



# Synchronous counters

## Exercises Digital Design

### Solution vs. Hints:



While not every response provided herein constitutes a comprehensive solution, some serve as helpful hints intended to guide you toward discovering the solution independently. In certain instances, only a portion of the solution is presented.

## 1 | CNT - Counters by a power of 2

### 1.1 Downwards Counter

$$\begin{aligned} D_0 &= \overline{Q_0} \\ D_1 &= Q_1 \oplus \overline{Q_0} \\ D_2 &= Q_2 \oplus \overline{Q_1} \overline{Q_0} \\ D_3 &= Q_3 \oplus \overline{Q_2} \overline{Q_1} \overline{Q_0} \end{aligned} \quad (1)$$

$$\begin{aligned} D_0 &= Q_0^+ = Q_0 \oplus 1 \\ D_1 &= Q_1^+ = Q_1 \oplus \overline{Q_0} \\ D_2 &= Q_2^+ = Q_2 \oplus \overline{Q_0} \overline{Q_1} \\ D_3 &= Q_3^+ = Q_3 \oplus \overline{Q_0} \overline{Q_1} \overline{Q_2} \end{aligned} \quad (2)$$

*cnt/pow2-01*

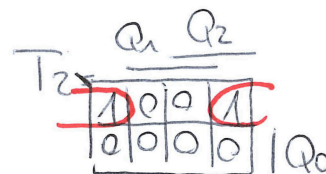
### 1.2 Downwards Counter

#### 1.2.1.1 Truth table

$Q_2 \dots Q_0$	$Q_2^+ \dots Q_0^+$	$T_2 \dots T_0$
000	111	111
001	000	001
010	001	011
011	010	001
100	011	111
101	100	001
110	101	011
111	110	001

#### 1.2.1.2 Equations

$$\begin{aligned} T_0 &= 1 \\ T_1 &= \overline{Q_0} \\ T_2 &= \overline{Q_1} \overline{Q_0} \end{aligned} \quad (3)$$



*cnt/cnt-pow2-02*



## 2 | CNT - Counters by any number

### 2.1 Downwards Counter

#### 2.1.1.1 Equations

$$D_0 = Q_0^+ = \overline{Q_0}$$

$$D_1 = Q_1^+ = Q_3 \overline{Q_0} + Q_2 \overline{Q_1} \overline{Q_0} + Q_1 Q_0 \quad (4)$$

$$D_2 = Q_2^+ = Q_3 \overline{Q_0} + Q_2 Q_1 + Q_2 Q_0$$

$$D_3 = Q_3^+ = Q_3 Q_0 + \overline{Q_3} \overline{Q_2} \overline{Q_1} \overline{Q_0}$$

#### 2.1.1.2 Sequence

$$9 \Rightarrow 8 \Rightarrow 7 \Rightarrow 6 \Rightarrow \dots 3 \Rightarrow 2 \Rightarrow 1 \Rightarrow 0 \Rightarrow 9 \Rightarrow 8 \Rightarrow \dots$$

