



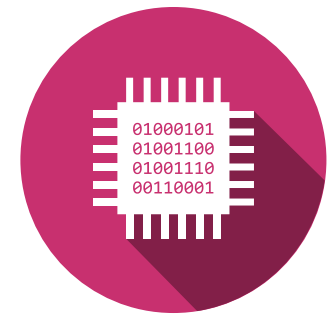
Digitales Design (DiD)

Kombinatorische Logikfunktionen

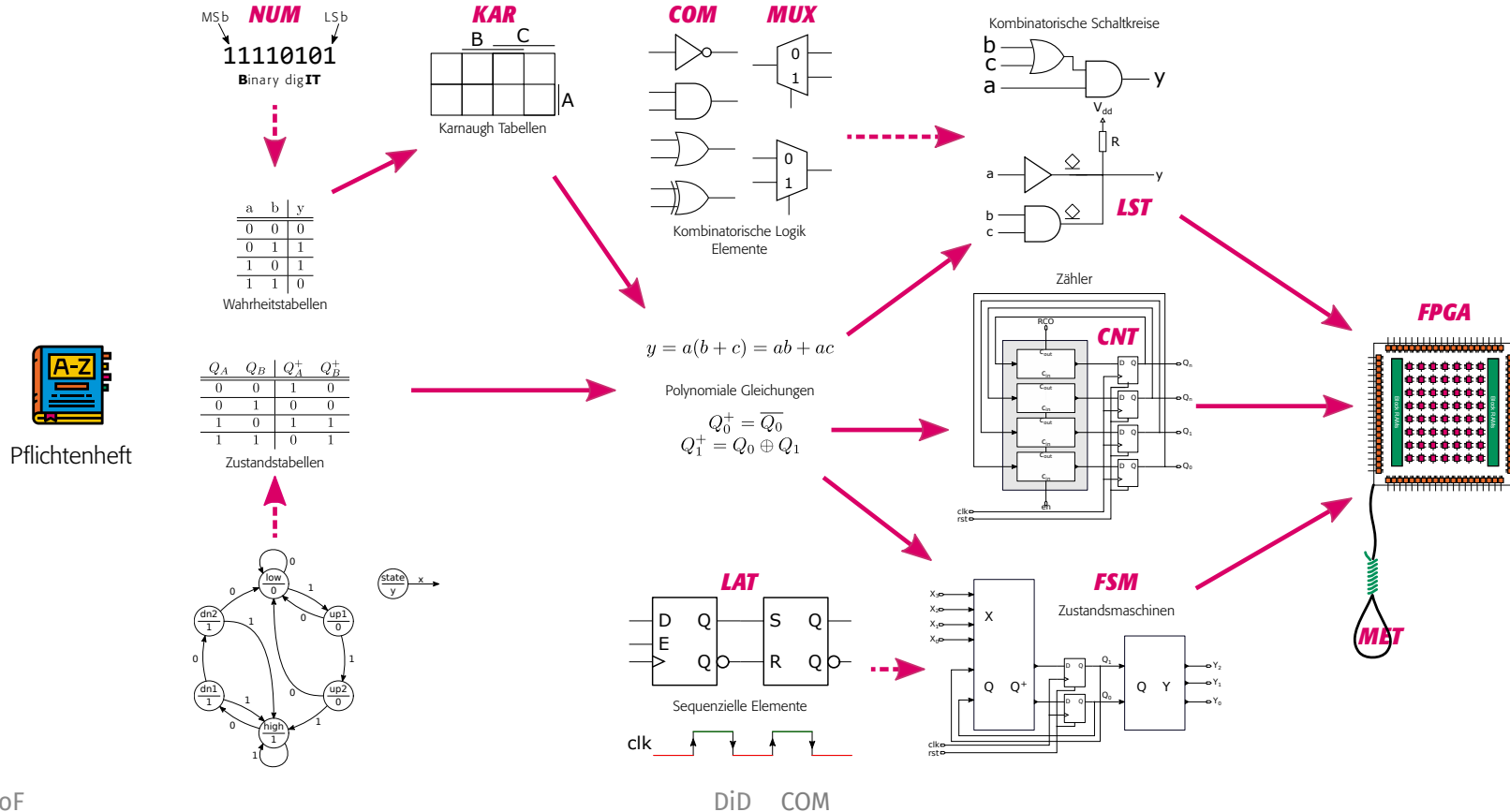
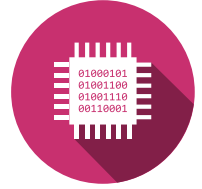
COM

Studiengang Systemtechnik
Studiengang Energie und Umwelttechnik
Studiengang Informatik und Kommunikationssysteme

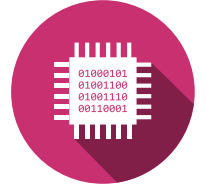
Silvan Zahno silvan.zahno@hevs.ch
Christophe Bianchi christophe.bianchi@hevs.ch
François Corthay francois.corthay@hevs.ch



Aktueller Inhalt des Themas im Kurs

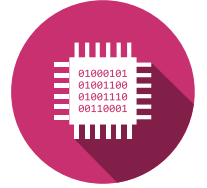


Inhalt



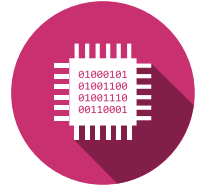
- **Darstellung von kombinatorischen Funktionen**
 - Wahrheitstabelle
 - Ablaufdiagramm
 - Venn-Diagramm
- Elementare Logikfunktionen
 - Puffer (Buffer)
 - Inverter (Inverter)
 - UND (AND)
 - ODER (OR)
 - Exklusiv-ODER (XOR)
- Boolesche Algebra
- Vollständige Operatoren

Beispiel einer Logikfunktion



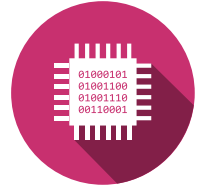
- Ein Lüftungssystem dient zur Regulierung der Temperatur eines Raumes.
- Als allgemeine Regel gilt: Wenn die $T^{\circ}_{\text{ausßen}} > T^{\circ}_{\text{lokal}}$ ist, wird die Lüftungsanlage eingeschaltet, um den Raum zu erwärmen.
- Im Sommer und tagsüber wird jedoch die Lüftung zur Kühlung des Raumes eingesetzt: Ist die $T^{\circ}_{\text{ausßen}} < T^{\circ}_{\text{lokal}}$, wird auch die Lüftung eingeschaltet.

Wahrheitstabelle (Truth Table)



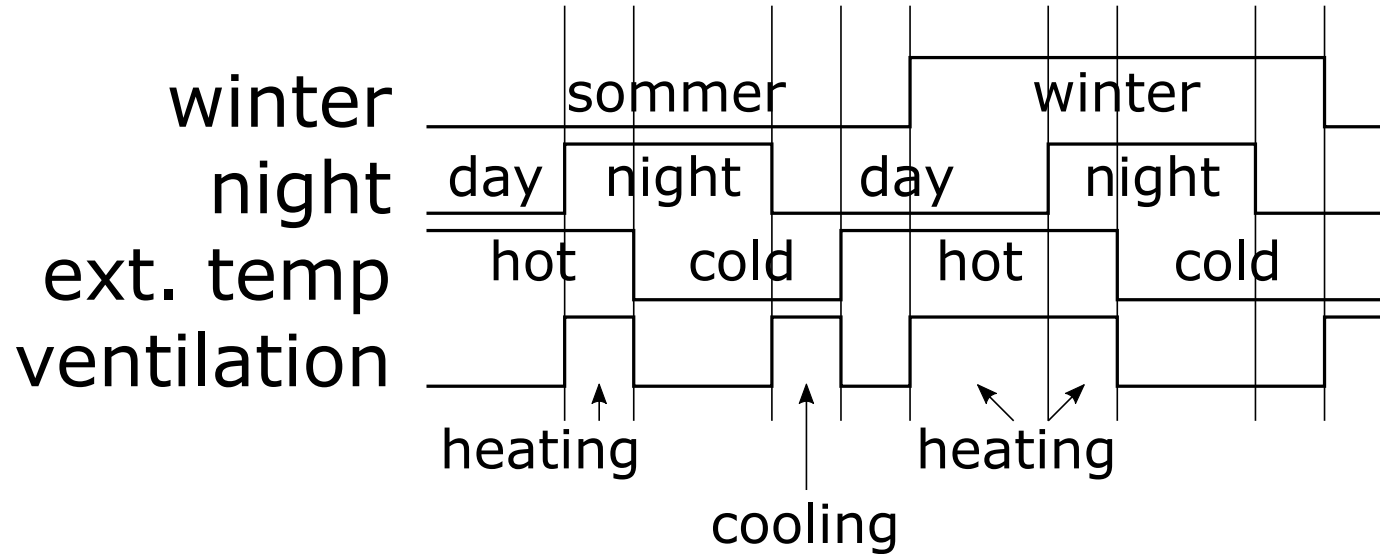
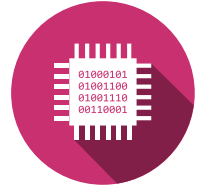
Winter	Nacht	Aussen wärmer	Ventilation
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Wahrheitstabelle (Truth Table)

