Synchronous counters

Exercises Digital Design

Solution vs. Hints:
While not every response provided herein constitutes a comprehensive solution, some serve as helpful hints intended to guide you toward discovering the solution independently. In certain instances, only a portion of the solution is presented.

1 CNT - Counters by a power of 2

1.1 Downwards Counter

$$\begin{aligned} D_0 &= \overline{Q_0} \\ D_1 &= Q_1 \oplus \overline{Q_0} \\ D_2 &= Q_2 \oplus \overline{Q_1}\,\overline{Q_0} \\ D_3 &= Q_3 \oplus \overline{Q_2}\,\overline{Q_1}\,\overline{Q_0} \end{aligned} \tag{1}$$

$$\begin{aligned} D_0 &= Q_0^+ = Q_0 \oplus 1 \\ D_1 &= Q_1^+ = Q_1 \oplus \overline{Q_0} \\ D_2 &= Q_2^+ = Q_2 \oplus \overline{Q_0}\,\overline{Q_1} \\ D_3 &= Q_3^+ = Q_3 \oplus \overline{Q_0}\,\overline{Q_1}\,\overline{Q_2} \end{aligned} \tag{2}$$

cnt/pow2-01

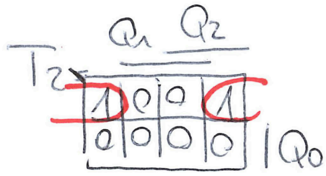
1.2 Downwards Counter

1.2.1.1 Truth table

$Q_2 \ldots Q_0$	$Q_2^+ \ldots Q_0^+$	$T_2 \ldots T_0$
000	111	111
001	000	001
010	001	011
011	010	001
100	011	111
101	100	001
110	101	011
111	110	001

1.2.1.2 Equations

$$\begin{aligned} T_0 &= 1 \\ T_1 &= \overline{Q_0} \\ T_2 &= \overline{Q_1}\,\overline{Q_0} \end{aligned} \tag{3}$$



cnt/cnt-pow2-02

2 | CNT - Counters by any number

2.1 Downwards Counter

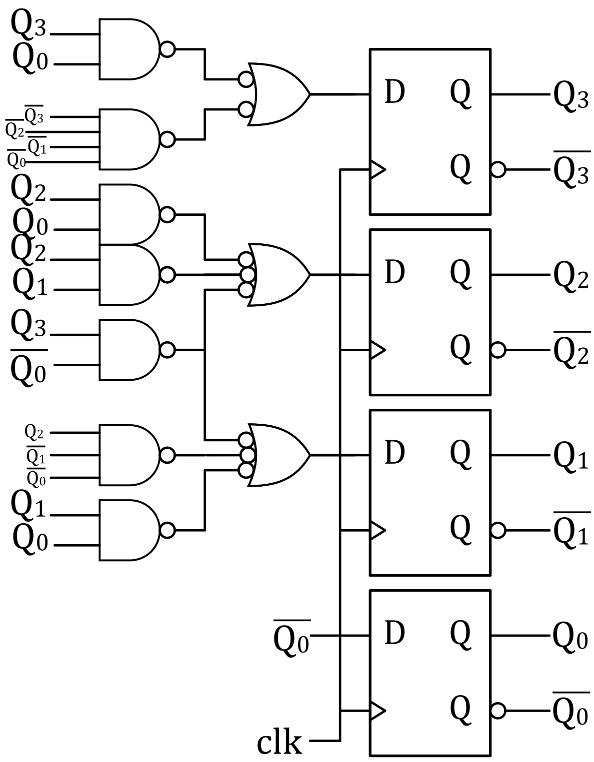
2.1.1.1 Equations

$$\begin{aligned} D_0 &= Q_0^+ = \overline{Q_0} \\ D_1 &= Q_1^+ = Q_3\overline{Q_0} + Q_2\overline{Q_1}\,\overline{Q_0} + Q_1Q_0 \\ D_2 &= Q_2^+ = Q_3\overline{Q_0} + Q_2Q_1 + Q_2Q_0 \\ D_3 &= Q_3^+ = Q_3Q_0 + \overline{Q_3}\,\overline{Q_2}\,\overline{Q_1}\,\overline{Q_0} \end{aligned} \tag{4}$$

