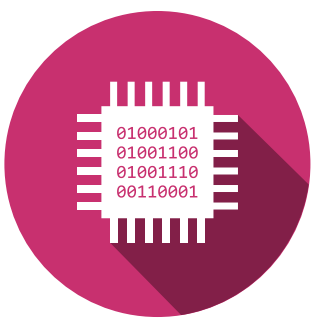
Hes·so VALAIS WALLIS
Haute Ecole d'Ingénierie
Hochschule für Ingenieurwissenschaften

Conception numérique (DiD)

Simplification par table de Karnaugh

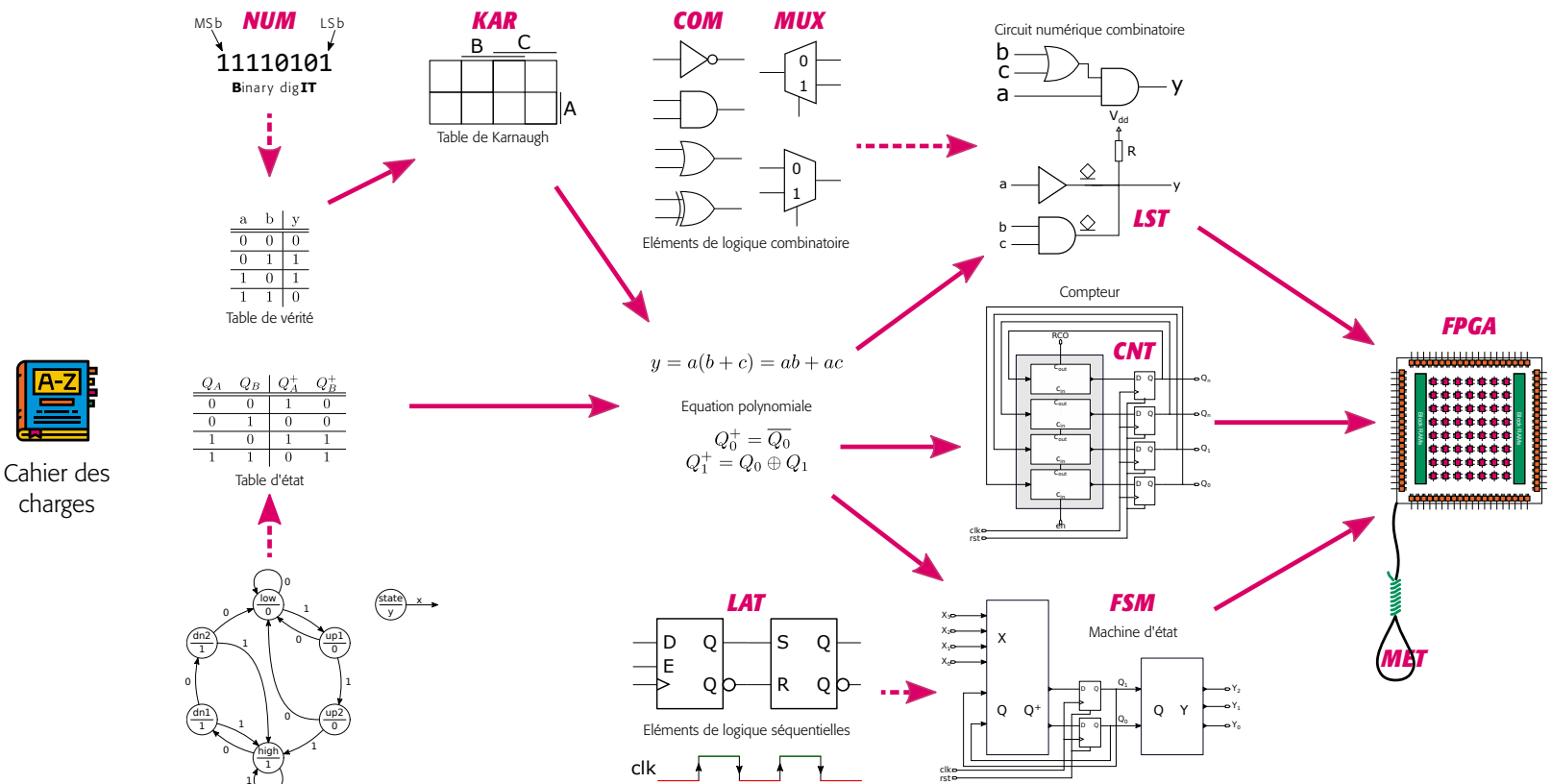
KAR

Filière Systèmes industriels
Filière Energie et techniques environmentales
Filière Informatique et systèmes de communications

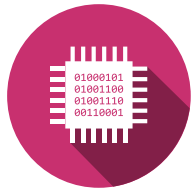


Silvan Zahno silvan.zahno@hevs.ch
Christophe Bianchi christophe.bianchi@hevs.ch
François Corthay francois.corthay@hevs.ch

Situation du thème dans le cours



Contenu

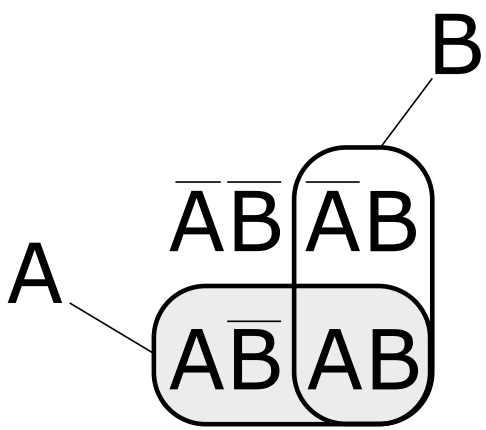


- **Tables de Karnaugh**
  - à 2 variables
  - à 3 variables
  - à 4 variables
  - à plus de 4 variables
- Simplification sous forme de sommes de produits
- Simplification de fonctions OU-exclusif
- Fonctions avec un nombre élevé d’entrées

Table de Karnaugh

à 2 variables

Est un diagramme de Venn



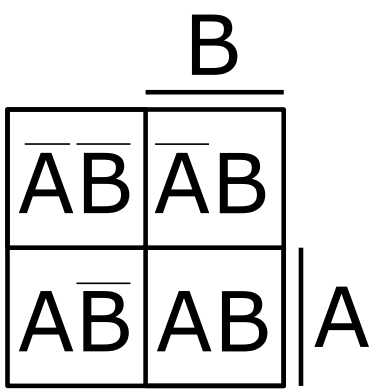
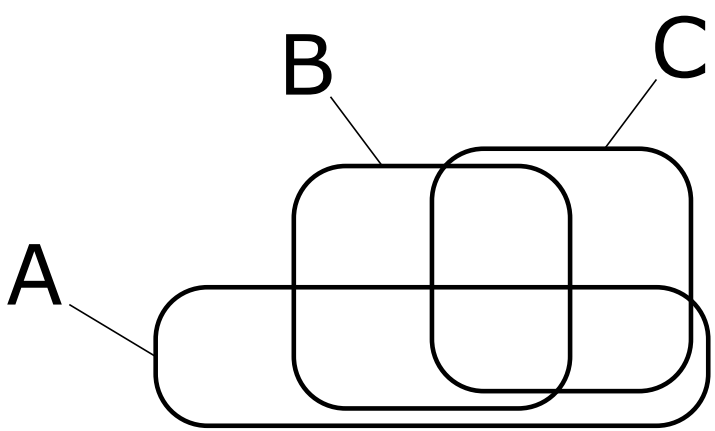


Table de Karnaugh

à 3 variables



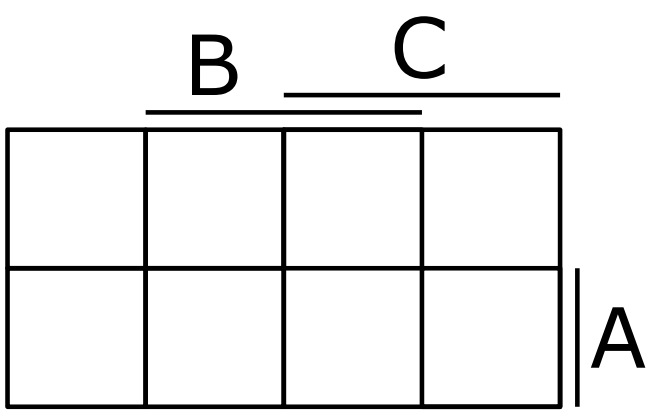
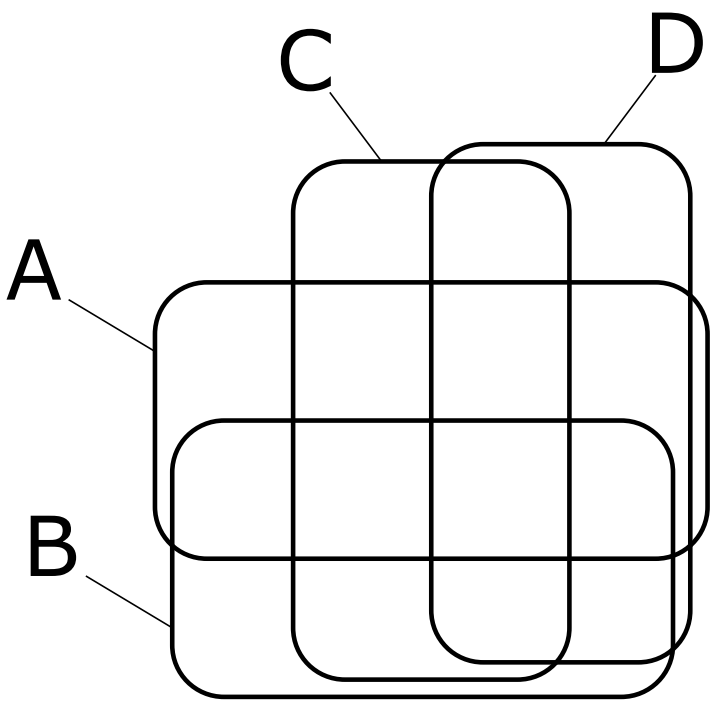