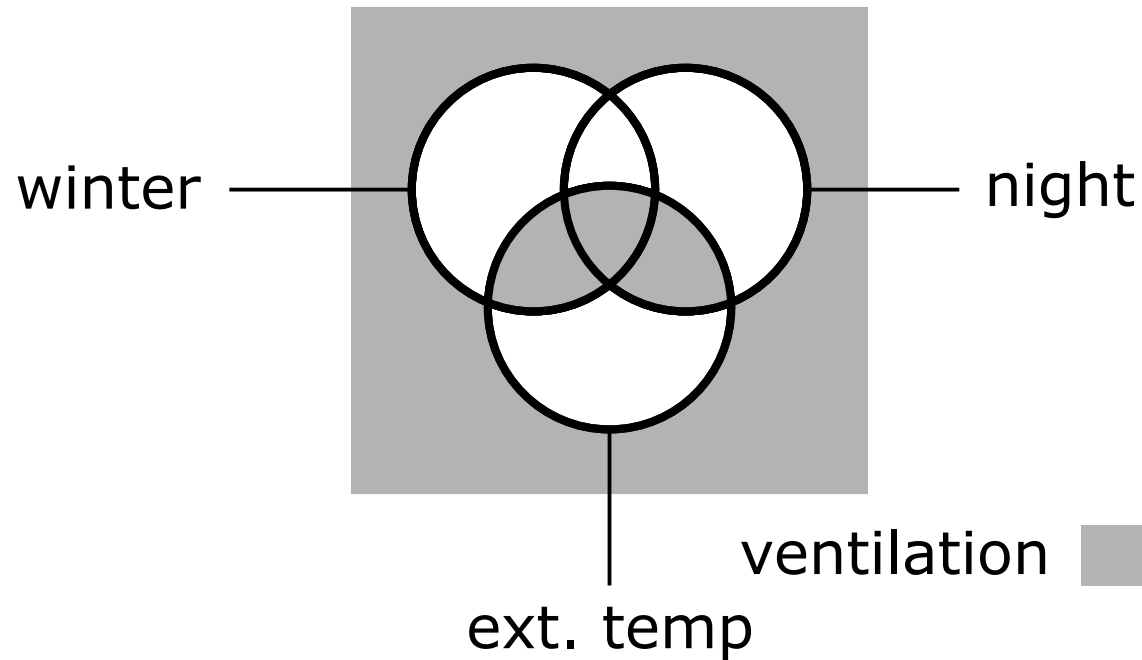
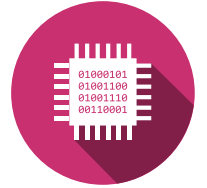


Winter	Nacht	Aussen wärmer	Ventilation
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

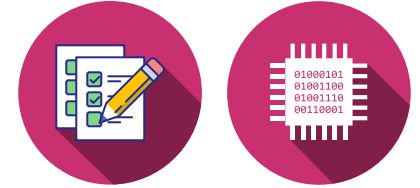
Ablaufdiagramm (Chronogram)



Venn-Diagramm (Venn Diagram)



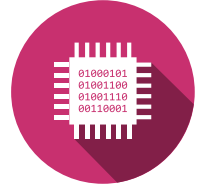
Aufgabe 2.1



- Erstellen Sie die Wahrheitstafel einer Steuerschaltung für einen Stockwerk eines Lifts. Das System beträgt folgende Eingänge:
 - $Tür_{offen}$: ist dieses Signal auf 1, so soll der Lift stehen bleiben,
 - $Anruf$: ist dieses Signal auf 1, so soll der Lift zum Stockwerk,
 - $tiefer$: im Falle eines Anrufs soll der Lift aufsteigen,
 - $höher$: im Falle eines Anrufs soll der Lift hinabsteigen.
- Sind $tiefer$ und $höher$ beide auf 0, heisst das, dass der Lift am Stockwerk ist
- Das System erstellt folgende Ausgänge:
 - $Motor_{ein}$: ist dieses Signal auf 1, so ist läuft der Motor,
 - auf : ist dieses Signal auf 1 und läuft der Motor, so steigt der Lift auf; ist dieses Signal auf 0 und läuft der Motor, so steigt der Lift hinab.

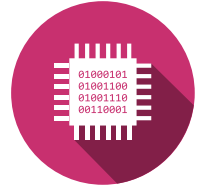
Das hier analysierte System nimmt den Fall nicht im Kauf, wo der Lift schon läuft.

Inhalt

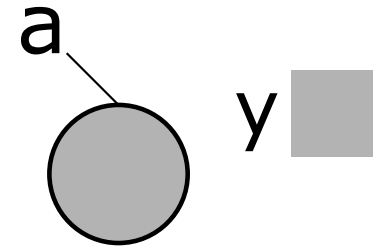
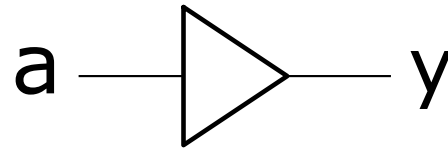


- Darstellung von kombinatorischen Funktionen
- **Elementare Logikfunktionen**
 - Puffer (Buffer)
 - Inverter (Inverter)
 - UND (AND)
 - ODER (OR)
 - Exklusiv-ODER (XOR)
- Boolesche Algebra
- Vollständige Operatoren

Puffer

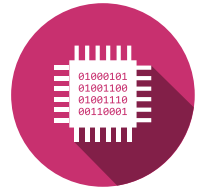


a	y
0	0
1	1

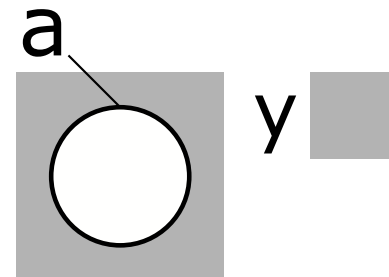


$$y = a$$

Inverter

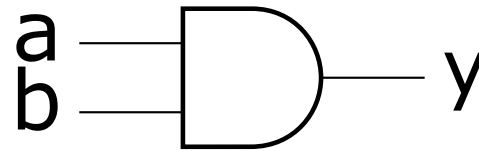
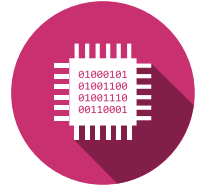


a	y
0	1
1	0

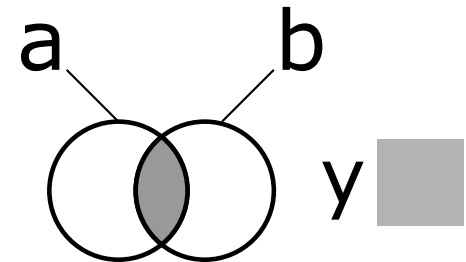


$$y = \bar{a}$$

Und (AND)

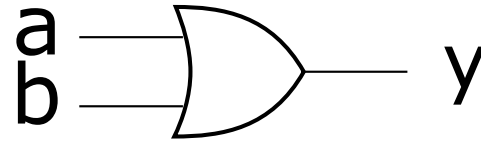
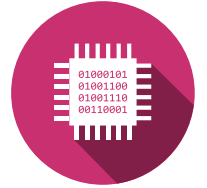


a	b	y
0	0	0
0	1	0
1	0	0
1	1	1

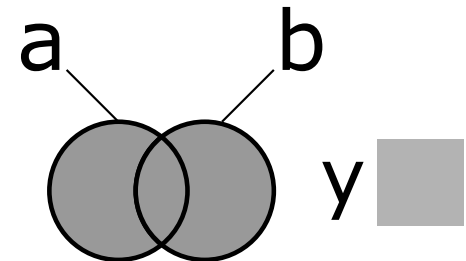


$$y = a * b$$

Oder (OR)

