Coneris:

Modular Reasoning about Error Bounds for Concurrent Probabilistic Programs

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A simple probability problem (based on real events)

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What is the probability I will not have a single internship offer 😭 ?

A sequential probabilistic program

$$\begin{aligned}
&\text{let } l = \text{ref } 0 \text{ in} \\
&l \leftarrow (!\, l + \text{coin}_{3/4}); \\
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&\text{let } x = !\, l \text{ in} \\
&\text{assert}(x > 0) \qquad\qquad //\text{fails if both coin}_{3/4} \text{ returns } 0
\end{aligned}$$

coin_p returns 1 with probability p and 0 with probability $1 - p$.

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coin_p returns 1 with probability p and 0 with probability $1 - p$.

Aim: show *twoAdd* **crashes with probability at most** $1/16$

Previous work: Eris (ICFP 24)

- *Eris*: a separation logic for proving error bounds of sequential probabilistic programs

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Previous work: Eris (ICFP 24)

- *Eris*: a separation logic for proving error bounds of sequential probabilistic programs
- **KEY IDEA: internalize error as a separation logic resource, aka *error credit***
- $\lightning(\varepsilon)$ asserts ownership of ε error credits, with $\varepsilon \in [0, 1]$

Theorem (Adequacy of Eris)

If $\{\lightning(\varepsilon)\}\, e\, \{v.\phi(v)\}$ *then* $\Pr[e \Downarrow v \wedge v \notin \phi] \leqslant \varepsilon.$

Eris rules

$$\lightning(\varepsilon_1) * \lightning(\varepsilon_2) \dashv\vdash \lightning(\varepsilon_1 + \varepsilon_2)$$

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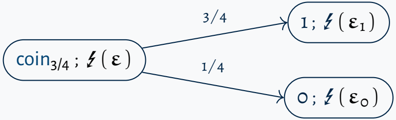
$$\frac{3/4 \cdot \varepsilon_1 + 1/4 \cdot \varepsilon_0 \leqslant \varepsilon}{\vdash \{\lightning(\varepsilon)\}\ \mathsf{coin}_{3/4}\ \{n\,.\ \lightning(\varepsilon_n)\}}\ \textsc{ht-coin-exp}$$



$$3/4 \cdot \varepsilon_1 + 1/4 \cdot \varepsilon_0 \leqslant \varepsilon$$

Eris in action

$$
\begin{array}{l}
\{\lightning(1/16)\} \\
\qquad \mathsf{let}\ l = \mathsf{ref}\ 0\ \mathsf{in} \\
\qquad l \leftarrow (!\,l + \mathsf{coin}_{3/4}); \\
\qquad l \leftarrow (!\,l + \mathsf{coin}_{3/4}); \\
\qquad \mathsf{let}\ x = !\,l\ \mathsf{in} \\
\qquad \mathsf{assert}(x > 0) \\
\{\mathsf{True}\}
\end{array}
$$

By adequacy, the probability of the program crashes is at most 1/16.

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$$\{\lightning(1/16) \ast l \mapsto 0\}$$

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$$\{\mathrm{True}\}$$

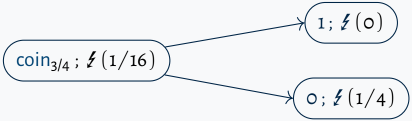
Allocate reference

Eris in action

$$\left\{ \begin{array}{l} \lightning(\text{if } x = 0 \text{ then } 1/4 \text{ else } 0) \;\; * \\ l \mapsto 0 \end{array} \right\}$$

$$\begin{array}{l} l \leftarrow (!\,l + x); \\ l \leftarrow (!\,l + \mathsf{coin}_{3/4}); \\ \text{let } x = !\,l \text{ in} \\ \mathsf{assert}(x > 0) \end{array}$$

$\{\text{True}\}$



$$3/4 \cdot 0 + 1/4 \cdot 1/4 \leqslant \frac{1}{16}$$

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$$\{ \lightning(1/4) \ast l \mapsto 0 \}$$

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$$\{\mathsf{True}\}$$

We continue with the case $x = 0$, otherwise it is trivial

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More steps...

