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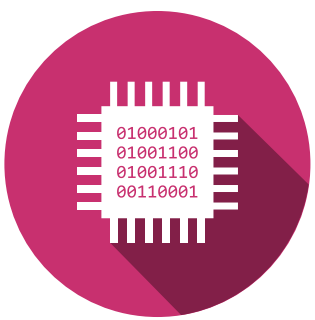
Digitales Design (DiD)

Kombinatorische Logikfunktionen

COM

Studiengang Systemtechnik
Studiengang Energie und Umwelttechnik
Studiengang Informatik und Kommunikationssysteme



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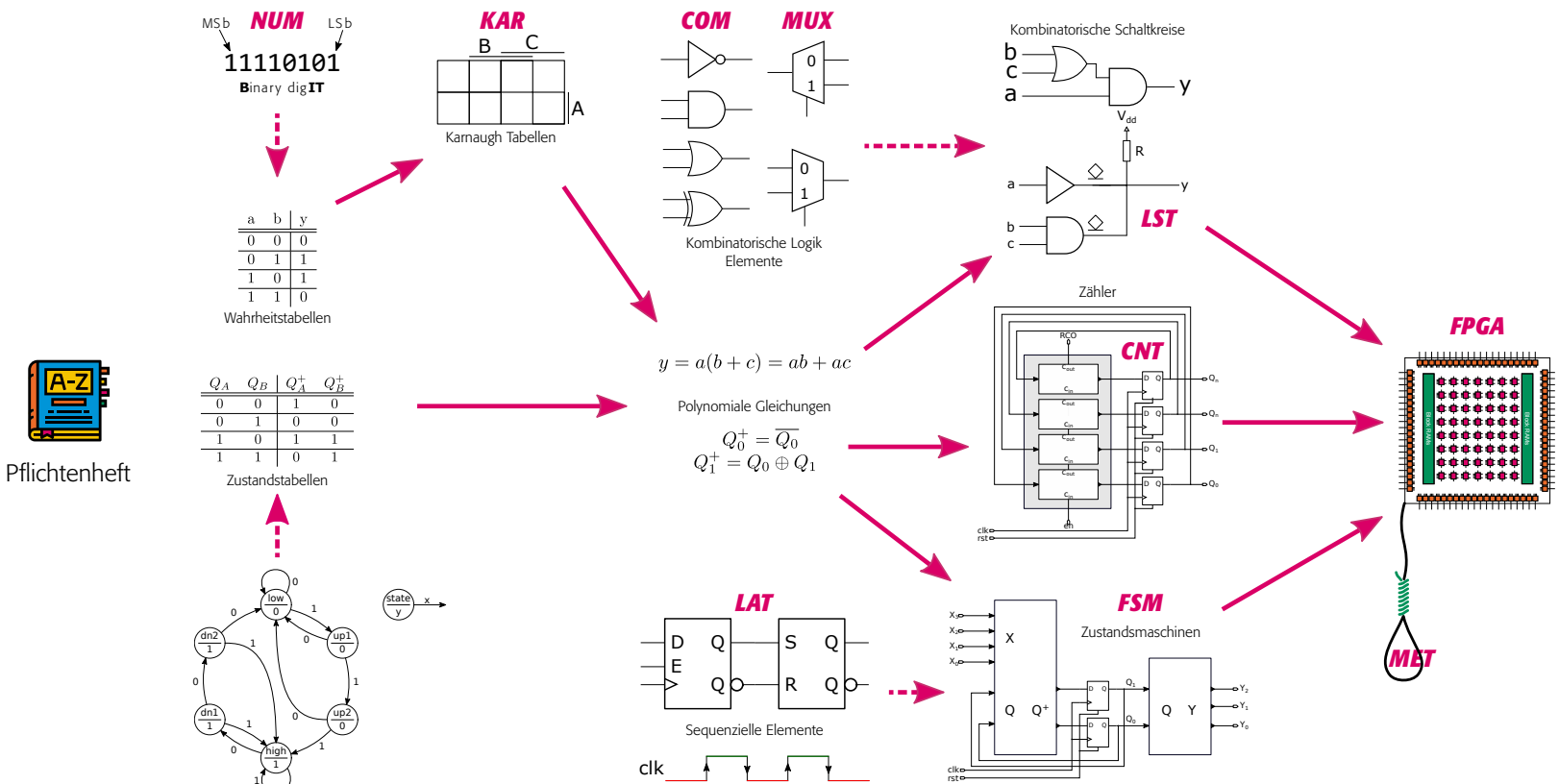
Aktueller Inhalt des Themas im Kurs



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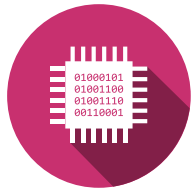
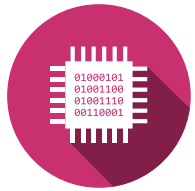


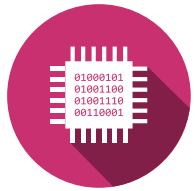
- **Darstellung von kombinatorischen Funktionen**
  - Wahrheitstabelle
  - Ablaufdiagramm
  - Venn-Diagramm
- Elementare Logikfunktionen
  - Puffer (Buffer)
  - Inverter (Inverter)
  - UND (AND)
  - ODER (OR)
  - Exklusic-ODER (XOR)
- Boolsche Algebra
- Vollständige Operatoren

Beispiel einer Logikfunktion



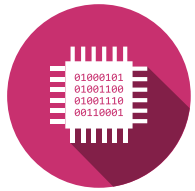
- Ein Lüftungssystem dient zur Regulierung der Temperatur eines Raumes.
- Als allgemeine Regel gilt: Wenn die $T°_{aussen} > T°_{lokal}$ ist, wird die Lüftungsanlage eingeschaltet, um den Raum zu erwärmen.
- Im Sommer und tagsüber wird jedoch die Lüftung zur Kühlung des Raumes eingesetzt: Ist die $T°_{aussen} < T°_{lokal}$, wird auch die Lüftung eingeschaltet.

Wahrheitstabelle (Truth Table)



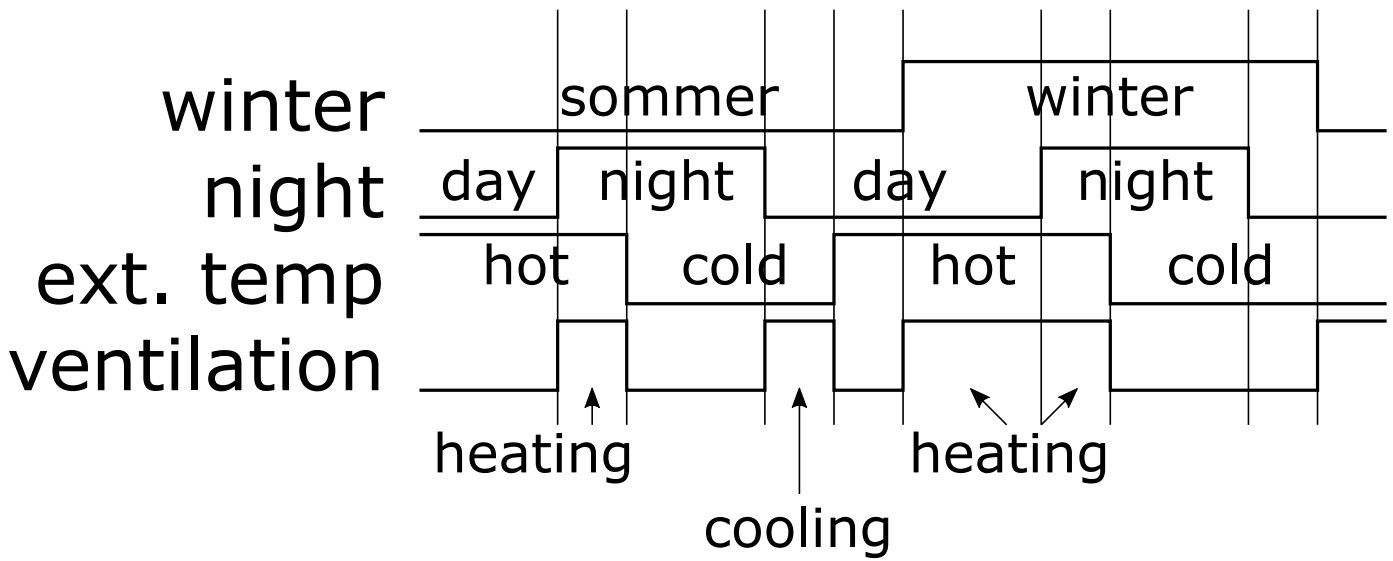
Winter	Nacht	Aussen wärmer	Ventilation
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Wahrheitstabelle (Truth Table)

