Simplification par tables de Karnaugh

Exercices Conception numérique

Solution vs. Hints:
Toutes les réponses fournies ici ne sont pas des solutions complètes. Certaines ne sont que des indices pour vous aider à trouver la solution vous-même. Dans d'autres cas, seule une partie de la solution est fournie.

1 KAR - Tables de Karnaugh

1.1 Représentation de monômes

y_1

	C	D	
0	0	0	0
1	1	1	1
0	0	0	0
0	0	0	0

A / B

y_3

	C	D	
0	0	0	0
0	0	0	0
0	1	0	0
0	1	0	0

A / B

y_5

	C	D	
1	0	0	1
0	0	0	0
0	0	0	0
0	0	0	0

A / B

y_2

	C	D	
1	1	0	0
0	0	0	0
0	0	0	0
1	1	0	0

A / B

y_4

	C	D	
0	0	0	0
0	0	0	0
0	0	1	1
0	0	0	0

A / B

y_6

	C	D	
0	0	0	0
0	0	0	0
0	0	0	0
0	0	1	0

A / B

kar/karnaugh-01

1.2 Monômes

$$y_1 = \overline{B}\ \overline{D} \qquad y_2 = \overline{A}\ \overline{C} \tag{1}$$

$$y_3 = \overline{B}C\overline{D} \qquad y_4 = \overline{A}C\overline{D} \tag{2}$$

1.3 Représentation de polynômes

y_1

	C		D
1	1	1	1
1	1	1	1
0	1	1	0
0	0	0	0

A
B

y_3

	C		D
0	0	0	1
0	0	0	1
0	1	0	0
0	1	0	0

A
B

y_5

	C		D
1	0	0	1
1	0	0	1
0	0	0	0
0	0	0	0

A
B

y_2

	C		D
1	1	1	1
1	1	0	0
1	1	1	0
1	1	1	1

A
B

y_4

	C		D
0	0	0	0
0	0	0	0
1	1	1	1
0	0	1	1

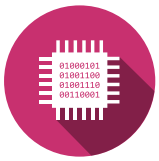
A
B

y_6

	C		D
0	0	0	0
0	0	0	0
0	0	0	0
0	1	1	0

A
B

kar/karnaugh-03



2 | KAR - Simplification sous forme de somme de produits

2.1 Table de Karnaugh à 4 variables

$$D\overline{B}A^* + \overline{D}\ \overline{C}^* + B\overline{A}^* + \begin{cases}\overline{C}\ \overline{B} \\ \overline{C}\ \overline{A}\end{cases} \tag{3}$$

kar/productsum-01

2.2 Table de Karnaugh à 5 variables

$$\overline{E}\ \overline{D}\ \overline{B}^* + C\overline{B}A^* + D\overline{C}BA^* + \overline{D}\ \overline{B}A^* + E\overline{D}CA^* + EDC\overline{A} \tag{4}$$

kar/productsum-02

2.3 Table de Karnaugh à 5 variables

$$\overline{E}\ \overline{D}\ \overline{C}^* + \overline{E}\ \overline{C}\ \overline{A}^* + \overline{E}\ \overline{D}\ \overline{B}\ \overline{A}^* + DCBA^* + ECB^* + \begin{cases}EB\overline{A} \\ \overline{C}B\overline{A}\end{cases} \tag{5}$$

kar/productsum-03

2.4 Table de Karnaugh à 5 variables

$$\overline{E}DB^* + \overline{C}\ \overline{B}\ \overline{A}^* + DBA^* + E\overline{D}CA + \overline{D}\ \overline{C}\ \overline{B} + \overline{E}\ \overline{D}\ \overline{C} \tag{6}$$

kar/productsum-04

2.5 Table de Karnaugh à 5 variables

$$\overline{E}C\overline{A}^* + \overline{E}B\overline{A}^* + E\ \overline{C}\ \overline{B}\ \overline{A}^* + DA^* + ECB^* + D\overline{B} \tag{7}$$

kar/productsum-05

2.6 Table de Karnaugh à 5 variables

$$\overline{C}\ \overline{B}^* + \overline{D}\ \overline{C}\ A^* + DCBA^* + \overline{E}CB + \overline{E}B\overline{A} \tag{8}$$

or

$$\overline{C}\ \overline{B}^* + \overline{D}\ \overline{C}\ A^* + DCBA^* + \overline{E}CB + \overline{E}\ \overline{C}\ \overline{A} \tag{9}$$