Table de Karnaugh

à 4 variables



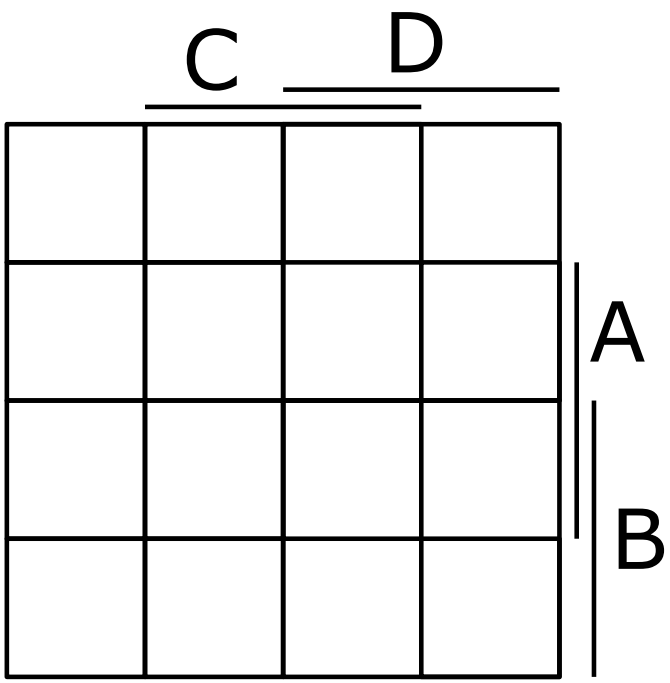
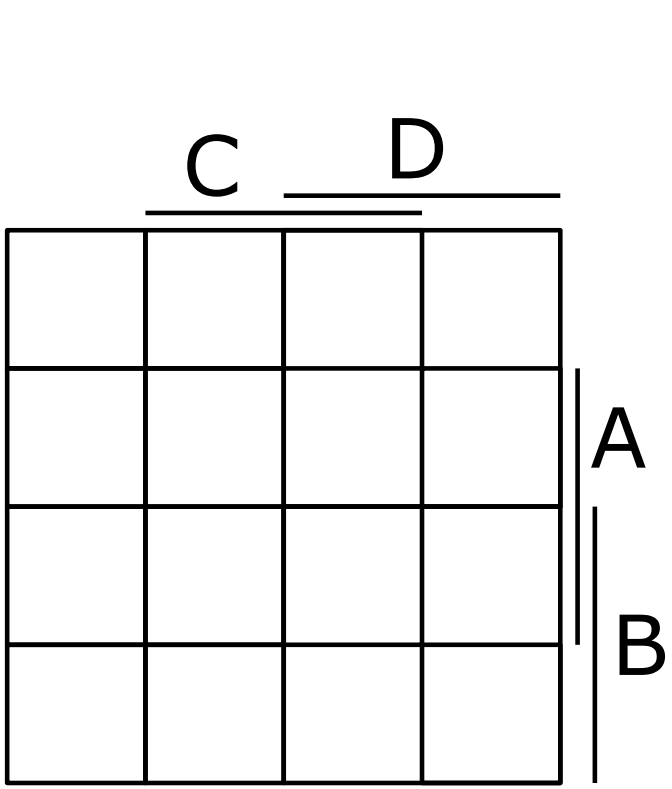
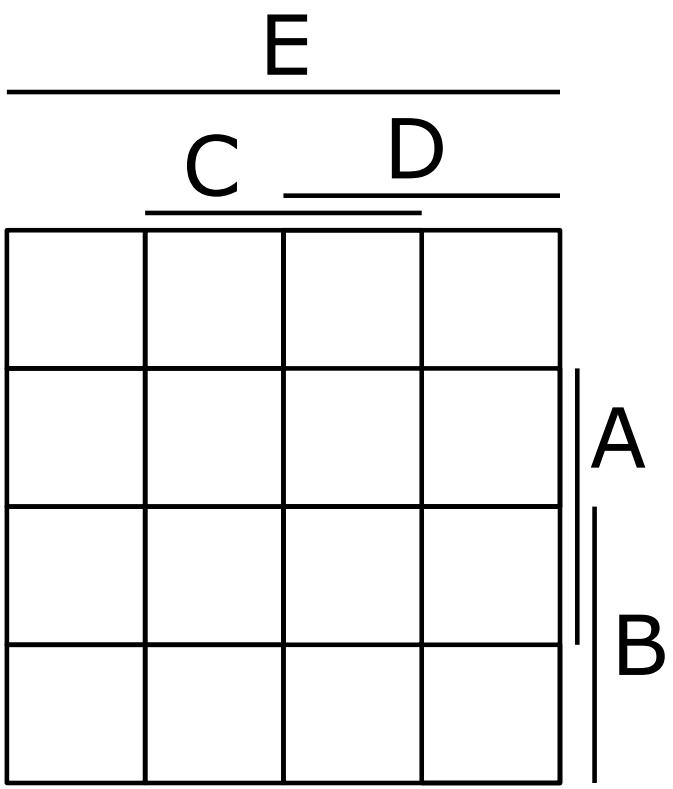


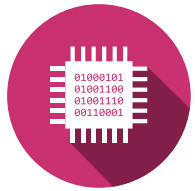
Table de Karnaugh

à 5 variables





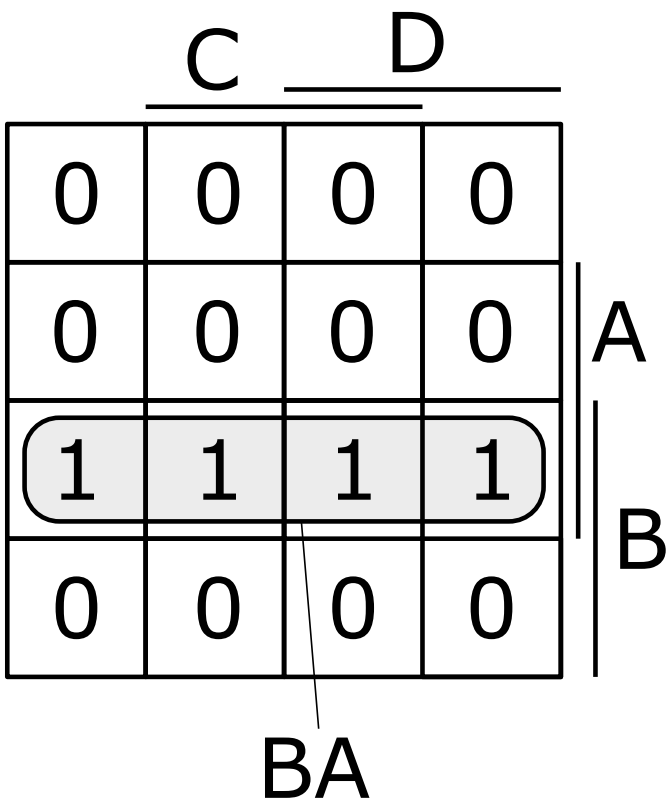
Contenu

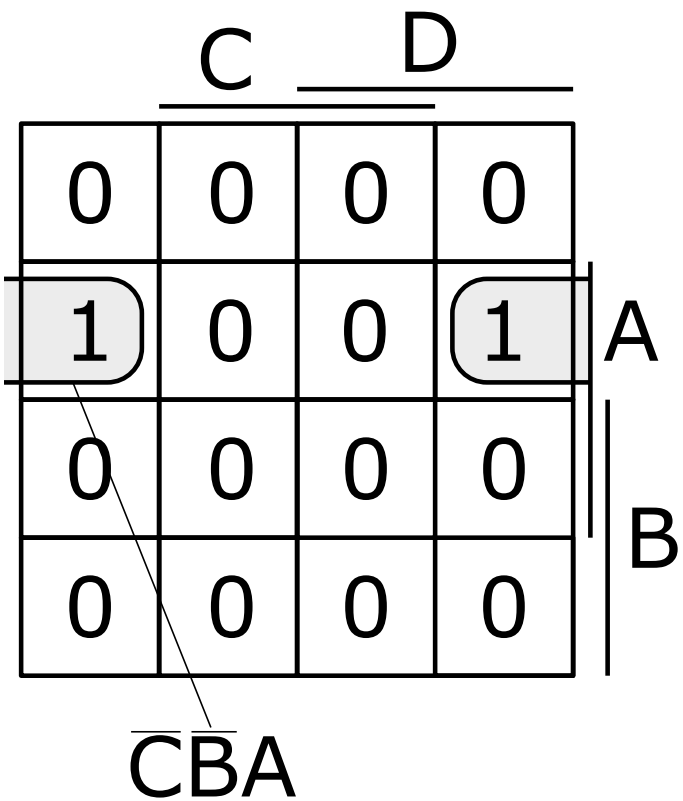


- Tables de Karnaugh
- **Simplification sous forme de sommes de produits**
  - Monôme
  - Polynôme
  - Impliquant premier, impliquant premier essentiel
  - Simplification
  - Fonction incomplètement définie
- Simplification de fonctions OU-exclusif
- Fonctions avec un nombre élevé d’entrées

