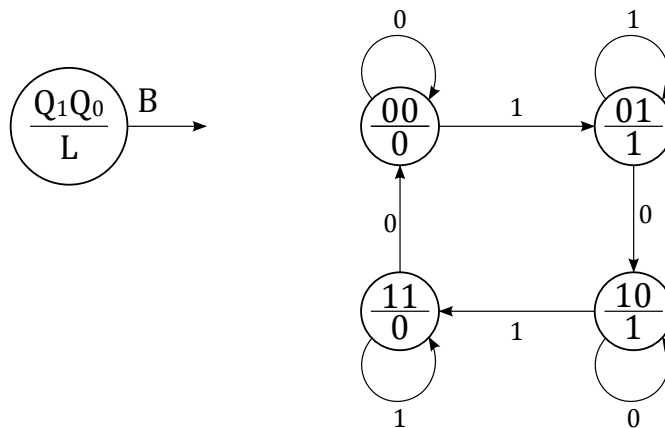# Machines d'états

Exercices Conception numérique

> **Solution vs. Hints:**
> Toutes les réponses fournies ici ne sont pas des solutions complètes. Certaines ne sont que des indices pour vous aider à trouver la solution vous-même. Dans d'autres cas, seule une partie de la solution est fournie.
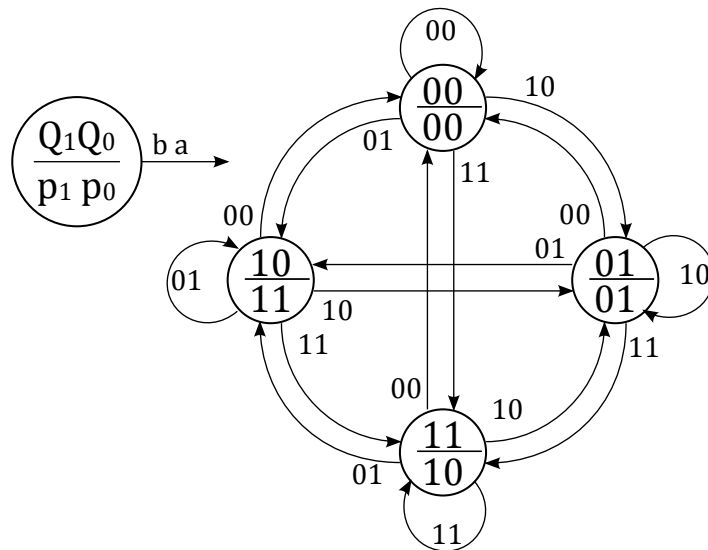
1 | FSM - Machines de Moore

1.1 Graphe d'une machine d'états

fsm/moore-01

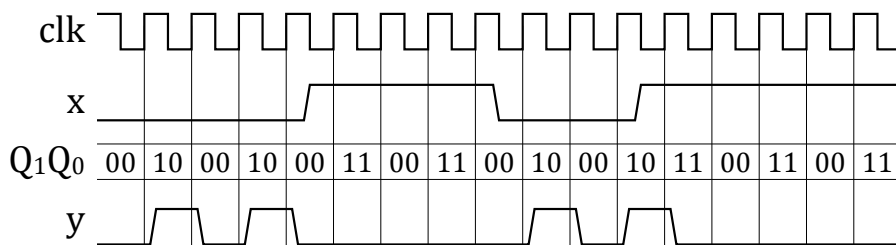## 1.2 Graphe d'une machine d'états

fsm/moore-02

1.3 Séquence d'un compteur

$$... \Rightarrow 0 \Rightarrow 1 \Rightarrow 3 \Rightarrow 2 \Rightarrow 6 \Rightarrow 7 \Rightarrow 5 \Rightarrow 4 \Rightarrow 0 \Rightarrow ... \tag{1}$$
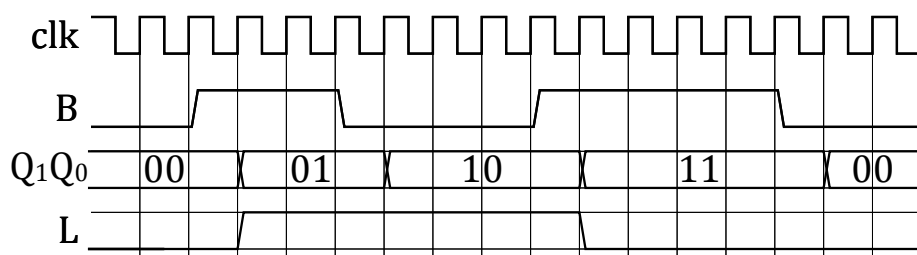
fsm/moore-03