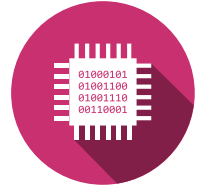


a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

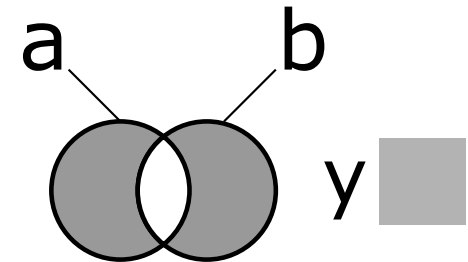
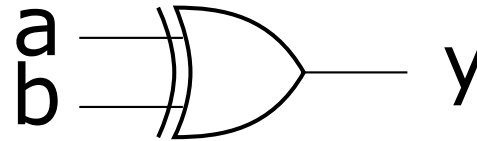


$$y = a + b$$

Exklusiv-Oder (XOR)

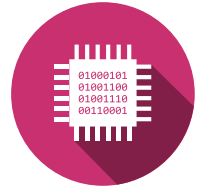


a	b	y
0	0	0
0	1	1
1	0	1
1	1	0



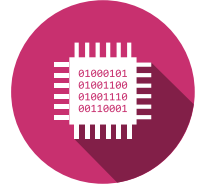
$$y = a \oplus b$$

Aufgabe 3.2



C	B	A	CA	CB	BA
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

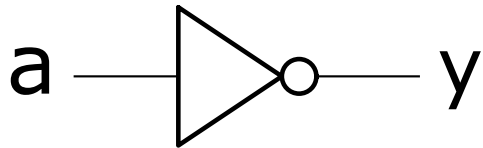
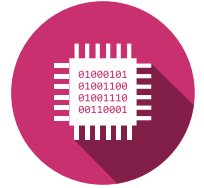
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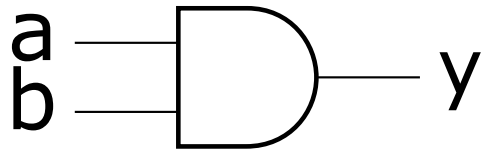
- Darstellung von kombinatorischen Funktionen
- Elementare Logikfunktionen
- **Boolsche Algebra**
 - Algebraische Operationen
 - Gesetz von De Morgan
 - XOR und inverter
 - Polynomische Form
- Vollständige Operatoren

Boolsche Algebra

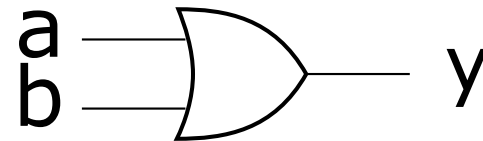
Operatoren



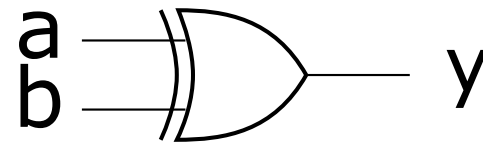
$$y = \bar{a}$$



$$y = a * b$$



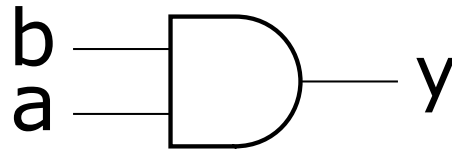
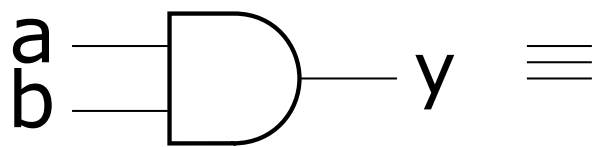
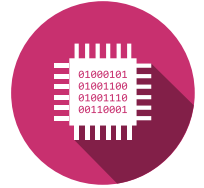
$$y = a + b$$



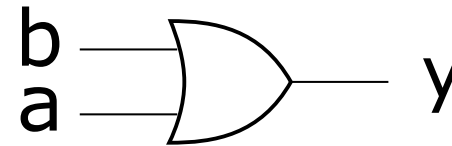
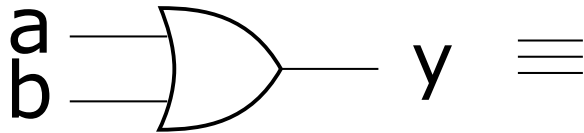
$$y = a \oplus b$$

Boolsche Algebra

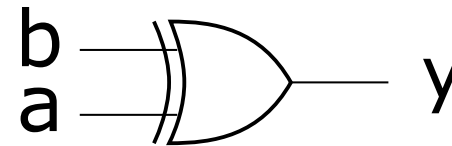
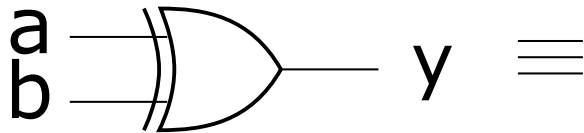
Kommutativität



$$a * b = b * a$$



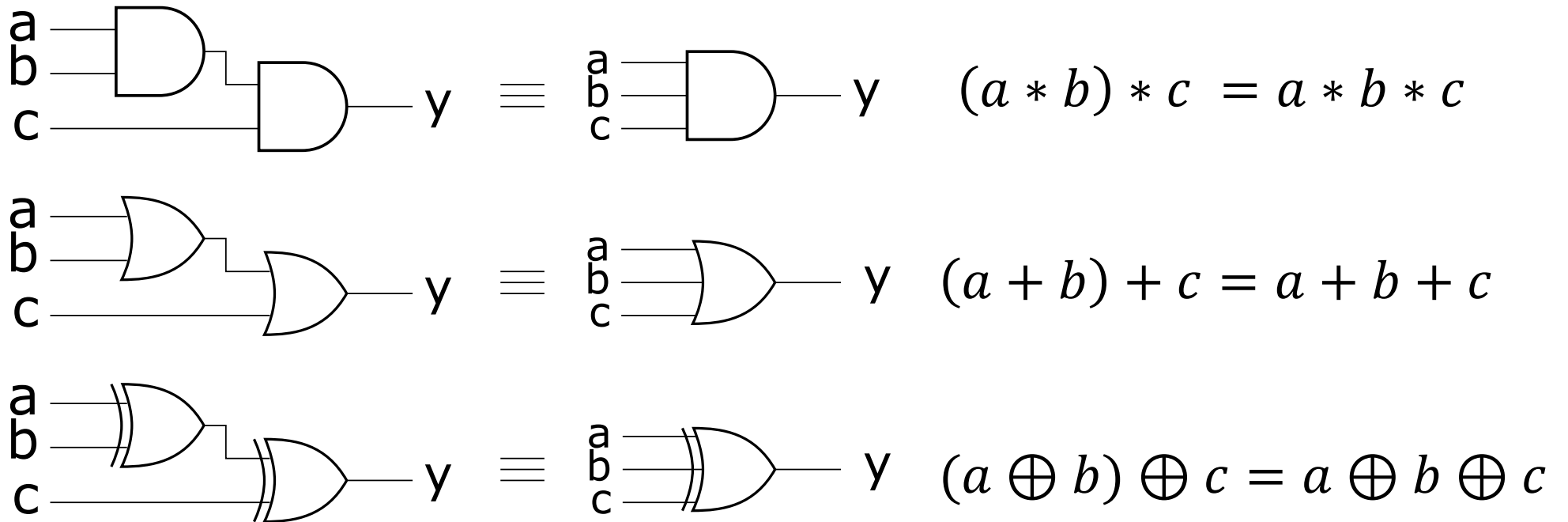
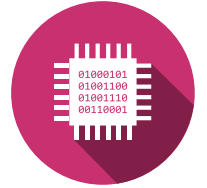
$$a + b = b + a$$



$$a \oplus b = b \oplus a$$

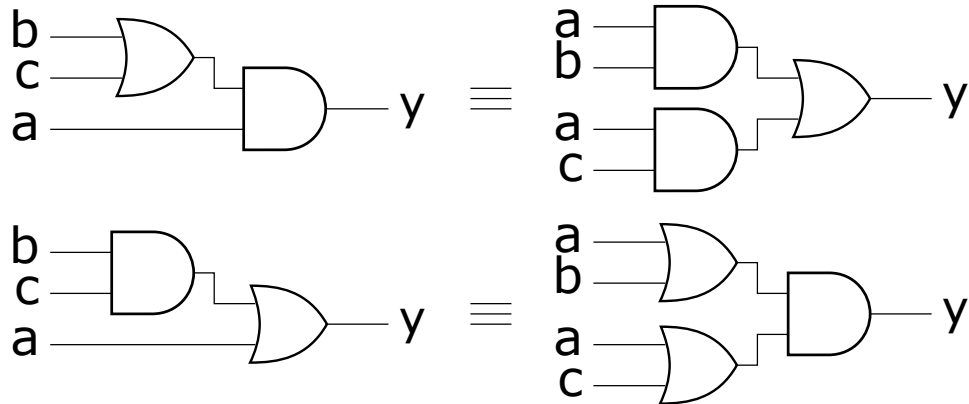
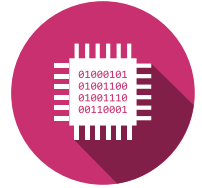
Boolsche Algebra