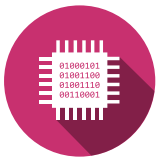
or

$$\overline{C}\ \overline{B}^* + \overline{D}\ \overline{C}\ A^* + DCBA^* + \overline{E}\ \overline{D}\ B + \overline{E}B\overline{A} \tag{10}$$

kar/productsum-06

2.7 Forme polynomiale minimale

$$\overline{x_3}\ x_2\ \overline{x_0}^* + \overline{x_2}\ x_0^* + x_1\ x_0^* + \overline{x_2}\ x_1^* \tag{11}$$



kar/productsum-07

2.8 Fonction inverse

$$\overline{E}\ C\ \overline{A}^* + CB^* + DBA^* + E\ \overline{B}\ A^* \tag{12}$$

kar/productsum-08

2.9 Forme polynomiale minimale

y: 5 termes; $\overline{y}$: 4 termes

kar/productsum-09

2.10 Fonction de 5 variables

$$DCA^* + DCB^* + CBA^* + DBA^* + EDC^* + EBA^* + ECA^* + EDA^* + ECB^* + EDB^*$$

kar/productsum-10

2.11 Fonction incomplètement définie

$$x_4\ x_3 + x_2\ x_1 \tag{13}$$

or

$$x_3\ x_1 + x_4\ x_2 \tag{14}$$

or

$$x_4\ x_1 + x_3\ x_2 \tag{15}$$

kar/productsum-11

2.12 Fonction incomplètement définie

$$\begin{aligned}
A &= \overline{B_2}\ \overline{B_0}^* + B_2\ B_0^* + B_1 + B_3^* \\
B &= \overline{B_2}^* + \overline{B_1}\ \overline{B_0}^* + B_1\ B_0 * \\
C &= B_2^* + \overline{B_1}^* + B_0 \\
D &= \overline{B_2}\ \overline{B_0}^* + B_3^* + B_2\ \overline{B_1}\ B_0^* + \overline{B_2}\ B_1^* + B_1\ \overline{B_0} \\
E &= \overline{B_2}\ \overline{B_0}^* + B_1\ \overline{B_0}^* \\
F &= B_3^* + B_2\ \overline{B_1}^* + B_2\ \overline{B_0}^* + \overline{B_1}\ \overline{B_0}^* \\
G &= B_3^* + B_2\ \overline{B_1} + \overline{B_0}\ B_1^* + B_2\ \overline{B_1}
\end{aligned} \tag{16}$$

kar/productsum-12

3 | KAR - Simplification de fonctions OU-Exclusif

3.1 Représentation de fonctions OU-exclusif

y_1

A \ B	CD=00	CD=01	CD=11	CD=10
0	0	0	0	0
1	1	1	1	1

y_3

A \ B	CD=00	CD=01	CD=11	CD=10
0	0	1	0	1
1	1	0	1	0

$y_{7\&8\&9}$

A \ B	CD=00	CD=01	CD=11	CD=10
0	0	0	1	1
1	0	0	1	1
0	1	1	0	0
1	1	1	0	0

y_2

A \ B	CD=00	CD=01	CD=11	CD=10
0	0	1	1	0
1	1	0	0	1
0	0	1	1	0
1	1	0	0	1

$y_{4\&5\&6}$

A \ B	CD=00	CD=01	CD=11	CD=10
1	1	1	1	1
0	0	0	0	0
1	1	1	1	1
0	0	0	0	0

kar/xor-01

3.2 Forme polynomiale minimale

$y = x_1 x_0^* + \overline{x_2} x_0^* + \overline{x_2} x_1^* + \overline{x_3} x_2 \overline{x_0}^*$

kar/xor-02

3.3 Forme polynomiale minimale

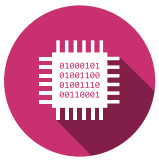
$\overline{E}\ \overline{D}\ C^* + \overline{E}\ \overline{C}\ \overline{B}\ \overline{A}^* + \overline{E}DBA^* + \overline{E}\ \overline{D}\ \overline{A}^* + \overline{E}CB^* + E\ \overline{D}\ \overline{C}\ A^* + EDC\overline{B}^* + ED\overline{C}B\overline{A}^* + \begin{cases} ED\overline{B}A \\ E\ \overline{C}\ \overline{B}A \end{cases}$

kar/xor-03

3.4 Forme ou-exclusif de produits

Many possible solutions e.g.: $Y = \overline{D}\ \overline{B} \oplus \overline{D}\ A \oplus C\ A \oplus C\ B$

kar/xor-04



3.5 Forme ou-exclusif de produits

Many possible solutions e.g.: $y = x_0 \oplus x_1\ \overline{x_0} \oplus x_2\ \overline{x_0} \oplus x_3\ x_2\ \overline{x_1}$

kar/xor-05

3.6 Additionneur

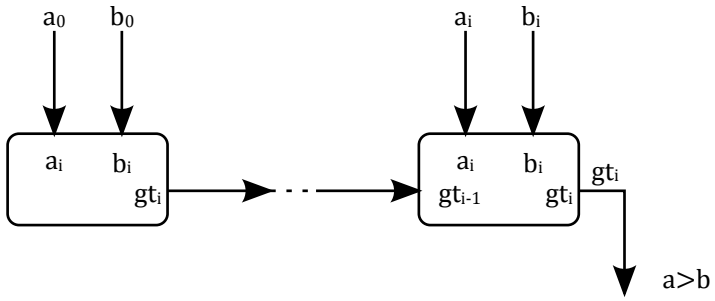
$$
\begin{aligned}
s_0 &= a_0 \oplus b_0 \\
s_1 &= a_1 \oplus b_1 \oplus a_0 b_0 \\
s_1 &= \overline{a_1} b_1 \oplus a_1 \overline{b_1} \oplus a_0 b_0 \\
s_2 &= a_1 b_1 \oplus \begin{cases} a_0 \overline{b_1} b_0 \oplus \overline{a_1} a_0 b_0 \\ a_0 b_1 b_0 \oplus a_1 a_0 b_0 \end{cases}
\end{aligned}
\tag{17}
$$