1.4 Comportement temporel d'une machine d'états

fsm/moore-04

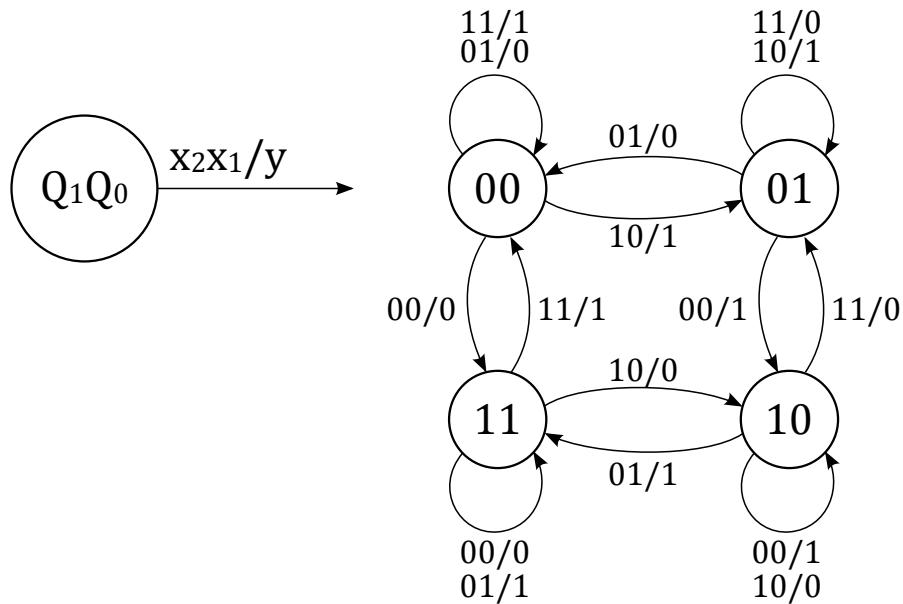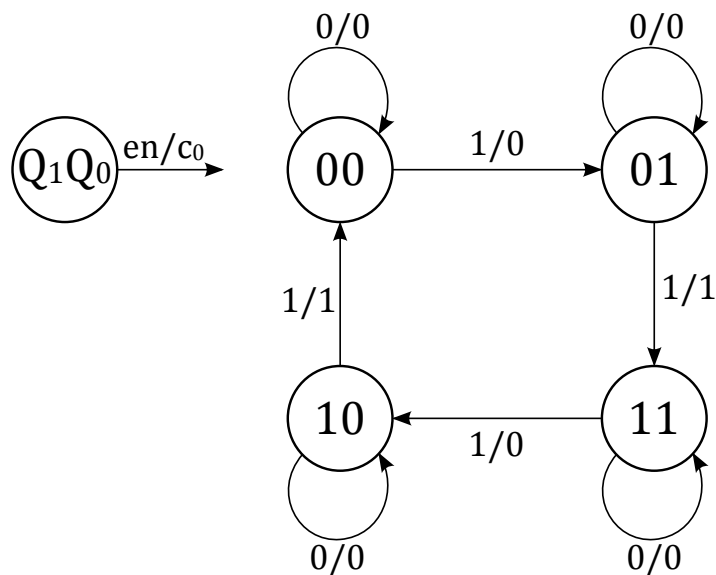## 1.5 Comportement temporel d'une machine d'états

fsm/moore-05

2 | FSM - Machines de Mealy

2.1 Graphe d'une machine d'états

fsm/mealy-01

2.2 Graphe d'une machine d'états

fsm/mealy-02

2.3 Comportement temporel d'une machine d'états

2.3.0.1 Initial State

$x = 0 \Rightarrow Q = \text{"00"}$

2.3.0.2 Outputs

$$
\begin{aligned}
y_1 = 1 \Rightarrow &\begin{cases} Q = \text{"10"} \ \& \ x = 1 \\ Q = \text{"11"} \ \& \ x = 1 \mid x = 0 \end{cases} \\
y_0 = 1 \Rightarrow &\begin{cases} Q = \text{"01"} \ \& \ x = 1 \\ Q = \text{"11"} \\ Q = \text{"10"} \ \& \ x = 0 \end{cases}
\end{aligned} \tag{2}
$$