Assoziativität



Boolsche Algebra

Distributivität

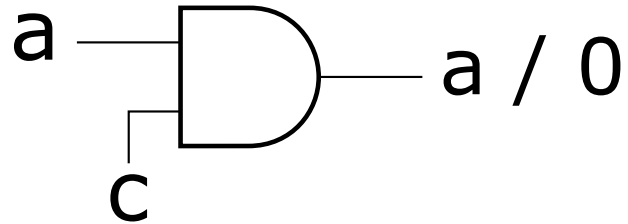
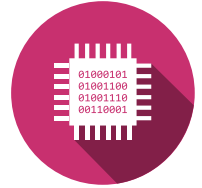


$$a * (b + c) = a * b + a * c$$

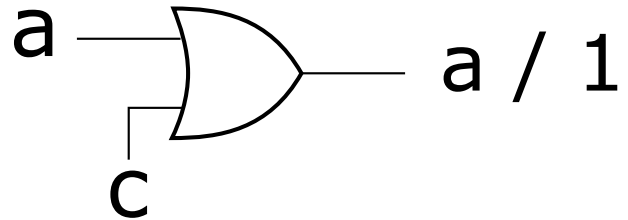
$$a + (b * c) = (a + b) * (a + c)$$

Boolsche Algebra

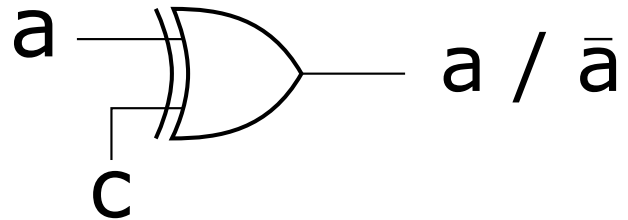
Neutrale und absorbierende Elemente



$$a \cdot 1 = a$$
$$a \cdot 0 = 0$$



$$a + 0 = a$$
$$a + 1 = 1$$

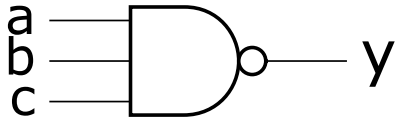
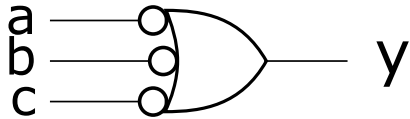


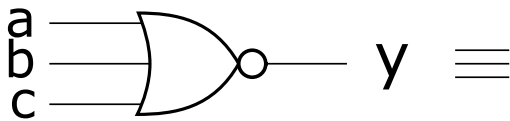
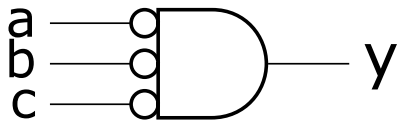
$$a \oplus 0 = a$$
$$a \oplus 1 = \bar{a}$$

Boolsche Algebra

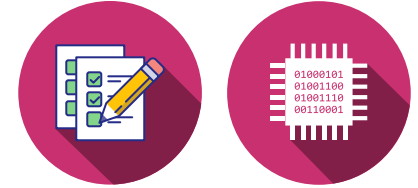
Theoreme von De Morgan



 \equiv  $\overline{a * b * c} = \bar{a} + \bar{b} + \bar{c}$

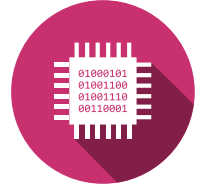
 \equiv  $\overline{a + b + c} = \bar{a} * \bar{b} * \bar{c}$

Aufgabe 4.3



- Ein System soll 2 Bits a und b übermitteln. Aus Sicherheitsgründen übermittelt er noch ein zusätzliches Bit, gegeben durch die Gleichung $y = a \oplus b$.
- Zeigen Sie, dass es möglich ist, wenn das Bit a während der Übermittlung verloren gegangen ist, es mit Hilfe von y und b wiederherzustellen.

Polynomische Formen



- Monome
- Polynom
- Summen Produkte
Product of OR
- Exklusiv-ODER Produkte
XOR of Products

$$\bar{a}c$$

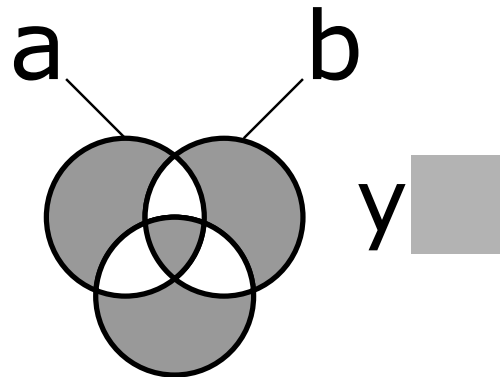
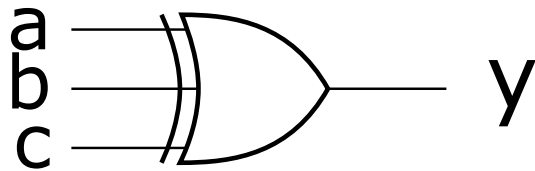
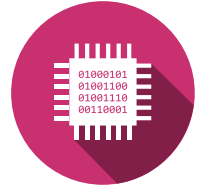
$$ab + \bar{a}c + b\bar{c}$$

$$(a + b) * (\bar{a} + c) * (b + \bar{c})$$

$$a\bar{b} \oplus ac \oplus \bar{b}c$$

Polynomische Formen

Exklusiv-Oder (XOR)



$$001 \Rightarrow \bar{a}\bar{b}c$$

$$010 \Rightarrow \bar{a}b\bar{c}$$

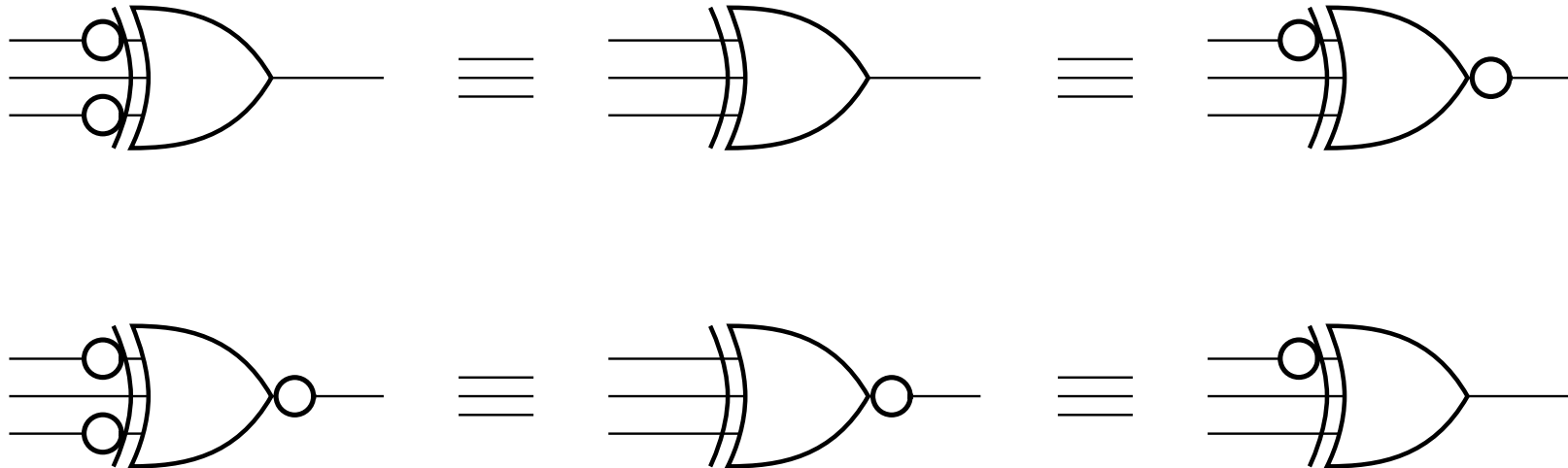
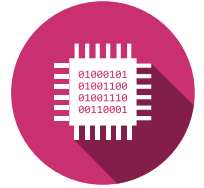
$$100 \Rightarrow a\bar{b}\bar{c}$$

$$111 \Rightarrow abc$$

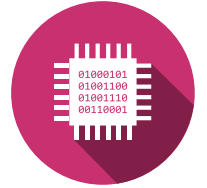
$$a \oplus b \oplus c$$

Polynomische Formen

Exklusiv-Oder (XOR) und Umkehrung



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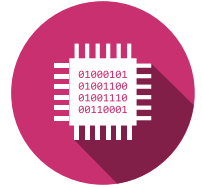
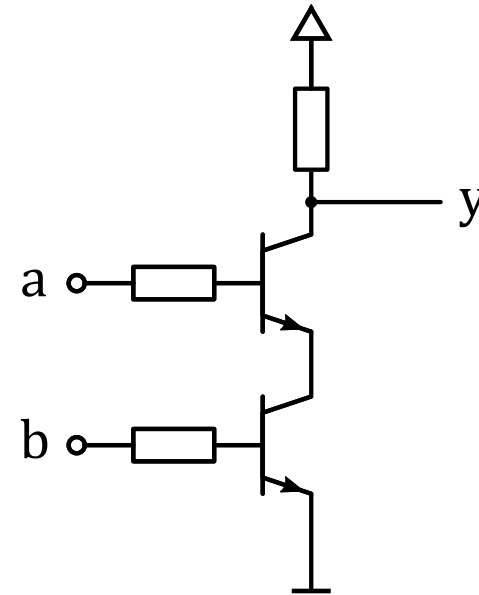
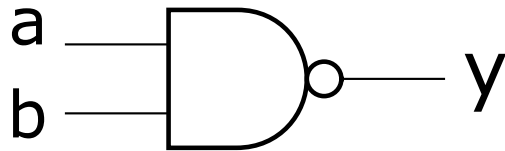


