# VPP Design Problem Formulation

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This document presents the formulation of the VPP design problem as well as an algorithm for solving it. Section 1 first provides an overview of the problem and proposed algorithm. Section 2 describes the selected case studies. Section 3 contains the problem formulation, including the objective and constraints. Section 4 clarifies the next steps. Finally, Appendices A and B give the details of linearization technique for constraints of hydro power (HP) plants and a summary of problem formulation, respectively.

## 1 Overview of Problem

When we combine small HP plants, photovoltaics (PVs), wind turbines (WTs), battery energy storage systems (BESs), and pumps (PMs) into a single portfolio and centrally manage them with a VPP, there are several advantages. *Firstly*, by integrating small HP plants into a VPP scheme, it is possible to optimize their production and increase their potential profit in the day-ahead market. *Secondly*, the less reliable flexibility of the HP plants with small reservoir capacity can be combined with the more reliable flexibility of the battery energy storage systems, allowing them to meet the requirements of the flexibility market as well as environmental constraints (e.g., limits related to hydro-peaking). *Thirdly*, the flexibility of the small HP plants can be used to compensate for PV and WT production imbalances due to forecast errors, resulting in increased profits in the day-ahead and intraday markets. *Fourthly*, the pumps can be used to transfer water multiple times to higher altitude HP plants, allowing them to provide and sell more flexibility if the price of flexibility is higher than the cost of energy in the day ahead.

As a result, we consider a VPP that focuses on three distinct electricity markets: day-ahead, intraday balancing, and flexibility (i.e., ancillary service) markets. The day-ahead market for energy production operates on a daily basis for each hour. Profit is maximized by forecasting day-ahead electricity prices and production of VPP components. The generation forecast is reviewed and the energy required to balance the energy transacted in the day-ahead market is exchanged in the intraday balancing market.

The forecast error of PV and wind generation will be balanced in intraday market. Transmission system operators handle the flexibility (i.e., ancillary service) market, in which it will exchange a variety of flexibility products[1], including (i) frequency containment reserve (FCR), also known as primary control reserve; (ii) automatic or manual frequency restoration reserve (a-FRR or m-FRR), also known as secondary control reserve; and (iii) positive or negative replacement reserve (RR), also known as tertiary control reserve.

In addition to the markets under consideration, we take into account other assumptions for the problem and the proposed VPP design algorithm. The following are the main assumptions underlying the problem formulation and proposed algorithm:

A1. The VPP will focus on three distinct electricity markets: day-ahead, intraday balancing, and flexibility (i.e., ancillary service).

A2. The number of VPP design schemes is limited. As a result, we can calculate the profit and risk of each design scheme individually.

A3. The VPP might consist of a number of existing and potential small HP plants, pumps, PVs, wind turbines, and battery energy storage systems.

A4. There could be a hydrological link constraint between small HP plants and pumps. FMV must be contacted for additional information on the configuration of HP plants.

A5. Small HP plants, pumps, and energy storage systems will participate in the flexibility market to provide flexibility products. Because of practical reasons, PVs and wind turbines do not curtail their power to participate in the flexibility market.

A6. The VPP will optimize its resources in day-ahead and flexibility markets knowing the forecast with lead time of 24h. Furthermore, the VPP will participate in intraday balancing market knowing the short term forecast with lead time of 1h.

To optimally design the VPP, we must determine the optimal operation of the VPP components over various time horizons, considering the three mentioned markets. However, if we consider all time horizons in one optimization problem, it would be so large that it would be impossible to solve if in a tractable manner. We employ a hierarchical approach to resolve the issue of tractability by estimating the optimal operation and designing the

[1]A suitable product from the daily flexibility market, out of FCR, RR, a-FRR, and m-FRR, will be selected based on its requirements and the physical constraints of VPP components. Our primary analysis is that the FCR will be a suitable product considering the limited reservoirs of small HP units.

VPP components, keeping in mind that the number of VPP design schemes is limited.

The flowchart in Figure 1 describes the process of designing the VPP. The process begins by generating a set of long-term annual scenarios, each of which represents a different possible hydro-meteorological condition (this will be done in WP 4). The next step is to optimize the VPP operation for each scenario. This is done using a three-level optimization framework including a long-term (annual) optimization model, a day-ahead optimization model, and a real-time optimization model. The long-term optimization model takes an annual scenario and determines the optimal weekly water discharge and average head of each reservoir for each week. The day-ahead optimization model determines the optimal power output of the VPP for the next day, given the forecast for the day. The real-time optimization model determines the optimal power output of the VPP for the current hour given the hour-ahead forecast. The process is repeated for all scenarios[2]. Once the VPP has been optimized for all scenarios, the profit and risk of all VPP schemes are calculated. The best scheme is then selected. The overall objective of the proposed algorithm is to identify the following: (a) The optimal VPP design scheme including the optimal capacity of its generation and storage units; (b) The optimal trade-off between profit and risk level of the VPP; (c) The optimal operating schedule for each generation and storage unit regarding various scenarios.