Eris in action

$$\{\lightning(\mathsf{if}\ x = 0\ \mathsf{then}\ 1\ \mathsf{else}\ 0) \ * l \mapsto 0\}$$

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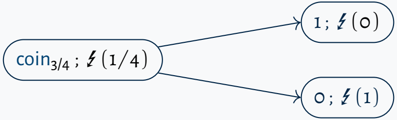

$$3/4 \cdot 0 + 1/4 \cdot 1 \leqslant \frac{1}{4}$$

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$$\{\mathsf{True}\}$$

$$\begin{tikzcd} & \boxed{1 \,;\, \lightning\,(0)} \\ \boxed{\mathsf{coin}_{3/4} \,;\, \lightning\,(1/4)} \arrow[ur] \arrow[dr] & \\ & \boxed{0 \,;\, \lightning\,(1)} \arrow[r, Rightarrow] & \checkmark \end{tikzcd}$$

$$3/4 \cdot 0 + 1/4 \cdot 1 \leqslant \frac{1}{4}$$

Eris in action

$$\{\lightning(0) \ast l \mapsto 0\}$$

$$l \leftarrow (!\,l + 1);$$
$$\mathsf{let}\; x = !\,l\; \mathsf{in}$$
$$\mathsf{assert}(x > 0)$$

$$\{\mathsf{True}\}$$

We sampled a 1.

Eris in action

$$\{\lightning(0) \ast l \mapsto 1\}$$

$$\quad \mathsf{assert}(1 > 0)$$

$$\{\mathsf{True}\}$$

And the assert goes through!

We can reason about error bounds of sequential **probabilistic** programs.

Nobody applies for internships sequentially.

A concurrent probabilistic program

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&\mathsf{let}\ l = \mathsf{ref}\ 0\ \mathsf{in} \\
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$\mathsf{faa}\ l\ x$ reads from reference l and increments it by x *atomically*

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Operational semantics of language extended to thread pools;
decision of which thread to step is decided by a scheduler

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Operational semantics of language extended to thread pools;
decision of which thread to step is decided by a scheduler

Theorem (Adequacy of Coneris)

If $\{\lightning(\varepsilon)\}\, e\, \{v.\phi(v)\}$ *then* ***for all possible schedulers*** $\mathfrak{s}$,
$\Pr[e \Downarrow_{\mathfrak{s}} v \wedge v \notin \phi] \leq \varepsilon$

First attempt in verifying *conTwoAdd*

$\{\lightning(1/16)\}$

$\quad \mathsf{let}\ l = \mathsf{ref}\ 0\ \mathsf{in}$
$\quad (\mathsf{faa}\ l\ (\mathsf{coin}_{3/4}) \mid\mid\mid \mathsf{faa}\ l\ (\mathsf{coin}_{3/4}))\ ;$
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By adequacy, for all possible interleavings, probability of the program crashing is at most 1/16.

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HT-PAR-COMP

$$\frac{\{P_1\}\, e_1\, \{v_1.\, Q_1\, v_1\} \qquad \{P_2\}\, e_2\, \{v_2.\, Q_2\, v_2\}}{\{P_1 \ast P_2\}\, e_1 \mathbin{|||} e_2\, \{(v_1, v_2).\, Q_1\, v_1 \ast Q_2\, v_2\}}$$

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$$\textsc{ht-par-comp}\ \dfrac{\{P_1\}\ e_1\ \{v_1.\ Q_1\ v_1\} \qquad \{P_2\}\ e_2\ \{v_2.\ Q_2\ v_2\}}{\{P_1 * P_2\}\ e_1\ |||\ e_2\ \{(v_1, v_2).\ Q_1\ v_1 * Q_2\ v_2\}}$$

1. We need to share the $l \mapsto 0$ resource between the two threads

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1. We need to share the $l \mapsto 0$ resource between the two threads
2. Splitting $\lightning(1/16)$ into $\lightning(1/32) \ast \lightning(1/32)$ is not enough for each thread to avoid sampling 0

Invariant

Invariants allow us to share resources between threads

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$$\frac{\textsc{ht-inv-open} \quad e \text{ atomic} \qquad \{I * P\}\, e\, \{I * Q\}}{\{\boxed{I} * P\}\, e\, \{Q\}}$$

We can access resources in invariants across atomic steps ...

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We can access resources in invariants across atomic steps ...

