



# Simplification par tables de Karnaugh

## Exercices Conception numérique



### Solution vs. Hints:

Toutes les réponses fournies ici ne sont pas des solutions complètes. Certaines ne sont que des indices pour vous aider à trouver la solution vous-même. Dans d'autres cas, seule une partie de la solution est fournie.

## 1 | KAR - Tables de Karnaugh

### 1.1 Représentation de monômes

$$y_1 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 0 & 0 & 0 & 0 & \\ 1 & 1 & 1 & 1 & A \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & B \end{array}$$

$$y_3 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & A \\ 0 & 1 & 0 & 0 & \\ 0 & 1 & 0 & 0 & B \end{array}$$

$$y_5 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 1 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & A \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & B \end{array}$$

$$y_2 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 1 & 1 & 0 & 0 & \\ 0 & 0 & 0 & 0 & A \\ 0 & 0 & 0 & 0 & \\ 1 & 1 & 0 & 0 & B \end{array}$$

$$y_4 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & A \\ 0 & 0 & 1 & 1 & \\ 0 & 0 & 0 & 0 & B \end{array}$$

$$y_6 \quad \begin{array}{c|cc|cc} & \text{C} & \text{D} & & \\ \hline & \text{C} & \text{D} & & \\ \hline 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & A \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & B \end{array}$$

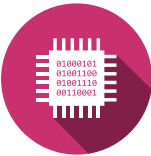
kar/karnaugh-01

### 1.2 Monômes

$$\begin{aligned} y_1 &= \overline{BD} \\ y_2 &= \overline{AC} \end{aligned} \quad (1)$$

$$\begin{aligned} y_3 &= \overline{BCD} \\ y_4 &= \overline{ACD} \end{aligned} \quad (2)$$

kar/karnaugh-02



### 1.3 Représentation de polynômes

$y_1$

C		D		
1	1	1	1	A B
1	1	1	1	
0	1	1	0	
0	0	0	0	

$y_3$

C		D		
0	0	0	1	A B
0	0	0	1	
0	1	0	0	
0	1	0	0	

$y_5$

C		D		
1	0	0	1	A B
1	0	0	1	
0	0	0	0	
0	0	0	0	

$y_2$

C		D		
1	1	1	1	A B
1	1	0	0	
1	1	1	0	
1	1	1	1	

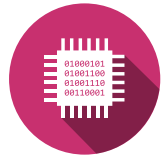
$y_4$

C		D		
0	0	0	0	A B
0	0	0	0	
1	1	1	1	
0	0	1	1	

$y_6$

C		D		
0	0	0	0	A B
0	0	0	0	
0	0	0	0	
0	1	1	0	

kar/karnaugh-03



## 2 | KAR - Simplification sous forme de somme de produits

### 2.1 Table de Karnaugh à 4 variables

$$D\overline{B}A^* + \overline{D}\overline{C}^* + B\overline{A}^* + \left\{ \begin{array}{l} \overline{C}\overline{B} \\ \overline{C}A \end{array} \right. \quad (3)$$

*kar/productsun-01*

### 2.2 Table de Karnaugh à 5 variables

$$\overline{E} \overline{D} \overline{B}^* + C\overline{B}A^* + D\overline{C}BA^* + \overline{D} \overline{B}A^* + E\overline{D}CA^* + EDC\overline{A} \quad (4)$$

*kar/productsun-02*

### 2.3 Table de Karnaugh à 5 variables

$$\overline{E} \overline{D} \overline{C}^* + \overline{E} \overline{C} \overline{A}^* + \overline{E} \overline{D} \overline{B} \overline{A}^* + DCBA^* + ECB^* + \left\{ \begin{array}{l} E\overline{B}\overline{A} \\ \overline{C}\overline{B}\overline{A} \end{array} \right. \quad (5)$$

