Simplification with Karnaugh tables

Exercises Digital Design

Solution vs. Hints:
While not every response provided herein constitutes a comprehensive solution, some serve as helpful hints intended to guide you toward discovering the solution independently. In certain instances, only a portion of the solution is presented.

1 | KAR - Karnaugh tables

1.1 Representation of Monoms

y_1

A \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

y_3

A \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	1	0	0
10	0	1	0	0

y_5

A \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

y_2

A \ CD	00	01	11	10
00	1	1	0	0
01	0	0	0	0
11	0	0	0	0
10	1	1	0	0

y_4

A \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	0	0

y_6

A \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	1	0

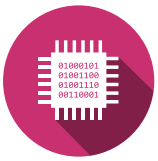
kar/karnaugh-01

1.2 Monoms

$$\begin{aligned} y_1 &= \overline{BD} \\ y_2 &= \overline{AC} \end{aligned} \tag{1}$$

$$\begin{aligned} y_3 &= \overline{B}C\overline{D} \\ y_4 &= \overline{A}C\overline{D} \end{aligned} \tag{2}$$

kar/karnaugh-02



1.3 Representation of Polynomials

y_1

C \ D	1	1	0	0
1	1	1	1	1
1	1	1	1	1
0	1	1	0	0
0	0	0	0	0

A, B

y_3

C \ D	1	1	0	0
1	0	0	0	1
1	0	0	0	1
0	1	0	0	0
0	1	0	0	0

A, B

y_5

C \ D	1	1	0	0
1	1	0	0	1
1	1	0	0	1
0	0	0	0	0
0	0	0	0	0

A, B

y_2

C \ D	1	1	0	0
1	1	1	1	1
1	1	1	0	0
1	1	1	1	0
1	1	1	1	1

A, B

y_4

C \ D	1	1	0	0
1	0	0	0	0
1	0	0	0	0
0	1	1	1	1
0	0	1	1	0

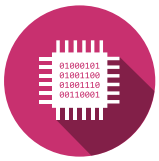
A, B

y_6

C \ D	1	1	0	0
1	0	0	0	0
1	0	0	0	0
0	0	0	0	0
0	1	1	0	0

A, B

kar/karnaugh-03



2 | KAR - Sum-of-products simplification

2.1 Karnaugh table with 4 variables

$$D\overline{B}A^* + \overline{D}\overline{C}^* + B\overline{A}^* + \begin{cases} \overline{C}B \\ \overline{C}A \end{cases} \tag{3}$$

kar/productsum-01

2.2 Karnaugh table with 5 variables

$$\overline{E}\ \overline{D}\ \overline{B}^* + C\overline{B}A^* + D\overline{C}BA^* + \overline{D}\ \overline{B}A^* + E\overline{D}CA^* + EDC\overline{A} \tag{4}$$

kar/productsum-02

2.3 Karnaugh table with 5 variables

$$\overline{E}\ \overline{D}\ \overline{C}^* + \overline{E}\ \overline{C}\ \overline{A}^* + \overline{E}\ \overline{D}\ \overline{B}\ \overline{A}^* + DCBA^* + ECB^* + \begin{cases} EB\overline{A} \\ \overline{C}B\overline{A} \end{cases} \tag{5}$$

kar/productsum-03

2.4 Karnaugh table with 5 variables

$$\overline{E}DB^* + \overline{C}\ \overline{B}\ \overline{A}^* + DBA^* + E\overline{D}CA + \overline{D}\ \overline{C}\ \overline{B} + \overline{E}\ \overline{D}\ \overline{C} \tag{6}$$

kar/productsum-04

2.5 Karnaugh table with 5 variables

$$\overline{E}C\overline{A}^* + \overline{E}B\overline{A}^* + E\ \overline{C}\ \overline{B}\ \overline{A}^* + DA^* + ECB^* + D\overline{B} \tag{7}$$

kar/productsum-05

2.6 Karnaugh table with 5 variables

$$\overline{C}\ \overline{B}^* + \overline{D}\ \overline{C}\ A^* + DCBA^* + \overline{E}CB + \overline{E}B\overline{A} \tag{8}$$

or

$$\overline{C}\ \overline{B}^* + \overline{D}\ \overline{C}\ A^* + DCBA^* + \overline{E}CB + \overline{E}\ \overline{C}\ \overline{A} \tag{9}$$

or

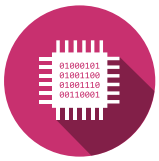
$$\overline{C}\ \overline{B}^* + \overline{D}\ \overline{C}\ A^* + DCBA^* + \overline{E}\ \overline{D}\ B + \overline{E}B\overline{A} \tag{10}$$

kar/productsum-06

2.7 Minimal Polynomialform

$$\overline{x_3}\ x_2\ \overline{x_0}^* + \overline{x_2}\ x_0^* + x_1\ x_0^* + \overline{x_2}\ x_1^* \tag{11}$$

kar/productsum-07



2.8 Inverse Function

$$\overline{E}\ C\ \overline{A}^{*} + CB^{*} + DBA^{*} + E\ \overline{B}\ A^{*} \tag{12}$$