The schematically scenario generation algorithm (the steps i, vi, and viii in Figure 1) is presented in Figure 2. The process begins by collecting historical time-series data, such as solar irradiation, water discharge, wind speed, and electricity market prices. The data are then preprocessed, which may involve removing outliers, selecting features, and clustering the data. Once the data has been pre-processed, a model is trained. The model calculates the statistics of each cluster and the transition matrices for each cluster. The transition matrices describe the probability of moving from one state to another. The next step is to generate scenarios. This is done by generating samples for the daily average of each time series. The cluster of each day is then determined, and a Markov chain state time series is generated on the basis of the associated transition matrix. Furthermore, using short-term forecasting model with lead time of 24 and 1 hour, the day-ahead and hour-ahead forecast will be generated, respectively. The final step is to reduce the number of scenarios. This can be done by using a variety of methods, such as clustering the scenarios or selecting the most representative scenarios. It should be noted that the output contains a number of scenarios, each of which represents one hydro-meteorological year.

[2]We will use aggregation and parallelization techniques to reduce computational time. Aggregation is accomplished by combining day-ahead and real-time problems from one week into a single problem, and parallelization is accomplished by running long-term problems from various scenarios on multiple parallel processors.

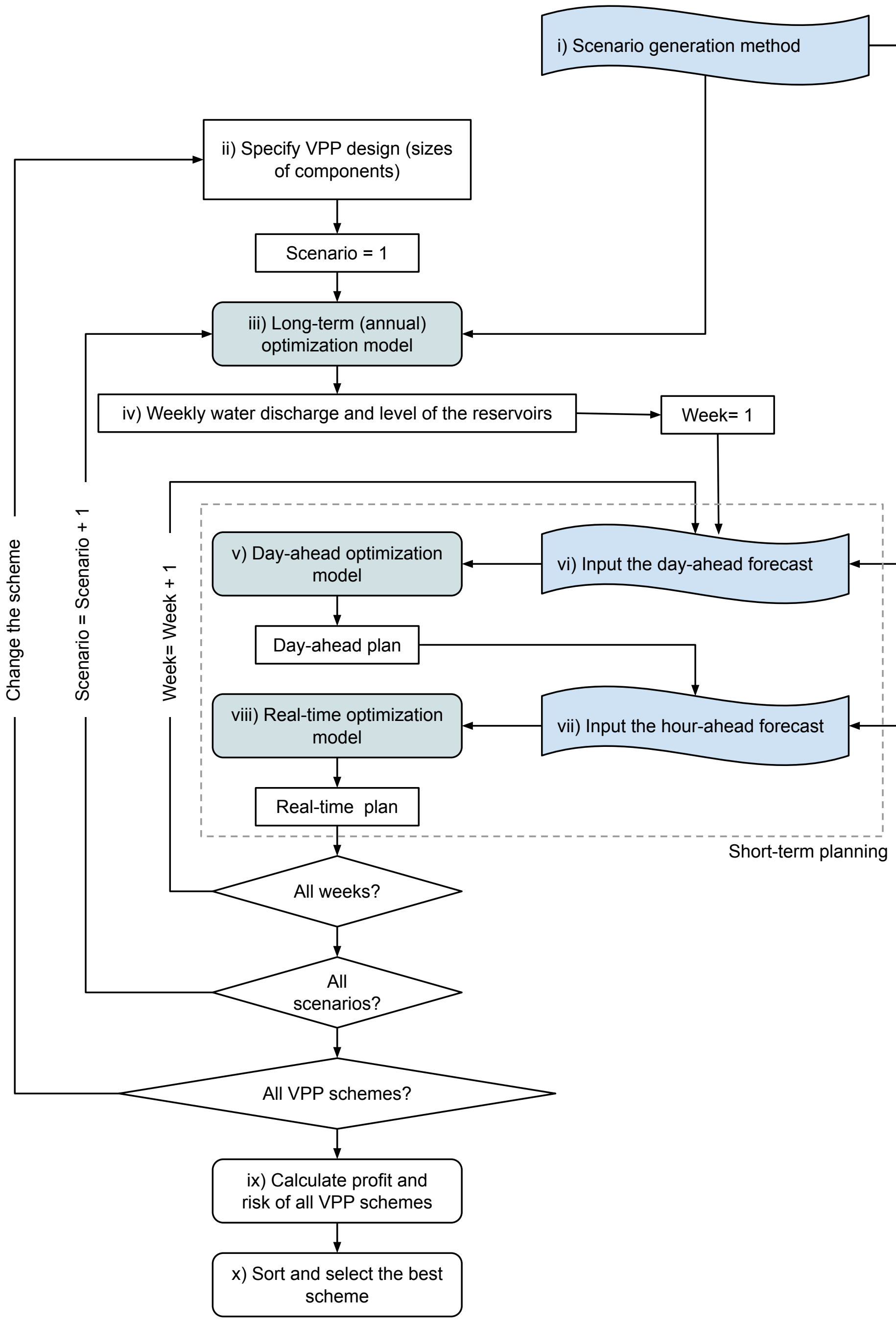


Figure 1: Proposed algorithm for VPP design problem.

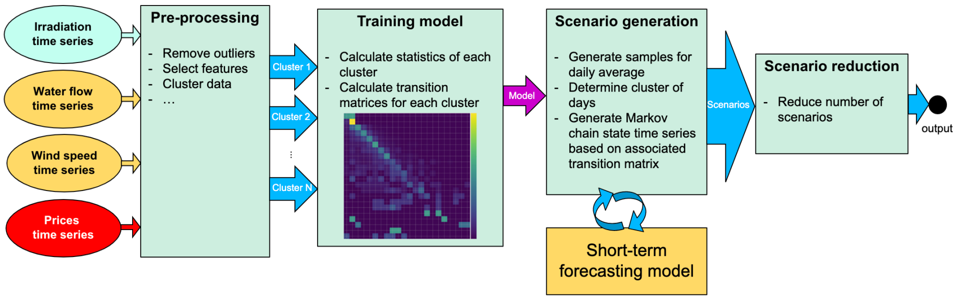


Figure 2: Scenario generation for planning problem.

## 2 Definition of Case Studies

In step ii of the proposed algorithm in Figure 1, we must define a number of VPP design schemes. These schemes will be defined based on the geographical and technical limitations that exist for the renovation of small HP units and the building of other distributed resources. We will consider the following four schemes for the VPP design[3]:

- Case 1: All interested small HPs in the Goms region (at least 7 units) will be integrated.
- Case 2: One small HP "KW Gletsch-Oberwald" will be integrated with a PV like "Griessee Solaranlage auf der Krone" and a WT like "Windpark und Griessee".
- Case 3: One small HP "KW Gletsch-Oberwald" will be integrated with other distributed resources, including a PV like "Griessee Solaranlage auf der Krone", a WT like "Windpark und Griessee", and an ESS with a predetermined capacity.
- Case 4: All interested small HPs in the Goms region (at least 7 units) will be integrated with distributed resources, including a pump like "Altstafel" (10 MW), a PV like "Griessee Solaranlage auf der Krone", a WT like "Windpark und Griessee", and an ESS with a predetermined capacity. A map of all interested HPs is provided in Figure 3 (in green and blue).

[3]This is a preliminary list of case studies. It will be extended or modified throughout the project by analyzing real-world data and hydro and meteorological historical time series related to the resources of the VPP.

| | | |
|---|---|---|
| 1 | KW Aegina AG | Altstafel |
| 2 | KW Gletsch-Oberwald | |
| 3 | KW Merezenbach AG | |
| 4 | KW Blinnenwerk AG | Reckingen (Wannebode) |
| 5 | KW Walibach | Walibach |
| 6 | Gommerkraftwerke AG | Rappental (GKW1) |
| 7 | Gommerkraftwerke AG | Saflisch (GKW2) |
| 8 | Gommerkraftwerke AG | Heiligkreuz (GKW2) |
| 9 | Gommerkraftwerke AG | Fieschertal (GKW3) |
| 10 | Gommerkraftwerke AG | Wysswasser |
| 11 | Rhonewerke AG | Ernen |
| 12 | Rhonewerke AG | Mörel |
| 13 | Aletsch AG | Mörel |
| 14 | KW Massaboden | Bitsch |
| 15 | KW Obergoms AG | Gere |
| 16 | KW Obergoms AG | Ulrichen |
| 17 | KW Obergoms AG | Niderbach |
| 18 | KW Obergoms AG | Münstigerbach |
| 19 | Rhonewerke AG | Binn |
| 20 | Gommerkraftwerke AG | Neubrigg (GKW1) |
| 21 | KW Bitsch | Electra Massa |

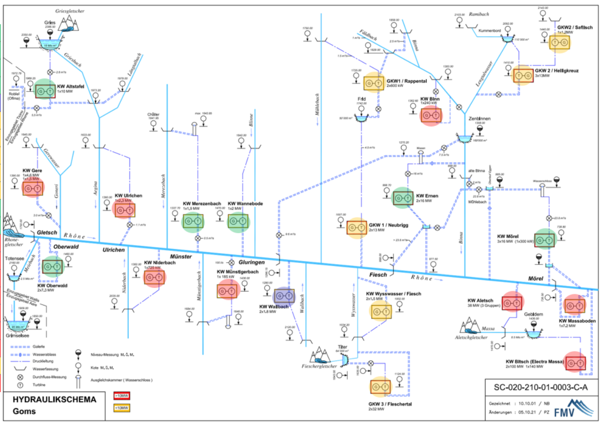


Figure 3: Small HPs in Goms region.