*kar/productsun-03*

### 2.4 Table de Karnaugh à 5 variables

$$\overline{E}DB^* + \overline{C} \overline{B} \overline{A}^* + DBA^* + E\overline{D}CA + \overline{D} \overline{C} \overline{B} + \overline{E} \overline{D} \overline{C} \quad (6)$$

*kar/productsun-04*

### 2.5 Table de Karnaugh à 5 variables

$$\overline{E}C\overline{A}^* + \overline{E}B\overline{A}^* + E \overline{C} \overline{B} \overline{A}^* + DA^* + ECB^* + D\overline{B} \quad (7)$$

*kar/productsun-05*

### 2.6 Table de Karnaugh à 5 variables

$$\overline{C} \overline{B}^* + \overline{D} \overline{C} A^* + DCBA^* + \overline{E}CB + \overline{E}B\overline{A} \quad (8)$$

or

$$\overline{C} \overline{B}^* + \overline{D} \overline{C} A^* + DCBA^* + \overline{E}CB + \overline{E} \overline{C} \overline{A} \quad (9)$$

or

$$\overline{C} \overline{B}^* + \overline{D} \overline{C} A^* + DCBA^* + \overline{E} \overline{D} B + \overline{E}B\overline{A} \quad (10)$$

*kar/productsun-06*

### 2.7 Forme polynomiale minimale

$$\overline{x}_3 \overline{x}_2 \overline{x}_0^* + \overline{x}_2 x_0^* + x_1 x_0^* + \overline{x}_2 x_1^* \quad (11)$$

*kar/productsun-07*



## 2.8 Fonction inverse

$$\overline{E} C \overline{A}^* + CB^* + DBA^* + E \overline{B} A^* \quad (12)$$

*kar/productsum-08*

## 2.9 Forme polynomiale minimale

$y$ : 5 termes;  $\overline{y}$ : 4 termes

*kar/productsum-09*

## 2.10 Fonction de 5 variables

$$DCA^* + DCB^* + CBA^* + DBA^* + EDC^* + EBA^* + ECA^* + EDA^* + ECB^* + EDB^*$$

*kar/productsum-10*

## 2.11 Fonction incomplètement définie

$$x_4 x_3 + x_2 x_1 \quad (13)$$

or

$$x_3 x_1 + x_4 x_2 \quad (14)$$

or

$$x_4 x_1 + x_3 x_2 \quad (15)$$

*kar/productsum-11*

## 2.12 Fonction incomplètement définie

$$A = \overline{B}_2 \overline{B}_0^* + B_2 B_0^* + B_1 + B_3^*$$

$$B = \overline{B}_2^* + \overline{B}_1 \overline{B}_0^* + B_1 B_0^*$$

$$C = B_2^* + \overline{B}_1^* + B_0$$

$$D = \overline{B}_2 \overline{B}_0^* + B_3^* + B_2 \overline{B}_1 B_0^* + \overline{B}_2 B_1^* + B_1 \overline{B}_0 \quad (16)$$

$$E = \overline{B}_2 \overline{B}_0^* + B_1 \overline{B}_0^*$$

$$F = B_3^* + B_2 \overline{B}_1^* + B_2 \overline{B}_0^* + \overline{B}_1 \overline{B}_0^*$$

$$G = B_3^* + B_2 \overline{B}_1 + \overline{B}_0 B_1^* + B_2 \overline{B}_1$$

*kar/productsum-12*



### 3 | KAR - Simplification de fonctions OU-Exclusif

#### 3.1 Représentation de fonctions OU-exclusif

$$y_1 \begin{array}{c|cc} & \overline{C} & D \\ \hline 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array} \left| \begin{array}{c} A \\ B \end{array} \right.$$

$$y_3 \begin{array}{c|cc} & \overline{C} & D \\ \hline 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{array} \left| \begin{array}{c} A \\ B \end{array} \right.$$

$$y_{7\&8\&9} \begin{array}{c|cc} & \overline{C} & D \\ \hline 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array} \left| \begin{array}{c} A \\ B \end{array} \right.$$