- Darstellung von kombinatorischen Funktionen
- Elementare Logikfunktionen
- Boolesche Algebra
- **Vollständige Operatoren**
 - NAND Operator
 - Transformation NAND
 - NOR Operator

Vollständige Operatoren

NAND Operator

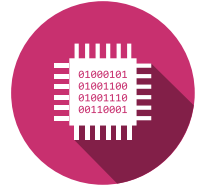
- Transistor-Schaltung



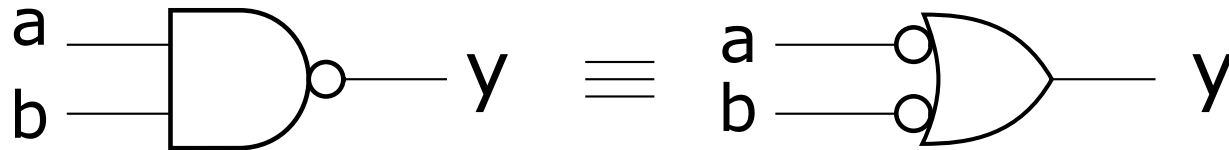
Vollständige Operatoren

NAND Operator

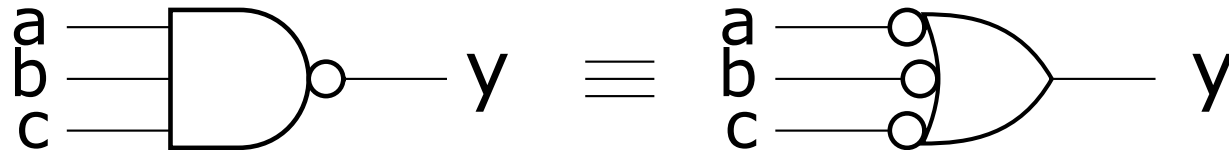
- Zwei Vertretungen nach De Morgans Recht



$$\overline{(a * b)} = \bar{a} + \bar{b}$$
$$\overline{(a + b)} = \bar{a} * \bar{b}$$

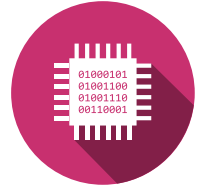
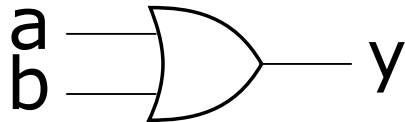
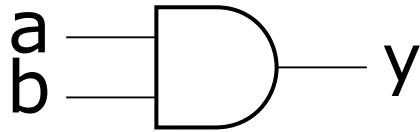
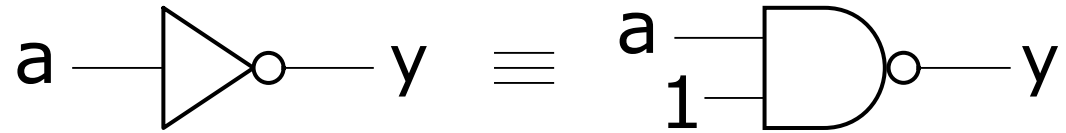


$$\overline{(a * b * c)} = \bar{a} + \bar{b} + \bar{c}$$
$$\overline{(a + b + c)} = \bar{a} * \bar{b} * \bar{c}$$



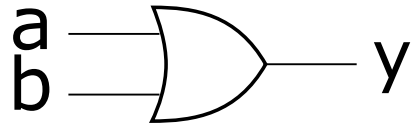
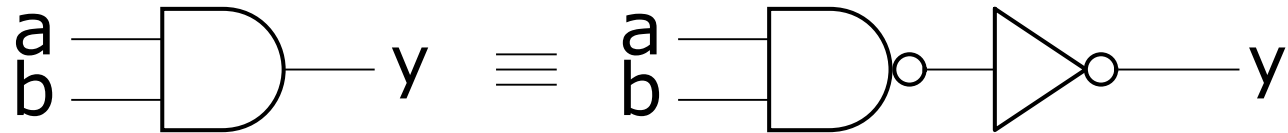
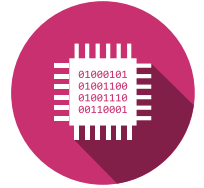
Vollständige Operatoren

NAND Operator



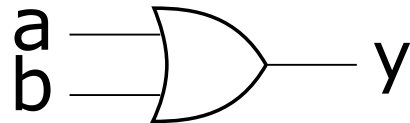
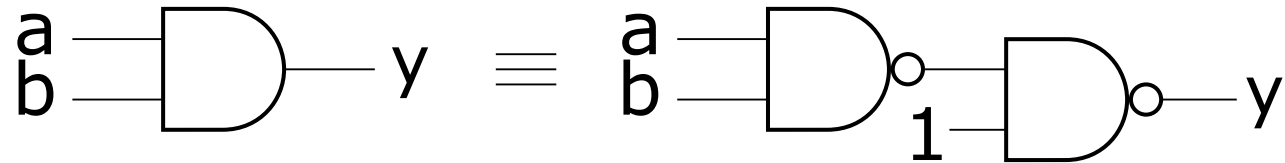
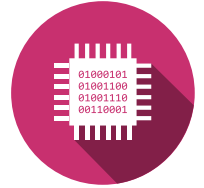
Vollständige Operatoren

NAND Operator



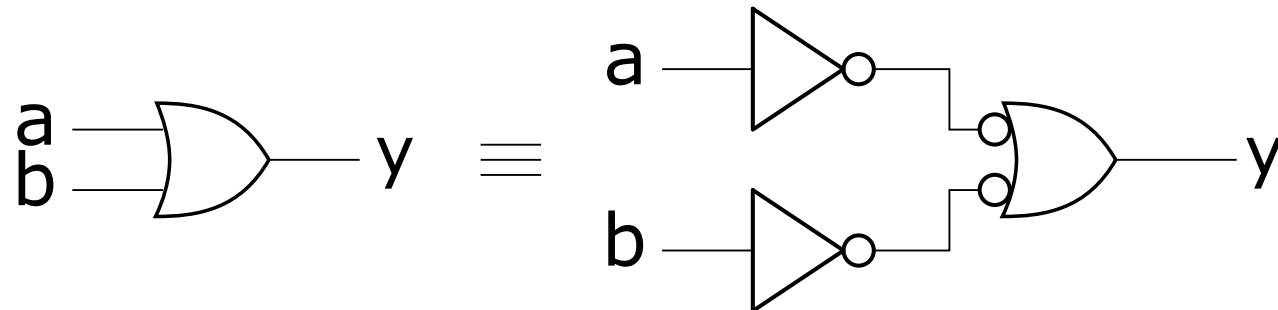
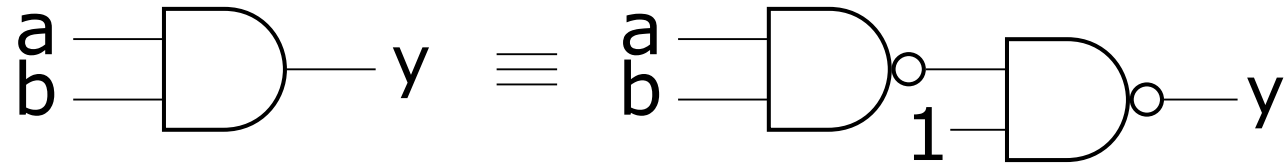
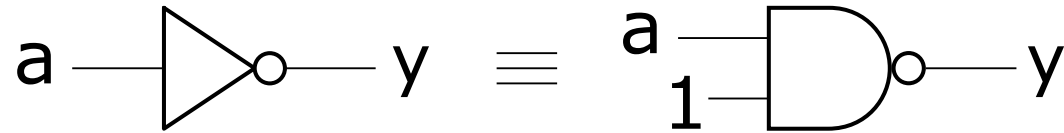
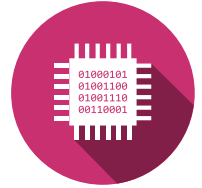
Vollständige Operatoren