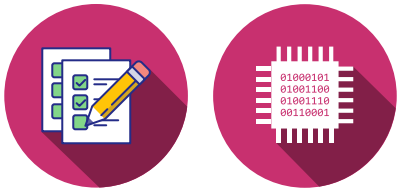
Table de Karnaugh

Monôme





Exercise 1.1 *(kar/karnaugh-01)*

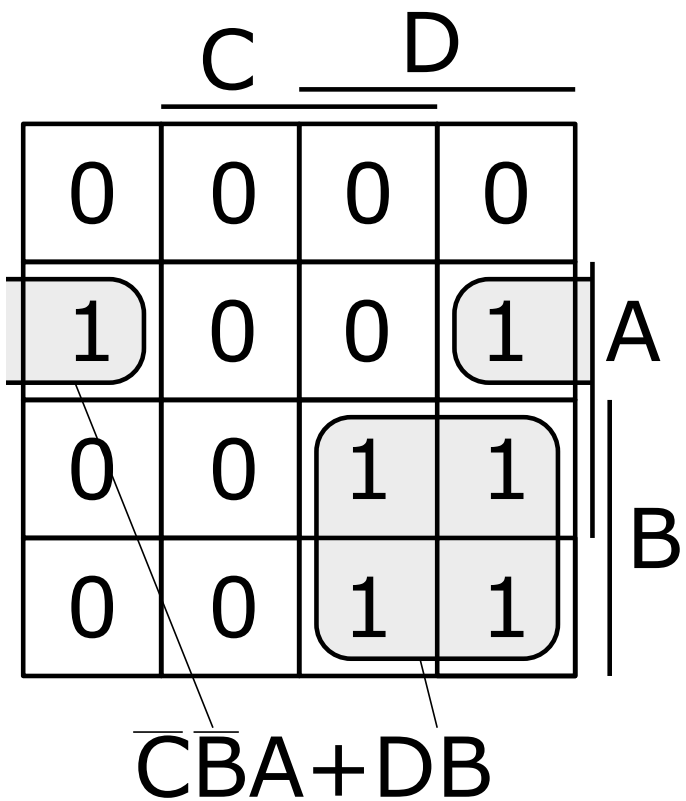


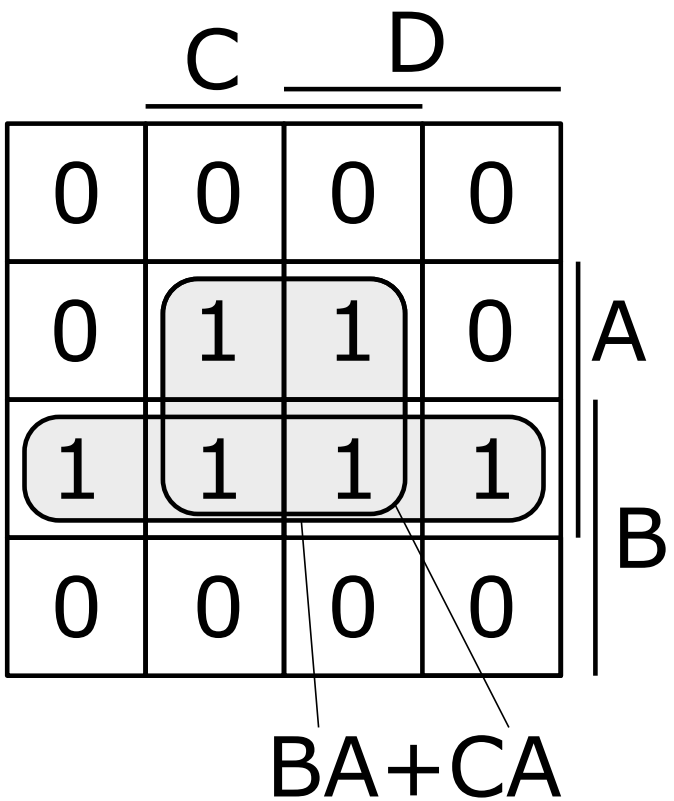
Représenter, dans une table de Karnaugh à 4 variables, les monômes suivants:

$$y_1 = \bar{b}a$$
$$y_2 = \bar{d}\bar{a}$$
$$y_3 = \bar{d}cb$$
$$y_4 = dba$$
$$y_5 = \bar{c}\bar{b}\bar{a}$$
$$y_6 = dcb\bar{a}$$

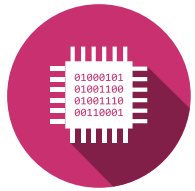
Table de Karnaugh

Polynôme





Exercise 1.3 *(kar/karnaugh-03)*



Répresenter, dans une table de Karnaugh à 4 variables, les polynômes suivants:

$$y_1 = \bar{b} + ac$$
$$y_2 = \bar{d} + \bar{a} + bc$$
$$y_3 = \bar{d}cb + d\bar{c}\bar{b}$$
$$y_4 = db + ab$$
$$y_5 = \bar{c}\bar{b}\bar{a} + \bar{c}\bar{b}$$
$$y_6 = dcb\bar{a} + \bar{d}cb\bar{a}$$