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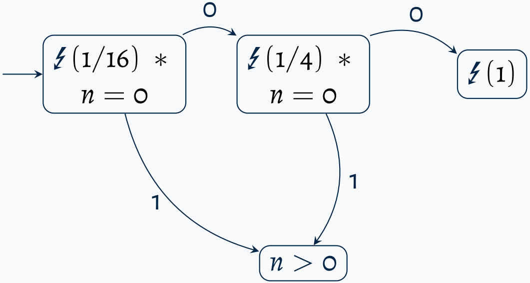
Key idea: we store both error credit and reference resource in an *invariant*

Coming up with an invariant

$I \triangleq \exists n. l \mapsto n \ast$



$\{\boxed{I} \ast \ldots\}$

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We can reason about error bounds of ~~sequential~~ **concurrent** probabilistic programs.

Now let's refactor stuff into a randomized concurrent counter

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\begin{array}{l}
\mathsf{let}\ l = \mathsf{ref}\ 0\ \mathsf{in} \\
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Problem 1: interaction with invariants

$$\left\{ \boxed{I} \ast \ldots \right\}$$

$$\quad \mathit{incr}\, l \triangleq (\ \lambda l.\ \mathsf{faa}\, l\, (\mathsf{coin}_{3/4})\ )\ l$$

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Challenge: specification should be expressive enough to open invariants (twice!)

Linearizability

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Intuitively, all operations appear to take place atomically.

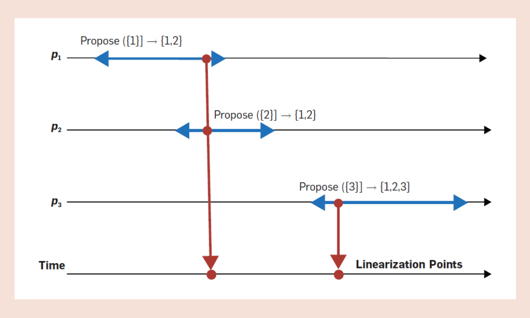

Figure from "A Linearizability-based Hierarchy for Concurrent Specifications"

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But this is not enough when we also have probability...

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$$\begin{aligned} incr_3 \triangleq{} & \\ & \mathsf{rec}\, f\, l = \mathsf{let}\, x = \mathsf{unif}\{0, \ldots, 7\}\ \mathsf{in} \\ & \quad \mathsf{if}\, x < 4\ \mathsf{then}\ (\mathsf{if}\, x > 0\ \mathsf{then}\ 1\ \mathsf{else}\ 0)\ \mathsf{else}\, f l \end{aligned}$$

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Challenge: our specification should be satisfied by all three implementations above

We need to capture “randomized logical atomicity”

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Here in Coneris, we capture this notion with a novel probabilistic update modality $⩪ P$;
it allows you to update error credits in addition to opening invariants

Case study

We implement a thread-safe idealized hash function:

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$$\{\mathsf{hashKey}\ h\ k\ v\}\ h\ k\ \{w.\ w = v\}$$

We use this to implement a concurrent Bloom filter and
we prove tight bound on probability of false positives (new result)

Take home message

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- Other examples: lazy sampler, concurrent amortized collision-free hash

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P.P.S. My advisor, Lars, is recruiting interns / PhD students / postdocs :)

Presampling tapes I

$$\sigma \in \mathit{State} \triangleq (\mathit{Loc} \xrightarrow{\text{fin}} \mathit{Val}) \times (\mathit{Label} \xrightarrow{\text{fin}} \mathit{Tape})$$
$$t \in \mathit{Tape} \triangleq \{(N, \vec{n}) \mid N \in \mathbb{N} \wedge \vec{n} \in \mathbb{N}^{*}_{\leqslant N}\}$$

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$$\mathrm{step}(\mathrm{tape}\ N, \sigma) = \mathrm{ret}(\kappa, \sigma[\kappa := (N, \epsilon)], []) \quad \text{(where } \kappa \text{ is fresh w.r.t. } \sigma\text{)}$$
$$\mathrm{step}(\mathrm{rand}\ \kappa\ N, \sigma) = \lambda(n, \sigma, []) \,.\ \tfrac{1}{N+1} \quad \text{if } \sigma[\kappa] = (N, \epsilon) \wedge n \in \{0, \ldots, N\} \quad \text{and} \quad 0 \text{ otherwise}$$

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There are no steps in operational semantics to *write* contents into a tape!

