



Simplification with Karnaugh tables

Exercises Digital Design



Solution vs. Hints:

While not every response provided herein constitutes a comprehensive solution, some serve as helpful hints intended to guide you toward discovering the solution independently. In certain instances, only a portion of the solution is presented.

1 | KAR - Karnaugh tables

1.1 Representation of Monoms

$$y_1 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & \text{C} & \text{D} \\ \hline 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_3 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & \text{C} & \text{D} \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_5 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & \text{C} & \text{D} \\ \hline 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_2 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & \text{C} & \text{D} \\ \hline 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_4 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & \text{C} & \text{D} \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_6 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & \text{C} & \text{D} \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

kar/karnaugh-01

1.2 Monoms

$$\begin{aligned} y_1 &= \overline{B} \overline{D} \\ y_2 &= \overline{A} \overline{C} \end{aligned} \quad (1)$$

$$\begin{aligned} y_3 &= \overline{B} C \overline{D} \\ y_4 &= \overline{A} C \overline{D} \end{aligned} \quad (2)$$



kar/karnaugh-02



1.3 Representation of Polynomials

$$y_1 \quad \begin{array}{c|c|c|c|} \hline & \overline{C} & \overline{D} & \\ \hline 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 \\ \hline 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_3 \quad \begin{array}{c|c|c|c|} \hline & \overline{C} & \overline{D} & \\ \hline 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ \hline \end{array} \begin{array}{l} A \\ B \end{array}$$

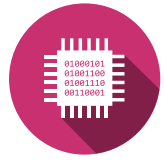
$$y_5 \quad \begin{array}{c|c|c|c|} \hline & \overline{C} & \overline{D} & \\ \hline 1 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_2 \quad \begin{array}{c|c|c|c|} \hline & \overline{C} & \overline{D} & \\ \hline 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 0 & 0 \\ \hline 1 & 1 & 1 & 0 \\ \hline 1 & 1 & 1 & 1 \\ \hline \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_4 \quad \begin{array}{c|c|c|c|} \hline & \overline{C} & \overline{D} & \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 1 & 1 \\ \hline \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_6 \quad \begin{array}{c|c|c|c|} \hline & \overline{C} & \overline{D} & \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 1 & 0 \\ \hline \end{array} \begin{array}{l} A \\ B \end{array}$$

kar/karnaugh-03



2 | KAR - Sum-of-products simplification

2.1 Karnaugh table with 4 variables

$$D\bar{B}A^* + \bar{D}\bar{C}^* + B\bar{A}^* + \left\{ \frac{\bar{C}\bar{B}}{\bar{C}A} \right\} \quad (3)$$

kar/productsum-01

2.2 Karnaugh table with 5 variables

$$\bar{E}\bar{D}\bar{B}^* + C\bar{B}A^* + D\bar{C}BA^* + \bar{D}\bar{B}A^* + E\bar{D}CA^* + EDC\bar{A} \quad (4)$$

kar/productsum-02

2.3 Karnaugh table with 5 variables

$$\bar{E}\bar{D}\bar{C}^* + \bar{E}\bar{C}\bar{A}^* + \bar{E}\bar{D}\bar{B}\bar{A}^* + DCBA^* + ECB^* + \left\{ \frac{E\bar{B}\bar{A}}{\bar{C}B\bar{A}} \right\} \quad (5)$$

kar/productsum-03

2.4 Karnaugh table with 5 variables

$$\bar{E}DB^* + \bar{C}\bar{B}\bar{A}^* + DBA^* + E\bar{D}CA + \bar{D}\bar{C}\bar{B} + \bar{E}\bar{D}\bar{C} \quad (6)$$

kar/productsum-04

2.5 Karnaugh table with 5 variables

$$\bar{E}C\bar{A}^* + \bar{E}B\bar{A}^* + E\bar{C}\bar{B}\bar{A}^* + DA^* + ECB^* + D\bar{B} \quad (7)$$