Table de Karnaugh

Impliquant premier

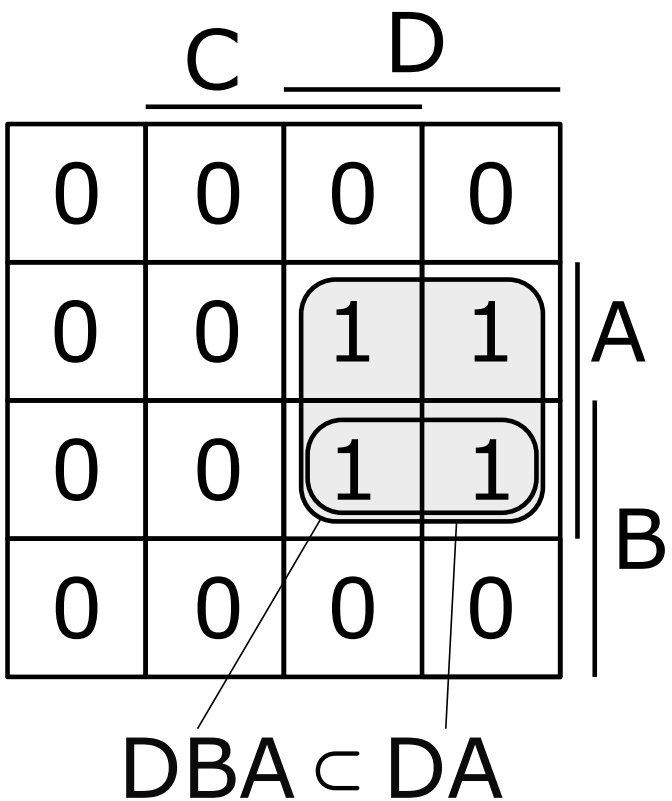


Table de Karnaugh

Impliquant premier essentiel

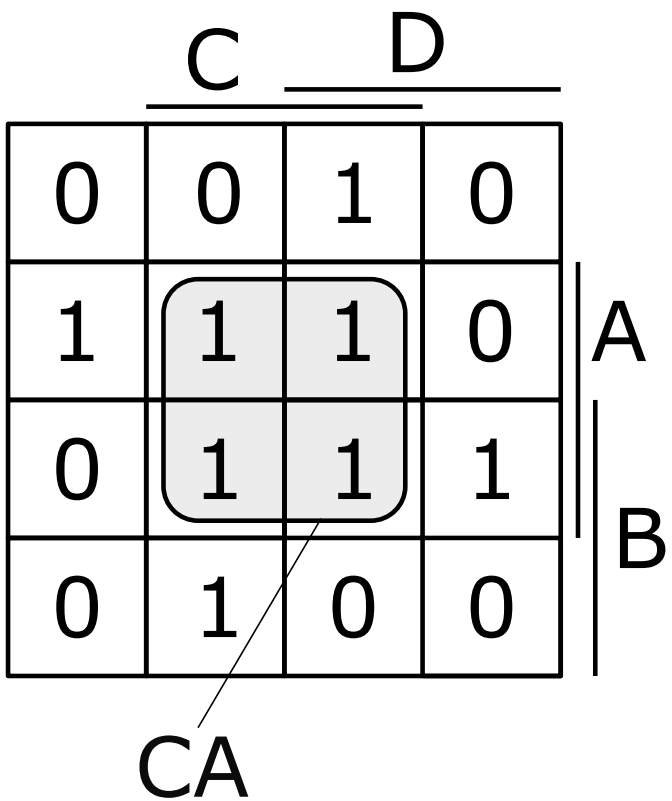


Table de Karnaugh

Impliquant premier essentiel

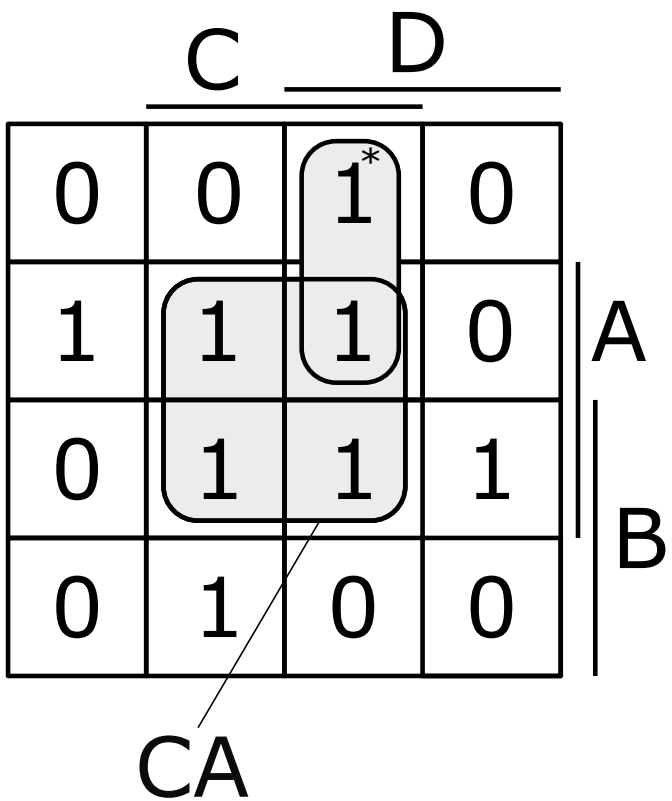


Table de Karnaugh

Impliquant premier essentiel

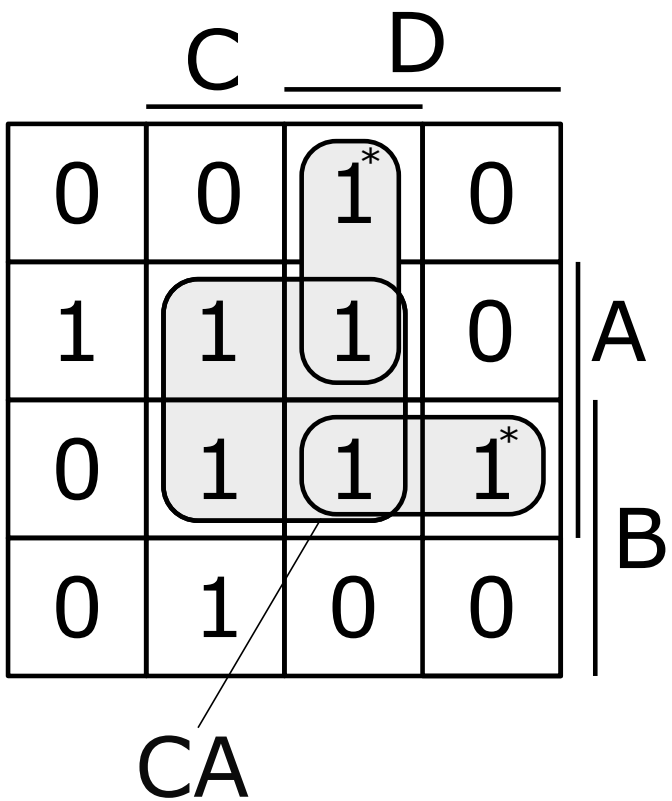


Table de Karnaugh

Impliquant premier essentiel

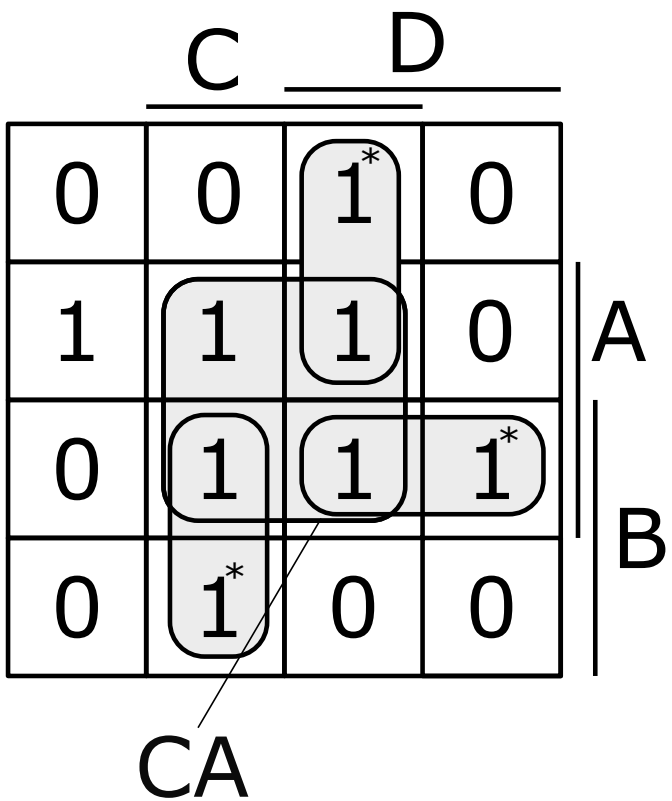


Table de Karnaugh

Impliquant premier essentiel

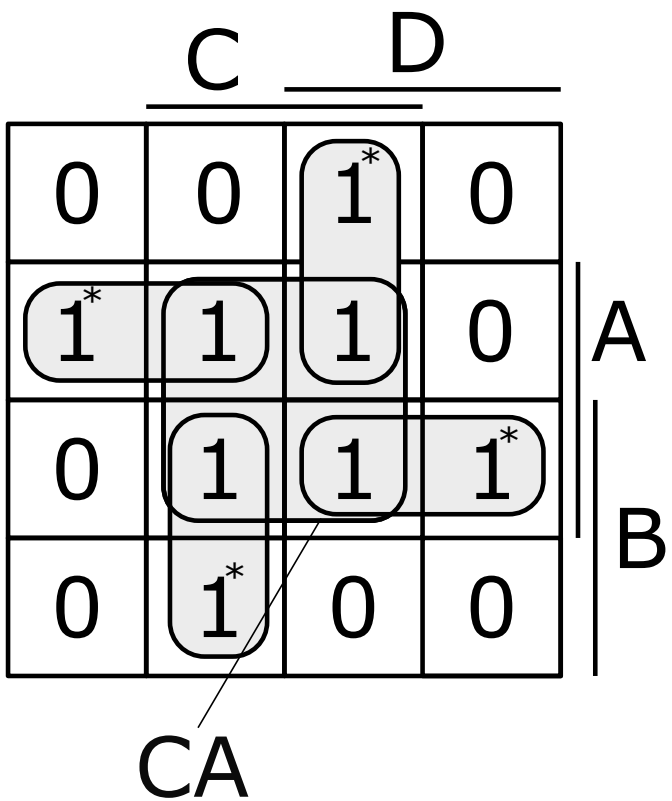


Table de Karnaugh

Impliquant premier essentiel

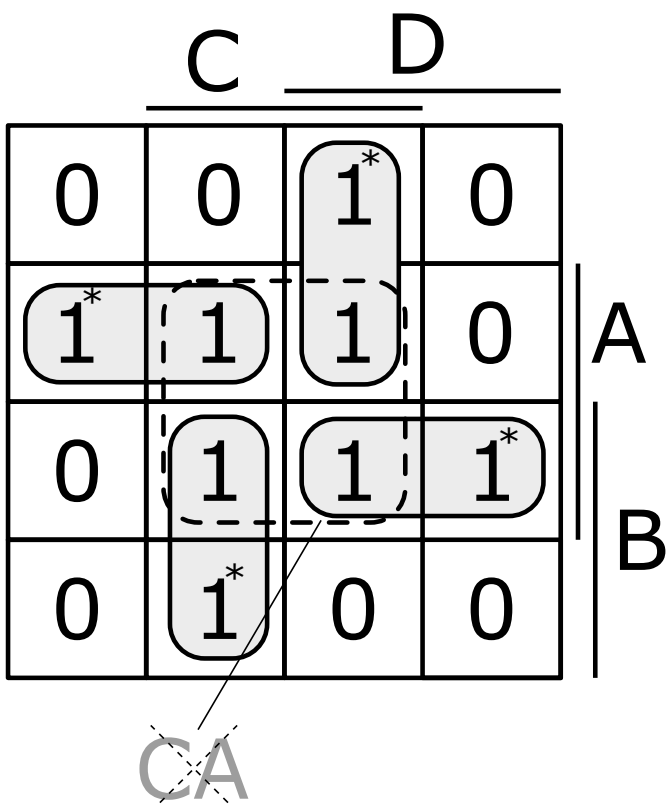
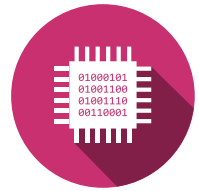


Table de Karnaugh

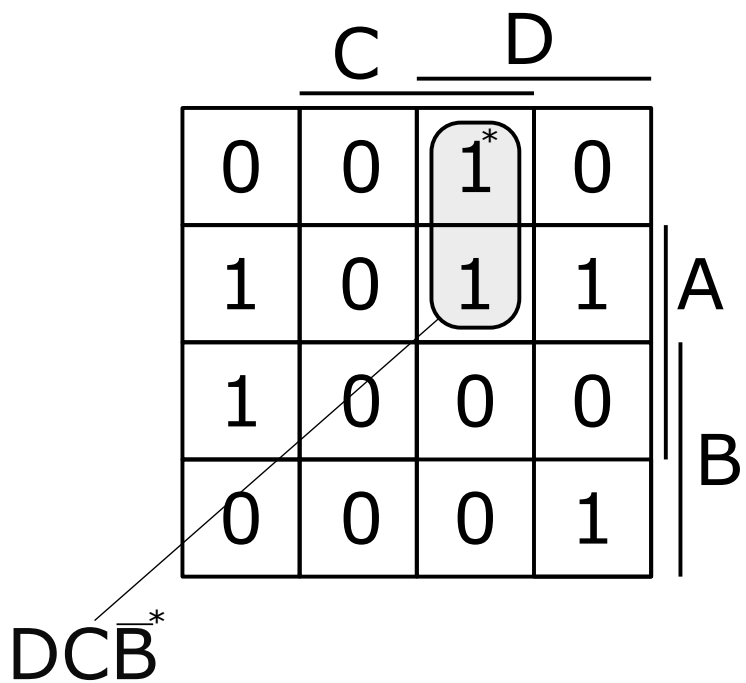
Déterminer l’equation polynomiale minimale



	C		D	
0	0	1	0	
1	0	1	1	A
1	0	0	0	
0	0	0	1	B

Table de Karnaugh

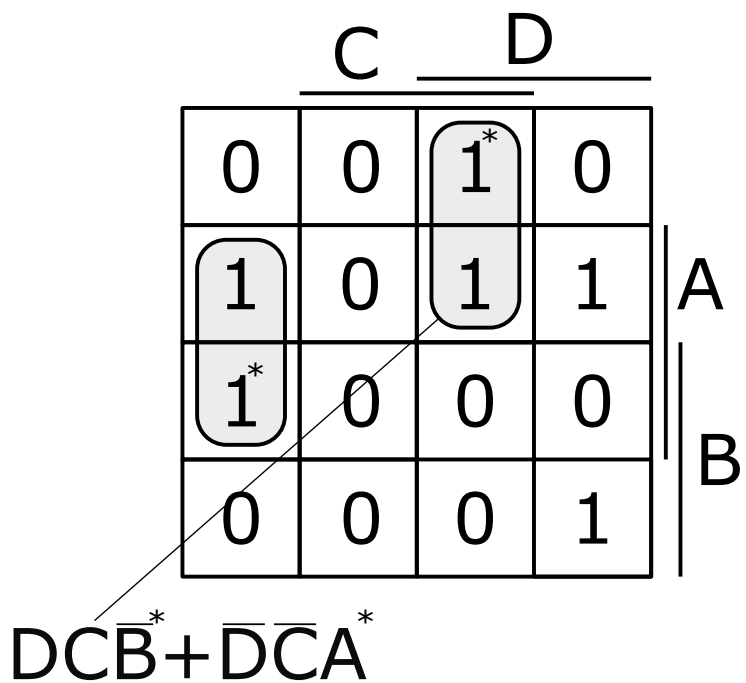
Déterminer l’equation polynomiale minimale



→Recherche des impliquants premiers essentiels

Table de Karnaugh

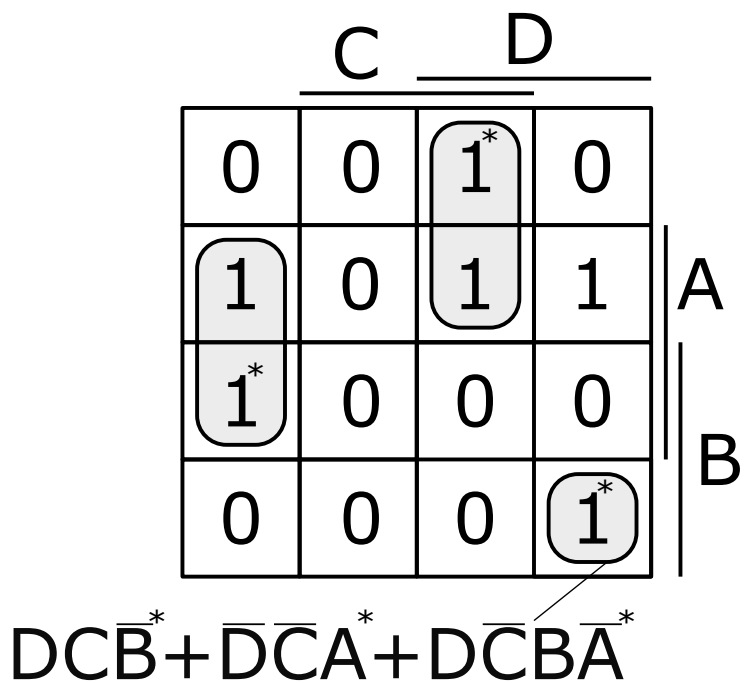
Déterminer l’equation polynomiale minimale



