



# Vereinfachung mit Hilfe von Karnaugh-Tafeln

## Übungen Digitales Design



### Lösung vs. Hinweise:

Nicht alle hier gegebenen Antworten sind vollständige Lösungen. Einige dienen lediglich als Hinweise, um Ihnen bei der eigenständigen Lösungsfindung zu helfen. In anderen Fällen wird nur ein Teil der Lösung präsentiert.

## 1 | KAR - Karnaugh-Tafel

### 1.1 Darstellung von Monomen

$$y_1 \quad \begin{array}{c|cc|cc} & \overline{C} & C & \overline{D} & D \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_3 \quad \begin{array}{c|cc|cc} & \overline{C} & C & \overline{D} & D \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_5 \quad \begin{array}{c|cc|cc} & \overline{C} & C & \overline{D} & D \\ \hline 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_2 \quad \begin{array}{c|cc|cc} & \overline{C} & C & \overline{D} & D \\ \hline 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_4 \quad \begin{array}{c|cc|cc} & \overline{C} & C & \overline{D} & D \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_6 \quad \begin{array}{c|cc|cc} & \overline{C} & C & \overline{D} & D \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array} \begin{array}{l} A \\ B \end{array}$$

kar/karnaugh-01

### 1.2 Monome

$$\begin{aligned} y_1 &= \overline{B}\overline{D} \\ y_2 &= \overline{A}\overline{C} \end{aligned} \quad (1)$$

$$\begin{aligned} y_3 &= \overline{B}C\overline{D} \\ y_4 &= \overline{A}C\overline{D} \end{aligned} \quad (2)$$

kar/karnaugh-02



### 1.3 Darstellung von Polynomen

$$y_1 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 1 & 1 & 1 & 1 & \\ \hline 1 & 1 & 1 & 1 & \\ \hline 0 & 1 & 1 & 0 & \\ \hline 0 & 0 & 0 & 0 & \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_3 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 0 & 0 & 0 & 1 & \\ \hline 0 & 0 & 0 & 1 & \\ \hline 0 & 1 & 0 & 0 & \\ \hline 0 & 1 & 0 & 0 & \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_5 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 1 & 0 & 0 & 1 & \\ \hline 1 & 0 & 0 & 1 & \\ \hline 0 & 0 & 0 & 0 & \\ \hline 0 & 0 & 0 & 0 & \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_2 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 1 & 1 & 1 & 1 & \\ \hline 1 & 1 & 0 & 0 & \\ \hline 1 & 1 & 1 & 0 & \\ \hline 1 & 1 & 1 & 1 & \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_4 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 0 & 0 & 0 & 0 & \\ \hline 0 & 0 & 0 & 0 & \\ \hline 1 & 1 & 1 & 1 & \\ \hline 0 & 0 & 1 & 1 & \end{array} \begin{array}{l} A \\ B \end{array}$$

$$y_6 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 0 & 0 & 0 & 0 & \\ \hline 0 & 0 & 0 & 0 & \\ \hline 0 & 0 & 0 & 0 & \\ \hline 0 & 1 & 1 & 0 & \end{array} \begin{array}{l} A \\ B \end{array}$$

*kar/karnaugh-03*



## 2 | KAR - Vereinfachung in der Form von Produktsumme

### 2.1 Karnaugh-Tafel mit 4 Variablen

$$D\bar{B}A^* + \overline{DC}^* + B\bar{A}^* + \left\{ \frac{\overline{CB}}{\overline{CA}} \right. \quad (3)$$

*kar/productsun-01*

### 2.2 Karnaugh-Tafel mit 5 Variablen

$$\bar{E} \bar{D} \bar{B}^* + C\bar{B}A^* + D\bar{C}BA^* + \bar{D} \bar{B}A^* + E\bar{D}CA^* + EDC\bar{A} \quad (4)$$

*kar/productsun-02*

### 2.3 Karnaugh-Tafel mit 5 Variablen

$$\bar{E} \bar{D} \bar{C}^* + \bar{E} \bar{C} \bar{A}^* + \bar{E} \bar{D} \bar{B} \bar{A}^* + DCBA^* + ECB^* + \left\{ \frac{E\bar{B}\bar{A}}{\bar{C}\bar{B}\bar{A}} \right. \quad (5)$$

