Simplification with Karnaugh tables

Exercises Digital Design

1 | KAR - Karnaugh tables

1.1 Representation of Monoms

Represent the following monomials in a Karnaugh table with 4 variables.

$$y_1 = \overline{b}a \quad (1) \qquad y_3 = \overline{d}cb \quad (3) \qquad y_5 = \overline{c}\ \overline{b}\ \overline{a} \quad (5)$$

$$y_2 = \overline{d}\ \overline{a} \quad (2) \qquad y_4 = dba \quad (4) \qquad y_6 = dcb\overline{a} \quad (6)$$

kar/karnaugh-01

1.2 Monoms

Give the algebraic form of the monomials of the following Karnaugh tables.

y_1

	C		D
1	1	0	0
1	1	0	0
0	0	0	0
0	0	0	0

A
B

y_3

	C		D
0	1	0	0
0	1	0	0
0	0	0	0
0	0	0	0

A
B

y_2

	C		D
1	0	0	1
0	0	0	0
0	0	0	0
1	0	0	1

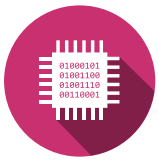
A
B

y_4

	C		D
0	1	0	0
0	0	0	0
0	0	0	0
0	1	0	0

A
B

kar/karnaugh-02



1.3 Representation of Polynomials

Represent the following polynomials in a Karnaugh table with 4 variables.

$$y_1 = \overline{b} + ac \quad (7)$$

$$y_2 = \overline{d} + \overline{a} + bc \quad (8)$$

$$y_3 = \overline{d}cb + d\overline{c}\,\overline{b} \quad (9)$$

$$y_4 = db + ab \quad (10)$$

$$y_5 = \overline{c}\,\overline{b}\overline{a} + \overline{c}\,\overline{b} \quad (11)$$

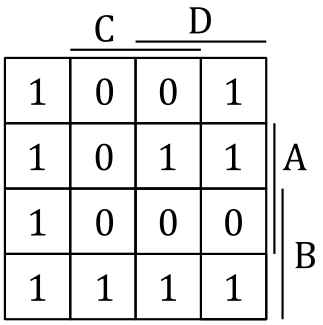
$$y_6 = dcb\overline{a} + \overline{d}cb\overline{a} \quad (12)$$

kar/karnaugh-03

2 KAR - Sum-of-products simplification

2.1 Karnaugh table with 4 variables

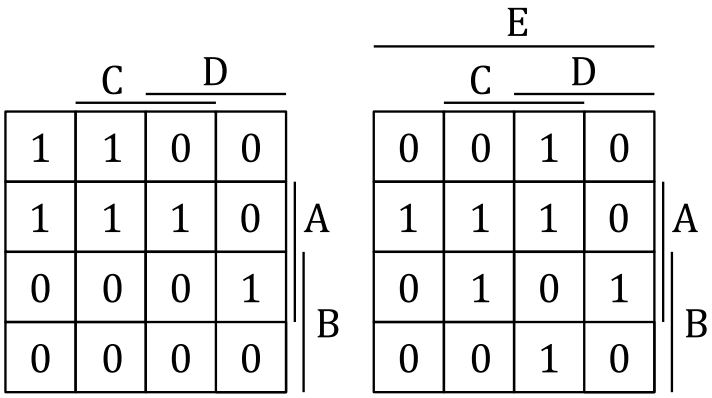
Determine the minimum polynomial form of the function shown in the Karnaugh chart of the following figure.



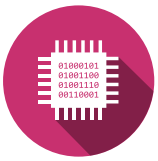
kar/productsum-01

2.2 Karnaugh table with 5 variables

Determine the minimum polynomial form of the function shown in the Karnaugh chart of the following figure.