NAND Operator



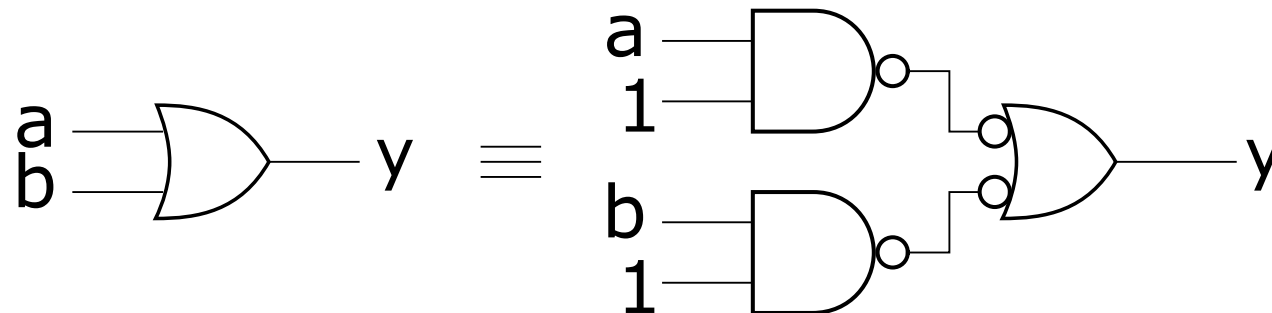
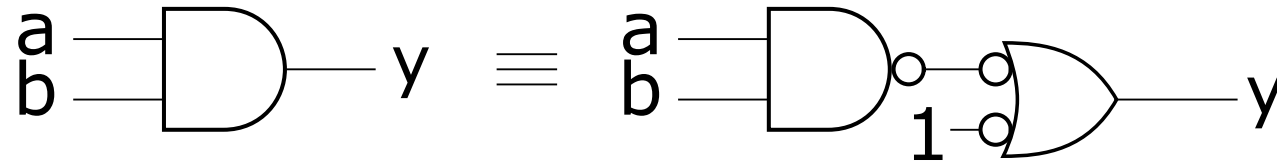
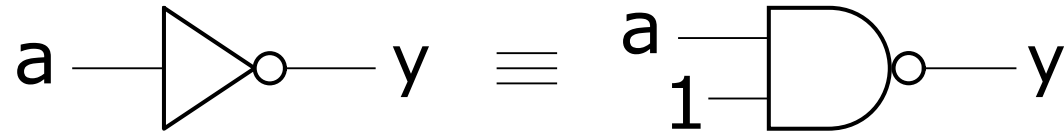
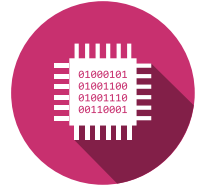
Vollständige Operatoren

NAND Operator



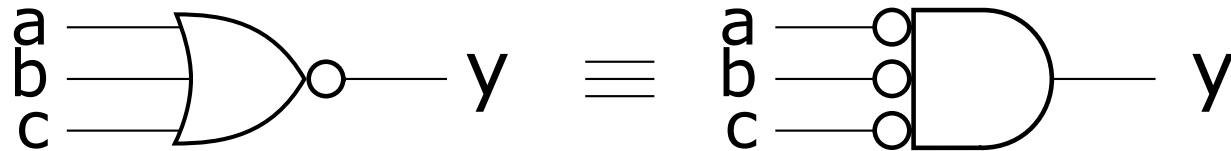
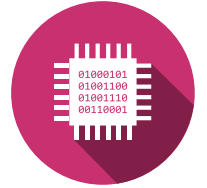
Vollständige Operatoren

NAND Operator

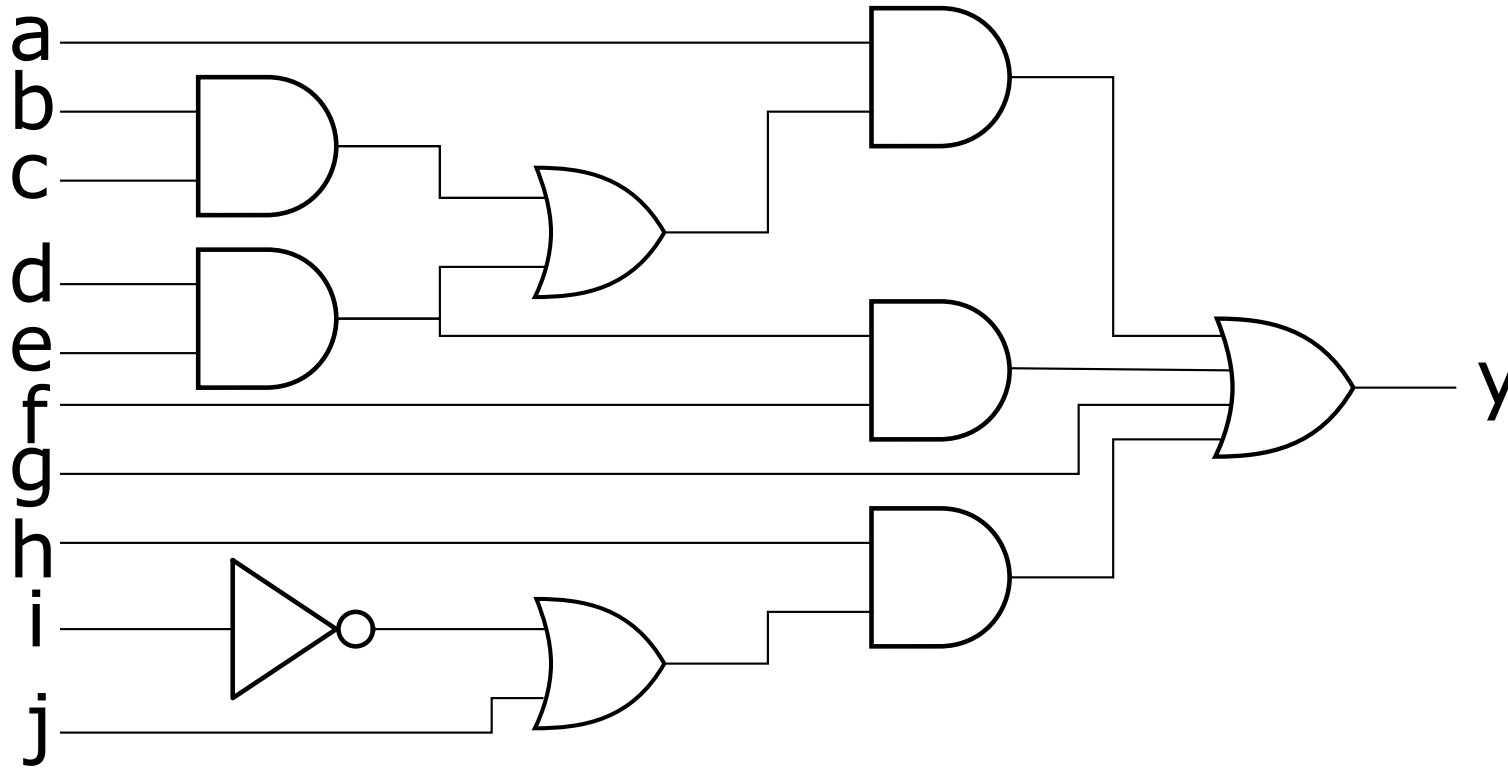
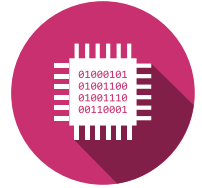