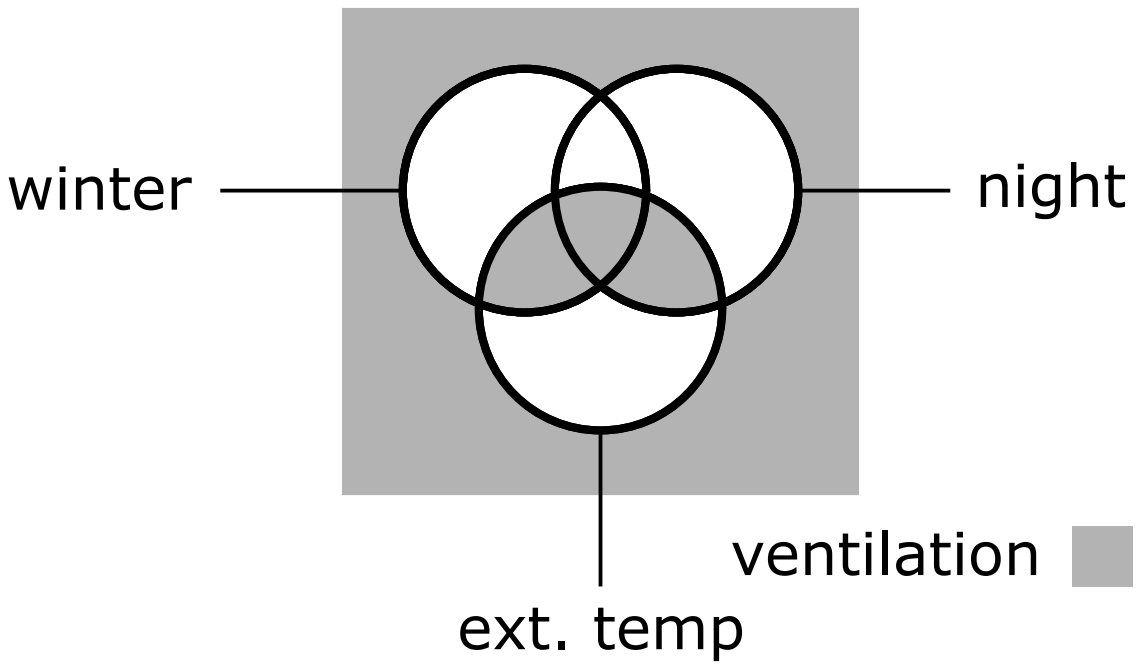


Winter	Nacht	Aussen wärmer	Ventilation
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

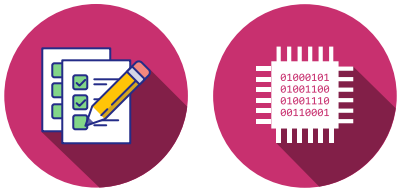
Ablaufdiagramm (Chronogram)



Venn-Diagramm (Venn Diagram)



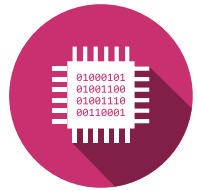
Aufgabe 1.1 *(com/representation-01)*



- Erstellen Sie die Wahrheitstafel einer Steuerschaltung für einen Stockwerk eines Lifts. Das System beträgt folgende Eingänge:
  - $\texttt{Tür}_{\texttt{offen}}$: ist dieses Signal auf 1, so soll der Lift stehen bleiben,
  - `Anruf` : ist dieses Signal auf 1, so soll der Lift zum Stockwerk,
  - `tiefer` : im Falle eines Anrufs soll der Lift aufsteigen,
  - `höher` : im Falle eines Anrufs soll der Lift hinabsteigen.
- Sind tiefer und höher beide auf `0`, heisst das, dass der Lift am Stockwerk ist
- Das System erstellt folgende Ausgänge:
  - $\texttt{Motor}_{\texttt{ein}}$: ist dieses Signal auf 1, so ist läuft der Motor,
  - auf : ist dieses Signal auf 1 und läuft der Motor, so steigt der Lift auf; ist dieses Signal auf `0` und läuft der Motor, so steigt der Lift hinab.

Das hier analysierte System nimmt den Fall nicht im Kauf, wo der Lift schon läuft.

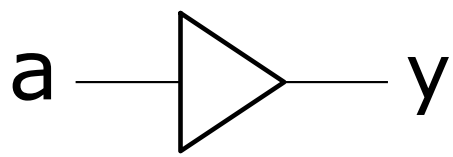
Inhalt

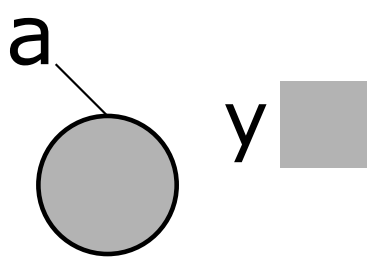


- Darstellung von kombinatorischen Funktionen
- **Elementare Logikfunktionen**
  - Puffer (Buffer)
  - Inverter (Inverter)
  - UND (AND)
  - ODER (OR)
  - Exklusic-ODER (XOR)
- Boolsche Algebra
- Vollständige Operatoren

Puffer

a	y
0	0
1	1

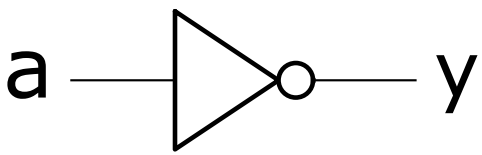


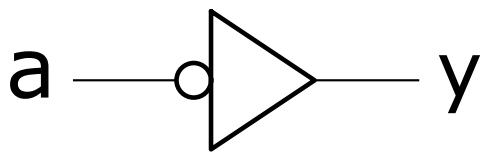


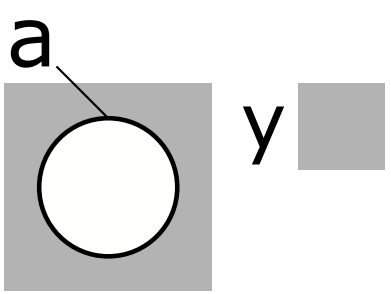
$$y = a$$

Inverter

a	y
0	1
1	0



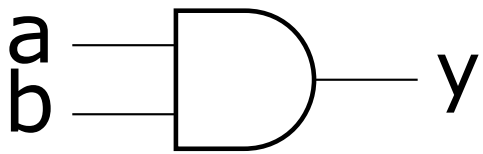


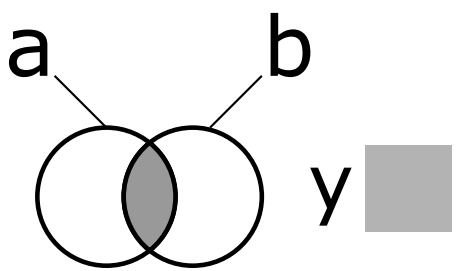


$$y = \bar{a}$$

Und (AND)

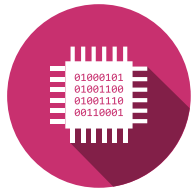
a	b	y
0	0	0
0	1	0
1	0	0
1	1	1



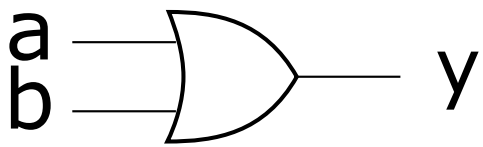


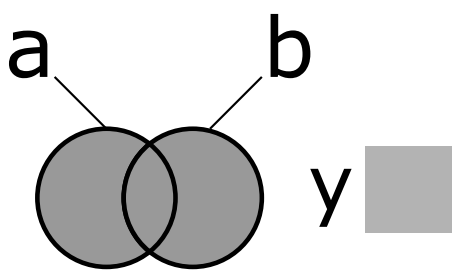
$$y = a * b$$

Oder (OR)



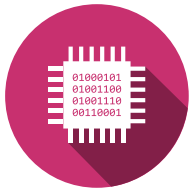
a	b	y
0	0	0
0	1	1
1	0	1
1	1	1



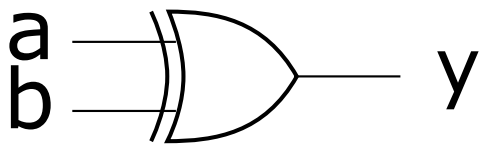


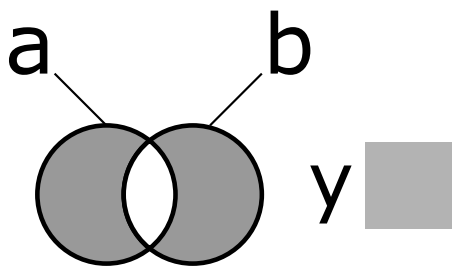
$$y = a + b$$

Exklusiv-Oder (XOR)



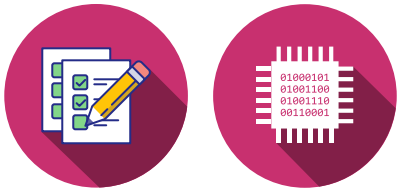
a	b	y
0	0	0
0	1	1
1	0	1
1	1	0





$$y = a \oplus b$$

Aufgabe 2.2 *(com/logic-functions-02)*



- Ergänzen Sie die nebenstehende Wahrheitstabelle.
- Vergleichen Sie das Ergebnis mit der Wahrheitstabelle der UND-Funktion von 2 Eingängen.