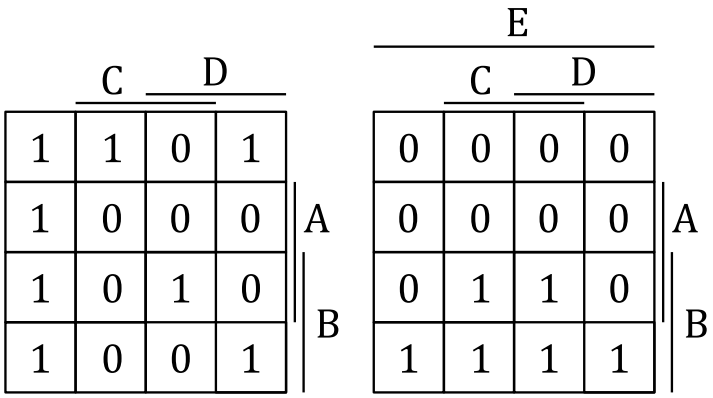


kar/productsum-02



2.3 Karnaugh table with 5 variables

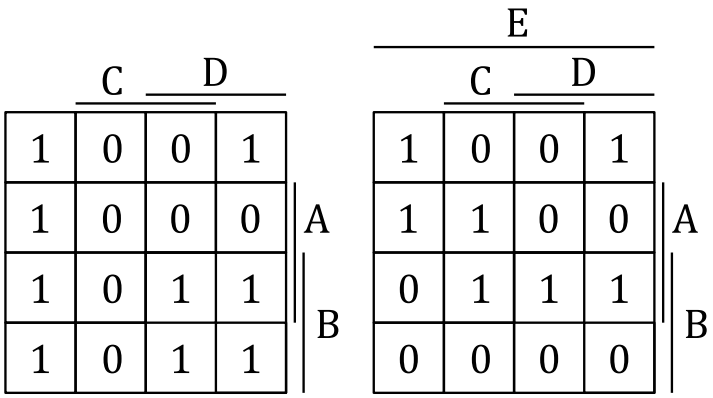
Determine the minimum polynomial form of the function shown in the Karnaugh chart of the following figure.



kar/productsum-03

2.4 Karnaugh table with 5 variables

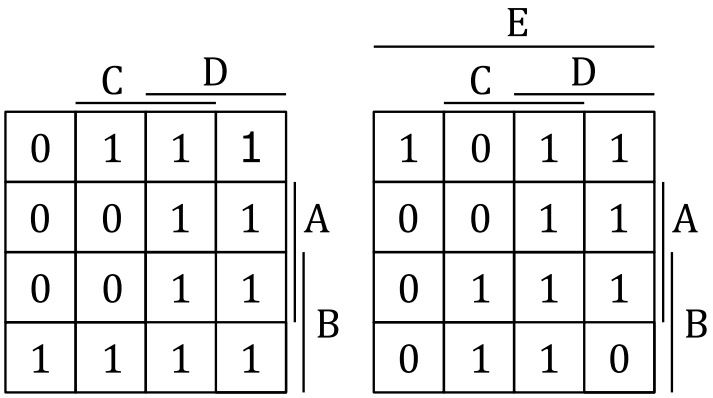
Determine the minimum polynomial form of the function shown in the Karnaugh chart of the following figure.



kar/productsum-04

2.5 Karnaugh table with 5 variables

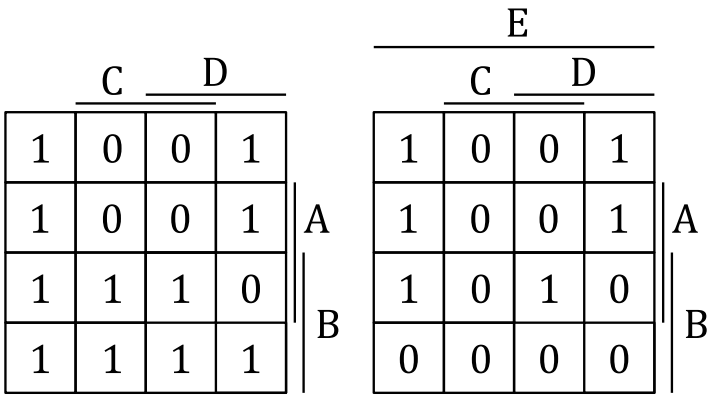
Determine the minimum polynomial form of the function shown in the Karnaugh chart of the following figure.