Vollständige Operatoren

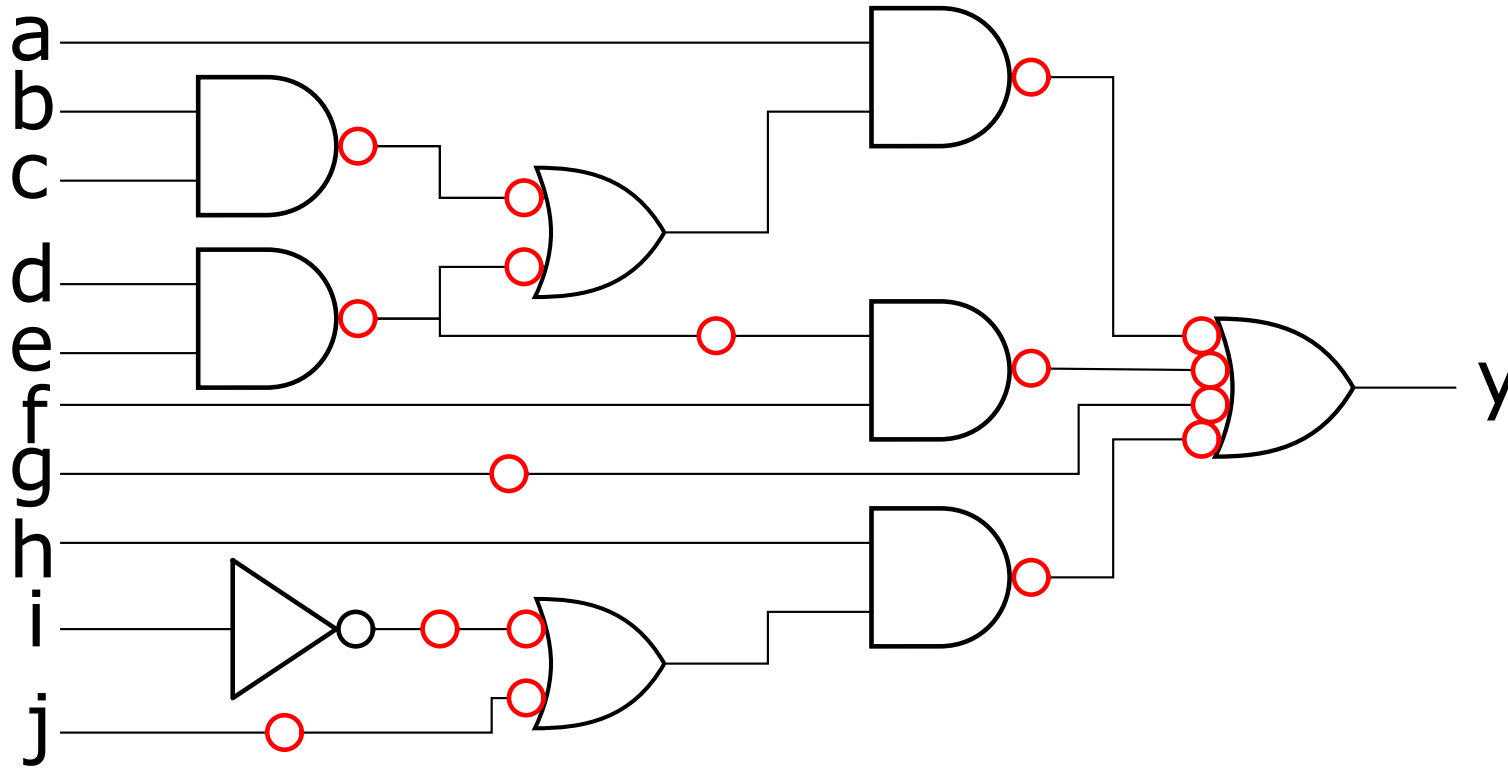
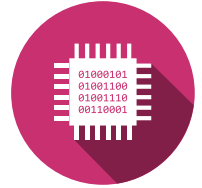
NOR Operator



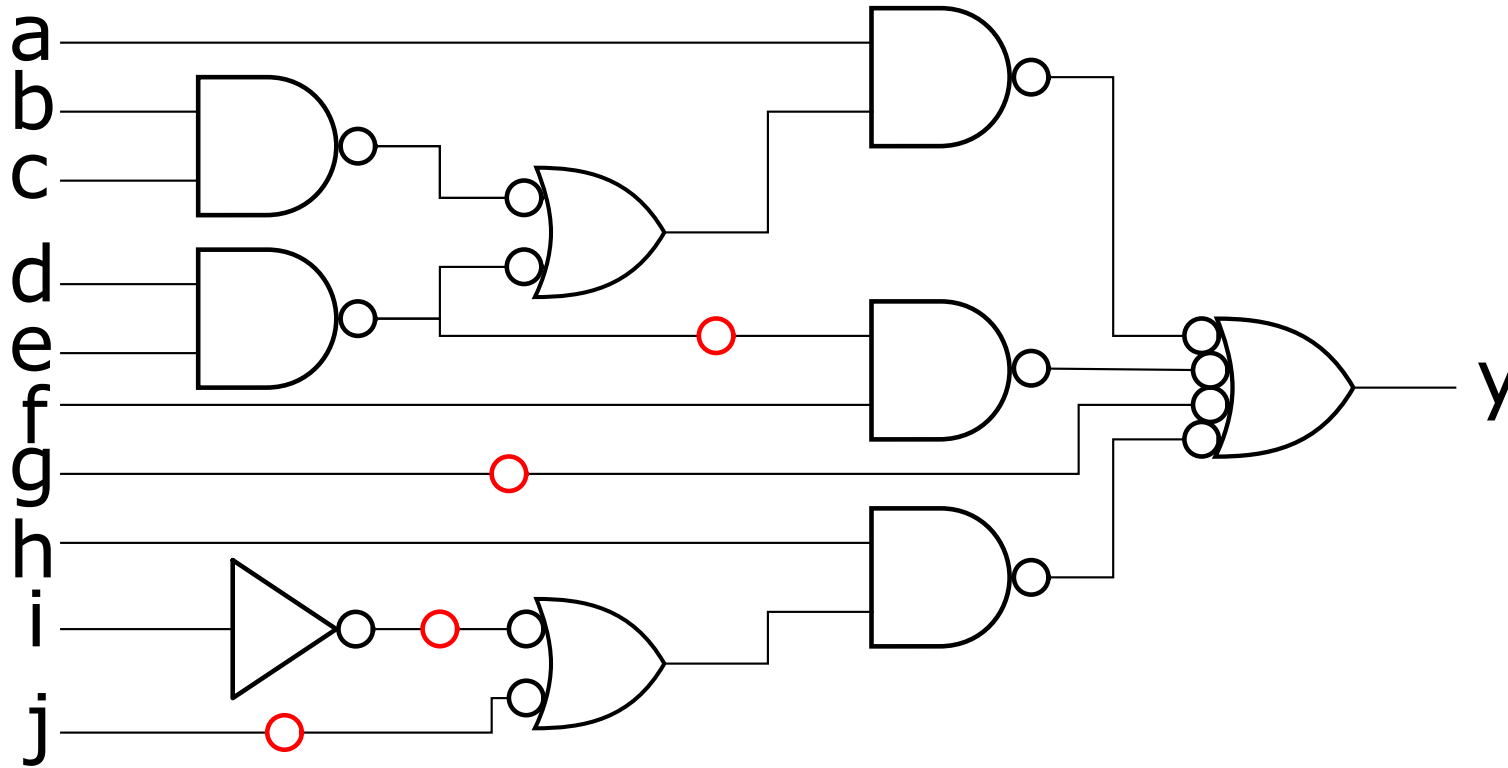
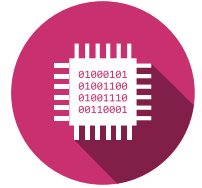
Vollständige Operatoren Transformation