kar/productsum-05

2.6 Karnaugh table with 5 variables

$$\bar{C}\bar{B}^* + \bar{D}\bar{C}A^* + DCBA^* + \bar{E}CB + \bar{E}B\bar{A} \quad (8)$$

or

$$\bar{C}\bar{B}^* + \bar{D}\bar{C}A^* + DCBA^* + \bar{E}CB + \bar{E}\bar{C}\bar{A} \quad (9)$$

or

$$\bar{C}\bar{B}^* + \bar{D}\bar{C}A^* + DCBA^* + \bar{E}\bar{D}B + \bar{E}B\bar{A} \quad (10)$$

kar/productsum-06

2.7 Minimal Polynomialform

$$\bar{x}_3 x_2 \bar{x}_0^* + \bar{x}_2 x_0^* + x_1 x_0^* + \bar{x}_2 x_1^* \quad (11)$$

kar/productsum-07



2.8 Inverse Function

$$\overline{E} C \overline{A}^* + CB^* + DBA^* + E \overline{B} A^* \quad (12)$$

kar/productsun-08

2.9 Minimum Polynomial Form

y : 5 termes; \overline{y} : 4 termes

kar/productsun-09

2.10 Function of 5 Variables

$$DCA^* + DCB^* + CBA^* + DBA^* + EDC^* + EBA^* + ECA^* + EDA^* + ECB^* + EDB^*$$

kar/productsun-10

2.11 Incompletely defined Function

$$x_4 x_3 + x_2 x_1 \quad (13)$$

or

$$x_3 x_1 + x_4 x_2 \quad (14)$$

or

$$x_4 x_1 + x_3 x_2 \quad (15)$$

kar/productsun-11

2.12 Incompletely defined function

$$\begin{aligned} A &= \overline{B_2} \overline{B_0}^* + B_2 B_0^* + B_1 + B_3^* \\ B &= \overline{B_2}^* + \overline{B_1} \overline{B_0}^* + B_1 B_0^* \\ C &= B_2^* + \overline{B_1}^* + B_0 \\ D &= \overline{B_2} \overline{B_0}^* + B_3^* + B_2 \overline{B_1} B_0^* + \overline{B_2} B_1^* + B_1 \overline{B_0} \\ E &= \overline{B_2} \overline{B_0}^* + B_1 \overline{B_0}^* \\ F &= B_3^* + B_2 \overline{B_1}^* + B_2 \overline{B_0}^* + \overline{B_1} \overline{B_0}^* \\ G &= B_3^* + B_2 \overline{B_1} + \overline{B_0} B_1^* + B_2 \overline{B_1} \end{aligned} \quad (16)$$

kar/productsun-12



3 | KAR - XOR function simplification

3.1 Representation of XOR Functions

$$y_1 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline 0 & 0 & 0 & 0 & \\ \hline 1 & 1 & 1 & 1 & \\ \hline 0 & 0 & 0 & 0 & \\ \hline 1 & 1 & 1 & 1 & \end{array} \left| \begin{array}{l} A \\ B \end{array} \right.$$

$$y_3 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline 0 & 1 & 0 & 1 & \\ \hline 1 & 0 & 1 & 0 & \\ \hline 0 & 1 & 0 & 1 & \\ \hline 1 & 0 & 1 & 0 & \end{array} \left| \begin{array}{l} A \\ B \end{array} \right.$$

$$y_{7\&8\&9} \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline 0 & 0 & 1 & 1 & \\ \hline 0 & 0 & 1 & 1 & \\ \hline 1 & 1 & 0 & 0 & \\ \hline 1 & 1 & 0 & 0 & \end{array} \left| \begin{array}{l} A \\ B \end{array} \right.$$

