Combinatorial logic functions

Exercises Digital Design

1 | COM - Combinatorial function representations

1.1 Truth Table

Create the truth table of a control circuit for one floor of an elevator. The system is the following inputs:

- **`door_open`** : if this signal is **`'1'`**, the lift shall stop,
- **`call`** : if this signal is **`'1'`**, the lift should go to the floor,
- **`lower`** : in case of a call, the lift shall rise,
- **`higher`** : in case of a call, the lift shall descend.

If **`lower`** and **`higher`** are both at **`'0'`**, it means that the lift is at the floor. The system creates the following outputs:

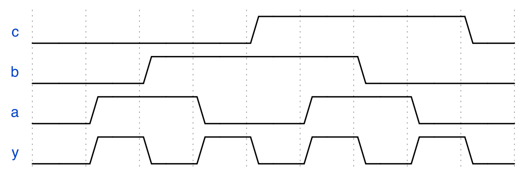
- **`motor_en`** : if this signal is at **`'1'`**, the motor is running,
- **`up`** : if this signal is at **`'1'`** and the motor is running, the lift is ascending; if this signal is at **`'0'`** and the motor is running, the lift is descending.

The system analyzed here does not take the case where the lift is already running.

com/representation-01

1.2 Truth Table from a Chonogram

Create the truth table of the function determined by the adjacent time graph.



com/representation-02

1.3 Representation by Venn diagram

Represent the following functions in a Venn diagram.

$y_1 = a\overline{b}$

$y_2 = \overline{a}\;\overline{b}\;c$

$y_3 = \overline{a}\;\overline{b}\;\overline{c}$

$y_4 = \mathrm{ab} + \overline{b}\;c$

$y_5 = \overline{a}\;\overline{b} + \overline{b}\;\overline{c}$

$y_6 = a + \mathrm{ab}$

com/representation-03

1.4 Simplification by Venn diagram

Plot the function $\mathrm{ab} + a\overline{c} + \mathrm{bc}$ on a Venn diagram. Show that one term of the polynomial is redundant.

com/representation-04

2 COM - Elementary logic functions

2.1 Switch Circuits

Determine the relationship between the control signals a and b and the output l of each circuit in the following figure.

The switches are closed when the control signal is `'1'`.

com/logic-functions-01

2.2 Truth table of Elementary Functions

Complete the adjacent truth table.
Compare the result with the truth table of the AND function of 2 inputs.

C	B	A	CA	CB	BA
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

com/logic-functions-02

2.3 Elementary Functions in a Truth Table

Write an algebraic form of the functions defined in the adjacent truth table.

C	B	A	Y_1	Y_2	Y_3
0	0	0	0	0	1
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	0	0	1
1	1	0	0	0	0
1	1	1	0	1	0

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2.4 Number Decoding

To warn motorists stopped at a red light of the imminent arrival of a green light, design a small circuit to momentarily activate an orange light.
To do this, we have a clock that is activated when the light change process is due to start. For each second, this clock delivers a binary code ranging from 000 to 111, and stops after 7 seconds.
The 3 output signals (**`red`**, **`orange`** and **`green`**) are specified as follows:

- The red light remains on up to and including the 5th second.
- The orange light comes on as soon as the 2nd second has passed, and goes off at the same time as the red light.
- The green light comes on as soon as the red light goes out.

Draw up the truth table and combinatorial circuit corresponding to the activation of the 3 lights.

com/logic-functions-04

3 COM - Boolean algebra

3.1 Proofs

Prove the following relationships:

$$a + \overline{a}b = a + b \quad (1)$$

$$a * (a + b) = a \quad (2)$$

$$a + \mathrm{ab} = a \quad (3)$$

com/algebra-01

3.2 De Morgan

$\overline{a + b + \overline{c}d}$

com/algebra-02

3.3 Redundancy with the XOR function

A system is to transmit 2 bits, a and b . For security reasons, it transmits one additional bit given by the equation $y = a \oplus b$.
Show that if bit a is lost during transmission, it is possible to recover it using b and y.

com/algebra-03

3.4 XOR function

Write the function $\overline{a \oplus b}$ to polynomial form.

com/algebra-04

3.5 Polynomial Form

Determine the polynomial form of the function
$\overline{\overline{a}\overline{b} + \overline{b}\overline{c} + \overline{c}\ \overline{a}}$

com/algebra-05

4 | COM - Complete operators

4.1 Create a function using NAND gates

Using only NAND gates, draw the complete schematic of a circuit that creates the following function.

$$y = (ab + cd + e + f * (\overline{g} + h)) * \overline{i} \tag{4}$$

com/operators-01

4.2 Create a function using NAND gates

$$y = \overline{a}bc + \overline{c}\overline{d}f + ae \tag{5}$$

com/operators-02

4.3 Create a function using NAND gates

Using only 2-input NAND gates, draw the complete schematic of a circuit that XORs 4 inputs.

com/operators-03

4.4 NOR-Operator

Prove that a NOR gate is a complete operator.

com/operators-04

4.5 Create a function using NOR gates

Using only NOR gates, draw the complete schematic of a circuit that creates the following function

$$y = (ab + cd + e + f * (\overline{g} + h)) * \overline{i} \tag{6}$$

com/operators-05

4.6 Create a function using inverting gates

In integrated circuits, you prefer to use NAND and NOR gates rather than AND and OR gates. Using inverters, NAND and NOR gates draw the complete schematic of a circuit which creates the following function.

$$y = (ab + cd + e + f * (\overline{g} + h)) * \overline{i} \tag{7}$$

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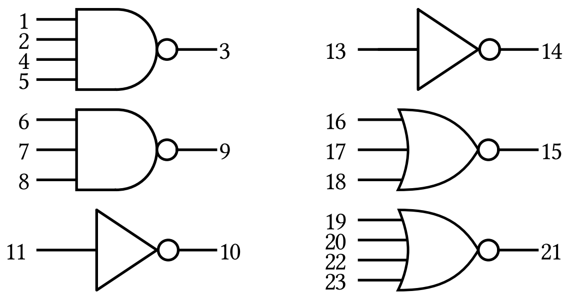
4.7 Versatile circuit

The circuit 74HC7006 is used to have a wide choice of functions created. Show that this circuit can be used to create AND, NAND, OR, and NOR circuits to 9 inputs.



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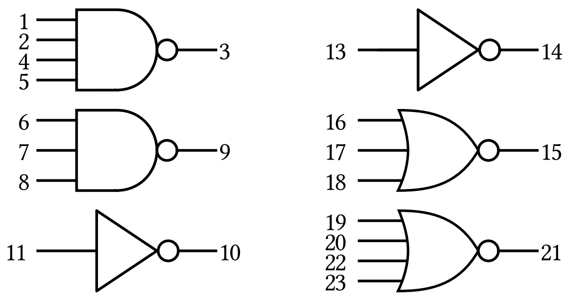
4.8 Creation of an XOR function

The circuit 74HC7006 is used to let create a wide choice of functions. Use it to realize an XOR function with 3 inputs. Use a minimum number of these blocks.



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