→Recherche des impliquants premiers essentiels

Table de Karnaugh

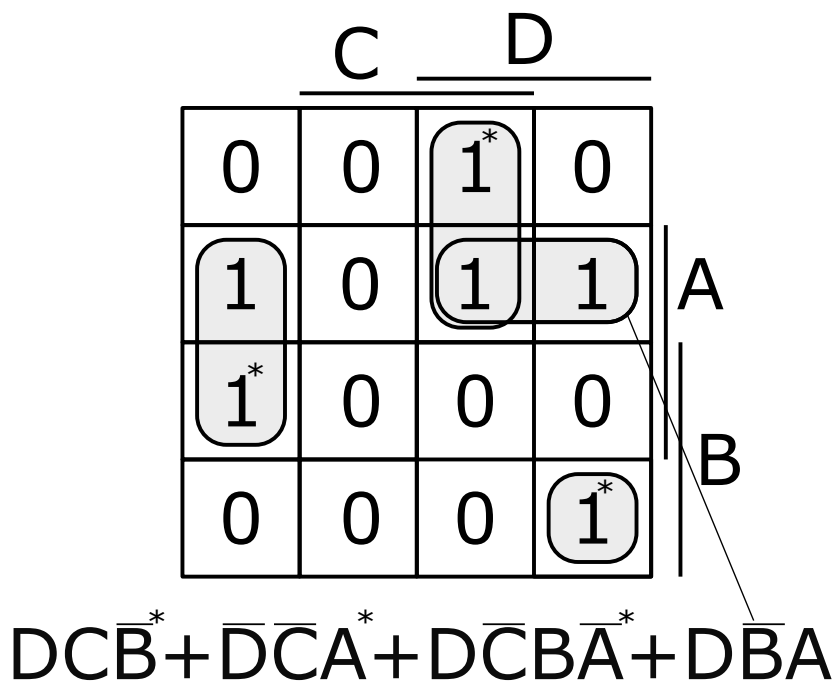
Déterminer l’equation polynomiale minimale



→Recherche des impliquants premiers essentiels

Table de Karnaugh

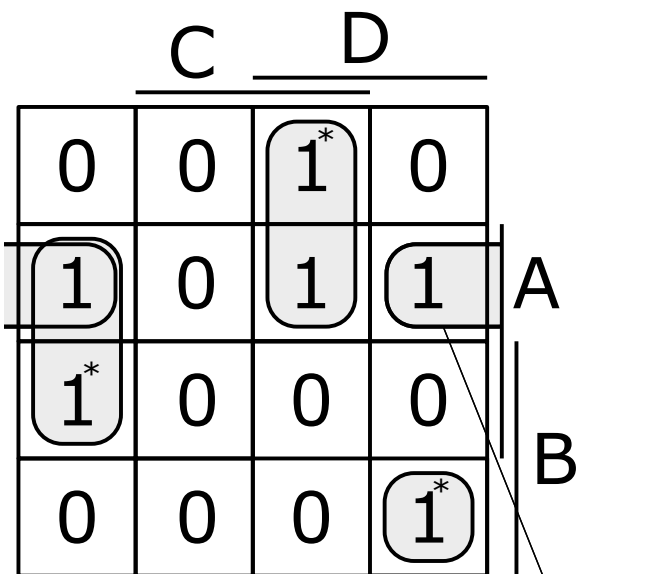
Déterminer l’equation polynomiale minimale



→Recherche des impliquants premiers essentiels puis des impliquants premiers restants

Table de Karnaugh

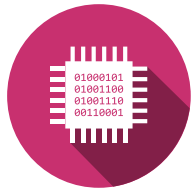
Déterminer l'equation polynomiale minimale



$$DC\overline{B}^{*}+\overline{D}\,\overline{C}A^{*}+DC\overline{B}\,\overline{A}^{*}+D\overline{B}A$$
$$+\overline{C}\,\overline{B}A$$

→Recherche des impliquants premiers essentiels puis des impliquants premiers restants

Exercise 2.1 *(kar/productsum-01)*

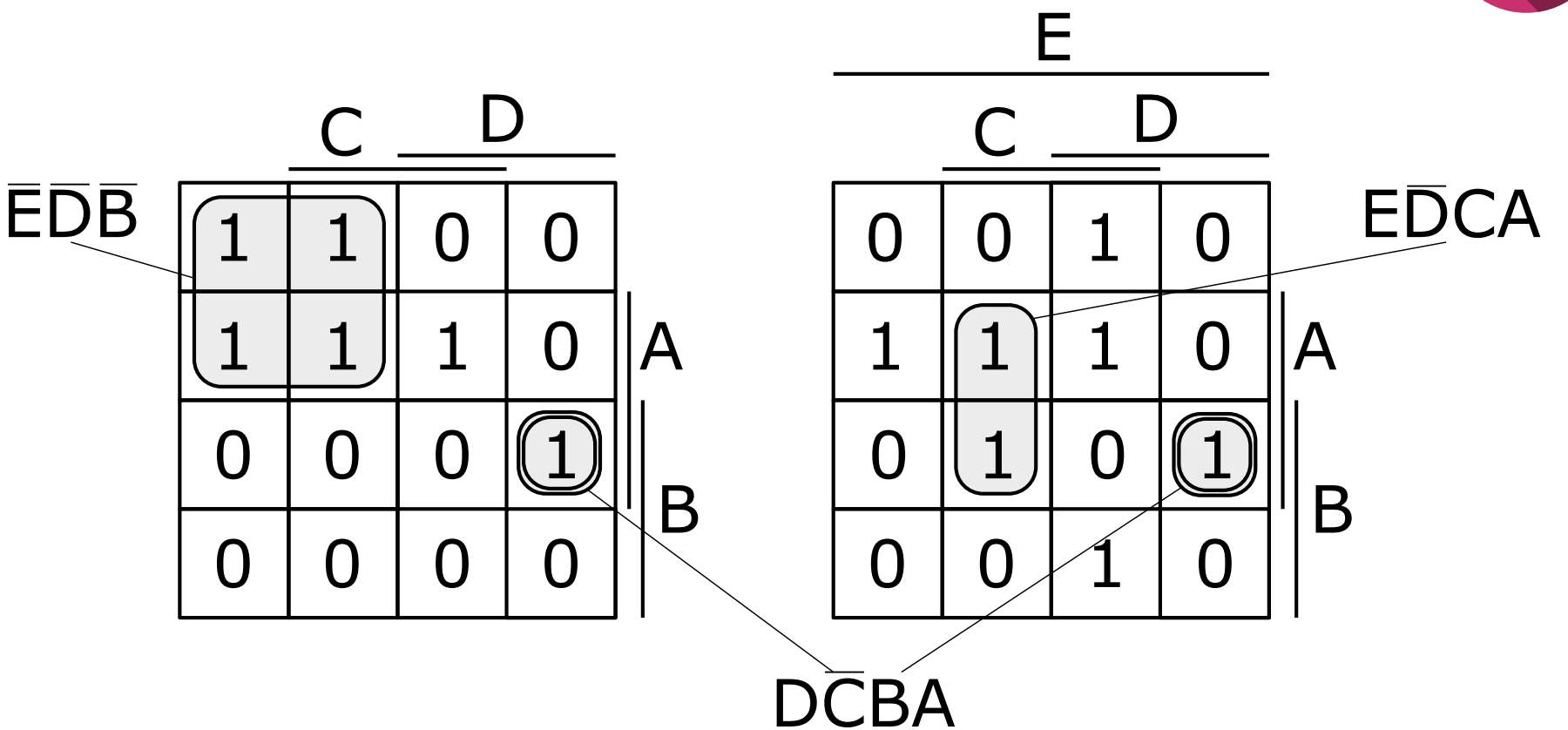


Déterminer la forme polynomiale minimale d‘une table de Karnaugh à 4 variables

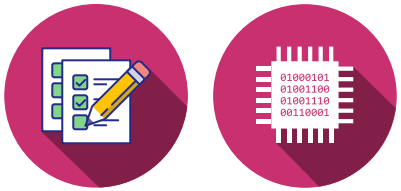
	C		D	
	1	0	0	1
	1	0	1	1
A	1	0	0	0
B	1	1	1	1

Table de Karnaugh

Simplification avec 5 variables



Exercise 2.2 *(kar/productsum-02)*



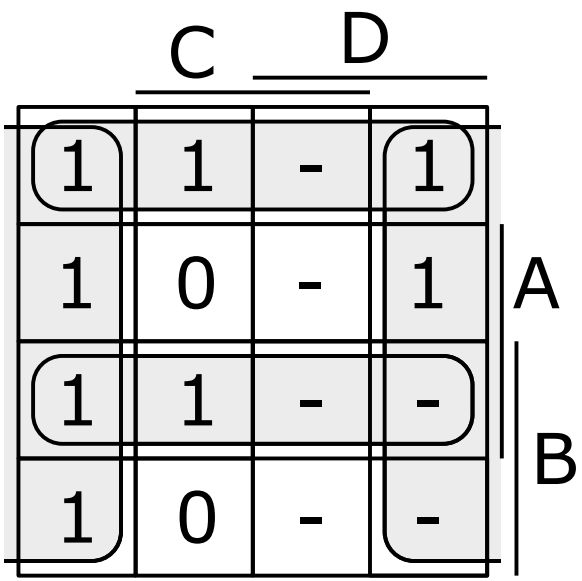
Forme polynomiale minimal d‘une table de Karnaugh à 5 variables

1	1	0	0
1	1	1	0
0	0	0	1
0	0	0	0

	E			
		C		D
	0	0	1	0
A	1	1	1	0
	0	1	0	1
B	0	0	1	0

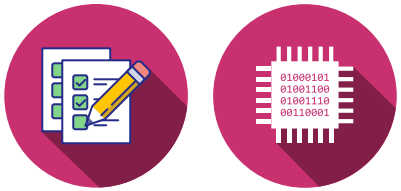
Table de Karnaugh

Fonction incomplètement définie



$$\overline{C} + \overline{B}\overline{A} + BA$$

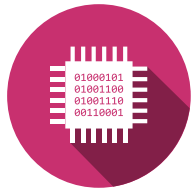
Exercise 2.11 *(kar/productsum-11)*



Déterminer l'expression polynomiale minimale de la fonction majorité à 4 entrées

- Plus que le moitie à '0' => '0'
- Plus que la moitie à '1' => '1'
- Même nombre de '0' et '1' le sortie est indéfinie

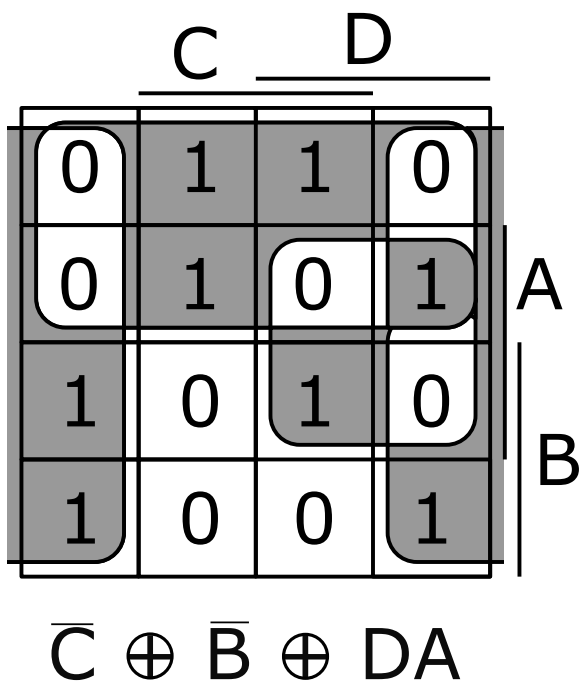
Contenu



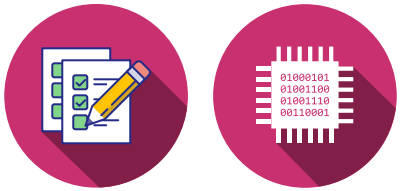
- Tables de Karnaugh
- Simplification sous forme de sommes de produits
- **Simplification de fonctions OU-exclusif**
  - Représentation
  - Simplification
- Fonctions avec un nombre élevé d’entrées