fsm/mealy-03

2.4 Compteur itératif

Mealy-Machine since c_2 depends on c_0 & Q_0 & Q_1.
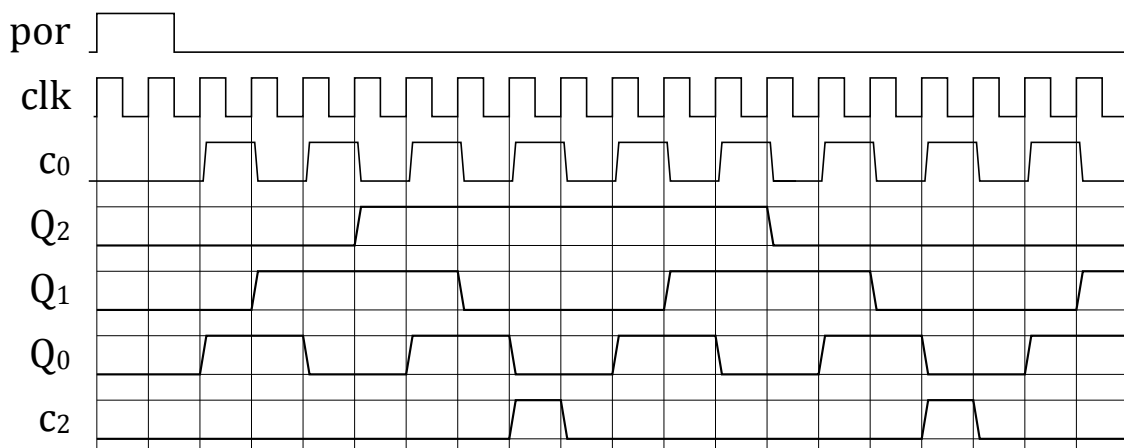
fsm/mealy-04

2.5 Comportement temporel d'une machine d'états
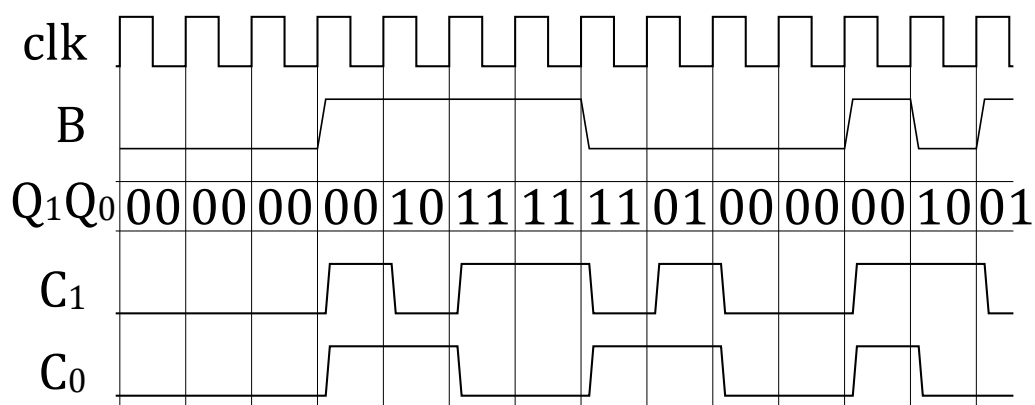
3 | FSM - Établissement du graphe des états