kar/productsum-05

2.6 Karnaugh table with 5 variables

Determine the minimum polynomial form of the function shown in the Karnaugh chart of the following figure. s



kar/productsum-06

2.7 Minimal Polynomialform

Determine the minimum polynomial form of the function

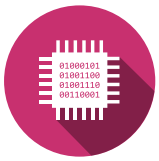
$$y = \overline{x_3\ x_2\ \overline{x_0} + \overline{x_2}\ \overline{x_1}\ \overline{x_0} + x_2\ \overline{x_1}\ x_0} \tag{13}$$

kar/productsum-07

2.8 Inverse Function

Determine the minimum polynomial form of the function

$$y = \overline{a}\ \overline{c} + a\ \overline{b}\ \overline{e} + b\ \overline{c}\ \overline{d} + \overline{a}\ \overline{b}\ e \tag{14}$$



kar/productsum-08

2.9 Minimum Polynomial Form

For the function given in the adjacent panel, determine which form has the smallest number of terms: that of the function Y or that of the function $\overline{Y}$.

D	C	B	A	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

kar/productsum-09

2.10 Function of 5 Variables

Determine the minimum polynomial form of the majority function with 5 inputs:

- the function yields a **`'0'`** if more than half of the inputs are at **`'0'`**.
- the function yields a **`'1'`** if more than half of the inputs are at **`'1'`**.

kar/productsum-10

2.11 Incompletely defined Function

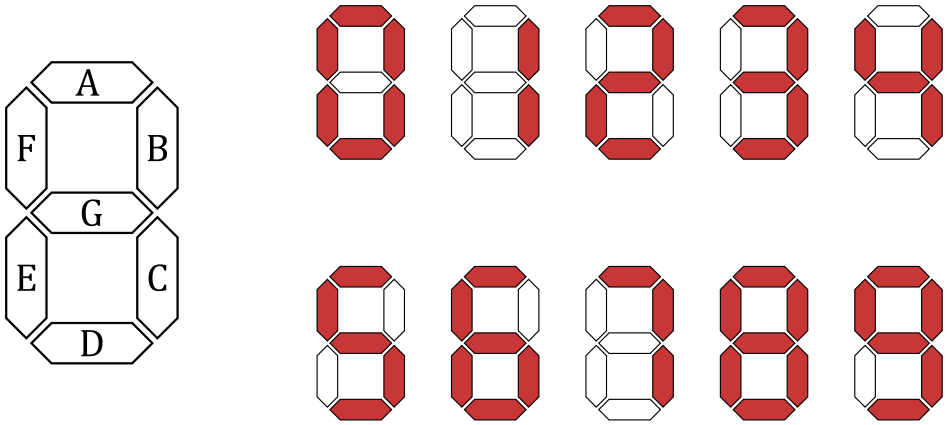
Determine the minimal polynomial expression of the majority function with 4 inputs:

- the function returns a **`'0'`** when more than half the inputs are at **`'0'`**,
- the function returns a **`'1'`** when more than half the inputs are at **`'1'`**,
- when the number of entries at **`'0'`** is identical to the number of entries at **`'1'`**, the result can take any value.

kar/productsum-11

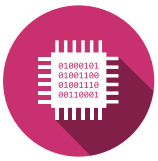
2.12 Incompletely defined function

A 7-segment display is applied to specify a decimal digit.



Create a combinational logic circuit that converts a binary number encoded to 4 bits to the 7 control signals used to illuminate the 7 segments.
In this system, the input number can only hold the binary values corresponding to the decimal digits from 0_d to 9_d.

kar/productsum-12



3 KAR - XOR function simplification

3.1 Representation of XOR Functions

In each Karnaugh table of 4 variables, represent the following functions:

$y_1 = a \oplus b$
$y_2 = a \oplus b \oplus c$
$y_3 = a \oplus b \oplus c \oplus d$
$y_4 = \overline{a \oplus b}$
$y_5 = \overline{a} \oplus b$
$y_6 = a \oplus \overline{b}$
$y_7 = d\overline{b} \oplus \overline{d}b$
$y_8 = d \oplus b$
$y_9 = \overline{b} \oplus \overline{d}$

kar/xor-01

3.2 Minimal Polynomial Form

Determine the minimum polynomial form of the function

$$y = x_1 \oplus \overline{x_1}x_0 \oplus x_2\overline{x_1} \oplus x_3x_2\overline{x_0} \tag{15}$$