Rewriting randomized concurrent counter module

$$\begin{aligned} \mathit{create} &\triangleq \lambda\_.\ \ \mathsf{ref}\, 0 \\ \mathit{read} &\triangleq \lambda l.\ \ !\, l \\ \mathit{incr} &\triangleq \lambda l.\ \ \mathsf{faa}\, l\, (\mathsf{rand}\, 3) \end{aligned}$$

$$\begin{aligned} \mathit{conTwoAdd} \triangleq\ & \mathsf{let}\, l = \mathit{create}\,()\ \mathsf{in} \\ & (\mathit{incr}\, l \mid\mid\mid \mathit{incr}\, l)\, ; \\ & \mathit{read}\, l \end{aligned}$$

Rewriting randomized concurrent counter module

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\end{aligned}$$

$$\Rightarrow$$

$$\begin{aligned}
\mathit{create} &\triangleq \lambda_.\ \mathsf{ref}\, 0\\
\mathit{read} &\triangleq \lambda l.\ !\, l\\
\mathit{createCtape} &\triangleq \lambda ().\ \mathsf{tape}\, 3\\
\mathit{incr} &\triangleq \lambda l\, \kappa.\ \mathsf{faa}\, l\, (\mathsf{rand}\, \kappa\, 3)
\end{aligned}$$

$$\begin{aligned}
\mathit{conTwoAdd} \triangleq\ & \mathsf{let}\ c = \mathit{create}()\ \mathsf{in}\\
& \left(\begin{array}{l} \mathsf{let}\ \kappa = \mathit{createCtape}()\ \mathsf{in}\\ \mathit{incr}\, c\, \kappa \end{array} \mid\mid\mid \ \ldots \right);\\
& \mathit{read}\, c
\end{aligned}$$

Presampling tapes II

HT-ALLOC-TAPE

$$\frac{}{\{\mathsf{True}\}\ \texttt{tape}\ N\,\{\kappa.\ \kappa \hookrightarrow (N, \epsilon)\}}$$

Presampling tapes II

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$$\frac{}{\{\mathsf{True}\}\ \mathsf{tape}\ N\,\{\kappa.\,\kappa \hookrightarrow (N, \epsilon)\}}$$

HT-RAND-TAPE

$$\frac{}{\{\kappa \hookrightarrow (N, n \cdot \vec{n})\}\ \mathsf{rand}\ \kappa\ N\,\{x.\,x = n * \kappa \hookrightarrow (N, \vec{n})\}}$$

Presampling tapes II

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Wait?! How do you presample onto a tape in the logic?

Presampling tapes II

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$$\frac{}{\{\kappa \hookrightarrow (N, n \cdot \vec{n})\}\ \mathsf{rand}\,\kappa\,N\,\{x.\,x = n * \kappa \hookrightarrow (N, \vec{n})\}}$$

Wait?! How do you presample onto a tape in the logic?

We do it with the probabilistic update modality!

Rules of the probabilistic update modality

${}_{\mathcal{E}_1}{\Rrightarrow}_{\mathcal{E}_2} P$ denotes a resource together with the invariants in $\mathcal{E}_1$, can perform *a randomized logical atomic* operation and split into two parts: P and one satisfying invariants in $\mathcal{E}_2$

Rules of the probabilistic update modality

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$$\frac{P}{\rightsquigarrow\!\!\!\!\rightsquigarrow_{\mathcal{E}} P}\ \textsc{pupd-ret}$$

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PUPD-RET
$$\frac{P}{\rightsquigarrow_{\mathcal{E}} P}$$

PUPD-BIND
$$\frac{{}_{\mathcal{E}_1}\!\rightsquigarrow_{\mathcal{E}_2} P \qquad P \twoheadrightarrow {}_{\mathcal{E}_2}\!\rightsquigarrow_{\mathcal{E}_3} Q}{{}_{\mathcal{E}_1}\!\rightsquigarrow_{\mathcal{E}_3} Q}$$

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PUPD-FUPD

$$\frac{{}_{\mathcal{E}_1}{\mid\!\!\Rrightarrow}_{\mathcal{E}_2} P}{{}_{\mathcal{E}_1}{\mid\!\!\rightsquigarrow}_{\mathcal{E}_2} P}$$

