is obvious that for each seller $i$, $i$'s utility is non-negative.

**Buyers:** Assume a buyer $i \in M$ reports her valuation $b_i$ truthfully. We can get that $i$'s payment is $p_i^b = W_{-l_i}^q - (W_{-l_{c_i}}^q - \pi_i^b b_i)$ if $i \notin D_{\mathcal{W}}$. Hence $i$'s utility is $u_i = \pi_i^b b_i - p_i^b = W_{-l_{c_i}}^q - W_{-l_i}^q$. Since $l_i \supseteq l_{c_i}$ and then $M \setminus l_i \subseteq M \setminus l_{c_i}$, from the definition of $W_{-l_i}^q$ and $W_{-l_{c_i}}^q$, we can conclude that $W_{-l_{c_i}}^q \geq W_{-l_i}^q$ which means $i$'s utility is non-negative. If $i \in D_{\mathcal{W}}$, her utility is zero obviously. Therefore, for each buyer $i$, $i$'s utility is non-negative.

Therefore, the double network auction mechanism is individually rational. □

Next we prove that DNA is incentive compatible. Before that, we prove the following two lemma.

Lemma 1. *Given a sellers' action profile $s'$, if the efficient trade quantity is $q$ under $((b_i, r_i'), \theta_{-i}') \in \Theta$, then the efficient trade quantity is $q' \in \{q+1, q, q-1\}$ under any $((b_i', r_i'), \theta_{-i}') \in \Theta$.*

Proof. Since $r_i'$ is fixed which means the networks is fixed under $((b_i, r_i'), \theta_{-i}')$. We can get that for each buyer $i \in G_j$: Consider the worst case that $i$ can control her group's rank from 1 to $|A|$ as long as other buyers' valuations report in $G_j$ is small enough. Enumerate all these cases and we can easily conclude that $q' \in \{q+1, q, q-1\}$. □

More specifically, for a buyer $i \in M$, decrease her valuation reports can only reduce $q$ by at most 1 and increase her valuation reports can only increase $q$ by at most 1. This means that the ability of one buyer to influence $q$ through misreporting her valuation is limited.

For the same reason, for a seller $j \in N$, increase her valuation reports can only reduce $q$ by at most 1 and decrease her valuation reports can only increase $q$ by at most 1.

Lemma 2. *Given a sellers' action profile $s'$, if the efficient trade quantity is $q$ under $((b_i', r_i), \theta_{-i}') \in \Theta$, then the efficient trade quantity is $q' \in \{q, q-1\}$ under any $((b_i', r_i'), \theta_{-i}') \in \Theta$.*

Accordingly, it means that the ability of one buyer to influence $q$ through not inviting her neighbours is also limited.

Next, we prove that DNA is also incentive compatible.

Theorem 2. *The double network auction mechanism is incentive compatible.*

Proof. We prove the IC property for sellers and buyers respectively.

**Sellers:** For sellers' part, since sellers' allocation and payment policies are the same as the generalized McAfee's mechanism. Therefore, for each seller $i$, reporting her valuation truthfully is a dominant strategy.

**Buyers:** First if $i \in D_{\mathcal{W}}$ which means $i$'s utility is always 0 whatever action $\theta_i'$ she is. For these buyers, reporting their valuation and diffuse the information to all their neighbours is a dominant strategy.

Next we prove that for each buyer $i \notin D_{\mathcal{W}}$, reporting her valuation $b_i$ truthfully is a dominant strategy when $r_i'$ is determined (i.e. fixing the neighbours that $i$ is inviting).

When $i$ reports her valuation truthfully, her utility is $u_i = W_{-l_{c_i}}^q - W_{-l_i}^q$. On the contrary, when she reports $b_i'$, her utility is $u_i' = W'^{q'}_{-l_{c_i}} - \pi_i'^b b_i' + \pi_i'^b b_i - W^{q'}_{-l_i}$. Since $r_i'$ is determined which means $l_{c_i}$ is fixed, so $i$ can only change her utility by changing $\pi_i^b$ or $q$.

If $\pi_i^b = 1$ which means $i$ receives an item when she reports truthfully, increase $b_i$ cannot help anything (she will still receive the item). But when she decrease $b_i$ to $b_i'$ such that lose the item and her utility becomes $u_i' = W'^{q'}_{-l_{c_i}} - W^{q'}_{-l_i}$. Based on Lemma 1, let's discuss $q' = q$ or $q-1$ respectively. When $q' = q$: We can get $\Delta u_i = W'^{q}_{-l_{c_i}} - W^{q}_{-l_{c_i}} \leq 0$ which means her utility is decreased. When $q' = q-1$: We can get that $\Delta u_i = b^{-l_i}_{G_{q-1}} - b_i$. Since $b_i \geq b_{G_{q-1}} \geq b^{-l_i}_{G_{q-1}}$, her utility is decreased.