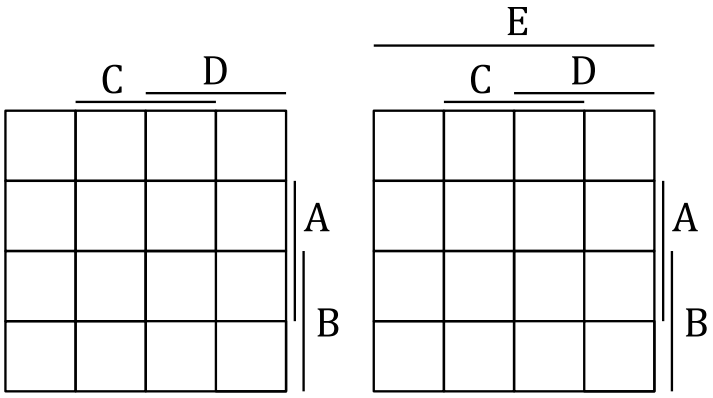
kar/xor-02

3.3 Minimal Polynomial Form

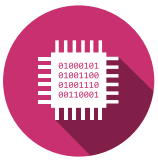
Determine the minimum polynomial form of the function

$$y = e \oplus \overline{d} \oplus dc \oplus d\overline{b} \oplus \overline{c}a \tag{16}$$

If you are working with a Karnaugh table, choose the same arrangement of variables as in the table of the following figure.



kar/xor-03



3.4 Representation in the form of XOR of Products

Write the equation of the function given in the following Karnaugh table in the form of XOR of products.

	C		D	
1	1	0	0	
0	1	1	0	A
1	1	0	0	
0	1	1	0	B

kar/xor-04

3.5 Representation in the form of XOR of Products

Rewrite the following function in the form of XOR of products.

$$y = x_1\ x_0 + \overline{x_2}\ x_1 + \overline{x_2}\ x_0 + \overline{x_3}\ x_2\ \overline{x_1} \tag{17}$$

kar/xor-05

3.6 Adder

Draw the scheme of a 2-bit adder using only AND and XOR gates.
The adder creates the sum of 2 numbers encoded on 2 bits. It gives a result on 3 bits.
Minimize the delay between inputs and outputs as a first priority. Consider that all gates have the same delay. Next, minimize the number of logic gates.

kar/xor-06

4 KAR - Functions with a large number of inputs

4.1 Number Comparison

Create a circuit that indicates whether a binary number A is greater than a binary number B. The numbers are coded to 8 bits. They correspond to positive integers.

kar/manyinputs-01

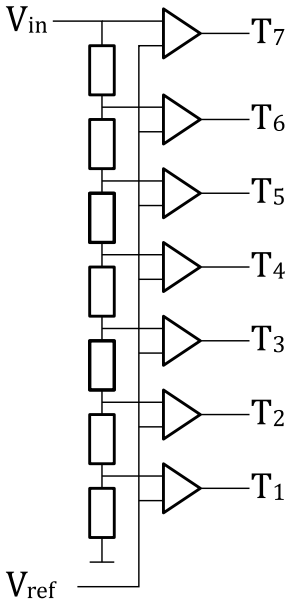
4.2 Binary Adder

Create a circuit that produces the sum of two 8-bit. The result should also be given on 8 bits. Make sure to minimize the number of logic gates.

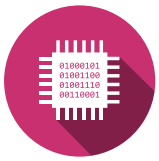
kar/manyinputs-02

4.3 Thermometer Code to Binary Code Conversion

A 3-bit analog/digital converter works as follows: The input voltage is divided by a chain of 7 resistors. Each intermediate voltage is compared to a reference voltage. The binary code thus generated is called the thermometer code. Create a circuit which converts this thermometer code to a binary code.



kar/manyinputs-03



4.4 Transmission based on Priority

Create a logical combinational circuit which passes only the one with the highest priority from all input signals.
The system is 8 inputs, I_1 onI_8, and 8 outputs, O_1 on O_8. The input I_8 has the highest priority.
The following panel gives some functional examples:

$I_1 ... I_8$	$O_1 ... O_8$
00000000	00000000
00010000	00010000
11000000	01000000
11010000	00010000
11010010	00000010

kar/manyinputs-04

4.5 Logic for Counter without decreasing to Zero

To design a 16-bit counter, create a combinational circuit that generates the next value of the counter, y, as a function of the current value, x.
The function to be generated is $y = x + 1$ as long as its maximum value, $x = \mathrm{FFFF}_h$ (all bits set to 1), is not reached. In this special case, the circuit generates the output value $y = \mathrm{FFFF}_h$ to avoid a reset to zero.

kar/manyinputs-05

4.6 Adder with Saturation

Propose a circuit of an adder whose inputs are coded to 16 bits as well as the output, and where exceeding the maximum value is treated with saturation. The operation takes place on positive numbers.
The saturation is defined as follows: if the sum of the 2 numbers exceeds the maximum value $y = \mathrm{FFFF}_h$ (all bits set to 1), the circuit produces this maximum value at the output.

kar/manyinputs-06

4.7 BCD coded numbers

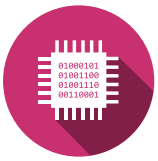
Propose a circuit that generates the sum of two BCD coded numbers (Binary Coded Decimal).
The 2 numbers are encoded on 3 BCD digits, which corresponds to 12 bits. The result is to be generated on 4 BCD digits, thus on 16 bits.

kar/manyinputs-07

4.8 Majority function with 7 inputs

Using inverters, AND, OR and Exclusive OR gates, draw the schematic of the circuit that determines the majority of 7 inputs.

kar/manyinputs-08



4.9 Arithmetic and logical unit

Draw the scheme of an Arithmetic and Logic Unit (ALU) of a microprocessor.
The ALU operates on 8 bits. The possible operations are:

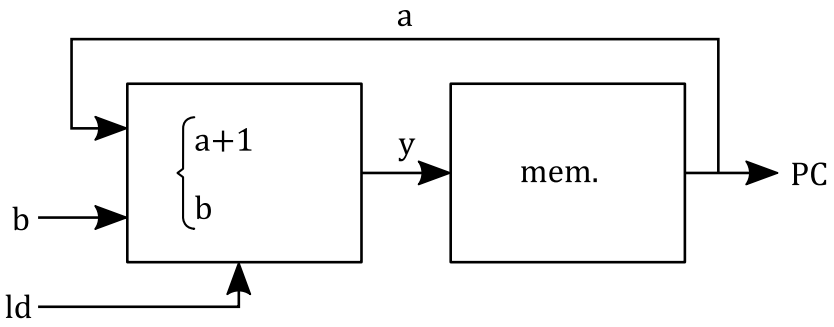
Command	Operation	
00	Addition	$y = a + b$
01	Substraction	$y = a - b$
10	AND	$y = a$ AND b
11	OR	$y = a$ OR b

This logical operations are performed bit by bit, example: $y_i = a_i$ AND $b_i, \forall i$.

kar/manyinputs-09

4.10 Logic for Program Counter

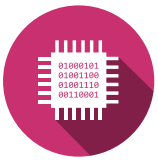
For the program counter (PC) of a microprocessor you need a block which either increments the counter (search for the next instruction) or loads it with a new value (jump).



Design a combinatorial circuit which calculates,

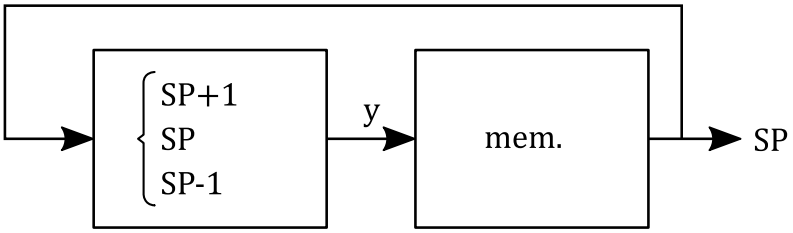
$$y = \begin{cases} \text{ld} = 0 \Rightarrow a + 1 \\ \text{ld} = 1 \Rightarrow b \end{cases} \tag{18}$$

kar/manyinputs-10



4.11 Logic for Stack Pointer

For the stack pointer (SP) of a microprocessor you need a block which either increments the pointer (a value is stacked up), decrements it (a value is stacked down) or keeps it the same (no operation with the stack).



The stack pointer is encoded to 16 bits.

kar/manyinputs-11