C	B	A	CA	CB	BA
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

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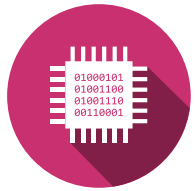
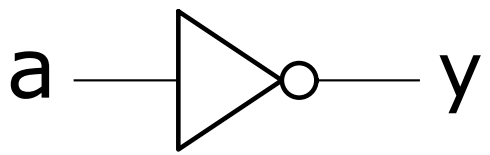


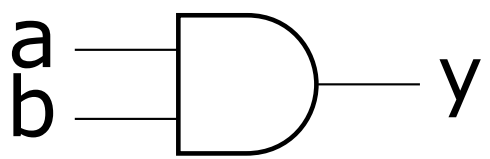
- Darstellung von kombinatorischen Funktionen
- Elementare Logikfunktionen
- **Boolsche Algebra**
  - Algebraische Operationen
  - Gesetz von De Morgan
  - XOR und inverter
  - Polynomische Form
- Vollständige Operatoren

Boolsche Algebra

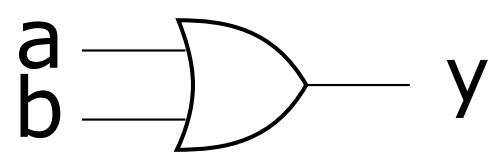
Operatoren



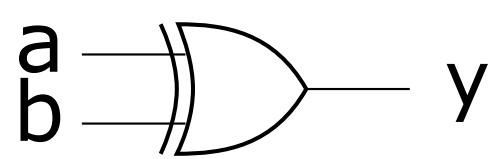
$$y = \bar{a}$$



$$y = a * b$$

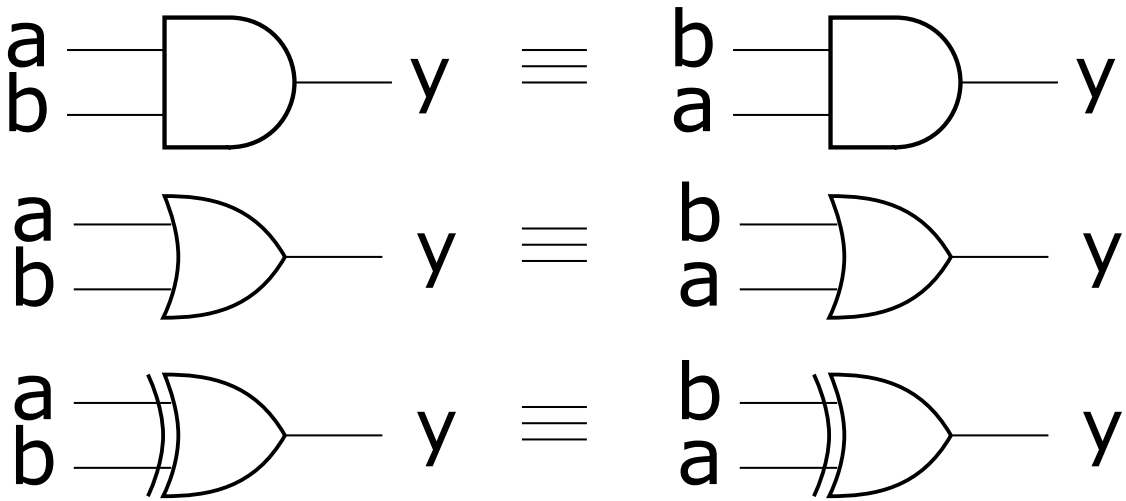


$$y = a + b$$



$$y = a \oplus b$$

Boolsche Algebra
Kommutivität



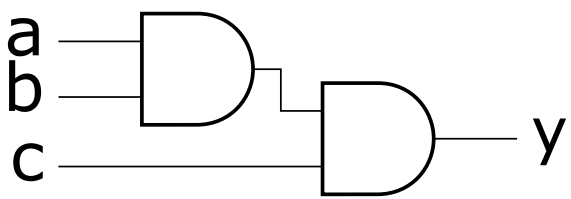
$$a * b = b * a$$

$$a + b = b + a$$

$$a \oplus b = b \oplus a$$

Boolsche Algebra

Assoziativität



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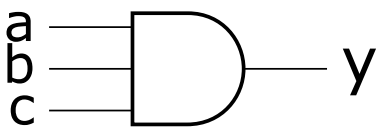
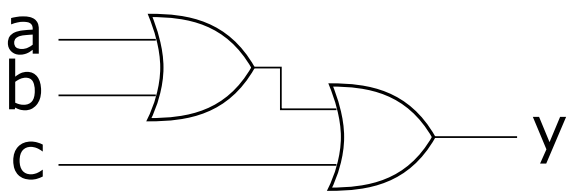


$$(a * b) * c = a * b * c$$



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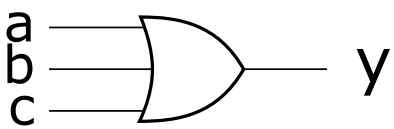
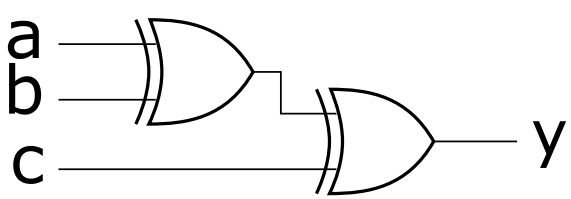


$$(a + b) + c = a + b + c$$



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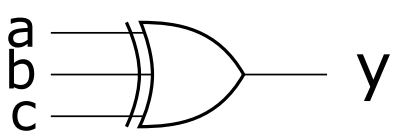
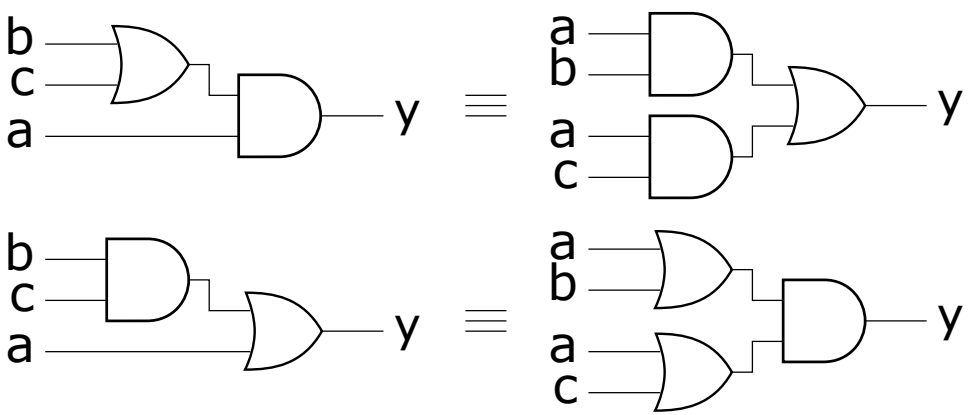


$$(a \oplus b) \oplus c = a \oplus b \oplus c$$

Boolsche Algebra
Distributivität

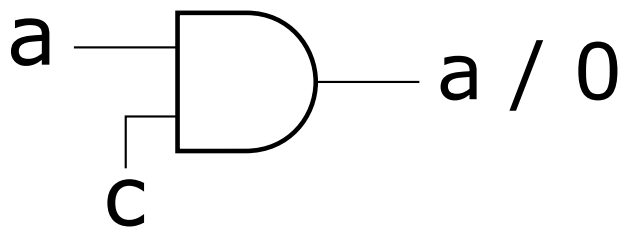


$$a * (b + c) = a * b + a * c$$

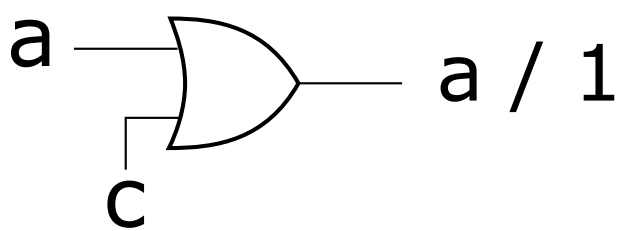
$$a + (b * c) = (a + b) * (a + c)$$

Boolsche Algebra

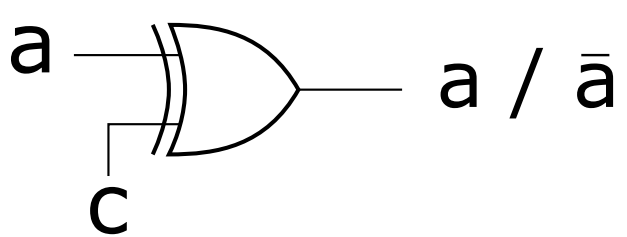
Neutrale und absorbierende Elemente



$$a * 1 = a$$
$$a * 0 = 0$$



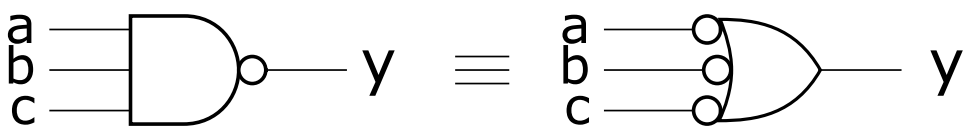
$$a + 0 = a$$
$$a + 1 = 1$$



$$a \oplus 0 = a$$
$$a \oplus 1 = \bar{a}$$

Boolsche Algebra

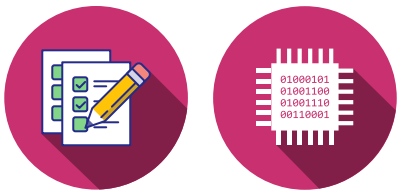
Theoreme von De Morgan



$$\overline{a * b * c} = \bar{a} + \bar{b} + \bar{c}$$

$$\overline{a + b + c} = \bar{a} * \bar{b} * \bar{c}$$

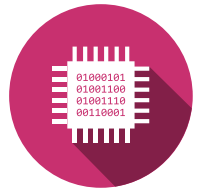
Aufgabe 3.3 *(com/algebra-03)*



- Ein System soll 2 Bits a und b übermitteln. Aus Sicherheitsgründen übermittelt er noch ein zusätzliches Bit, gegeben durch die Gleichung $y = a \oplus b$.