2.1.1.2 Sequence

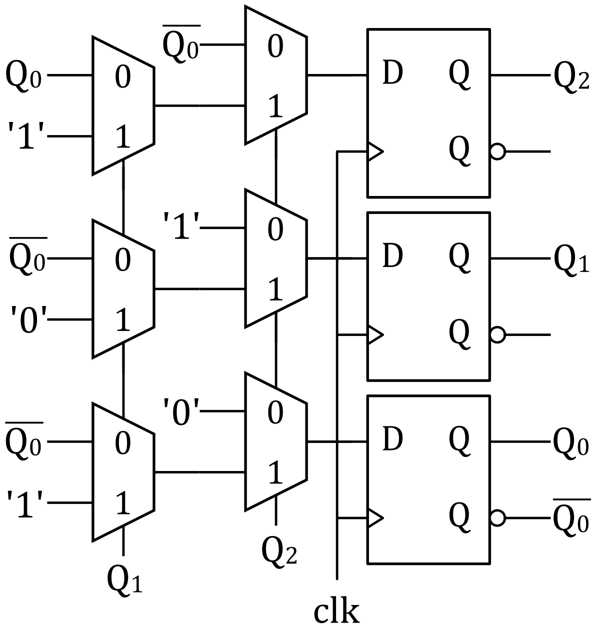
$$\begin{gathered} 9 \Rightarrow 8 \Rightarrow 7 \Rightarrow 6 \Rightarrow \ldots 3 \Rightarrow 2 \Rightarrow 1 \Rightarrow 0 \Rightarrow 9 \Rightarrow 8 \Rightarrow \ldots \\ 11 \Rightarrow 10 \Rightarrow 7 \\ 13 \Rightarrow 12 \Rightarrow 7 \\ 15 \Rightarrow 14 \Rightarrow 7 \end{gathered} \tag{5}$$

2.1.1.3 Circuit



cnt/cnt-01

2.2 Downwards Counter



cnt/cnt-02

2.3 Johnson Counter

$D_B = Q_A + \overline{Q_C}Q_B$

or

$D_B = \overline{Q_C}Q_A + Q_BQ_A$

cnt/cnt-03

3 CNT - Iterative circuits

3.1 Counter with Synchronous Zeroing

Equation of a “+1” counter:

$$\begin{aligned} Q^+ &= D = Q \oplus \text{en} \\ c_{\text{out}} &= Q * \text{en} \end{aligned} \tag{6}$$

The **`restart`** can be added with the help of a AND gate and an inverter.

cnt/cnt-iterativ-01

3.2 Counter with loading of a value

Equation of a “+1” counter:

$$\begin{aligned} Q^+ &= D = Q \oplus \text{en} \\ c_{\text{out}} &= Q * \text{en} \end{aligned} \tag{7}$$

The **`load`** can be added with the help of a Multiplexer 2-1.

cnt/cnt-iterativ-02

3.3 up-down counter

down-Counter

$$\begin{aligned} Q_i^+ &= Q_i \oplus c_i \\ c_{i+1} &= \overline{Q_i} * c_i \end{aligned} \tag{8}$$

up-Counter

$$\begin{aligned} Q_i^+ &= Q_i \oplus c_i \\ c_{i+1} &= Q_i * c_i \end{aligned} \tag{9}$$

up-down-Counter

$$\begin{aligned} Q_i^+ &= Q_i \oplus c_i \\ c_{i+1} &= \text{updown} Q_i * c_i + \overline{\text{updown}} \, \overline{Q_i} * c_i \end{aligned} \tag{10}$$

The difference of the down- vs the up-Counter is a XOR of Q_i

cnt/cnt-iterative-03

3.4 Programmable Counter

reset if $P = Q$
sequence $0 \Rightarrow 1 \Rightarrow \ldots \Rightarrow P \Rightarrow 0$
Sequence lenght = $P + 1$

cnt/cnt-iterativ-04