Table de Karnaugh

Représentation d'un Ou-exclusif (XOR)



Exercise 3.1 *(kar/xor-01)*



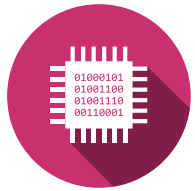
Représentation de fonctions XOR (utilisez des tables de Karnaugh avec 4 entrées)

$a \oplus b$

$a \oplus b \oplus c$

Table de Karnaugh

Simplification sous forme de Ou-exclusif (XOR)



	C		D	
0	1	1	1	
0	0	1	0	A
1	1	1	0	
0	1	1	1	B

Table de Karnaugh

Simplification sous forme de Ou-exclusif (XOR)

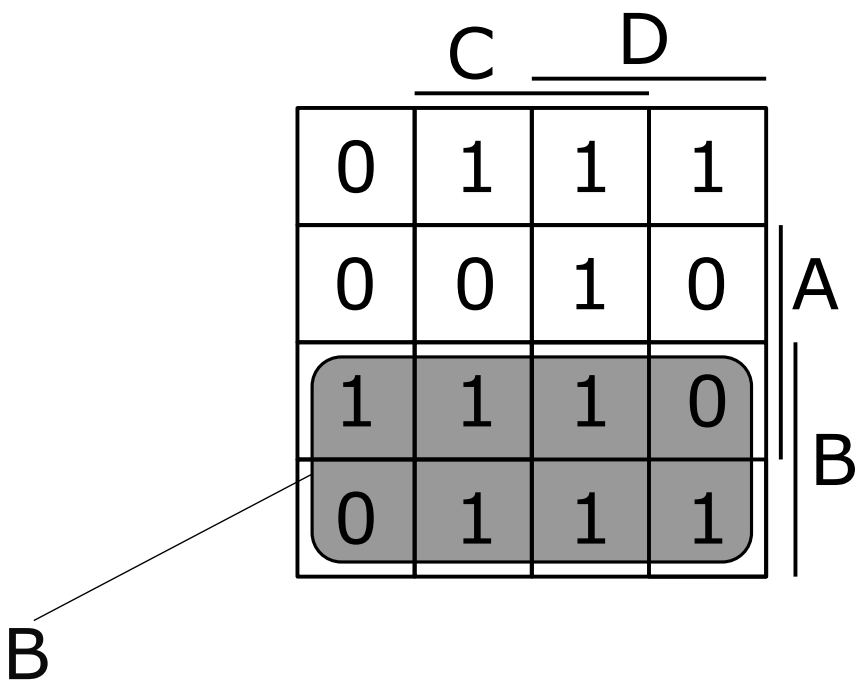


Table de Karnaugh

Simplification sous forme de Ou-exclusif (XOR)

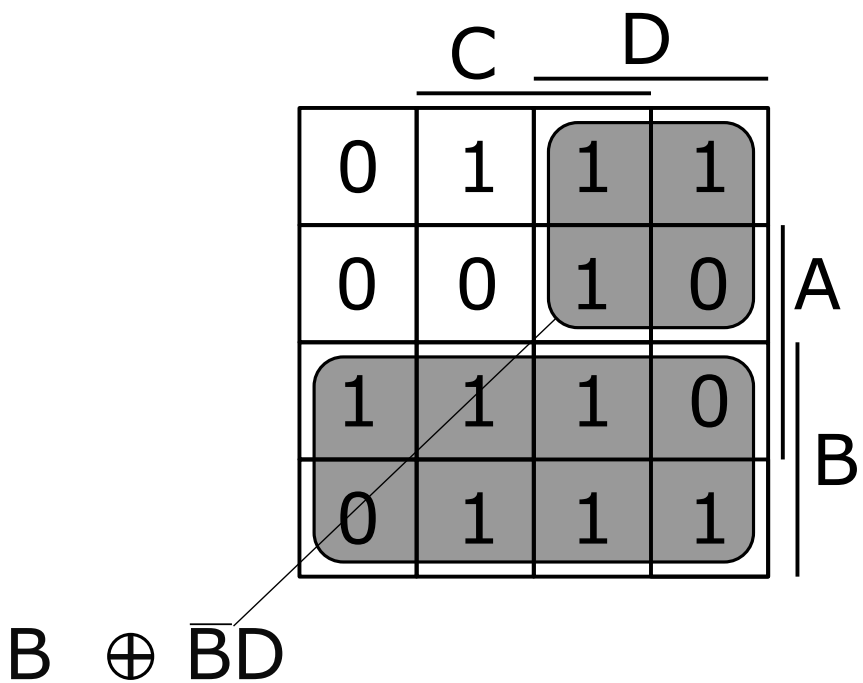


Table de Karnaugh

Simplification sous forme de Ou-exclusif (XOR)

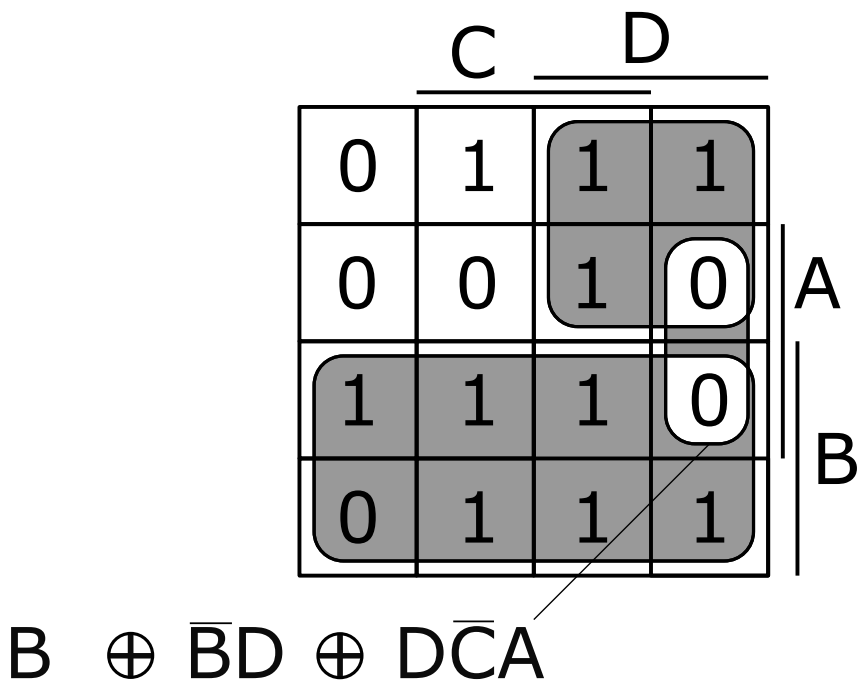
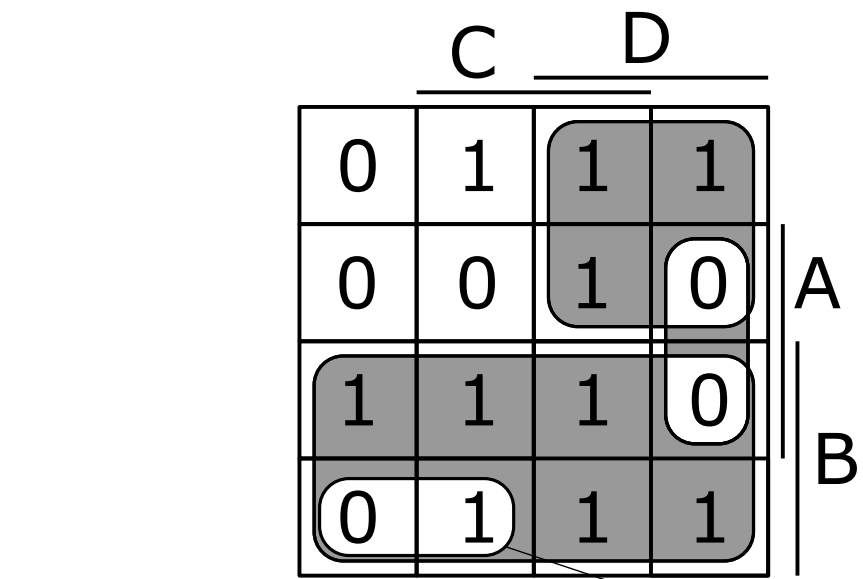


Table de Karnaugh

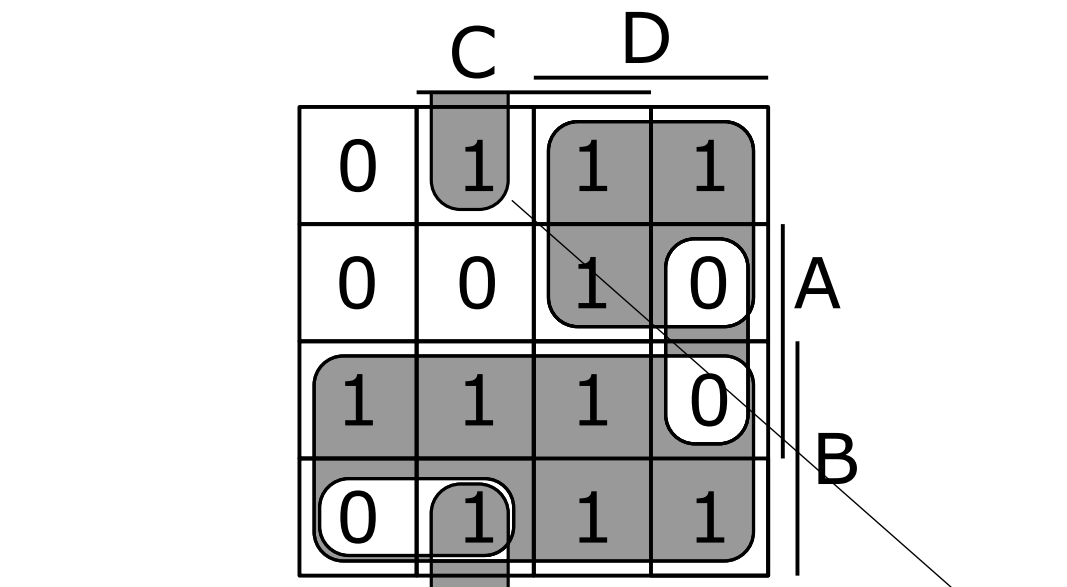
Simplification sous forme de Ou-exclusif (XOR)



$$B \oplus \overline{B}D \oplus D\overline{C}A \oplus \overline{D}B\overline{A}$$