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$$\textsc{pupd-fupd}\quad \frac{{}_{\mathcal{E}_1}{\Rrightarrow}_{\mathcal{E}_2}\ P}{{}_{\mathcal{E}_1}{\rightsquigarrow}_{\mathcal{E}_2}\ P}$$

$$\textsc{pupd-presample-exp}\quad \frac{\kappa \hookrightarrow (N, \vec{n}) \qquad ⚡(\varepsilon) \qquad \mathbb{E}_{\mathfrak{U}N}[\mathcal{F}] \leqslant \varepsilon}{{\rightsquigarrow}_{\mathcal{E}}\ (\exists n.\ \kappa \hookrightarrow (N, \vec{n} \cdot n) * ⚡(\mathcal{F}(n)))}$$

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PUPD-PRESAMPLE-EXP

$$\frac{\kappa \hookrightarrow (N, \vec{n}) \qquad \lightning(\varepsilon) \qquad \mathbb{E}_{\mathfrak{U}N}[\mathcal{F}] \leqslant \varepsilon}{\rightsquigarrow_{\mathcal{E}} (\exists n.\, \kappa \hookrightarrow (N, \vec{n} \cdot n) * \lightning(\mathcal{F}(n)))}$$

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$$\frac{{}_{\mathcal{E}_1}\!\Rrightarrow_{\mathcal{E}_2} P}{{}_{\mathcal{E}_1}\!\rightsquigarrow_{\mathcal{E}_2} P}$$

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New specification that exposes presampling

$$\forall \iota, c.\ \{\mathit{counter}\ \iota\ c\}\ \mathit{createCtape}(\,)\ \left\{\kappa.\ \mathit{ctape}\ \kappa\ \epsilon\right\}$$

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$$\begin{array}{l}
\forall \mathcal{E}, \iota, c, n, \vec{n}, Q. \\
\left\{\begin{array}{l} \mathit{counter}\ \iota\ c \ast \mathit{ctape}\ \kappa\ (n \cdot \vec{n}) \ast \\ \ (\forall z.\ \mathit{cauth}\ z \mathrel{-\!\!\ast} \Rrightarrow_{\mathcal{E}} \mathit{cauth}\ (z + n) \ast Q\, z) \end{array}\right\} \\
\quad \mathit{incr}\ c\ \kappa \\
\left\{z. \mathit{ctape}\ \kappa\ \vec{n} \ast Q\, z\right\}_{\mathcal{E} \uplus \{\iota\}}
\end{array}$$

New specification that exposes presampling

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$$\left\{\begin{array}{l} \mathit{counter}\ \iota\ c * \mathit{ctape}\ \kappa\ (n \cdot \vec{n}) * \\ \quad (\forall z.\ \mathit{cauth}\ z \mathrel{-\!\!*} \Rrightarrow_{\mathcal{E}} \mathit{cauth}\ (z+n) * Q\ z) \end{array}\right\}$$

$$\quad \mathit{incr}\ c\ \kappa$$

$$\left\{z. \mathit{ctape}\ \kappa\ \vec{n} * Q\ z\right\}_{\mathcal{E} \uplus \{\iota\}}$$

$$\forall \mathcal{E}, \varepsilon, \mathcal{F}, \vec{n}, \kappa.$$

$$(\lightning(\varepsilon) * (\mathbb{E}_{\mathcal{U}3}[\mathcal{F}] \leqslant \varepsilon) * \mathit{ctape}\ \kappa\ \vec{n} \mathrel{-\!\!*} \rightsquigarrow_{\mathcal{E}} \exists n \in \{0..3\}.\ \lightning(\mathcal{F}(n)) * \mathit{ctape}\ \kappa\ (\vec{n} \cdot [n]))$$

HOCAP specification of *create* and *read*

$$\{\mathsf{True}\}\ \mathit{create}(\,)\ \{c.\ \exists\iota.\ \mathit{counter}\ \iota\ c * \mathit{cfrag}\ 1\ 0\}$$

$$\begin{aligned}&\forall \mathcal{E}, \iota, c, Q.\\ &\quad \{\mathit{counter}\ \iota\ c * (\forall z.\ \mathit{cauth}\ z \twoheadrightarrow\!* \Rrightarrow_{\mathcal{E}} \mathit{cauth}\ z * Q\,z)\}\\ &\qquad \mathit{read}\ c\\ &\quad \{z.\ Q\,z\}_{\mathcal{E} \uplus \{\iota\}}\end{aligned}$$