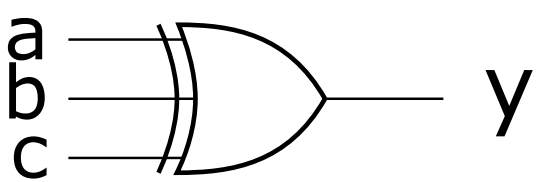
Zeigen Sie, dass es möglich ist, wenn das Bit a während der Übermittlung verloren gegangen ist, es mit Hilfe von y und b wiederherzustellen.

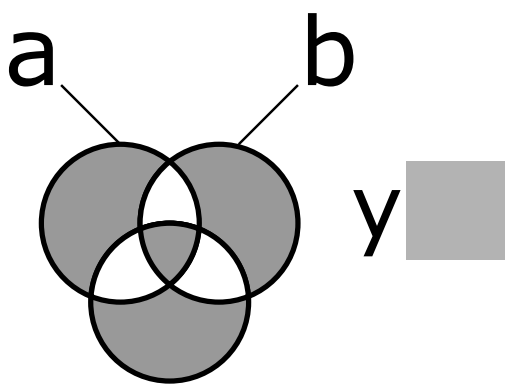
Polynomische Formen



- Monome $\bar{a}c$
- Polynom $ab + \bar{a}c + b\bar{c}$
- Summen Produkte
  Product of OR $(a + b) * (\bar{a} + c) * (b + \bar{c})$
- Exklusiv-ODER Produkte
  XOR of Products $a\bar{b} \oplus ac \oplus \bar{b}c$

Polynomische Formen
Exklusiv-Oder (XOR)



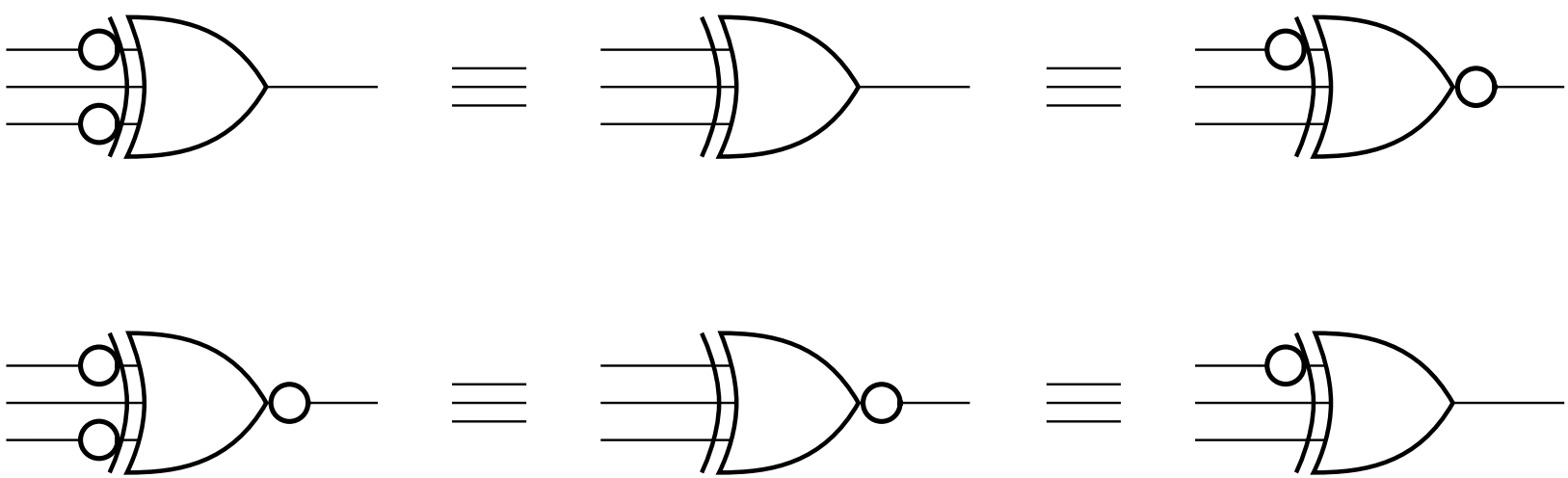


$$001 \Rightarrow \bar{a}\bar{b}c$$
$$010 \Rightarrow \bar{a}b\bar{c}$$
$$100 \Rightarrow a\bar{b}\bar{c}$$
$$111 \Rightarrow abc$$

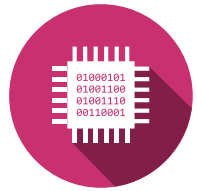
$$a \oplus b \oplus c$$

Polynomische Formen

Exklusiv-Oder (XOR) und Umkehrung



Inhalt

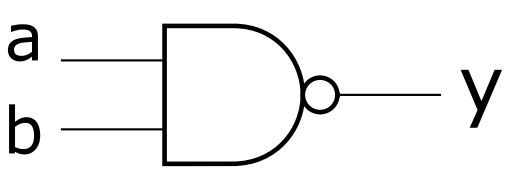


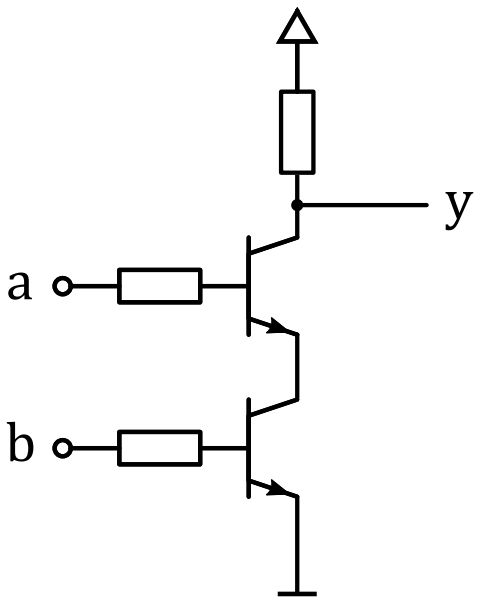
- Darstellung von kombinatorischen Funktionen
- Elementare Logikfunktionen
- Boolsche Algebra
- **Vollständige Operatoren**
  - NAND Operator
  - Transformation NAND
  - NOR Operator

Vollständige Operatoren

NAND Operator

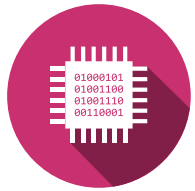
- Transistor-Schaltung





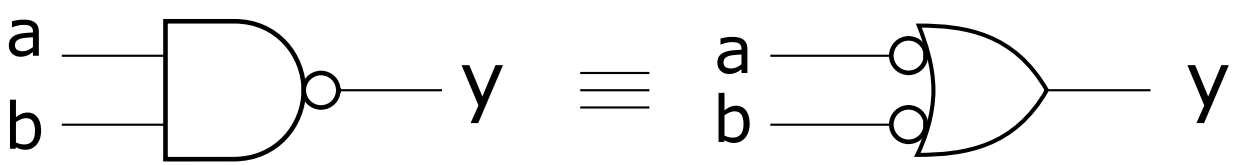
Vollständige Operatoren

NAND Operator

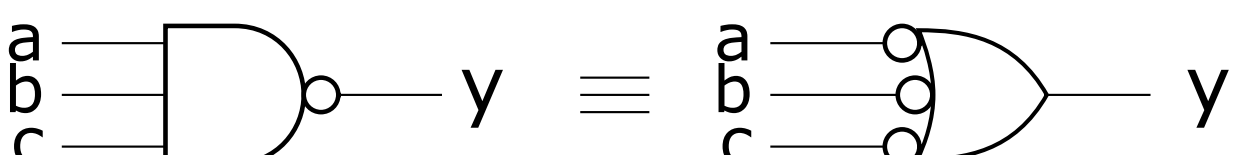


- Zwei Vertretungen nach De Morgans Recht

$$\overline{(a * b)} = \overline{a} + \overline{b}$$
$$\overline{(a + b)} = \overline{a} * \overline{b}$$