*kar/productsun-03*

### 2.4 Karnaugh-Tafel mit 5 Variablen

$$\bar{E}DB^* + \bar{C} \bar{B} \bar{A}^* + DBA^* + E\bar{D}CA + \bar{D} \bar{C} \bar{B} + \bar{E} \bar{D} \bar{C} \quad (6)$$

*kar/productsun-04*

### 2.5 Karnaugh-Tafel mit 5 Variablen

$$\bar{E}C\bar{A}^* + \bar{E}B\bar{A}^* + E \bar{C} \bar{B} \bar{A}^* + DA^* + ECB^* + D\bar{B} \quad (7)$$

*kar/productsun-05*

### 2.6 Karnaugh-Tafel mit 5 Variablen

$$\bar{C} \bar{B}^* + \bar{D} \bar{C} A^* + DCBA^* + \bar{E}CB + \bar{E}B\bar{A} \quad (8)$$

or

$$\bar{C} \bar{B}^* + \bar{D} \bar{C} A^* + DCBA^* + \bar{E}CB + \bar{E} \bar{C} \bar{A} \quad (9)$$

or

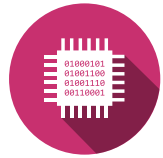
$$\bar{C} \bar{B}^* + \bar{D} \bar{C} A^* + DCBA^* + \bar{E} \bar{D} B + \bar{E}B\bar{A} \quad (10)$$

*kar/productsun-06*

### 2.7 Minimale Polynomialform

$$\bar{x}_3 x_2 \bar{x}_0^* + \bar{x}_2 x_0^* + x_1 x_0^* + \bar{x}_2 x_1^* \quad (11)$$

*kar/productsun-07*



## 2.8 Inverse Funktion

$$\overline{E} C \overline{A}^* + CB^* + DBA^* + E \overline{B} A^* \quad (12)$$

*kar/productsun-08*

## 2.9 Minimale Polynomialform

$y$ : 5 termes;  $\overline{y}$ : 4 termes

*kar/productsun-09*

## 2.10 Funktion von 5 Variablen

$$DCA^* + DCB^* + CBA^* + DBA^* + EDC^* + EBA^* + ECA^* + EDA^* + ECB^* + EDB^*$$

*kar/productsun-10*

## 2.11 Unvollständig definierte Funktion

$$x_4 x_3 + x_2 x_1 \quad (13)$$

or

$$x_3 x_1 + x_4 x_2 \quad (14)$$

or

$$x_4 x_1 + x_3 x_2 \quad (15)$$

*kar/productsun-11*

## 2.12 Unvollständig definierte Funktion

$$A = \overline{B_2} \overline{B_0}^* + B_2 B_0^* + B_1 + B_3^*$$

$$B = \overline{B_2}^* + \overline{B_1} \overline{B_0}^* + B_1 B_0^*$$

$$C = B_2^* + \overline{B_1}^* + B_0$$

$$D = \overline{B_2} \overline{B_0}^* + B_3^* + B_2 \overline{B_1} B_0^* + \overline{B_2} B_1^* + B_1 \overline{B_0} \quad (16)$$

$$E = \overline{B_2} \overline{B_0}^* + B_1 \overline{B_0}^*$$

$$F = B_3^* + B_2 \overline{B_1}^* + B_2 \overline{B_0}^* + \overline{B_1} \overline{B_0}^*$$

$$G = B_3^* + B_2 \overline{B_1} + \overline{B_0} B_1^* + B_2 \overline{B_1}$$

*kar/productsun-12*



### 3 | KAR - Vereinfachung von XOR-Funktionen

#### 3.1 Darstellung von XOR-Funktionen

$$y_1 \begin{array}{c|cc} & \overline{C} & D \\ \hline 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array} \left| \begin{array}{l} A \\ B \end{array} \right.$$

$$y_3 \begin{array}{c|cc} & \overline{C} & D \\ \hline 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{array} \left| \begin{array}{l} A \\ B \end{array} \right.$$