kar/productsum-08

2.9 Minimum Polynomial Form

y: 5 termes; $\overline{y}$: 4 termes

kar/productsum-09

2.10 Function of 5 Variables

$$DCA^{*} + DCB^{*} + CBA^{*} + DBA^{*} + EDC^{*} + EBA^{*} + ECA^{*} + EDA^{*} + ECB^{*} + EDB^{*}$$

kar/productsum-10

2.11 Incompletely defined Function

$$x_4\ x_3 + x_2\ x_1 \tag{13}$$

or

$$x_3\ x_1 + x_4\ x_2 \tag{14}$$

or

$$x_4\ x_1 + x_3\ x_2 \tag{15}$$

kar/productsum-11

2.12 Incompletely defined function

$$\begin{aligned}
A &= \overline{B_2}\ \overline{B_0}^{*} + B_2\ B_0^{*} + B_1 + B_3^{*} \\
B &= \overline{B_2}^{*} + \overline{B_1}\ \overline{B_0}^{*} + B_1\ B_0\ * \\
C &= B_2^{*} + \overline{B_1}^{*} + B_0 \\
D &= \overline{B_2}\ \overline{B_0}^{*} + B_3^{*} + B_2\ \overline{B_1}\ B_0^{*} + \overline{B_2}\ B_1^{*} + B_1\ \overline{B_0} \\
E &= \overline{B_2}\ \overline{B_0}^{*} + B_1\ \overline{B_0}^{*} \\
F &= B_3^{*} + B_2\ \overline{B_1}^{*} + B_2\ \overline{B_0}^{*} + \overline{B_1}\ \overline{B_0}^{*} \\
G &= B_3^{*} + B_2\ \overline{B_1} + \overline{B_0}\ B_1^{*} + B_2\ \overline{B_1}
\end{aligned} \tag{16}$$

kar/productsum-12

3 KAR - XOR function simplification

3.1 Representation of XOR Functions

y_1

C \ D	0	1	1	0
0	0	1	0	1
1	1	1	0	0
0	0	0	0	0
1	1	1	0	0

A / B

y_3

C \ D	0	1	1	0
0	0	1	0	1
1	1	0	1	0
0	0	1	0	1
1	1	0	1	0

A / B

$y_{7\&8\&9}$

C \ D	0	1	1	0
0	0	0	1	1
0	0	0	1	1
1	1	1	0	0
1	1	1	0	0

A / B

y_2

C \ D	0	1	1	0
0	0	1	1	0
1	1	0	0	1
0	0	1	1	0
1	1	0	0	1

A / B

$y_{4\&5\&6}$

C \ D	0	1	1	0
1	1	1	1	1
0	0	0	0	0
1	1	1	1	1
0	0	0	0	0

A / B

kar/xor-01

3.2 Minimal Polynomial Form

$y = x_1 x_0^* + \overline{x_2} x_0^* + \overline{x_2} x_1^* + \overline{x_3} x_2 \overline{x_0}^*$

kar/xor-02

3.3 Minimal Polynomial Form

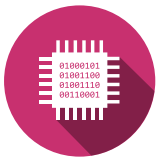
$\overline{E}\ \overline{D}\ C^* + \overline{E}\ \overline{C}\ \overline{B}\ \overline{A}^* + \overline{E}DBA^* + \overline{E}\ \overline{D}\ \overline{A}^* + \overline{E}CB^* + E\ \overline{D}\ \overline{C}\ A^* + EDC\overline{B}^* + ED\overline{C}B\overline{A}^* + \begin{cases} ED\overline{B}A \\ E\ \overline{C}\ \overline{B}A \end{cases}$

kar/xor-03

3.4 Representation in the form of XOR of Products

Many possible solutions e.g.: $Y = \overline{D}\ \overline{B} \oplus \overline{D}\ A \oplus C\ A \oplus C\ B$

kar/xor-04



3.5 Representation in the form of XOR of Products

Many possible solutions e.g.: $y = x_0 \oplus x_1\ \overline{x_0} \oplus x_2\ \overline{x_0} \oplus x_3\ x_2\ \overline{x_1}$

kar/xor-05

3.6 Adder

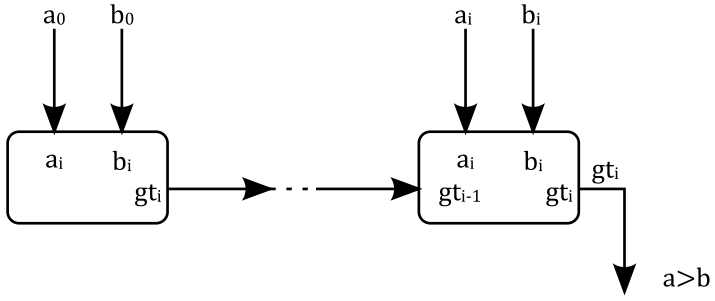
$$
\begin{aligned}
s_0 &= a_0 \oplus b_0 \\
s_1 &= a_1 \oplus b_1 \oplus a_0 b_0 \\
s_1 &= \overline{a_1} b_1 \oplus a_1 \overline{b_1} \oplus a_0 b_0 \\
s_2 &= a_1 b_1 \oplus \begin{cases} a_0 \overline{b_1} b_0 \oplus \overline{a_1} a_0 b_0 \\ a_0 b_1 b_0 \oplus a_1 a_0 b_0 \end{cases}
\end{aligned}
\tag{17}
$$