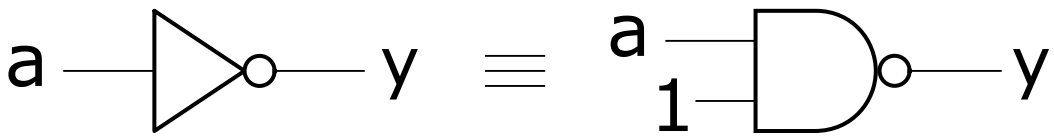


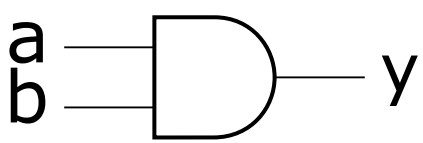
$$\overline{(a * b * c)} = \overline{a} + \overline{b} + \overline{c}$$
$$\overline{(a + b + c)} = \overline{a} * \overline{b} * \overline{c}$$

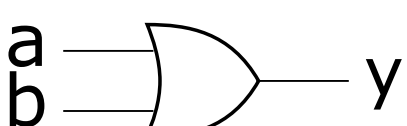


Vollständige Operatoren

NAND Operator

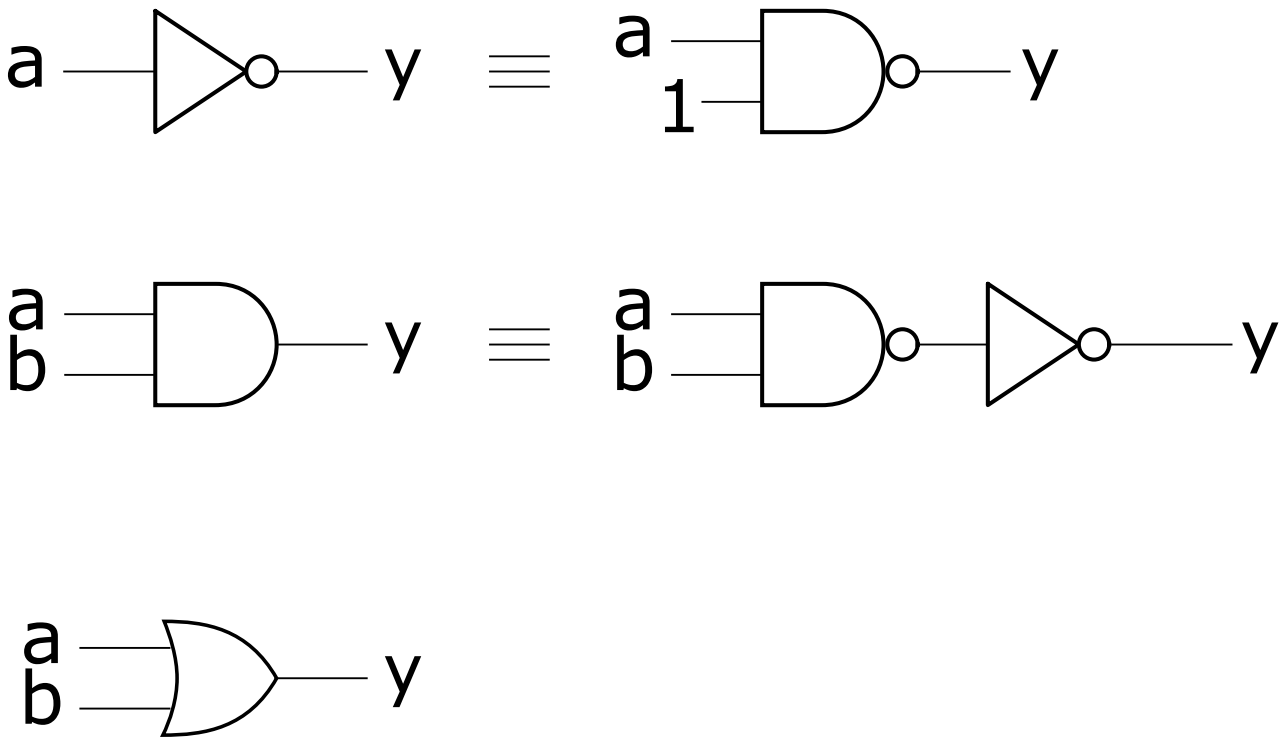






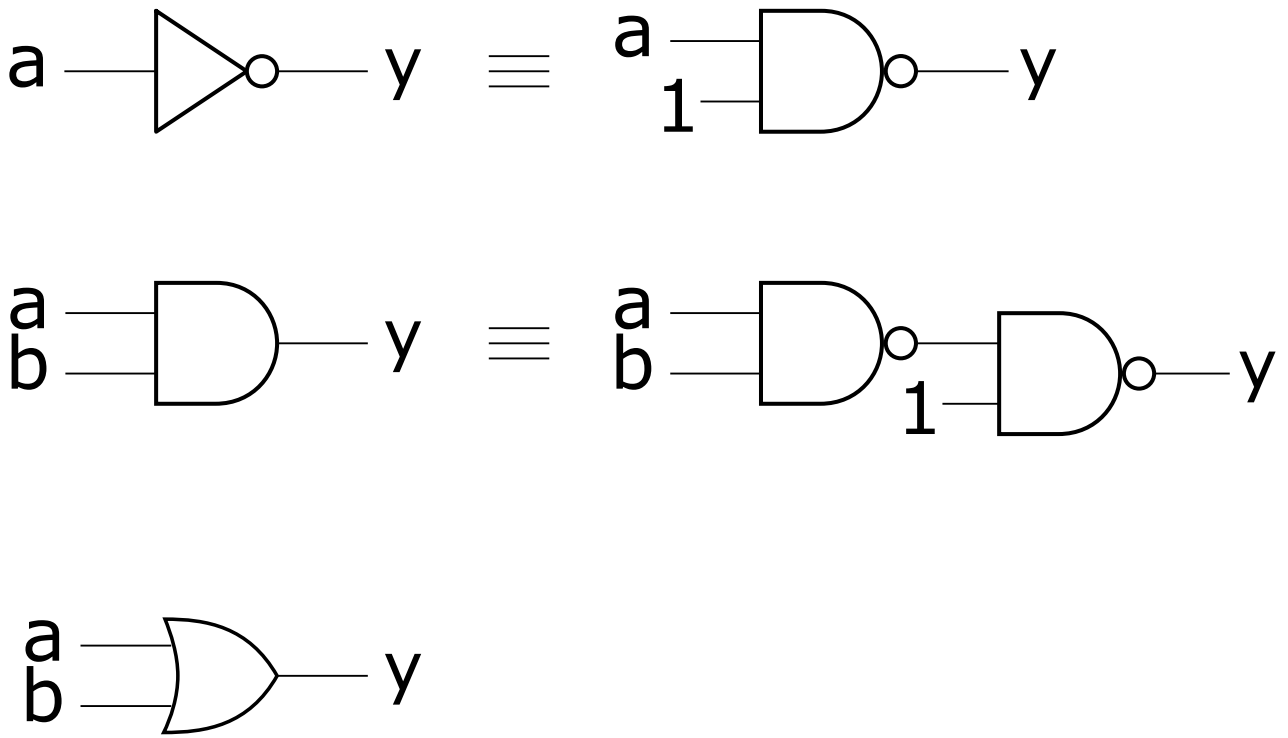
Vollständige Operatoren

NAND Operator



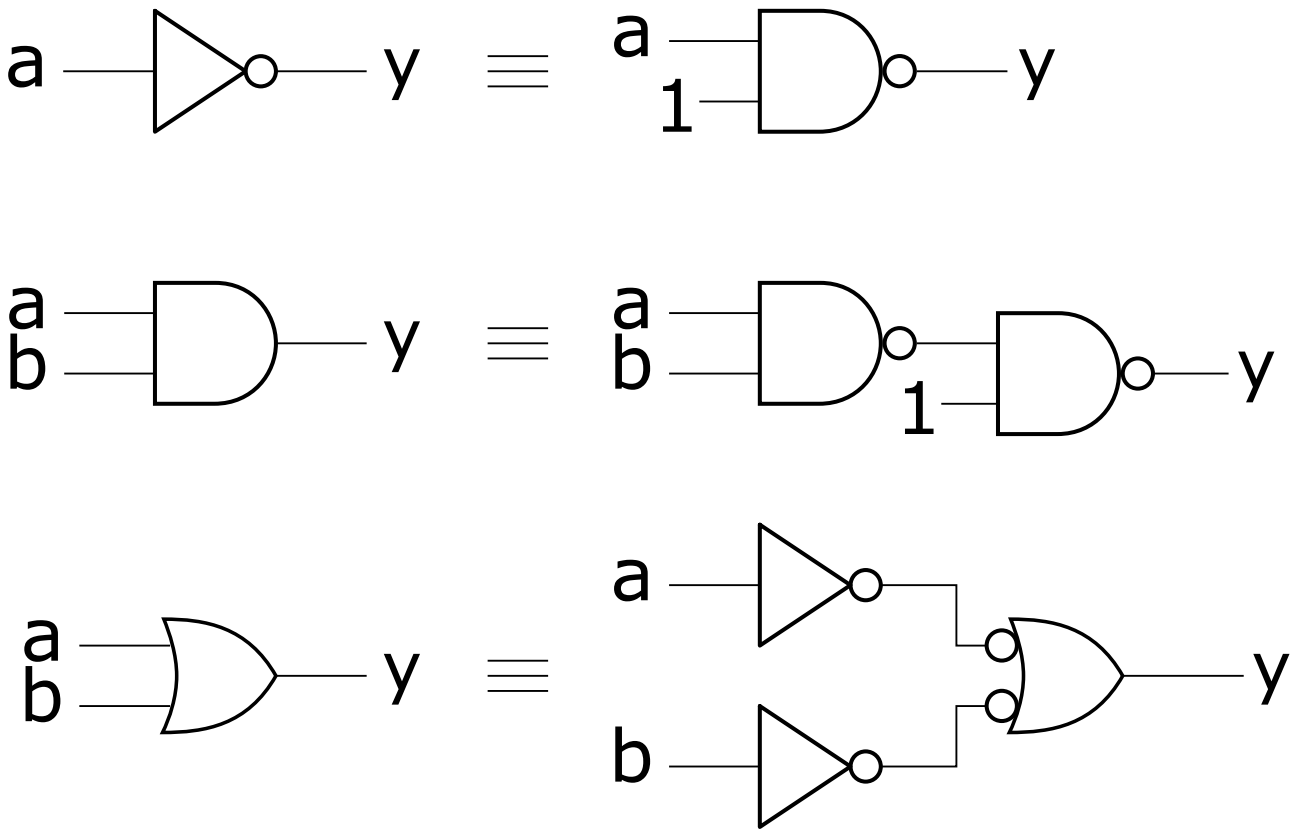
Vollständige Operatoren

NAND Operator



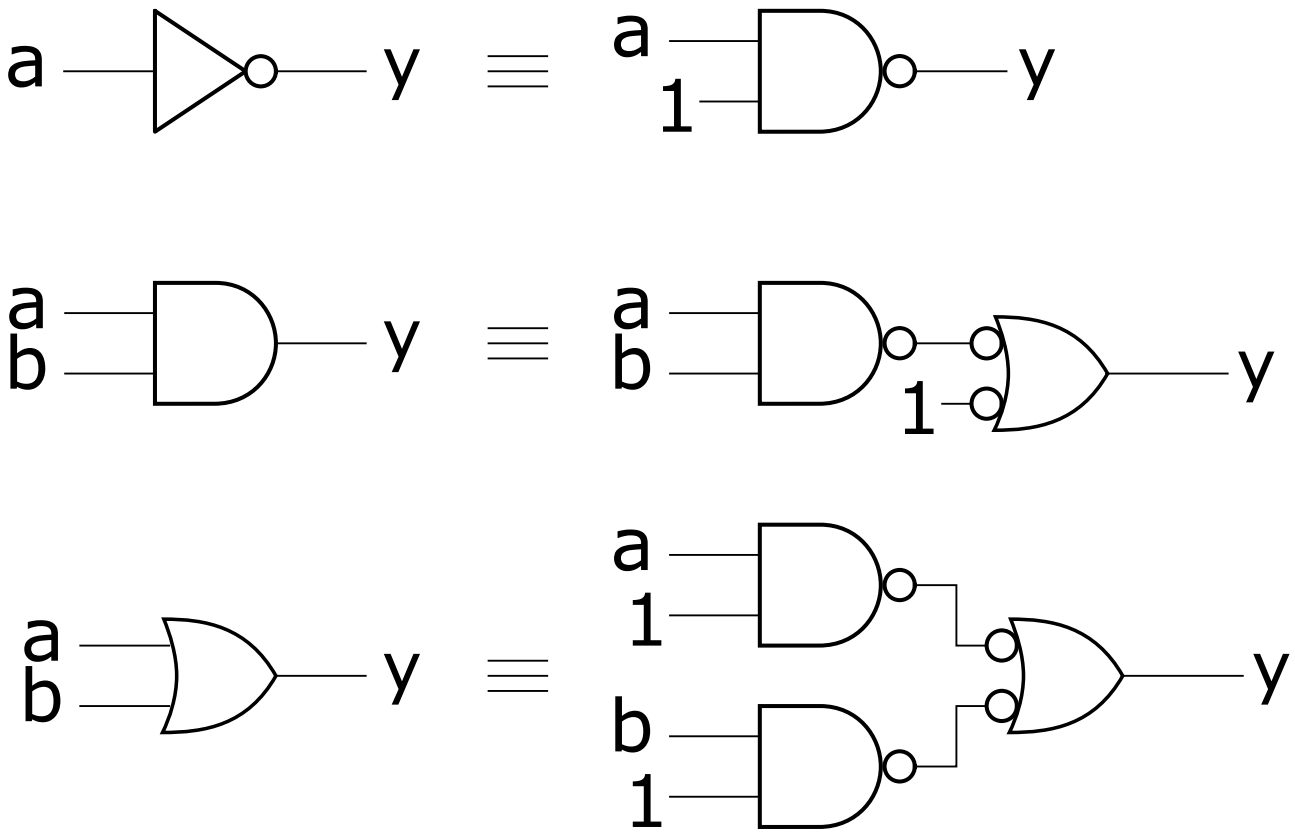
Vollständige Operatoren

NAND Operator



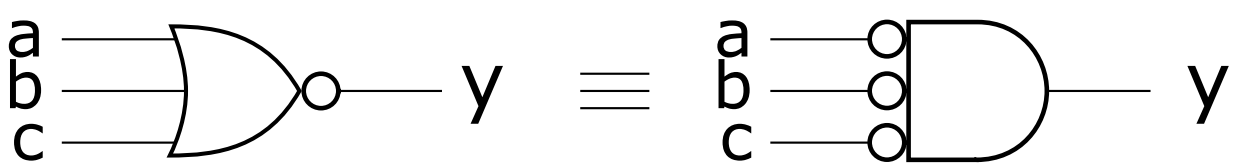
Vollständige Operatoren

NAND Operator



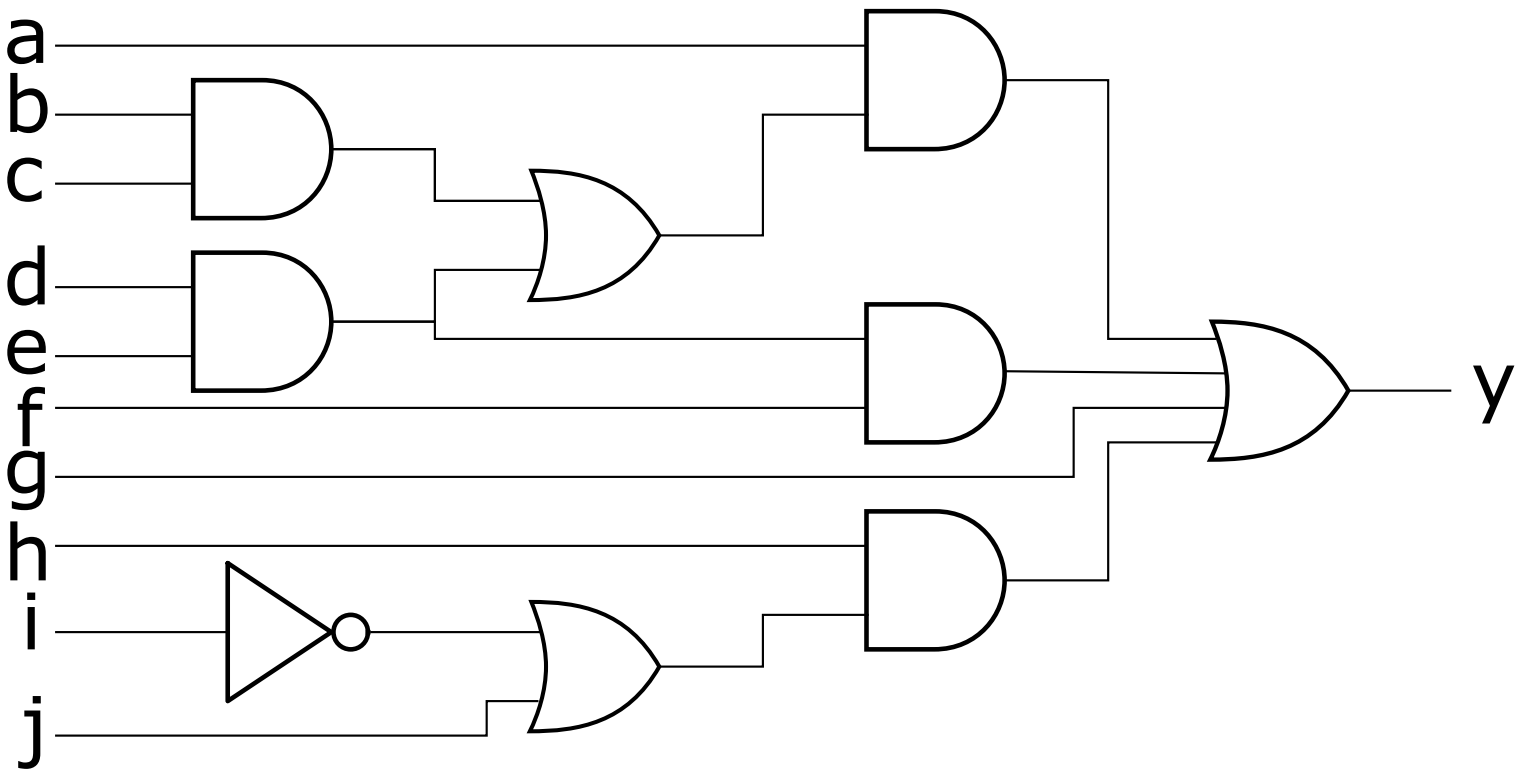
Vollständige Operatoren

NOR Operator



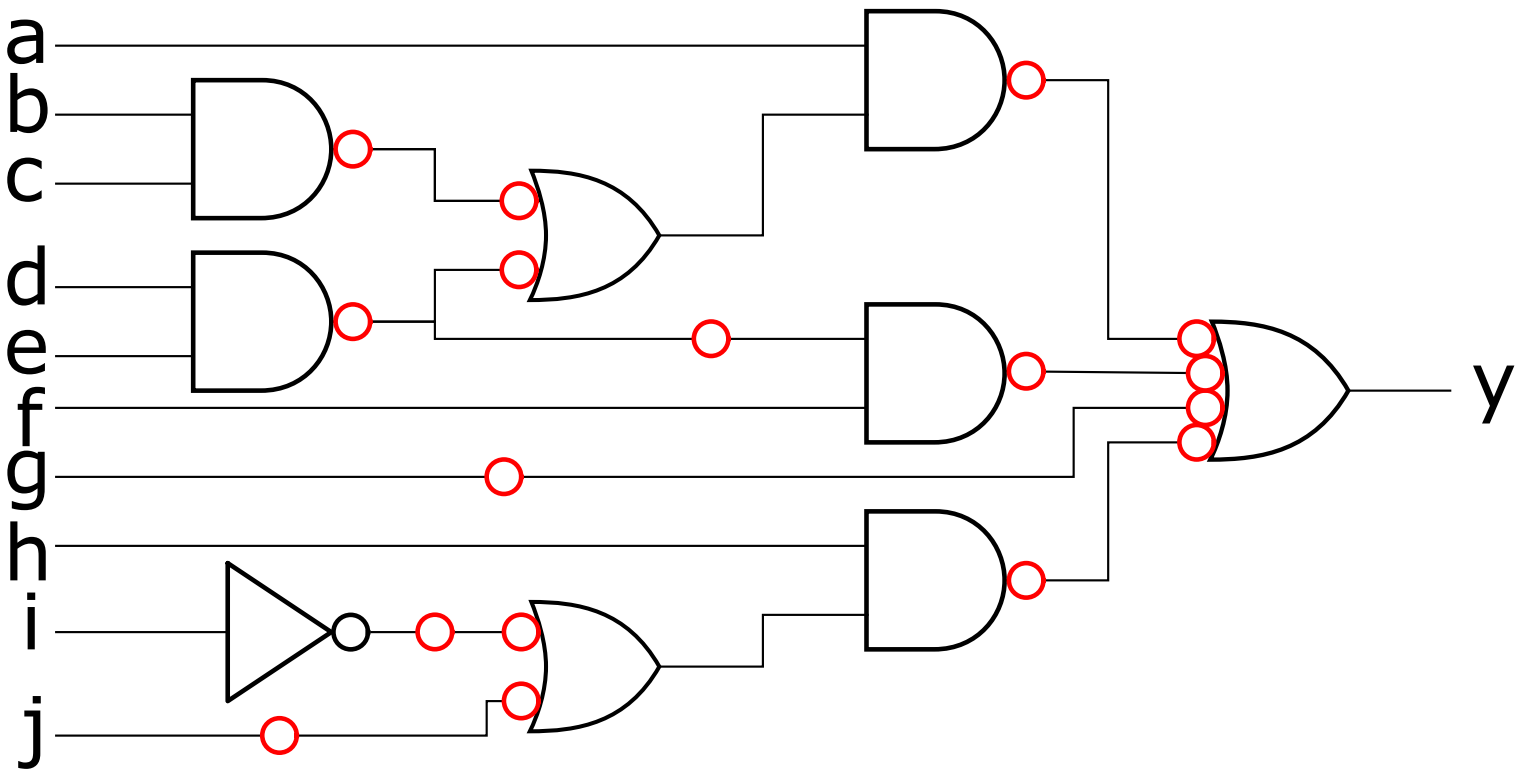
Vollständige Operatoren

Transformation



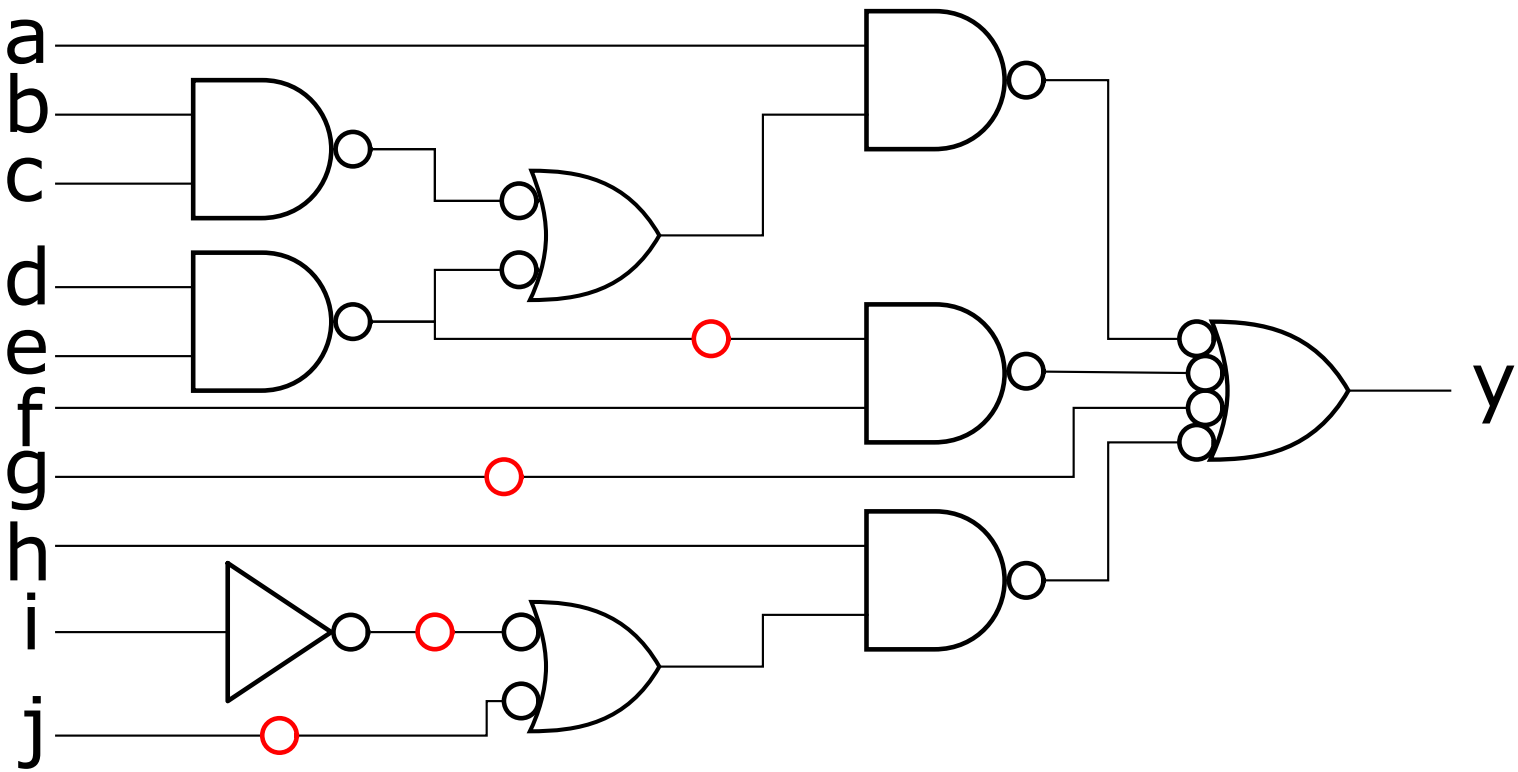
Vollständige Operatoren

Transformation



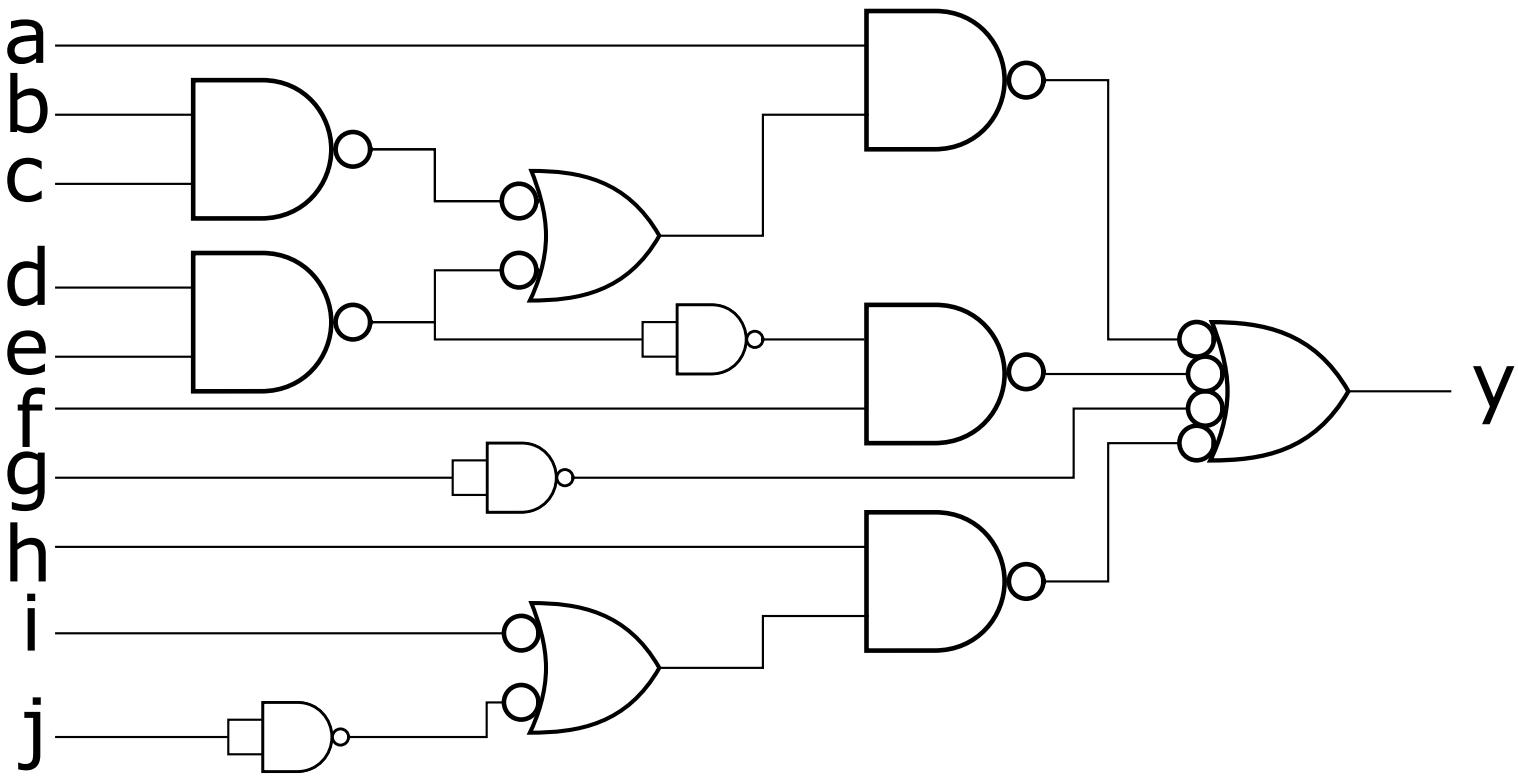
Vollständige Operatoren

Transformation

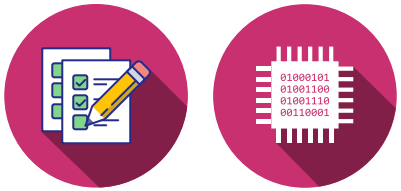


Vollständige Operatoren