$$y_{7\&8\&9} \begin{array}{c|cc} & \overline{C} & D \\ \hline 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array} \left| \begin{array}{l} A \\ B \end{array} \right.$$

$$y_2 \begin{array}{c|cc} & \overline{C} & D \\ \hline 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array} \left| \begin{array}{l} A \\ B \end{array} \right.$$

$$y_{4\&5\&6} \begin{array}{c|cc} & \overline{C} & D \\ \hline 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \left| \begin{array}{l} A \\ B \end{array} \right.$$

kar/xor-01

#### 3.2 Minimale Polynomialform

$$y = x_1 x_0^* + \overline{x_2} x_0^* + \overline{x_2} x_1^* + \overline{x_3} x_2 \overline{x_0}^*$$

kar/xor-02

#### 3.3 Minimale Polynomialform

$$\overline{E} \overline{D} C^* + \overline{E} \overline{C} \overline{B} \overline{A}^* + \overline{E} D B A^* + \overline{E} \overline{D} \overline{A}^* + \overline{E} C B^* + E \overline{D} \overline{C} A^* + E D C \overline{B}^* + E D \overline{C} B \overline{A}^* + \begin{cases} E D \overline{B} A \\ E \overline{C} \overline{B} A \end{cases}$$

kar/xor-03

#### 3.4 Darstellung in der Form von XOR von Produkten

Many possible solutions e.g.:  $Y = \overline{D} \overline{B} \oplus \overline{D} A \oplus C A \oplus C B$

kar/xor-04



### 3.5 Darstellung in der Form von XOR von Produkten

Many possible solutions e.g.:  $y = x_0 \oplus x_1 \overline{x_0} \oplus x_2 \overline{x_0} \oplus x_3 x_2 \overline{x_1}$

*kar/xor-05*

### 3.6 Addierer

$$s_0 = a_0 \oplus b_0$$

$$s_1 = a_1 \oplus b_1 \oplus a_0 b_0$$

$$s_1 = \overline{a_1} b_1 \oplus a_1 \overline{b_1} \oplus a_0 b_0 \quad (17)$$

$$s_2 = a_1 b_1 \oplus \begin{cases} a_0 \overline{b_1} b_0 \oplus \overline{a_1} a_0 b_0 \\ a_0 b_1 b_0 \oplus a_1 a_0 b_0 \end{cases}$$

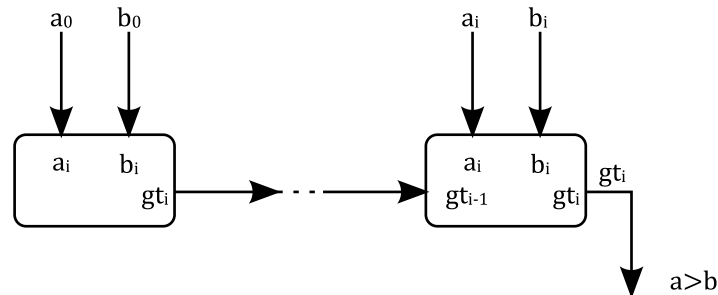
*kar/xor-06*



## 4 | KAR - Funktionen mit einer grossen Anzahl an Eingängen

### 4.1 Zahlenvergleich

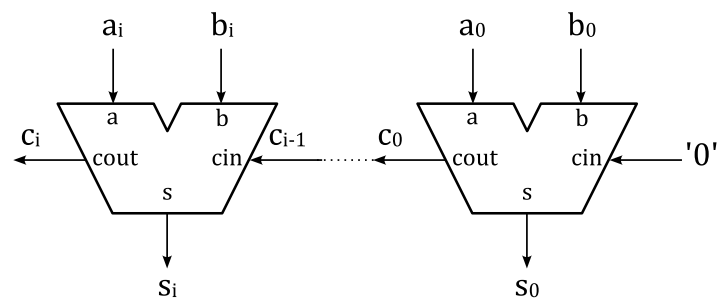
Possible with an iterative blocschema.