If $\pi^b_i = 0$ which means $i$ cannot receive the item when she reports truthfully, decrease $b_i$ cannot help anything. When $i$ increase her valuation to $b'_i$ such that she becomes a winner and receives an item. At this time, her utility becomes $u'_i = W'^{q'}_{-l_{c_i}} - b'_i + b_i - W^{q'}_{-l_i}$. Based on Lemma 1, let's discuss $q' = q$ or $q+1$ respectively. When $q' = q$: We can get $\Delta u_i = W'^{q}_{-l_{c_i}} - b'_i + b_i - W^{q}_{-l_{c_i}}$. We can easily get that her utility is decreased since $W^{q}_{-l_{c_i}} \geq W'^{q}_{-l_{c_i}} - b'_i + b_i$. When $q' = q+1$: In this case, we can conclude that $b_{G^{-l_i}_q} = b_{G_q} \geq b_i$ since $q'$ is increased. Then $\Delta u_i = b_i - b_{G^{-l_i}_q} \leq 0$ which means her utility is decreased. Therefore, reporting valuation truthfully maximizes $i$'s utility when $r'_i$ is determined.

Assume buyer $i \notin D_{\mathcal{W}}$ reports her valuation $b_i$ truthfully, then we prove that diffusing the information to all her neighbours is a dominant strategy.

For each buyer $i$, her utility is $u_i = W^{q}_{-l_{c_i}} - W^{q}_{-l_i}$ when she reports valuation truthfully no matter whether she is a winner. Since $r'_i$'s change will make $q$ change, based on Lemma 2, let's discuss $q' = q$ or $q-1$ under $r'_i$ respectively. When $q' = q$: Since $W^{q}_{-l_i}$ is independent of $i$, that is to say changing $r_i$ cannot make $W^{q}_{-l_i}$ change as long as $q$ is fixed. As for $W^{q}_{-l_{c_i}}$, diffusion the information to more neighbours would increase $W^{q}_{-l_{c_i}}$ when she has some potential high valuation neighbours. When $q' = q-1$: We can get that $\Delta u_i = W'^{q-1}_{-l_{c_i}} - W^{q-1}_{-l_i} - W^{q}_{-l_{c_i}} + W^{q}_{-l_i}$. Since $W^{q}_{-l_i} - W^{q-1}_{-l_i} = b_{G^{-l_i}_{q-1}}$, $W'^{q-1}_{-l_{c_i}} - W^{q}_{-l_{c_i}} \leq W'^{q-1}_{-l_{c_i}} - W'^{q}_{-l_{c_i}} \leq W^{q-1}_{-l_i} - W^{q}_{-l_i} = -b_{G^{-l_i}_{q-1}}$ which means $\Delta u_i \leq 0$, i.e. her utility is decreased.

Therefore, diffusing the information to all her neighbours is a dominant strategy. All the above together proves that the double network auction mechanism is incentive compatible. □

Theorem 3. *The double network auction mechanism is weakly budget-balanced.*

Proof. From the definition of revenue and the DNA payment policy, since in buyers part there are $|A|$ graphs $\{G_1, \cdots, G_{|A|}\}$ and does not intersect with each other, we can compute $R^{DNA}$ in $\{G_1, \cdots, G_{|A|}\}$ respectively and then add them together and minus the money paid to the seller.

$$R^{DNA} = \sum_{i \in G_1} p^b_i + \sum_{i \in G_2} p^b_i + \cdots + \sum_{i \in G_{|A|}} p^b_i - (q-1)s_q$$

Let's compute revenue in $G_1$ and other parts are similar.

$$\sum_{i \in G_1} p^b_i = \sum_{i \notin l_w} (W^{q}_{-l_i} - W^{q}_{-l_{c_i}}) + (W^{q}_{-l_w} - W^{q}_{-l_{c_w}} - b_w)$$
$$= W^{q}_{-l_k} - W^{q}_{-l_{c_w}} + b_w = b_{G_q}$$

Where $k \in A \cap G_1$.Therefore, the revenue is

$$R^{DNA} = (q-1)b_{G_q} - (q-1)s_{G_q} = (q-1)(b_{G_q} - s_{G_q}) \geq 0$$

□

Moreover, from Theorem 3, we can verify that the sum of each buyer's payment in any single group equals to $b_{G_q}$, which means the revenue of the market owner is equivalent to McAfee's trade reduction mechanism in the social networks described in Section 2.

# 4 CONCLUSION

In this paper, we consider the traditional double auctions (DAs) in a new view. We propose a new method to hold an incentive compatible auction in the double auction market, where the original buyers are willing to propagate the auction information to their neighbours to extend the market.

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