3.1 Superviseur de fonctionnement
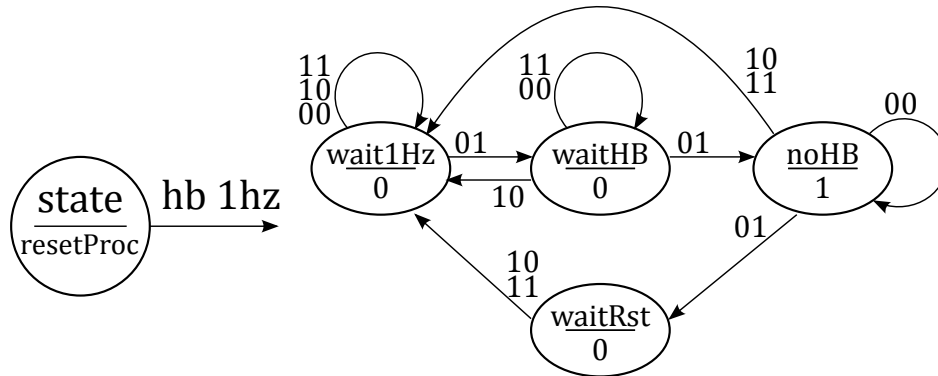
fsm/fsm-01

3.2 Générateur de commandes non-recouvrantes
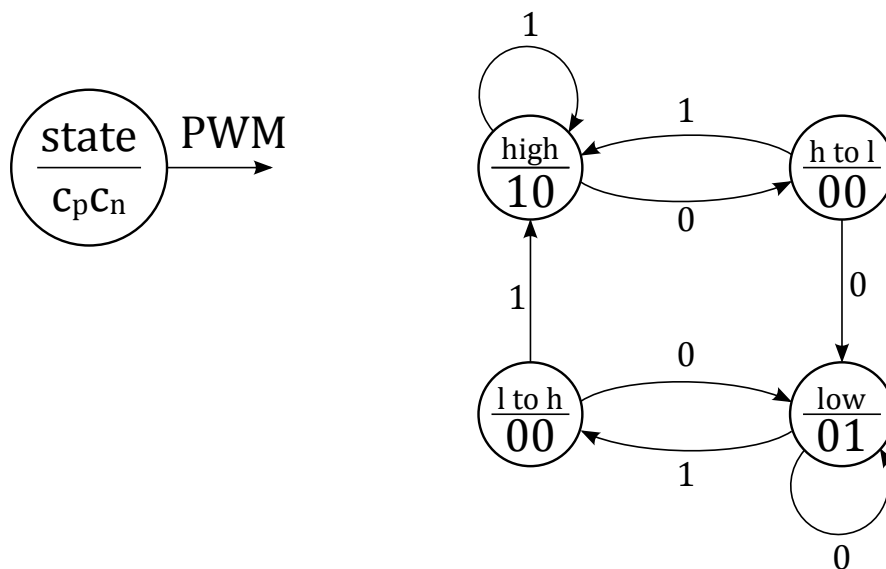
fsm/fsm-02

3.3 Commande de distributeur automatique

FSM-Type = Moore. There is no realtime action needed

$c_1 c_2 = $ "11" $\Rightarrow$ impossible

fsm/fsm-03

3.4 Commande de lumières

FSM Type = Moore. There is no realtime action needed.
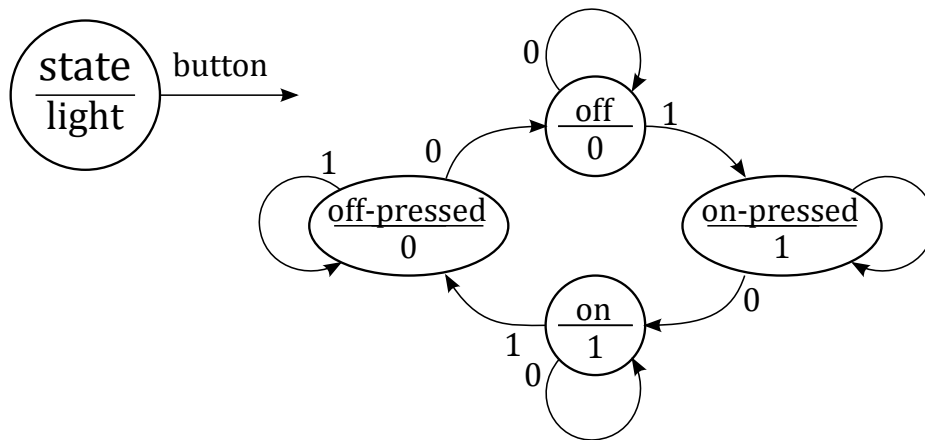
fsm/fsm-04

3.5 Détecteur de flanc montant

FSM Type = Moore and Mealy possible.