- *counter* ι c captures the fact that c is a counter with invariant name ι

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- *counter* ι c captures the fact that c is a counter with invariant name ι
- *cauth* and *cfrag* provides *authoritative* and *fragmental* views of the counter

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- $\mathit{counter}\ \iota\ c$ captures the fact that c is a counter with invariant name ι
- *cauth* and *cfrag* provides *authoritative* and *fragmental* views of the counter
- Many conditions of these abstract predicates not shown, e.g. $\mathit{cauth}\ n * \mathit{cfrag}\ q\ m \vdash \Rrightarrow_{\mathcal{E}} \mathit{cauth}\ (n+p) * \mathit{cfrag}\ q\ (m+p)$

Second attempt in verifying *conTwoAdd*

$$\{ \lightning(1/16) \ * l \mapsto 0 \}$$

$$(\mathsf{faa}\ l\ (\mathsf{rand}\ 3) \ ||| \ \mathsf{faa}\ l\ (\mathsf{rand}\ 3))\ ;$$
$$!\, l$$

$$\{ v.v > 0 \}$$

Where we left off...

Second attempt in verifying *conTwoAdd*

$$\left\{ \boxed{I(\gamma_1, \gamma_2)}^{\iota} * \boxed{\circ S_0}^{\gamma_1} * \boxed{\circ S_0}^{\gamma_2} \right\}$$

$$(\mathsf{faa}\ l\ (\mathsf{rand}\ 3) \mathbin{|||} \mathsf{faa}\ l\ (\mathsf{rand}\ 3))\ ;$$
$$!\,l$$

$$\{v.v > 0\}$$

Allocating invariants and resources:

$$\lightning(1/16) * l \mapsto 0 \twoheadrightarrow$$
$$\Rrightarrow \exists \gamma_1\, \gamma_2.\ I(\gamma_1, \gamma_2) * \boxed{\circ S_0}^{\gamma_1} * \boxed{\circ S_0}^{\gamma_2}$$

Second attempt in verifying *conTwoAdd*

$$\left\{ \boxed{I(\gamma_1, \gamma_2)}^{\iota} * \boxed{\circ S_0}^{\gamma_1} \right\} \ \mathsf{faa}\, l\, (\mathsf{rand}\, 3) \ \left\{ \exists n.\ \boxed{\circ S_2(n)}^{\gamma_1} \right\}$$

$$\left\{ \boxed{I(\gamma_1, \gamma_2)}^{\iota} * \boxed{\circ S_0}^{\gamma_2} \right\} \ \mathsf{faa}\, l\, (\mathsf{rand}\, 3) \ \left\{ \exists n.\ \boxed{\circ S_2(n)}^{\gamma_2} \right\}$$

$$\left\{ \boxed{I(\gamma_1, \gamma_2)}^{\iota} * \boxed{\circ S_2(n_1)}^{\gamma_1} * \boxed{\circ S_2(n_2)}^{\gamma_2} \right\} \ !\, l\, \{v.\ v > 0\}$$

Applying HT-PAR-COMP

Second attempt in verifying *conTwoAdd* – First two Hoare triples

$$\left\{ \boxed{I(\gamma_1, \gamma_2)}^{\iota} * \boxed{\circ S_\circ}^{\gamma_1} \right\}$$

$$\text{faa}\ l\ (\text{rand}\ 3)$$

$$\left\{ \exists n.\ \boxed{\circ S_2(n)}^{\gamma_1} \right\}$$

First Hoare triple

(second Hoare triple is proven similarly)

Recall invariant opening rule:

HT-INV-OPEN

$$\frac{e \text{ atomic} \qquad \{I * P\}\, e\, \{I * Q\}}{\left\{ \boxed{I} * P \right\} e \left\{ \boxed{I} * Q \right\}}$$