$$y_2 \begin{array}{c|cc} & \overline{C} & D \\ \hline 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array} \left| \begin{array}{c} A \\ B \end{array} \right.$$

$$y_{4\&5\&6} \begin{array}{c|cc} & \overline{C} & D \\ \hline 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \left| \begin{array}{c} A \\ B \end{array} \right.$$

kar/xor-01

#### 3.2 Forme polynomiale minimale

$$y = x_1 x_0^* + \overline{x_2} x_0^* + \overline{x_2} x_1^* + \overline{x_3} x_2 \overline{x_0}^*$$

kar/xor-02

#### 3.3 Forme polynomiale minimale

$$\overline{E} \overline{D} C^* + \overline{E} \overline{C} \overline{B} \overline{A}^* + \overline{E} D B A^* + \overline{E} \overline{D} \overline{A}^* + \overline{E} C B^* + E \overline{D} \overline{C} A^* + E D C \overline{B}^* + E D \overline{C} B \overline{A}^* + \begin{cases} E D \overline{B} A \\ E \overline{C} \overline{B} A \end{cases}$$

kar/xor-03

#### 3.4 Forme ou-exclusif de produits

$$\text{Many possible solutions e.g.: } Y = \overline{D} \overline{B} \oplus \overline{D} A \oplus C A \oplus C B$$

kar/xor-04



### 3.5 Forme ou-exclusif de produits

Many possible solutions e.g.:  $y = x_0 \oplus x_1 \overline{x_0} \oplus x_2 \overline{x_0} \oplus x_3 x_2 \overline{x_1}$

*kar/xor-05*

### 3.6 Additionneur

$$\begin{aligned} s_0 &= a_0 \oplus b_0 \\ s_1 &= a_1 \oplus b_1 \oplus a_0 b_0 \\ s_1 &= \overline{a_1} b_1 \oplus a_1 \overline{b_1} \oplus a_0 b_0 \\ s_2 &= a_1 b_1 \oplus \begin{cases} a_0 \overline{b_1} b_0 \oplus \overline{a_1} a_0 b_0 \\ a_0 b_1 b_0 \oplus a_1 a_0 b_0 \end{cases} \end{aligned} \quad (17)$$

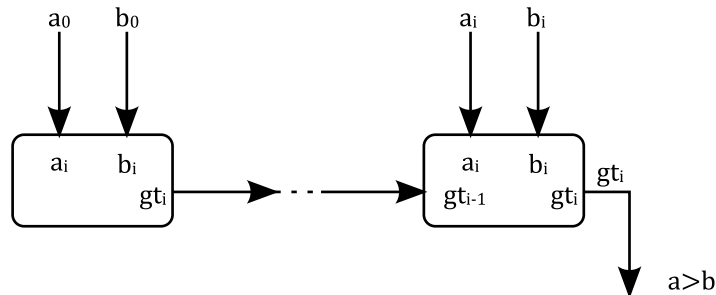
*kar/xor-06*



## 4 | KAR - Fonctions avec un nombre élevé d'entrées

### 4.1 Comparaison de nombres

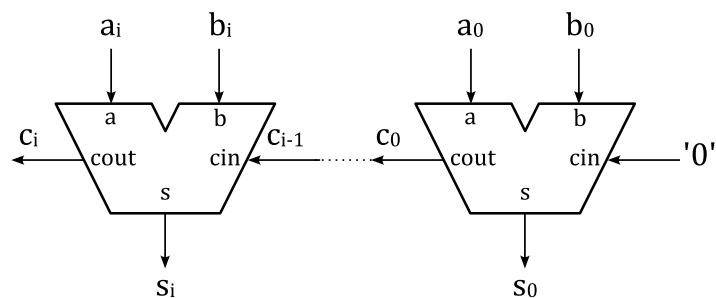
Possible with an iterative blocschema.



*kar/manyinputs-01*

### 4.2 Additionneur binaire

Possible with an iterative blocschema.