3.5.0.1 Timing Diagram
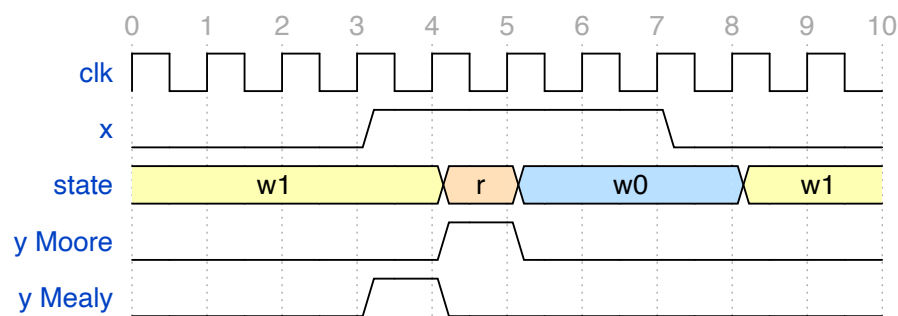
3.5.0.2 Grap

Moore FSM can be done with 3 states. Mealy FSM can be done with 2 states.

fsm/fsm-05

3.6 Détection de chaîne de caractères

FSM-Type = Mealy since an immediate response is needed.

3.6.0.1 Graph

fsm/fsm-06

3.7 Serrure électronique

FSM-Type = Moore. The output signal is during one clock period.
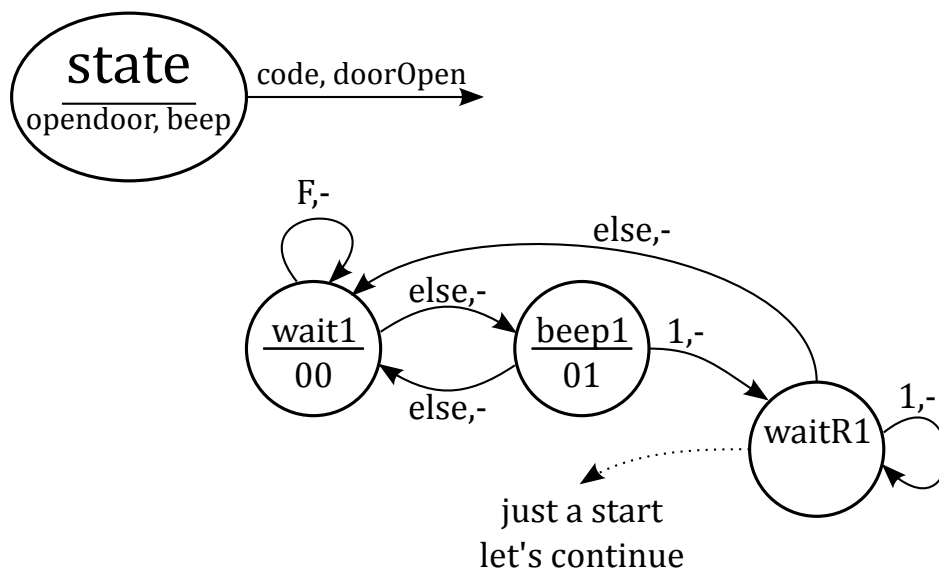
fsm/fsm-07

4 | FSM - Réduction de graphes

4.1 Réduction de graphe

4.1.0.1 Truth Table

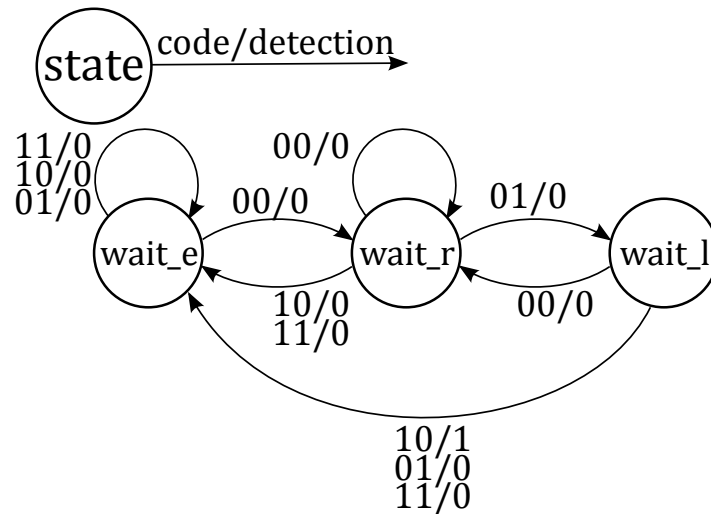| state \ x	0	1
st0	st0,0	st1,0
st1	st3,0	st2,0
st2	st3,0	st4,1
st3	st0,0	st1,0
st4	st5,1	st7,1
st5	st6,1	st7,1
st6	st0,0	st7,1
st7	st5,1	st4,1