Second attempt in verifying *conTwoAdd* – First two Hoare triples

$$\left\{ \boxed{I(\gamma_1, \gamma_2)}^{\iota} * \boxed{\circ S_1(n)}^{\gamma_1} \right\}$$

$$\quad \mathsf{faa}\ l\ n$$

$$\left\{ \exists n.\ \boxed{\circ S_2(n)}^{\gamma_1} \right\}$$

rand 3 is atomic

We can open invariants temporarily and update ghost resources to track n sampled

Second attempt in verifying *conTwoAdd* – First two Hoare triples

$$\left\{ \boxed{I(\gamma_1, \gamma_2)}^{\iota} * \boxed{\circ S_1(n)}^{\gamma_1} \right\}$$

$$\texttt{faa}\ l\ n$$

$$\left\{ \exists n.\ \boxed{\circ S_2(n)}^{\gamma_1} \right\}$$

rand 3 is atomic

We can open invariants temporarily and update ghost resources to track n sampled

We can do the same again with faa $l\ n$

Second attempt in verifying *conTwoAdd* – last Hoare triples

Last Hoare triple

$$\left\{ \boxed{I(\gamma_1, \gamma_2)}^{\iota} * \boxed{\circ S_2(n_1)}^{\gamma_1} * \boxed{\circ S_2(n_2)}^{\gamma_2} \right\}$$

$$!\, l$$

$$\{v.\, v > 0\}$$

Second attempt in verifying *conTwoAdd* – last Hoare triples

$$\left\{ \boxed{I(\gamma_1, \gamma_2)}^{\iota} * \boxed{\circ S_2(n_1)}^{\gamma_1} * \boxed{\circ S_2(n_2)}^{\gamma_2} \right\}$$

$$!\, l$$

$$\{v.\, v > 0\}$$

Last Hoare triple

- But nothing too surprising!

Second attempt in verifying *conTwoAdd* – last Hoare triples

$$\left\{ \boxed{I(\gamma_1, \gamma_2)}^{\iota} * \boxed{\circ S_2(n_1)}^{\gamma_1} * \boxed{\circ S_2(n_2)}^{\gamma_2} \right\}$$

$$! \, l$$

$$\{v.\, v > 0\}$$

Last Hoare triple

- But nothing too surprising!
- $!\,l$ is atomic, so we can open invariants and do a case split on value of n_1 and n_2.

Second attempt in verifying *conTwoAdd* – last Hoare triples

$$\left\{ \boxed{I(\gamma_1, \gamma_2)}^{\iota} * \boxed{\circ S_2(n_1)}^{\gamma_1} * \boxed{\circ S_2(n_2)}^{\gamma_2} \right\}$$

$$!\,l$$

$$\{v.\, v > 0\}$$

Last Hoare triple

- But nothing too surprising!
- $!\,l$ is atomic, so we can open invariants and do a case split on value of n_1 and n_2.
- If both are 0, we get $\lightning(1)$ and can derive $\bot$!

Weakest-pre

$$
\begin{aligned}
\mathsf{wp}\ e_1\ \{\Phi\} \triangleq \forall \sigma_1, \varepsilon_1.\ S(\sigma_1, \varepsilon_1) \mathrel{-\!\!*}\ {}_{\top}\!\Rrightarrow_{\emptyset}\ \mathsf{sstep}\ \sigma_1\ \varepsilon_1\ \{\sigma_2, \varepsilon_2.\\
&\big(e_1 \in \mathit{Val} * {}_{\emptyset}\!\Rrightarrow_{\top} S(\sigma_2, \varepsilon_2) * \Phi(e_1)\big) \vee \\
&\big(e_1 \notin \mathit{Val} * \mathsf{pstep}\ (e_1, \sigma_2)\ \varepsilon_2\ \{e_2, \sigma_3, l, \varepsilon_3.\\
&\quad \triangleright\, \mathsf{sstep}\ \sigma_3\ \varepsilon_3\ \{\sigma_4, \varepsilon_4.\ {}_{\emptyset}\!\Rrightarrow_{\top} S(\sigma_4, \varepsilon_4) * \mathsf{wp}\ e_2\ \{\Phi\} * \mathop{\bigstar}_{e' \in l} \mathsf{wp}\ e'\ \{\mathsf{True}\}\}\}\big)\}
\end{aligned}
$$