Vollständige Operatoren Transformation

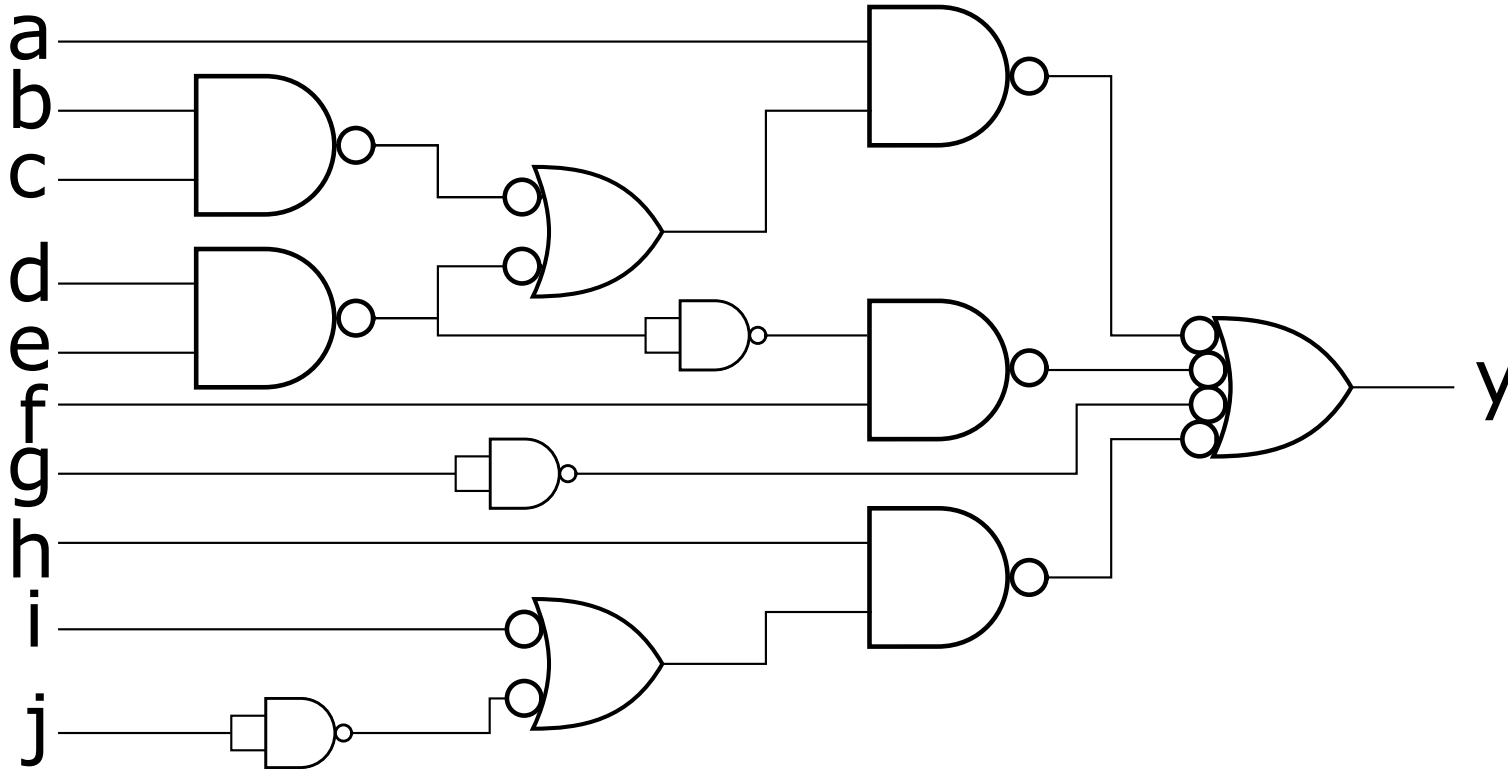
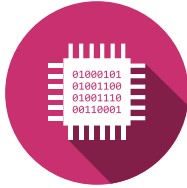


Vollständige Operatoren Transformation

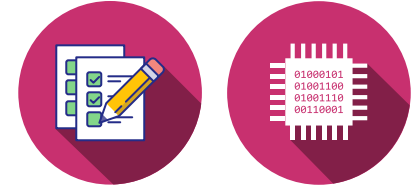


Vollständige Operatoren

Transformation



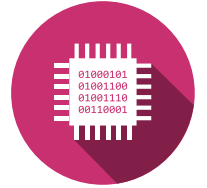
Aufgabe 5.1



Mit Hilfe nur von NAND-Gattern zeichnen Sie das vollständige Schema einer Schaltung, welche die folgende Funktion erstellt.

$$y = (ab + cd + e + f * (\bar{g} + h)) * \bar{i}$$

Referenzen



- [Max95] (Englisch) Einfach zum lesen
- [Man91] (Deutsch) Einfach zum lesen
- [Toc92] (Französisch) Einfach zum lesen
- [Kün97] (Deutsch) Benutzung von ISO Symbolen

WHY ARE THERE MIRRORS ABOVE BEDS

WHY DO I SAY UH

WHY IS SEA SALT BETTER

WHY ARE THERE TREES IN THE MIDDLE OF FIELDS

WHY IS THERE NOT A POKEMON MMO

WHY IS THERE LAUGHING IN TV SHOWS

WHY ARE THERE DOORS ON THE FREEWAY

WHY ARE THERE SO MANY SUCHOST-EXE RUNNING

WHY AREN'T ANY COUNTRIES IN ANTARCTICA

WHY ARE THERE SCARY SOUNDS IN MINECRAFT

WHY IS THERE KICKING IN MY STOMACH

WHY ARE THERE TWO SLASHES AFTER HTTP

WHY ARE THERE CELEBRITIES

WHY DO SNAKES EXIST

WHY DO OYSTERS HAVE PEARLS

WHY ARE DUCKS CALLED DUCKS

WHY DO THEY CALL IT THE CLAP

WHY ARE KYLE AND CARTMAN FRIENDS

WHY IS THERE AN ARROW ON AANG'S HEAD

WHY ARE TEXT MESSAGES BLUE

WHY ARE THERE MUSTACHES ON CLOTHES

WHY WUBA LUBBA DUB DUB MEANING

WHY IS THERE A WHALE AND A POT FALLING

WHY ARE THERE SO MANY BIRDS IN SWISS

WHY IS THERE SO LITTLE RAIN IN WALLIS

WHY IS WALLIS WEATHER FORECAST ALWAYS WRONG

WHY ARE THERE MALE AND FEMALE BIKES

WHY ARE THERE BRIDESMAIDS

WHY DO DYING PEOPLE REACH UP

HOW FAST IS LIGHTSPEED

WHY ARE OLD KLINGONS DIFFERENT

WHY ARE THERE SQUIRRELS

WHY ARE THERE TINY SPIDERS IN MY HOUSE

WHY DO SPIDERS COME INSIDE

WHY ARE THERE HUGE SPIDERS IN MY HOUSE

WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE

WHY ARE THERE SPIDERS IN MY ROOM

WHY ARE THERE SO MANY SPIDERS IN MY ROOM

WHY DO SPYDER BITES ITCH

WHY IS DYING SO SCARY

WHY IS THERE NO GPS IN LAPTOPS

WHY DO KNEES CLICK

WHY IS THERE CAFFEINE IN MY SHAMPOO

WHY HAVE DINOSAURS NO FUR

WHY DO IGUANAS DIE

WHY AREN'T ECONOMISTS RICH

WHY DO AMERICANS CALL IT SOCCER

WHY ARE MY EARS RINGING

WHY IS 42 THE ANSWER TO EVERYTHING

WHY CAN'T NOBODY ELSE LIFT THORS HAMMER

WHY IS MARVIN ALWAYS SO SAD

WHY ARE THERE ANTS IN MY LAPTOP

WHY IS EARTH TILTED

WHY IS SPACE BLACK

WHY IS OUTER SPACE SO COLD

WHY ARE THERE PYRAMIDS ON THE MOON

WHY IS NASA SHUTTING DOWN

WHY ARE THERE GHOSTS

WHY IS THERE AN OWL IN MY BACKYARD

WHY IS THERE AN OWL OUTSIDE MY WINDOW

WHY IS THERE AN OWL ON THE DOLLAR BILL

WHY DO OWLS ATTACK PEOPLE

WHY ARE FPGA's EVERYWHERE

WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE

WHY ARE THERE GODS

WHY ARE THERE TWO SPOCKS

WHY ARE MY BOOBS ITCHY

WHY ARE CIGARETTES LEGAL

WHY ARE THERE DUCKS IN MY POOL

WHY IS JESUS WHITE

WHY IS THERE LIQUID IN MY EAR

WHY DO Q TIPS FEEL GOOD

WHY DO PEOPLE DIE

WHY AREN'T THERE GUNS IN

WHY ARE THERE DOGS AFRAID OF FIRE

WHY IS THERE NO KING IN E

WHY ARE THERE NO GUNS IN

WHY ARE THERE NO GUNS IN

WHY ARE THERE NO GUNS IN

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WHY ARE THERE NO GUNS IN

WHY ARE THERE NO GUNS IN

WHY ARE THERE NO GUNS IN

WHY ARE THERE NO GUNS IN

WHY ARE THERE NO GUNS IN

QUESTIONS

CAN BE ASKED BY ANYONE ANYTIME

WHY AREN'T MY
ARMS GROWING



WHY ARE THERE
GHOSTS



WHY IS THERE AN OWL IN MY BACKYARD

WHY IS THERE AN OWL OUTSIDE MY WINDOW

WHY IS THERE AN OWL ON THE DOLLAR BILL

WHY DO OWLS ATTACK PEOPLE

WHY ARE FPGA's EVERYWHERE

WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE

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WHY AREN'T
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WHY ARE THERE
SQUIRRELS



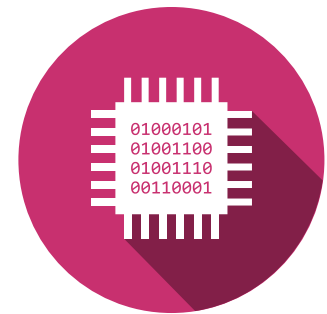
B



Hes·so  **VALAIS
WALLIS**



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