The blue and green states can be combined to new states e.g. **st03** and **st47**. Draw also the new graph.

fsm/reduction-01

4.2 Réduction de graphe

4.2.0.1 Truth Table

state \ $x_1 x_2$	00	01	10	11
st0	st0,0	st2,0	st1,0	st0,0
st1	st1,0	st2,0	st1,0	st3,0
st2	st2,0	st2,0	st1,0	st3,0
st3	st5,1	st4,1	st3,0	st3,0
st4	st4,1	st4,1	st0,0	st3,0
st5	st5,1	st5,1	st0,0	st3,0

The blue and green states can be combined to new states e.g. **st12** and **st45**. Draw also the new graph.

fsm/reduction-02

5 | FSM - Codage des états

5.1 Circuit logique

$$Q_2^+ = D_2 = \overline{x \oplus Q_2}$$
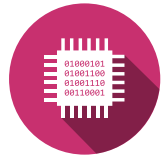$$Q_1^+ = D_1 = \overline{x} \ \overline{Q_2} \ \overline{Q_1} \ Q_0 + \overline{x} \ Q_2 \ Q_1 \ Q_0 + x\overline{Q_2} \ Q_1 \ \overline{Q_0} + x \ Q_2 \ \overline{Q_1} \ \overline{Q_0}$$
$$Q_0^+ = D_0 = \overline{x} \ \overline{Q_2} \ \overline{Q_1} \ \overline{Q_0} + \overline{x} \ Q_2 \ \overline{Q_1} \ Q_0 + x \ \overline{Q_2} \ \overline{Q_1} \ Q_0 + x \ Q_2 \ Q_1 \ \overline{Q_0} \qquad (3)$$
$$y_1 = Q_2$$
$$y_2 = Q_2 \ \overline{Q_1} \ \overline{Q_0}$$

fsm/coding-01

5.2 Circuit logique

$$Q_1^+ = x(Q_1 + Q_0)$$
$$Q_0^+ = xQ_1 + x\overline{Q_0}$$
$$y_1 = Q_1 Q_0 + xQ_1 \qquad (4)$$
$$y_0 = \overline{x}Q_1 + xQ_0$$

fsm/coding-02

5.3 Circuit logique

One-Hot Encoding Scheme was used.

$$\text{all arrows to a state} \begin{cases} D_0 = Q_0\overline{\text{step}} + Q_7\text{step cw} + Q_1\text{step } \overline{\text{cw}} \\ D_1 = Q_1\overline{\text{step}} + Q_0\text{step cw} + Q_2\text{step } \overline{\text{cw}} \\ D_2 = Q_2\overline{\text{step}} + Q_1\text{step cw} + Q_3\text{step } \overline{\text{cw}} \\ D_3 = Q_3\overline{\text{step}} + Q_2\text{step cw} + Q_4\text{step } \overline{\text{cw}} \\ D_4 = Q_4\overline{\text{step}} + Q_3\text{step cw} + Q_5\text{step } \overline{\text{cw}} \\ D_5 = Q_5\overline{\text{step}} + Q_4\text{step cw} + Q_6\text{step } \overline{\text{cw}} \\ D_6 = Q_6\overline{\text{step}} + Q_5\text{step cw} + Q_7\text{step } \overline{\text{cw}} \\ D_7 = Q_7\overline{\text{step}} + Q_6\text{step cw} + Q_0\text{step } \overline{\text{cw}} \end{cases} \qquad (5)$$

$$\text{states were the output is set} \begin{cases} c_1 = Q_0 + Q_1 + Q_7 \\ c_2 = Q_1 + Q_2 + Q_3 \\ c_3 = Q_3 + Q_4 + Q_5 \\ c_4 = Q_5 + Q_6 + Q_7 \end{cases}$$

fsm/coding-03

5.4 Circuit logique

Additional signal

The states Q_1 and Q_0 can distinguish 4 different clock periods. But the signal as 8 clockperiods repeating as a mirror.

$\Rightarrow$ An additional signal is needed, to differentiate.