$$y_2 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline 0 & 1 & 1 & 0 & \\ \hline 1 & 0 & 0 & 1 & \\ \hline 0 & 1 & 1 & 0 & \\ \hline 1 & 0 & 0 & 1 & \end{array} \left| \begin{array}{l} A \\ B \end{array} \right.$$

$$y_{4\&5\&6} \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline 1 & 1 & 1 & 1 & \\ \hline 0 & 0 & 0 & 0 & \\ \hline 1 & 1 & 1 & 1 & \\ \hline 0 & 0 & 0 & 0 & \end{array} \left| \begin{array}{l} A \\ B \end{array} \right.$$

kar/xor-01

3.2 Minimal Polynomial Form

$$y = x_1 x_0^* + \overline{x_2} x_0^* + \overline{x_2} x_1^* + \overline{x_3} x_2 x_0^*$$

kar/xor-02

3.3 Minimal Polynomial Form

$$\overline{E} \overline{D} C^* + \overline{E} \overline{C} \overline{B} \overline{A}^* + \overline{E} D B A^* + \overline{E} \overline{D} \overline{A}^* + \overline{E} C B^* + E \overline{D} \overline{C} A^* + E D C \overline{B}^* + E D \overline{C} B \overline{A}^* + \begin{cases} E D \overline{B} A \\ E \overline{C} \overline{B} A \end{cases}$$

kar/xor-03

3.4 Representation in the form of XOR of Products

$$\text{Many possible solutions e.g.: } Y = \overline{D} \overline{B} \oplus \overline{D} A \oplus C A \oplus C B$$

kar/xor-04



3.5 Representation in the form of XOR of Products

Many possible solutions e.g.: $y = x_0 \oplus x_1 \overline{x_0} \oplus x_2 \overline{x_0} \oplus x_3 x_2 \overline{x_1}$

kar/xor-05

3.6 Adder

$$s_0 = a_0 \oplus b_0$$

$$s_1 = a_1 \oplus b_1 \oplus a_0 b_0$$

$$s_1 = \overline{a_1} b_1 \oplus a_1 \overline{b_1} \oplus a_0 b_0 \quad (17)$$

$$s_2 = a_1 b_1 \oplus \begin{cases} a_0 \overline{b_1} b_0 \oplus \overline{a_1} a_0 b_0 \\ a_0 b_1 \overline{b_0} \oplus a_1 a_0 \overline{b_0} \end{cases}$$

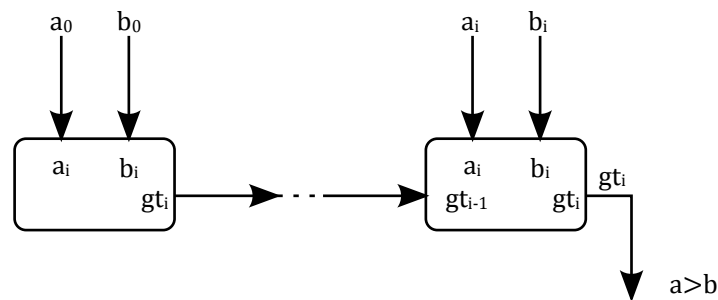
kar/xor-06



4 | KAR - Functions with a large number of inputs

4.1 Number Comparison

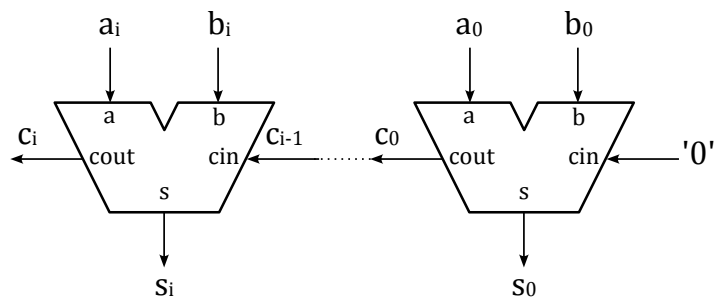
Possible with an iterative blocschema.



kar/manyinputs-01

4.2 Binary Adder

Possible with an iterative blocschema.



kar/manyinputs-02

4.3 Thermometer Code to Binary Code Conversion

One possible solution is.