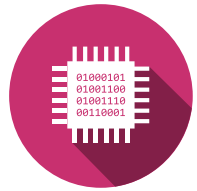
Table de Karnaugh

Simplification sous forme de Ou-exclusif (XOR)



$$B \oplus \overline{B}D \oplus D\overline{C}A \oplus \overline{D}B\overline{A} \oplus \overline{D}C\overline{A}$$

Contenu



Tables de Karnaugh

Simplification sous forme de sommes de produits

Simplification de fonctions OU-exclusif

Fonctions avec un nombre élevé d'entrées

- Simplification algébrique
- Réduction en blocs de taille maîtrisable
- Systèmes itératifs

Table de Karnaugh

Fonctions avec un nombre élevé d'entrées

Fonction à 6 entrées

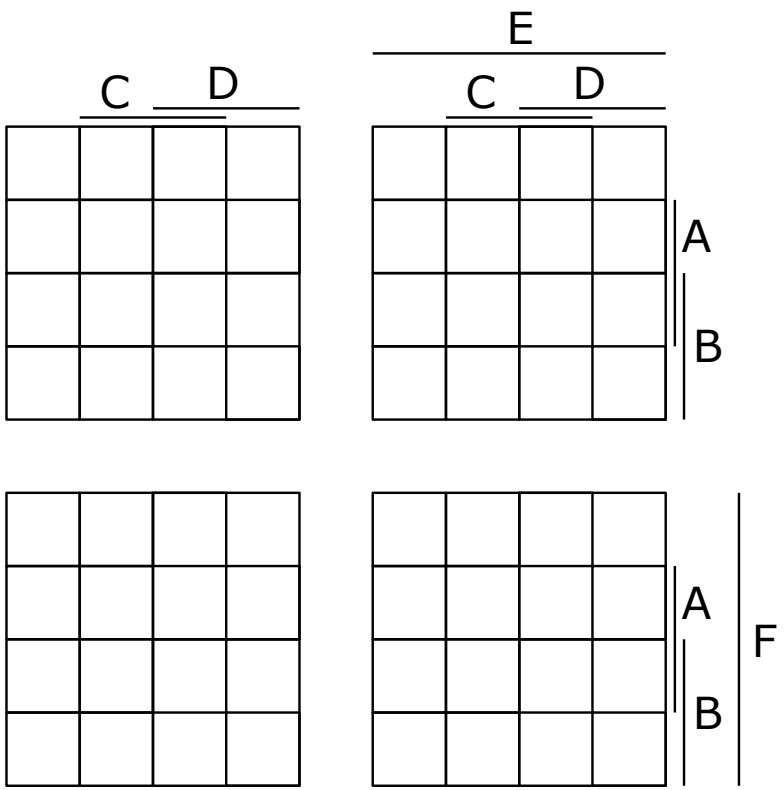


Table de Karnaugh

Décomposition par blocs

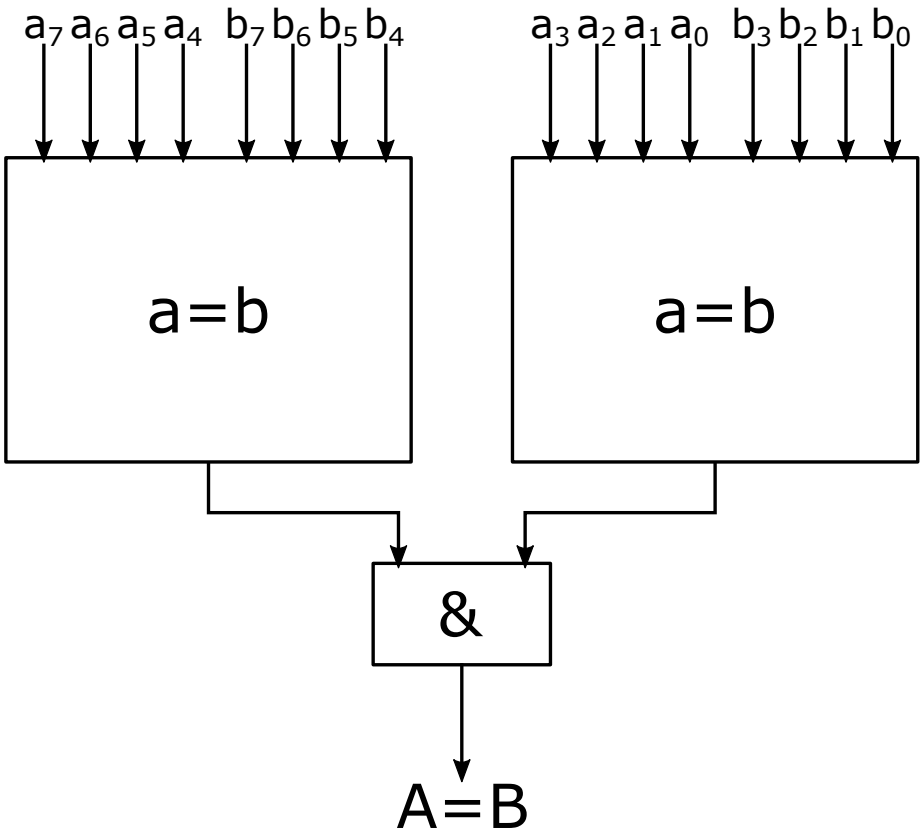


Table de Karnaugh

Décomposition par blocs

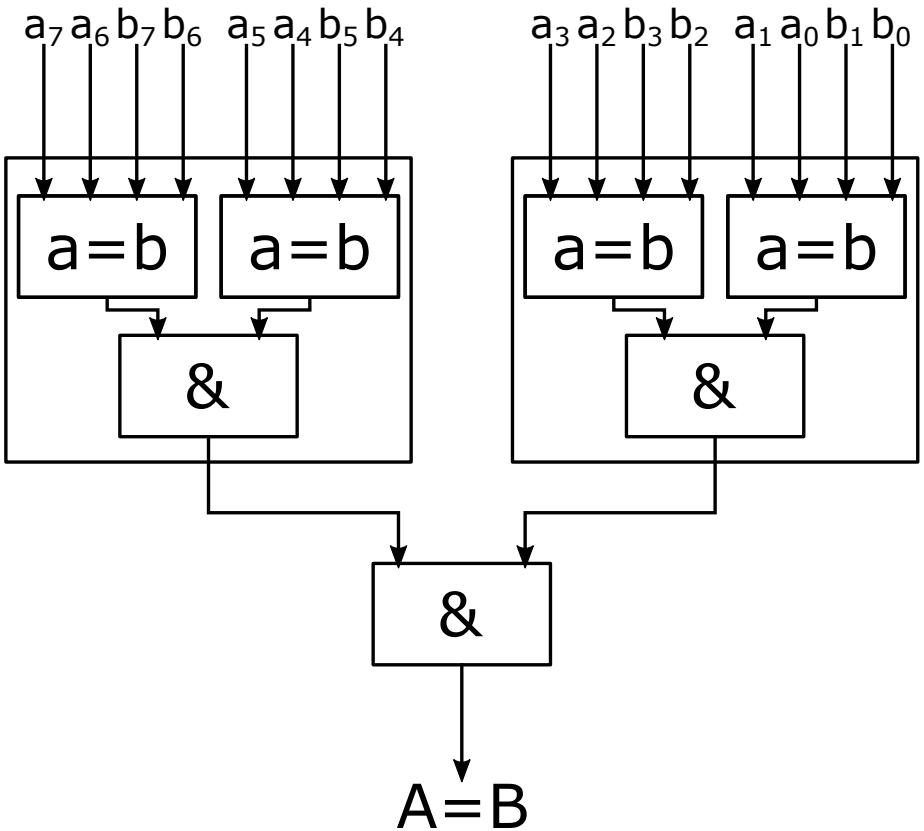
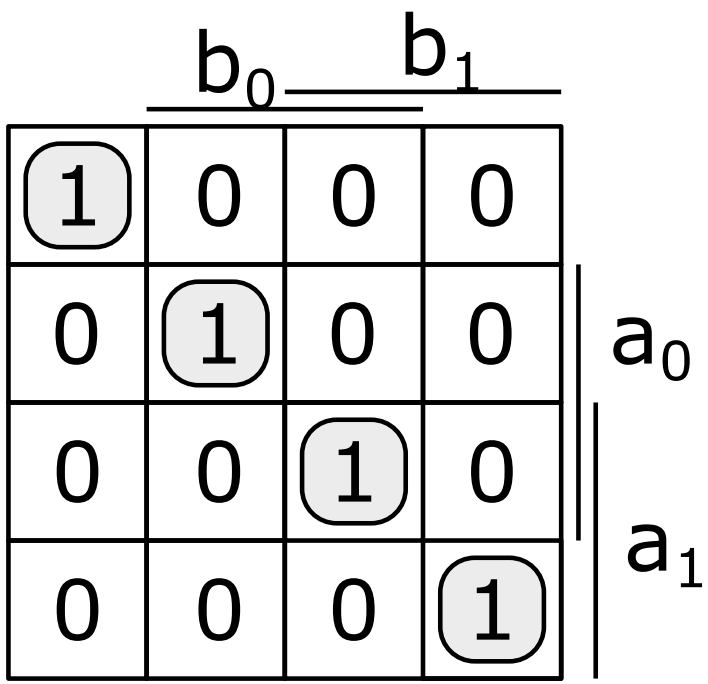
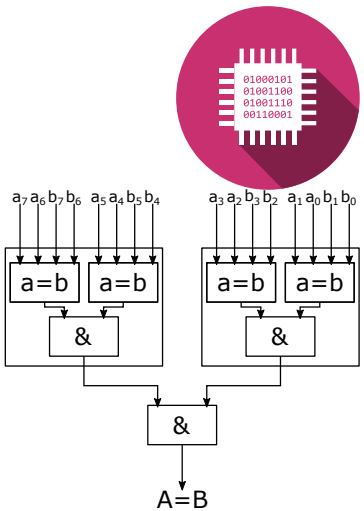


Table de Karnaugh

Décomposition par blocs

A=B, 2x2 Bits

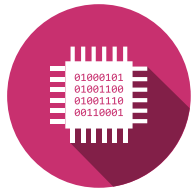




$$\overline{a_0}\,\overline{b_0}\,\overline{a_1}\,\overline{b_1} + a_0 b_0 \overline{a_1}\,\overline{b_1} + \overline{a_0}\,\overline{b_0} a_1 b_1 + a_0 b_0 a_1 b_1$$