5.4.0.1 Truth table

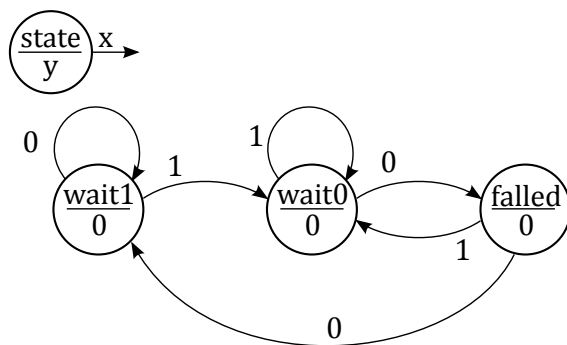| Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	c_1
0	0	0	0	0	1	0
0	0	1	0	1	1	0
0	1	0	1	1	0	0
0	1	1	0	1	0	0
1	0	0	0	0	0	1
1	0	1	1	0	0	0
1	1	0	1	1	1	0
1	1	1	1	0	1	0

5.4.0.3 Equations

$$D_2 = Q_0 Q_2 + \overline{Q_0} Q_1$$
$$D_1 = Q_0 \overline{Q_2} + \overline{Q_0} Q_1$$
$$D_0 = Q_1 \oplus \overline{Q_2} \tag{6}$$
$$c_1 = Q_2 \, \overline{Q_1} \, \overline{Q_0}$$

D_1 Karnaugh map (Q_2, Q_1 columns, Q_0 rows):

0	1	1	0
1	1	0	0

D_0 Karnaugh map:

1	0	1	0
1	0	1	0

5.4.0.2 Karnaugh Table

D_2 Karnaugh map (Q_2, Q_1 columns, Q_0 rows):

0	1	1	0
0	0	1	1

c_1 Karnaugh map:

0	0	0	1
0	0	0	0

fsm/coding-04

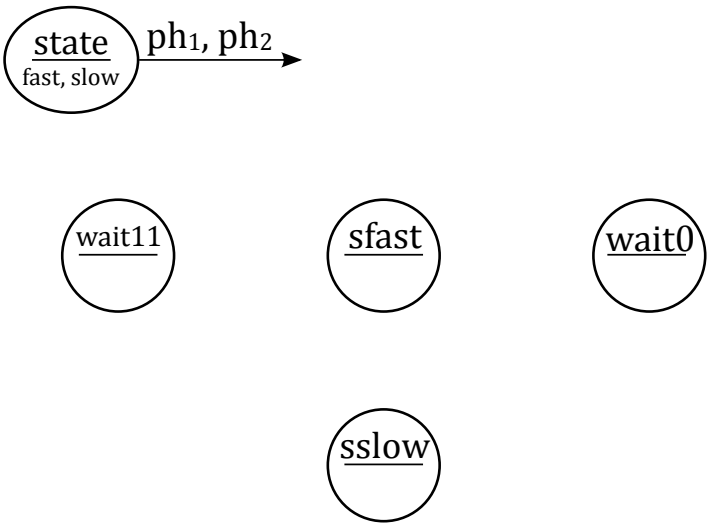## 5.5 Détecteur de flanc descendant

state	Q Encoding
wait1	10
wait0	00
falled	11

Next steps is to create the truth table and the Equations in order to draw the circuit.

fsm/coding-05

5.6 Détecteur de phase

state
fast, slow
ph₁, ph₂

wait11

sfast

wait0

sslow

5.6.0.1 State encoding (One-Hot)

state	Q Encoding
wait11	0001
sfast	0010
sslow	0100
wait0	1000

5.6.0.2 Equations

$$D_0 = \mathrm{ph}_1\mathrm{ph}_2$$
$$D_1 = (Q_0 + Q_1)\overline{\mathrm{ph}_1}\mathrm{ph}_2$$
$$D_2 = (Q_0 + Q_2)\mathrm{ph}_1\overline{\mathrm{ph}_2}$$
$$D_3 = \overline{\mathrm{ph}_1}\,\overline{\mathrm{ph}_2} + (Q_1)\mathrm{ph}_1\overline{\mathrm{ph}_2} + (Q_2)\overline{\mathrm{ph}_1}\mathrm{ph}_2 + Q_3(\mathrm{ph}_1 \oplus \mathrm{ph}_2)$$

$$(7)$$

$$\mathrm{fast} = Q_1$$
$$\mathrm{slow} = Q_2$$

fsm/coding-06