*kar/manyinputs-01*

### 4.2 Binäraddierer

Possible with an iterative blocschema.



*kar/manyinputs-02*

### 4.3 Thermometer-Code zu Binärcode Umwandlung

One possible solution is.

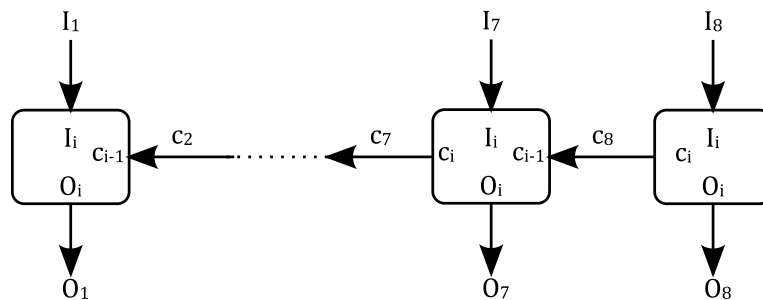
$$\begin{aligned} B_2 &= T_4 \\ B_1 &= T_2 + T_6 \overline{T_4} \\ B_0 &= ??? \end{aligned} \quad (18)$$

*kar/manyinputs-03*



#### 4.4 Übermittlung anhand der Priorität

Possible with an iterative blocschema.



*kar/manyinputs-04*

#### 4.5 Logik für Zähler ohne Rückgang auf Null

The solutions involves an Adder  $x+1$  and a Comparator to  $0xFFFF$

*kar/manyinputs-05*

#### 4.6 Addierer mit Sättigung

The output  $c_{out}$  of an iterative adder indicates an overflow.

*kar/manyinputs-06*

#### 4.7 BCD-codierte Zahlen

The BCD-Adder is a special case of a normal adder. Only the values between 0...9 exist. Therefore  $c_{out} = 1$  if the sum is  $> 10$ . In this case  $-10$  has to be subtracted from the output.

*kar/manyinputs-07*

#### 4.8 Mehrheitsfunktion mit 7 Eingängen

A concatenation of adders with a comparison

*kar/manyinputs-08*

#### 4.9 Arithmetische und logische Einheit

First determine the schema of an adder and that of a subtractor, tie them together and create the logical functions.

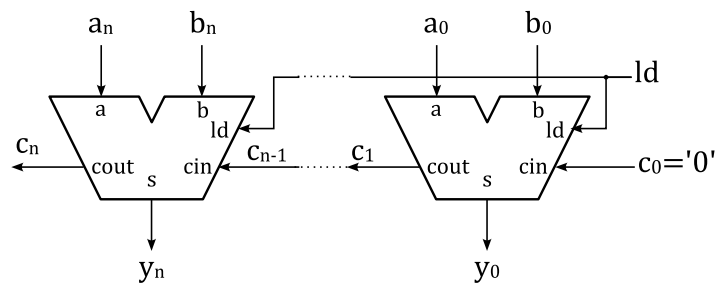
*kar/manyinputs-09*





#### 4.10 Logik für Programmzähler

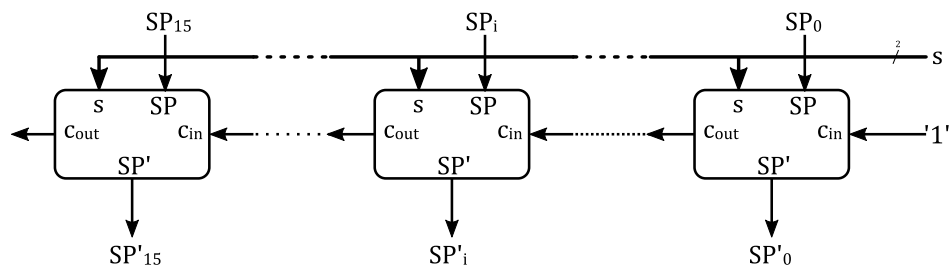
An iterative circuits with a load input.



*kar/manyinputs-10*

#### 4.11 Logik für Stackpointer

An iterative circuits with a 2bit selection input.



*kar/manyinputs-11*