*kar/manyinputs-02*

### 4.3 Conversion de code thermomètre en code binaire

One possible solution is.

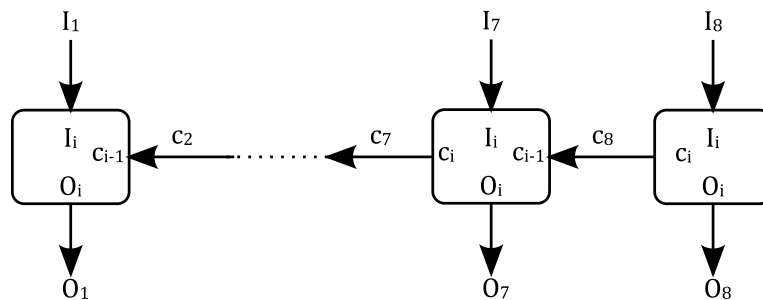
$$\begin{aligned} B_2 &= T_4 \\ B_1 &= T_2 + T_6 \overline{T_4} \\ B_0 &= ??? \end{aligned} \tag{18}$$

*kar/manyinputs-03*



#### 4.4 Transmission selon la priorité

Possible with an iterative blocschema.



*kar/manyinputs-04*

#### 4.5 Logique pour compteur sans retour à zéro

The solutions involves an Adder  $x+1$  and a Comparator to  $0xFFFF$

*kar/manyinputs-05*

#### 4.6 Additionneur avec saturation

The output  $c_{out}$  of an iterative adder indicates an overflow.

*kar/manyinputs-06*

#### 4.7 Nombres en code BCD

The BCD-Adder is a special case of a normal adder. Only the values between 0...9 exist. Therefore  $c_{out} = 1$  if the sum is  $> 10$ . In this case  $-10$  has to be subtracted from the output.

*kar/manyinputs-07*

#### 4.8 Fonction majorité à 7 entrées

A concatenation of adders with a comparison

*kar/manyinputs-08*

#### 4.9 Unité arithmétique et logique

First determine the schema of an adder and that of a subtractor, tie them together and create the logical functions.

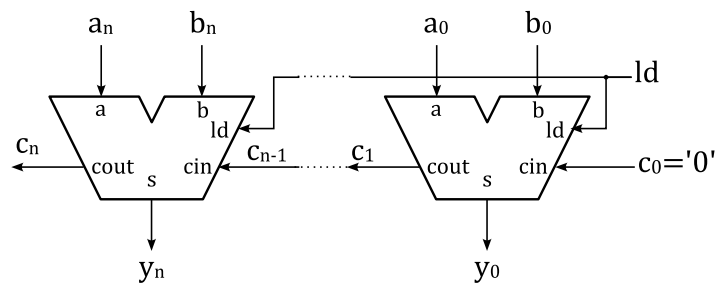
*kar/manyinputs-09*





#### 4.10 Logique pour compteur de programme

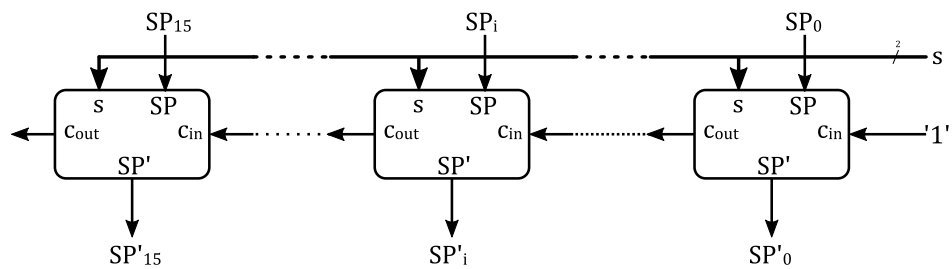
An iterative circuits with a load input.



*kar/manyinputs-10*

#### 4.11 Logique pour pointeur de pile

An iterative circuits with a 2bit selection input.



*kar/manyinputs-11*