Transformation



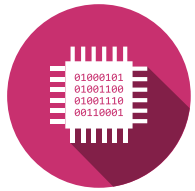
Aufgabe 4.1 *(com/operators-01)*



Mit Hilfe nur von NAND-Gattern zeichnen Sie das vollständige Schema einer Schaltung, welche die folgende Funktion erstellt.

$$y = \left(ab + cd + e + f * (\bar{g} + h)\right) * \bar{i}$$

Referenzen

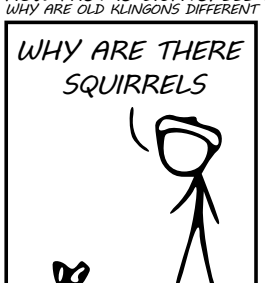


- [Max95] (Englisch) Einfach zum lesen
- [Man91] (Deutsch) Einfach zum lesen
- [Toc92] (Französisch) Einfach zum lesen
- [Kün97] (Deutsch) Benutzung von ISO Symbolen

QUESTIONS

CAN BE ASKED BY ANYONE ANYTIME

WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SVCHOST·EXE RUNNING
WHY AREN'T ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP
WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY WUBA LUBBA DUB DUB MEANING
WHY IS THERE A WHALE AND A POT FALLING
WHY ARE THERE SO MANY BIRDS IN SWISS
WHY IS THERE SO LITTLE RAIN IN WALLIS
WHY IS WALLIS WEATHER FORECAST ALWAYS WRONG
WHY ARE THERE MALE AND FEMALE BIKES
WHY ARE THERE BRIDESMAIDS
WHY DO DYING PEOPLE REACH UP
HOW FAST IS LIGHTSPEED
WHY ARE OLD KLINGONS DIFFERENT