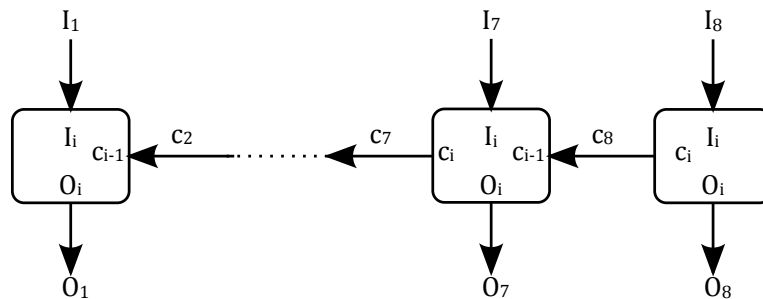
$$\begin{aligned} B_2 &= T_4 \\ B_1 &= T_2 + T_6 \overline{T_4} \\ B_0 &= ??? \end{aligned} \quad (18)$$

kar/manyinputs-03



4.4 Transmission based on Priority

Possible with an iterative blocschema.



kar/manyinputs-04

4.5 Logic for Counter without decreasing to Zero

The solutions involves an Adder $x+1$ and a Comparator to $0x\text{FFFF}$

kar/manyinputs-05

4.6 Adder with Saturation

The output c_{out} of an iterative adder indicates an overflow.

kar/manyinputs-06

4.7 BCD coded numbers

The BCD-Adder is a special case of a normal adder. Only the values between 0...9 exist. Therefore $c_{\text{out}} = 1$ if the sum is > 10 . In this case -10 has to be subtracted from the output.

kar/manyinputs-07

4.8 Majority function with 7 inputs

A concatenation of adders with a comparison

kar/manyinputs-08

4.9 Arithmetic and logical unit

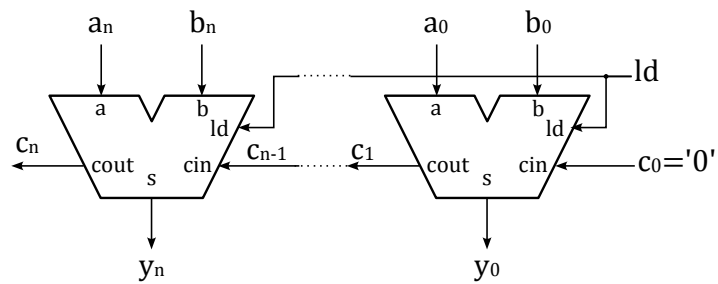
First determine the schema of an adder and that of a subtractor, tie them together and create the logical functions.

kar/manyinputs-09



4.10 Logic for Program Counter

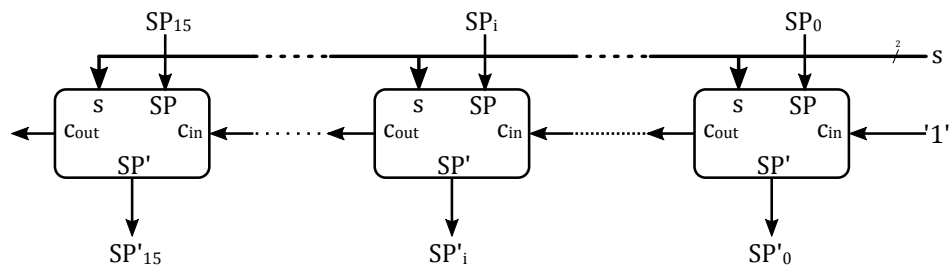
An iterative circuits with a load input.



kar/manyinputs-10

4.11 Logic for Stack Pointer

An iterative circuits with a 2bit selection input.



kar/manyinputs-11