State and program step precondition

$$\textsc{State-step-err-1}\quad \frac{1 \leqslant \varepsilon}{\mathsf{sstep}\ \sigma\ \varepsilon\ \{\Phi\}}$$

$$\textsc{State-step-ret}\quad \frac{\Phi(\sigma, \varepsilon)}{\mathsf{sstep}\ \sigma\ \varepsilon\ \{\Phi\}}$$

$$\textsc{State-step-continuous}\quad \frac{\forall \varepsilon'.\ \varepsilon < \varepsilon' \twoheadrightarrow \mathsf{sstep}\ \sigma\ \varepsilon'\ \{\Phi\}}{\mathsf{sstep}\ \sigma\ \varepsilon\ \{\Phi\}}$$

$$\textsc{State-step-exp}\quad \frac{\mathbb{E}_{\mu}[\mathcal{F}] \leqslant \varepsilon \qquad \mathsf{schErasable}(\mu, \sigma_1) \qquad \forall \sigma_2.\ 0 < \mu(\sigma_2) \twoheadrightarrow \mathsf{sstep}\ \sigma_2\ (\mathcal{F}(\sigma_2))\ \{\Phi\}}{\mathsf{sstep}\ \sigma_1\ \varepsilon\ \{\Phi\}}$$

$$\textsc{Prog-step-exp}\quad \frac{\begin{array}{c}\mathsf{red}(e_1, \sigma_1) \qquad \mathbb{E}_{\mathsf{step}(e_1,\sigma_1)}[\mathcal{F}] \leqslant \varepsilon \\ \forall (e_2, \sigma_2, l).\ 0 < \mathsf{step}(e_1, \sigma_1)(e_2, \sigma_2, l) \twoheadrightarrow \Phi(e_2, \sigma_2, l, \mathcal{F}(e_2, \sigma_2, l))\end{array}}{\mathsf{pstep}\ (e_1, \sigma_1)\ \varepsilon\ \{\Phi\}}$$

Probabilistic update modality

$$ {}_{\mathcal{E}_1}\!\mathrel{|\!\!\rightsquigarrow}_{\mathcal{E}_2} P \triangleq \forall \sigma_1, \varepsilon_1.\, S(\sigma_1, \varepsilon_1) \mathrel{-\!\!*} {}_{\mathcal{E}_1}\!\mathrel{|\!\!\Rrightarrow}_{\emptyset} \mathsf{sstep}\ \sigma_1\ \varepsilon_1 \{\sigma_2, \varepsilon_2.\ {}_{\emptyset}\!\mathrel{|\!\!\Rrightarrow}_{\mathcal{E}_2} S(\sigma_2, \varepsilon_2) * P\} $$

PUPD-ELIM

$$ \frac{\{P * Q\}\, e\, \{R\}_{\mathcal{E}}}{\{(\mathrel{|\!\!\rightsquigarrow}_{\mathcal{E}} P) * Q\}\, e\, \{R\}_{\mathcal{E}}} $$

PUPD-RET

$$ \frac{P}{\mathrel{|\!\!\rightsquigarrow}_{\mathcal{E}} P} $$

PUPD-BIND

$$ \frac{{}_{\mathcal{E}_1}\!\mathrel{|\!\!\rightsquigarrow}_{\mathcal{E}_2} P \qquad P \mathrel{-\!\!*} {}_{\mathcal{E}_2}\!\mathrel{|\!\!\rightsquigarrow}_{\mathcal{E}_3} Q}{{}_{\mathcal{E}_1}\!\mathrel{|\!\!\rightsquigarrow}_{\mathcal{E}_3} Q} $$

PUPD-FUPD

$$ \frac{{}_{\mathcal{E}_1}\!\mathrel{|\!\!\Rrightarrow}_{\mathcal{E}_2} P}{{}_{\mathcal{E}_1}\!\mathrel{|\!\!\rightsquigarrow}_{\mathcal{E}_2} P} $$

PUPD-PRESAMPLE-EXP

$$ \frac{\mathbb{E}_{\mathfrak{U}N}[\mathcal{F}] \leqslant \varepsilon \qquad ⚡(\varepsilon) \qquad \kappa \hookrightarrow (N, \vec{n})}{\mathrel{|\!\!\rightsquigarrow}_{\mathcal{E}} (\exists n.\, \kappa \hookrightarrow (N, \vec{n} \cdot n) * ⚡(\mathcal{F}(n)))} $$

PUPD-ERR

$$ \frac{}{\mathrel{|\!\!\rightsquigarrow}_{\mathcal{E}} (\exists \varepsilon.\, 0 < \varepsilon * ⚡(\varepsilon))} $$