Table de Karnaugh

Simplification algébrique



Comparaison de nombres, algébriquement

$$A = a_7 * 2^7 + a_6 * 2^6 + \cdots + a_1 * 2^1 + a_0 * 2^0$$
$$B = b_7 * 2^7 + b_6 * 2^6 + \cdots + b_1 * 2^1 + b_0 * 2^0$$

$$A = B \iff (a_7 = b_7) * (a_6 = b_6) * \ldots * (a_1 = b_1) * (a_0 = b_0)$$
$$A = B \iff \overline{(a_7 \oplus b_7)} * \overline{(a_6 \oplus b_6)} * \ldots * \overline{(a_1 \oplus b_1)} * \overline{(a_0 \oplus b_0)}$$
$$A = B \iff (a_7 b_7 + \overline{a_7}\,\overline{b_7}) * (a_6 b_6 + \overline{a_6}\,\overline{b_6}) * \ldots * (a_1 b_1 + \overline{a_1}\,\overline{b_1}) * (a_0 b_0 + \overline{a_0}\,\overline{b_0})$$

Table de Karnaugh

Système itératif

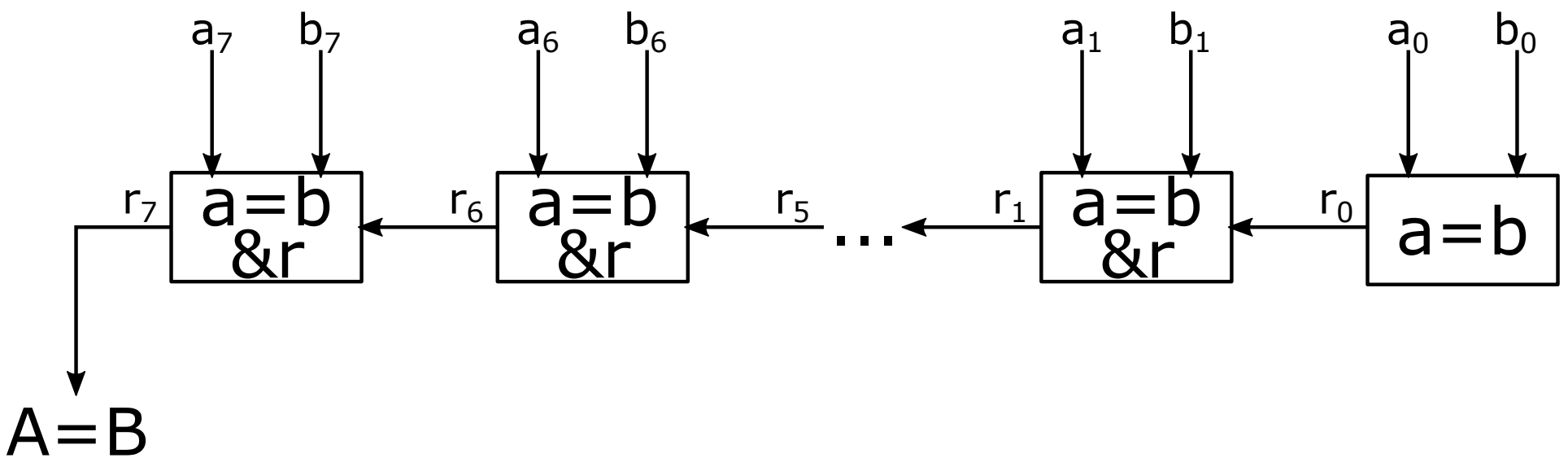
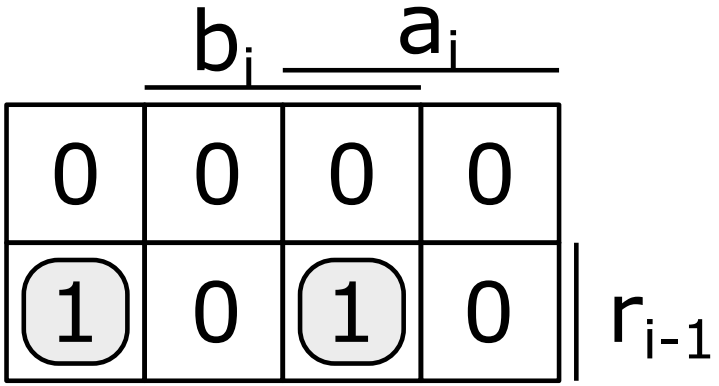


Table de Karnaugh

Système itératif

Bloc itératif

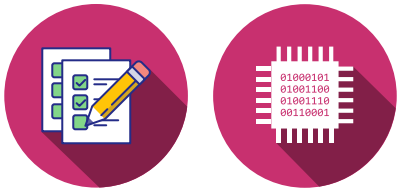


$\overline{a_i}\,\overline{b_i}\,r_{i-1} + a_i b_i r_{i-1}$

Premier bloc différent des autres: $r_0 = \overline{a_0} * \overline{b_0} + a_0 * b_0$

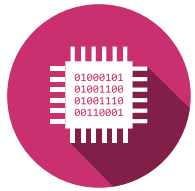
Premier bloc comme les autres: $r_{-1} = 1$

Exercise 4.1 *(kar/manyinputs-01)*



Comparaison de nombre (A > B)

Références

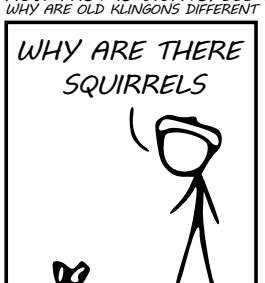


- [Man78] (français) Bonne présentation
- [Toc92] (français) Exemples complets
- [Max95] (anglais) Facile à lire
- [Wak00] (anglais) Bonne explication Karnaugh + algorithmes
- [Fle80] (anglais) Algorithmes de simplification
- [Kat94] (anglais) Algorithmes de simplification

QUESTIONS

CAN BE ASKED BY ANYONE ANYTIME

WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SVCHOST·EXE RUNNING
WHY AREN'T ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP
WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY WUBA LUBBA DUB DUB MEANING
WHY IS THERE A WHALE AND A POT FALLING
WHY ARE THERE SO MANY BIRDS IN SWISS
WHY IS THERE SO LITTLE RAIN IN WALLIS
WHY IS WALLIS WEATHER FORECAST ALWAYS WRONG
WHY ARE THERE MALE AND FEMALE BIKES
WHY ARE THERE BRIDESMAIDS
WHY DO DYING PEOPLE REACH UP
HOW FAST IS LIGHTSPEED
WHY ARE OLD KLINGONS DIFFERENT



WHY ARE THERE DINOSAUR GHOSTS
WHY IS THERE HELL IF

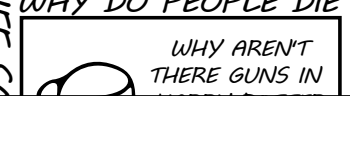
WHY DO IGUANAS DIE
WHY IS THERE CAFFEINE IN MY SHAMPOO
WHY HAVE DINOSAURS NO FUR
WHY DO TWINS HAVE DIFFERENT FINGERPRINTS
WHY ARE SWISS AFRAID OF DRAGONS
WHY AREN'T ECONOMISTS RICH
WHY DO AMERICANS CALL IT SOCCER
WHY ARE MY EARS RINGING
WHY IS 42 THE ANSWER TO EVERYTHING
WHY CAN'T NOBODY ELSE LIFT THORS HAMMER
WHY IS MARVIN ALWAYS SO SAD
WHY ARE THERE ANTS IN MY LAPTOP
WHY IS EARTH TILTED
WHY IS SPACE BLACK
WHY IS OUTER SPACE SO COLD
WHY ARE THERE PYRAMIDS ON THE MOON
WHY IS NASA SHUTTING DOWN
WHY ARE THERE TINY SPIDERS IN MY HOUSE
WHY DO SPIDERS COME INSIDE
WHY ARE THERE HUGE SPIDERS IN MY HOUSE
WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE
WHY ARE THERE SPIDERS IN MY ROOM
WHY ARE THERE SO MANY SPIDERS IN MY ROOM
WHY DO SPYDER BITES ITCH
WHY IS DYING SO SCARY
WHY IS THERE NO GPS IN LAPTOPS
WHY DO KNEES CLICK

WHY ARE THERE FEMALE

WHY IS HTTPS CROSSED OUT IN RED
WHY IS THERE A LINE THROUGH HTTPS
WHY IS THERE A RED LINE THROUGH HTTPS ON TWITTER
WHY IS HTTPS IMPORTANT

WHY IS YKK ON ALL ZIPPERS
WHY ARE THERE WEEKS
WHY DO I FEEL DIZZY
WHY IS THERE A SWARM OF ANTS
WHY IS THERE PILEGRIM
WHY IS THERE LAVA
WHY ARE THERE SO MANY CROWS IN ROCHESTER
WHY IS TO BE OR NOT TO BE FUNNY
WHY DO CHILDREN GET CANCER
WHY IS POSEIDON ANGRY WITH ODYSSEUS
WHY IS THERE ICE IN SPACE
WHY IS THERE AN OWL IN MY BACKYARD
WHY IS THERE AN OWL OUTSIDE MY WINDOW
WHY IS THERE AN OWL ON THE DOLLAR BILL
WHY DO OWLS ATTACK PEOPLE
WHY ARE FPGA's EVERYWHERE
WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE
WHY ARE THERE GODS
WHY ARE THERE TWO SPOCKS
WHAT IS https://xkcd.com/1256/
WHY DO THEY SAY T-MINUS
WHY ARE THERE OBELISKS
WHY ARE WRESTLERS ALWAYS WET

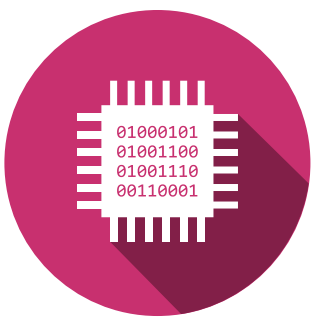
WHY IS LIFE SO
WHY ARE MY BOOBS ITCHY
WHY ARE CIGARETTES LEGAL
WHY ARE THERE DUCKS IN MY POOL
WHY IS JESUS WHITE
WHY IS THERE LIQUID IN MY EAR
WHY DO Q TIPS FEEL GOOD
WHY DO PEOPLE DIE



WHY ARE DOGS AFRAID OF FIR
WHY IS THERE NO KING IN E

Hes·so VALAIS WALLIS

Haute Ecole d'Ingénierie
Hochschule für Ingenieurwissenschaften



Silvan Zahno silvan.zahno@hevs.ch
Christophe Bianchi christophe.bianchi@hevs.ch
François Corthay francois.corthay@hevs.ch