kar/xor-06

4 KAR - Functions with a large number of inputs

4.1 Number Comparison

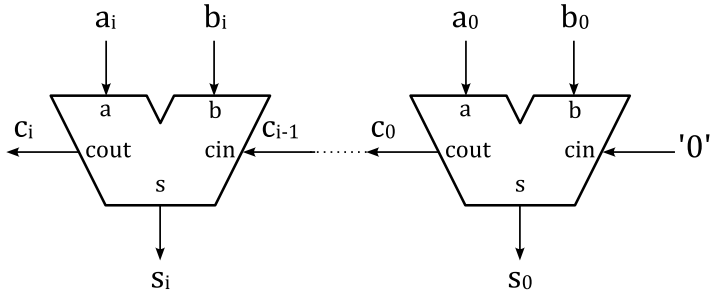
Possible with an iterative blocschema.



kar/manyinputs-01

4.2 Binary Adder

Possible with an iterative blocschema.



kar/manyinputs-02

4.3 Thermometer Code to Binary Code Conversion

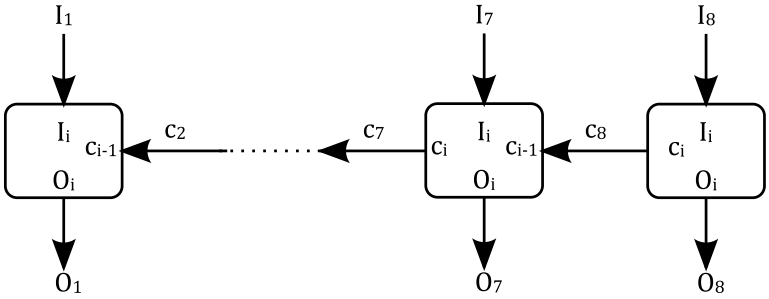
One possible solution is.

$$\begin{aligned} B_2 &= T_4 \\ B_1 &= T_2 + T_6\overline{T_4} \\ B_0 &= ??? \end{aligned} \tag{18}$$

kar/manyinputs-03

4.4 Transmission based on Priority

Possible with an iterative blocschema.



kar/manyinputs-04

4.5 Logic for Counter without decreasing to Zero

The solutions involves an Adder **`x+1`** and a Comparator to **`0xFFFF`**

kar/manyinputs-05

4.6 Adder with Saturation

The output c_{out} of an iterative adder indicates an overflow.

kar/manyinputs-06

4.7 BCD coded numbers

The BCD-Adder is a special case of a normal adder. Only the values between 0…9 exist. Therefore $c_{\text{out}} = 1$ if the sum is > 10. In this case -10 has to be substracted from the output.

kar/manyinputs-07

4.8 Majority function with 7 inputs

A concatenation of adders with a comparison

kar/manyinputs-08

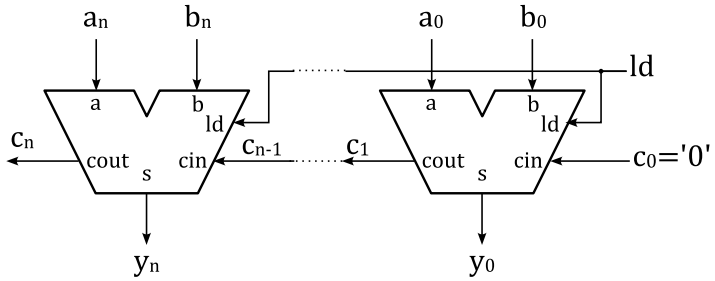
4.9 Arithmetic and logical unit

First determine the schema of an adder and that of a subtractor, tie them together and create the logical functions.

kar/manyinputs-09

4.10 Logic for Program Counter

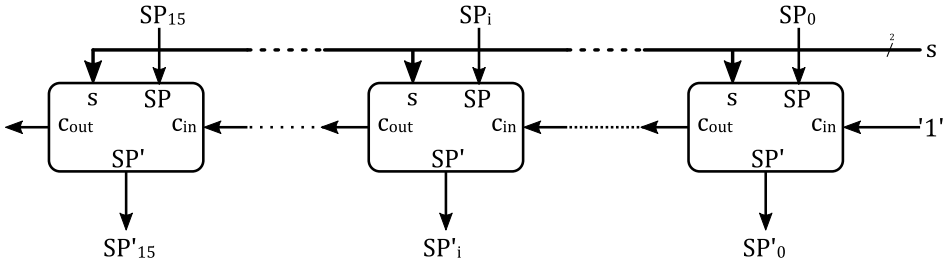
An iterative circuits with a load input.



kar/manyinputs-10

4.11 Logic for Stack Pointer

An iterative circuits with a 2bit selection input.



kar/manyinputs-11