WHY ARE THERE DINOSAUR GHOSTS
WHY IS THERE HELL IF

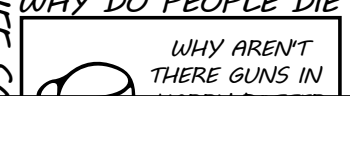
WHY DO IGUANAS DIE
WHY IS THERE CAFFEINE IN MY SHAMPOO
WHY HAVE DINOSAURS NO FUR
WHY DO TWINS HAVE DIFFERENT FINGERPRINTS
WHY ARE SWISS AFRAID OF DRAGONS
WHY AREN'T ECONOMISTS RICH
WHY DO AMERICANS CALL IT SOCCER
WHY ARE MY EARS RINGING
WHY IS 42 THE ANSWER TO EVERYTHING
WHY CAN'T NOBODY ELSE LIFT THORS HAMMER
WHY IS MARVIN ALWAYS SO SAD
WHY ARE THERE ANTS IN MY LAPTOP
WHY IS EARTH TILTED
WHY IS SPACE BLACK
WHY IS OUTER SPACE SO COLD
WHY ARE THERE PYRAMIDS ON THE MOON
WHY IS NASA SHUTTING DOWN
WHY ARE THERE TINY SPIDERS IN MY HOUSE
WHY DO SPIDERS COME INSIDE
WHY ARE THERE HUGE SPIDERS IN MY HOUSE
WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE
WHY ARE THERE SPIDERS IN MY ROOM
WHY ARE THERE SO MANY SPIDERS IN MY ROOM
WHY DO SPYDER BITES ITCH
WHY IS DYING SO SCARY
WHY IS THERE NO GPS IN LAPTOPS
WHY DO KNEES CLICK

WHY ARE THERE FEMALE

WHY IS HTTPS CROSSED OUT IN RED
WHY IS THERE A LINE THROUGH HTTPS
WHY IS THERE A RED LINE THROUGH HTTPS ON TWITTER
WHY IS HTTPS IMPORTANT

