1 General McCall Search Model

- Offer arriving rate: λ
- Separation rate: α
- Discount rate: β
- Wage offers: $\{w_1, w_2, \dots, w_n\}$ with probability $\{p_1, p_2, \dots, p_n\}$
- Unemployment benefit: c
- Life time utility

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t u(y_t)$$

The value functions are

$$V_s = u(w_s) + \beta\left[(1-\alpha)V_s + \alpha U\right]$$

$$U = u(c) + \beta(1-\lambda)U + \beta\lambda \sum_s p_s \max\{U, V_s\}$$

2 A Simple Example

We will make the following assumptions to make it easy to solve: the wage offers are $\{w_1, w_2\}$ with probability $\{p_1, p_2\}$, the period utility function $u(y_t) = y_t$, and the unemployment benefit $c = 0$. We also assume that $w_2 > w_1 > 0$.

With these assumption, the value functions become

$$V_1 = w_1 + \beta\left[(1-\alpha)V_1 + \alpha U\right] \tag{2.1}$$

$$V_2 = w_2 + \beta\left[(1-\alpha)V_2 + \alpha U\right] \tag{2.2}$$

$$U = \beta(1-\lambda)U + \beta\lambda p_1 \max\{U, V_1\} + \beta\lambda p_2 \max\{U, V_2\} \tag{2.3}$$

There are two possible scenarios: first, $U > V_1$; second, $U < V_1$. Let us guess that the first is true, and we will verify whether this guess is valid or not.

With the first guess, we have

$$V_1 = w_1 + \beta\left[(1-\alpha)V_1 + \alpha U\right] \tag{2.4}$$

$$V_2 = w_2 + \beta\left[(1-\alpha)V_2 + \alpha U\right] \tag{2.5}$$

$$U = \beta(1-\lambda)U + \beta\lambda p_1 U + \beta\lambda p_2 V_2 \tag{2.6}$$

It is easy to see the following is true

$$V_1 = V_2 - \frac{w_2 - w_1}{1-\beta(1-\alpha)} \tag{2.7}$$

and

$$U = \frac{\beta\lambda p_2}{1-\beta(1-\lambda)-\beta\lambda p_1} V_2 < V_2 \tag{2.8}$$

Then we have

$$V_2 = w_2 + \beta(1-\alpha)V_2 + \alpha\beta\frac{\beta\lambda p_2}{1-\beta(1-\lambda)-\beta\lambda p_1} V_2 \tag{2.9}$$

which leads to

$$V_2 = \frac{1}{1-\beta(1-\alpha)-\frac{\alpha\beta^2\lambda p_2}{1-\beta(1-\lambda)-\beta\lambda p_1}} w_2 = \frac{1-\beta(1-\lambda)-\beta\lambda p_1}{(1-\beta(1-\lambda p_2)+\alpha\beta)(1-\beta)} w_2 \equiv A w_2 \tag{2.10}$$

Then it follows

$$V_1 = V_2 - \frac{w_2-w_1}{1-\beta(1-\alpha)} = \left(A - \frac{1}{1-\beta(1-\alpha)}\right) w_2 + \frac{1}{1-\beta(1-\alpha)} w_1 \tag{2.11}$$

$$U = \frac{\beta\lambda p_2}{1-\beta(1-\lambda)-\beta\lambda p_1} A w_2 \tag{2.12}$$

To verify our initial guess, the following has to be true

$$U > V_1 \tag{2.13}$$

which is equivalent to

$$\frac{\beta\lambda p_2}{1-\beta(1-\lambda)-\beta\lambda p_1} A w_2 > \left(A - \frac{1}{1-\beta(1-\alpha)}\right) w_2 + \frac{1}{1-\beta(1-\alpha)} w_1 \tag{2.14}$$

which can be simplified to

$$\frac{1}{1-\beta(1-\alpha)}(w_2 - w_1) > \frac{1-\beta}{1-\beta(1-\lambda)-\beta\lambda p_1} A w_2 \tag{2.15}$$

After some algebra, the equation above simplifies to

$$\frac{1}{1-\beta+\alpha\beta}(w_2 - w_1) > \frac{1}{1-\beta(1-\lambda p_2)+\alpha\beta} w_2 \tag{2.16}$$

$$\beta\lambda p_2 w_2 > [1-\beta(1-\lambda p_2)+\alpha\beta] w_1 \tag{2.17}$$

The following intuitions should apply

- If w_1 is small enough, $U > V_1$
- If β is large, it is more likely to have $U > V_1$
- If p_2 is large, it is more likely to have $U > V_1$
- If λ is large, it is more likely to have $U > V_1$
- If α is smaller, it is more likely to have $U > V_1$