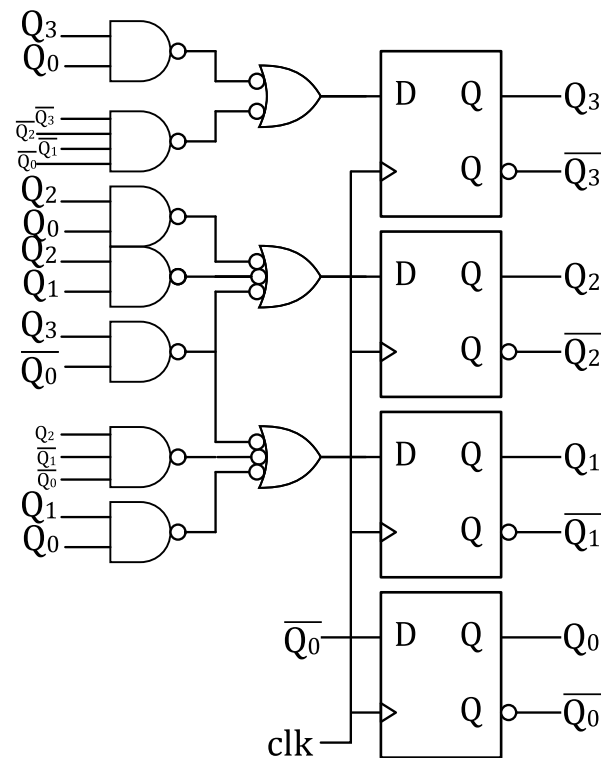
$$11 \Rightarrow 10 \Rightarrow 7$$

$$13 \Rightarrow 12 \Rightarrow 7$$

$$15 \Rightarrow 14 \Rightarrow 7$$

(5)

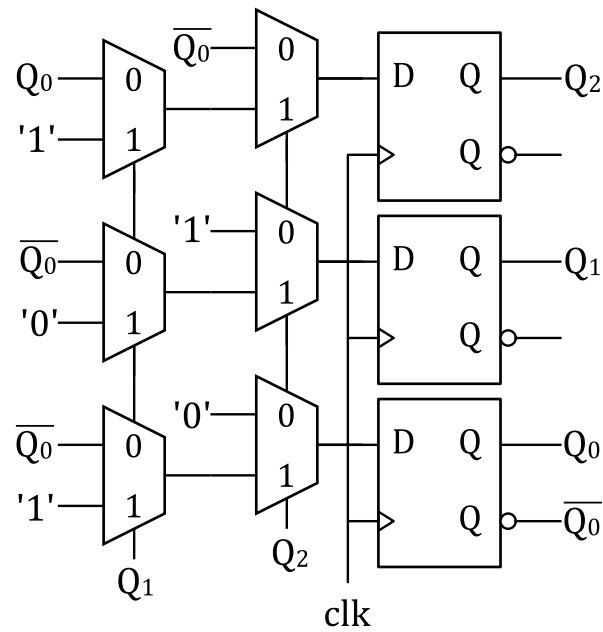
#### 2.1.1.3 Circuit



cnt/cnt-01



## 2.2 Downwards Counter



*cnt/cnt-02*

## 2.3 Johnson Counter

$$D_B = Q_A + \overline{Q_C}Q_B$$

or

$$D_B = \overline{Q_C}Q_A + Q_BQ_A$$

*cnt/cnt-03*



### 3 | CNT - Iterative circuits

#### 3.1 Counter with Synchronous Zeroing

Equation of a “+1” counter:

$$\begin{aligned} Q^+ &= D = Q \oplus \text{en} \\ c_{\text{out}} &= Q * \text{en} \end{aligned} \quad (6)$$

The **restart** can be added with the help of a AND gate and an inverter.

*cnt/cnt-iterativ-01*

#### 3.2 Counter with loading of a value

Equation of a “+1” counter:

$$\begin{aligned} Q^+ &= D = Q \oplus \text{en} \\ c_{\text{out}} &= Q * \text{en} \end{aligned} \quad (7)$$

The **load** can be added with the help of a Multiplexer 2-1.

*cnt/cnt-iterativ-02*

#### 3.3 up-down counter

**down-Counter**

$$\begin{aligned} Q_i^+ &= Q_i \oplus c_i \\ c_{i+1} &= \overline{Q_i} * c_i \end{aligned} \quad (8)$$

**up-Counter**

$$\begin{aligned} Q_i^+ &= Q_i \oplus c_i \\ c_{i+1} &= Q_i * c_i \end{aligned}$$

**up-down-Counter**

$$\begin{aligned} Q_i^+ &= Q_i \oplus c_i \\ c_{i+1} &= \text{updown} Q_i * c_i + \overline{\text{updown} Q_i} * c_i \end{aligned} \quad (9) \quad (10)$$

The difference of the down- vs the up-Counter is a XOR of  $Q_i$

*cnt/cnt-iterative-03*

#### 3.4 Programmable Counter

reset if  $P = Q$

sequence  $0 \Rightarrow 1 \Rightarrow \dots \Rightarrow P \Rightarrow 0$

Sequence lenght =  $P + 1$

*cnt/cnt-iterativ-04*