WHY IS YKK ON ALL ZIPPERS
WHY ARE THERE WEEKS
WHY DO I FEEL DIZZY
WHY IS THERE A SWARM OF ANTS
WHY IS THERE PILEGRIM
WHY IS THERE LAVA
WHY ARE THERE SO MANY CROWS IN ROCHESTER
WHY IS TO BE OR NOT TO BE FUNNY
WHY DO CHILDREN GET CANCER
WHY IS POSEIDON ANGRY WITH ODYSSEUS
WHY IS THERE ICE IN SPACE
WHY IS THERE AN OWL IN MY BACKYARD
WHY IS THERE AN OWL OUTSIDE MY WINDOW
WHY IS THERE AN OWL ON THE DOLLAR BILL
WHY DO OWLS ATTACK PEOPLE
WHY ARE FPGA's EVERYWHERE
WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE
WHY ARE THERE GODS
WHY ARE THERE TWO SPOCKS
WHAT IS https://xkcd.com/1256/
WHY DO THEY SAY T-MINUS
WHY ARE THERE OBELISKS
WHY ARE WRESTLERS ALWAYS WET

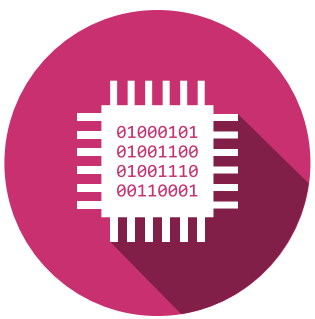
WHY IS LIFE SO
WHY ARE MY BOOBS ITCHY
WHY ARE CIGARETTES LEGAL
WHY ARE THERE DUCKS IN MY POOL
WHY IS JESUS WHITE
WHY IS THERE LIQUID IN MY EAR
WHY DO Q TIPS FEEL GOOD
WHY DO PEOPLE DIE



WHY ARE DOGS AFRAID OF FIR
WHY IS THERE NO KING IN E

Hes·so VALAIS WALLIS

Haute Ecole d'Ingénierie
Hochschule für Ingenieurwissenschaften



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