Homework

1. A firm produces both pencils and erasers in a ratio of 2:3. Its costs are:

$$C(q_1, q_2) = 3200 + 0.1q_1 + 2q_1^2 + 0.2q_2$$

where q_1 is the output of pencils and q_2 the output of erasers. What are its ray average costs, and what scale of production minimizes these? What would the minimum efficient scale be if the firm produced only pencils? What if it produced only erasers?

Answer:

Given that pencils and erasers are produced in a ratio of 2:3, we define a scale of output measure, q, such that $q_1 = 0.4q$ and $q_2 = 0.6q$. Substituting these expressions into the cost function and then dividing by q yields:

$$RAC = \frac{3200 + 0.04q + 0.32q^2 + 0.12q}{q}.$$

Setting the derivative of this expression with respect to q equal to zero, we find that RAC is minimized at $q = 100$, or 40 pencils and 60 erasers. If the firm produced only pencils, its average costs would be minimized at an output of 40 pencils as well. If it produced only erasers, it would have no minimum efficient scale, because average costs fall everywhere with output of erasers.

2. If the market demand curve is $Q = 100 - p$, what is the market price elasticity of demand? If the supply curve of individual firms is $q = p$ and there are 50 identical firms in the market, draw the residual demand facing any one firm. What is the residual demand elasticity facing one firm at the competitive equilibrium?

   Answer:

   The market price elasticity of demand is:

$$\frac{\Delta Q}{\Delta p}\frac{p}{Q} = -\frac{p}{Q} = \frac{Q-100}{Q} = 1 - \frac{100}{Q}.$$

   The residual demand curve facing any one firm is $Q_r(p) = Q(p) - 49q(p) = 100 - p - 49p = 100 - 50p$. At the competitive equilibrium, $p^* = q^* = \frac{100}{51}$, so the residual demand elasticity facing one firm is:

$$\frac{\Delta Q_r}{\Delta p}\frac{p}{Q_r} = -50$$

3. If the demand curve is:

$$Q(p) = p^{\varepsilon},$$

what is the elasticity of demand? If MC is \$1 and ε equals -2, what is the profit-maximizing price?

Answer:

The elasticity of demand is:

$$\frac{\partial Q}{\partial p}\frac{p}{Q} = \frac{\varepsilon p^{\varepsilon-1} p}{p\varepsilon} = \varepsilon$$

From equations in the text, we find that the profit-maximizing price is:

$$p = \frac{MC}{(1+\frac{1}{\varepsilon})} = \frac{1}{(1-\frac{1}{2})} = 2$$

4. An industry consists of eleven identical firms with costs:

$$c(q) = 4q + q^2,$$

Market demand is $Q = 100 - p$. What are the equilibrium price, output and profits of each firm if all eleven participate in a cartel? If a single firm cheated, what would its output and profits be, assuming the other firms maintain the cartel price?

Answer:

Under perfect collusion, the firms jointly produce the output that a monopoly would produce if it had the same costs as the eleven firms combined. Such a monopoly's costs would be

$$C(Q) = 11c(\frac{Q}{11}) = 11[4\frac{Q}{11} + (\frac{Q}{11})^2] = 4Q + \frac{Q^2}{11}$$

and its profits

$$\pi_m = (100 - Q)Q - 4Q - \frac{Q^2}{11}$$

Setting the derivative of these profits with respect to Q equal to zero, we find that $Q_m = 44$. Under perfect collusion, each firm therefore produces $q = \frac{Q_m}{11} = 4$, at the monopoly price $p_m = \$56$. The profits of each firm are \$192. If a single firm cheated, it would expand its output up to where the cartel price equals its marginal costs, or $56 = 4 + 2q$. It would therefore produce $q = 26$, earning profits of \$676.

5. An industry consists of two firms which can either "collude" or "compete" in each period. The per period payoffs of doing either are as follows:

	Firm 1			
Firm 2	Collude		Compete	
Collude	\$50	\$50	-\$10	\$75
Compete	\$75	-\$10	\$45	\$45

If the firms initially collude, either of them can earn \$25 extra by cheating on the agreement and competing instead. The other firm can retaliate, however, by refusing to collude in subsequent periods.

(i) For how many periods would such retaliation have to last to deter cheating if the interest rate is 0%, so that the firms do not discount future profits?

(ii) Would this minimum retaliation period be longer if the interest rate were positive? Why or why not?

(iii) What is the minimum retaliation period if the interest rate is 10%?

(iv) Is there an interest rate so high that no retaliation ever deters cheating? If so, what is it?

Answer:

(i) Cheating is deterred if the long-term loss from inviting retaliation outweighs the short-term gain from cheating. With zero discounting, cheating is deterred if $5t \geq 25$, where t is the number of periods of retaliation (assuming neither firm cheats if short-term gains just equal long-term losses). Thus, the minimum number of periods is 5.

(ii) Discounting reduces the weight given to long-term losses from retaliation, so that t must increase.

(iii) Let the discount factor be $\delta = \frac{1}{1+r}$, where *r* is the interest rate. The long-term loss from *t* periods of retaliation is:

$$5\delta + 5\delta^2 + ... + 5\delta^t = 5\delta(1 + \delta + ... + \delta^{t-1}) = 5\delta\frac{1-\delta^t}{1-\delta}$$

Cheating is deterred if:

$$5\delta\frac{1-\delta^t}{1-\delta} \geq 25.$$

Setting $\delta = \frac{10}{11}$ and solving for *t*, this condition is:

$$t \geq \frac{\ln(\frac{1}{2})}{\ln(\frac{10}{11})} \approx 7.27$$

so that the minimum number of periods is 8.

(iv) We can rewrite the general condition for deterrence by multiplying through by $\frac{\delta-1}{5\delta}$, adding 1 to both sides, and then multiplying through by δ to obtain:

$$\delta^{t-1} \leq 6\delta - 5$$

If $\delta < \frac{5}{6}$ (the interest rate is over 20%), this condition cannot be met for any *t*, because the right-hand side of this expression is negative, whereas the left-hand side is positive.