kar/xor-06

4 KAR - Fonctions avec un nombre élevé d'entrées

4.1 Comparaison de nombres

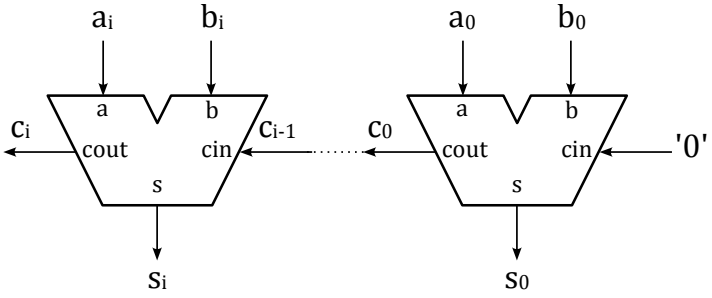
Possible with an iterative blocschema.



kar/manyinputs-01

4.2 Additionneur binaire

Possible with an iterative blocschema.



kar/manyinputs-02

4.3 Conversion de code thermomètre en code binaire

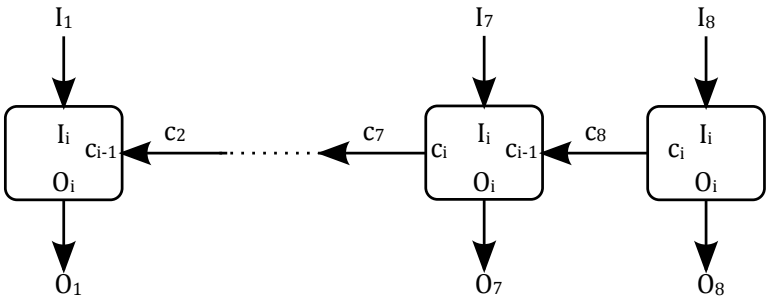
One possible solution is.

$$\begin{aligned} B_2 &= T_4 \\ B_1 &= T_2 + T_6\overline{T_4} \\ B_0 &= ??? \end{aligned} \tag{18}$$

kar/manyinputs-03

4.4 Transmission selon la priorité

Possible with an iterative blocschema.



kar/manyinputs-04

4.5 Logique pour compteur sans retour à zéro

The solutions involves an Adder **`x+1`** and a Comparator to **`0xFFFF`**

kar/manyinputs-05

4.6 Additionneur avec saturation

The output c_{out} of an iterative adder indicates an overflow.

kar/manyinputs-06

4.7 Nombres en code BCD

The BCD-Adder is a special case of a normal adder. Only the values between 0…9 exist. Therefore $c_{\text{out}} = 1$ if the sum is > 10. In this case -10 has to be substracted from the output.

kar/manyinputs-07

4.8 Fonction majorité à 7 entrées

A concatenation of adders with a comparison

kar/manyinputs-08

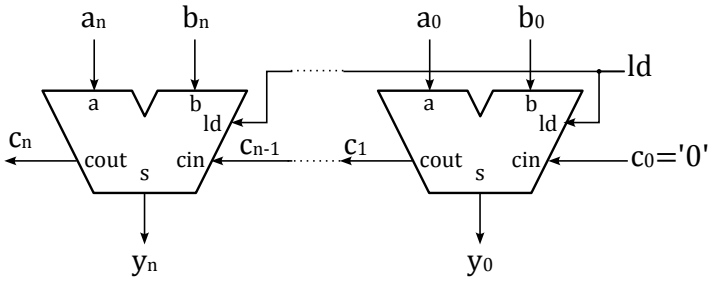
4.9 Unité arithmétique et logique

First determine the schema of an adder and that of a subtractor, tie them together and create the logical functions.

kar/manyinputs-09

4.10 Logique pour compteur de programme

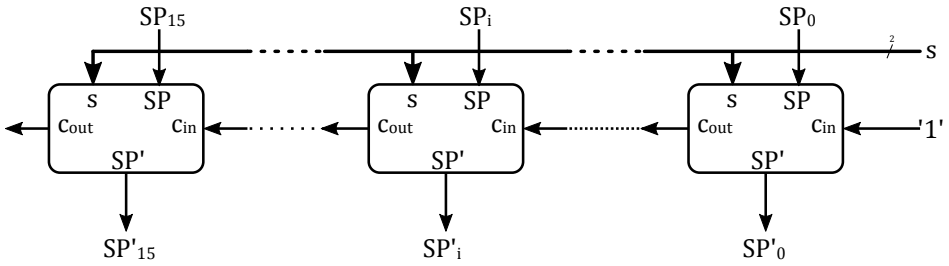
An iterative circuits with a load input.



kar/manyinputs-10

4.11 Logique pour pointeur de pile

An iterative circuits with a 2bit selection input.



kar/manyinputs-11