## 3 Problem Formulation

This section contains the models and formulations for steps iii, v, viii, ix, and x of Figure 1, including the long-term (annual), day-ahead, and real-time optimization models. These three optimization problems can be combined to form a larger problem. We will address the scalability and numerical issues by addressing these three problems with long-term and short-term optimization problems for various scenarios and VPP design schemes, which can be solved in parallel.

Each short-term and long-term optimization problem has its own objective and set of constraints. Specifically, the following constraints are considered:

- The flow rates of small HP plants and pumps limit their capacity for production. Additionally, the flow of small HP plants must respect environmental constraints (e.g., hydro-peaking and limits on the amount of stored water), which depend on input discharge and unit flow. Small HP plant minimum and maximum head (or minimum and maximum volume), ramp-rate restrictions for flexibility, and water flow dependence of cascaded units and pumps are also taken into consideration.

- The charging and discharging power of a battery energy storage system, as well as the minimum and maximum charging/discharging power, all have an impact on the state of energy of the battery energy storage system. The battery's ramp-rate for supplying flexibility is also considered.

- The production of PVs and wind turbines is limited by the irradiation and wind speed considering their production efficiencies.

- The VPP follows the rules and regulations of the power exchange and the ancillary service markets.

The constraints for the small HP plants and pumps are complex due to the non-linear relationship between production and flow. We used a step-wise linearization technique to approximate them. The constraints for the battery energy storage systems and PVs and wind turbines are more straightforward.

## 3.1 Main Notations

Let consider $\mathbb{E}(.)$ and $\mathbb{P}(.)$ be the expectation and probability function operators, respectively. Time is indexed by $t \in \mathcal{T}$, where $\mathcal{T} := \{1, 2, \ldots, T\}$ is the set of time index and $\delta t$ is the resolution of time steps[4]. Weeks of a year are indexed by $w \in \mathcal{W}$, where $\mathcal{W} := \{1, 2, \ldots, W\}$ is the set of weeks[5]. The prediction horizon is indexed by $z \in \mathcal{Z}$, where $\mathcal{Z} := \{\mathrm{RT}, \mathrm{DA}\}$ means that the prediction horizon is either real-time or day-ahead. Selected market products are indexed by $m \in \mathcal{M}$, where $\mathcal{M}$ is the set of suitable market products[6] for the VPP.

The VPP can be composed of a set of existing and potential resources $r \in \mathcal{R}$, where $\mathcal{R} := \{1, 2, \ldots, R\}$ includes all distributed resources and small HPs, i.e.,

$$\mathcal{R} = \cup_{g \in \{\mathrm{HP,PM,PV,WT,ES}\}} \mathcal{R}^{(\mathrm{g})},$$

where $\mathcal{R}^{(\mathrm{HP})}$, $\mathcal{R}^{(\mathrm{PM})}$, $\mathcal{R}^{(\mathrm{PV})}$, $\mathcal{R}^{(\mathrm{WT})}$, and $\mathcal{R}^{(\mathrm{ES})}$ are the sets of small HPs, PMs, PVs, WTs, and BESs, respectively. The design scheme $l \in \mathcal{L}$ of the VPP represents the case where a subset $\mathcal{R}_{\mathrm{l}} \subset \mathcal{R}$ of the resources is considered inside the portfolio of the VPP. The set $\mathcal{R}_{\mathrm{l}}$ is the union of all small HP units, PMs, PVs, WTs, and BESs in the design scheme $l$, i.e.,

$$\mathcal{R}_{\mathrm{l}} = \cup_{g \in \{\mathrm{HP,PM,PV,WT,ES}\}} \mathcal{R}_{\mathrm{l}}^{(\mathrm{g})}.$$

The scenarios of operation are indexed by $s \in \mathcal{S}$, where $\mathcal{S} := \{1, 2, \ldots, S\}$ and $\sum_{s \in \mathcal{S}} \mathbb{P}(s) = 1$. Each scenario $s \in \mathcal{S}$ represents:

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[4]The set $\mathcal{T}$ and the parameter $\delta t$ can be different for each optimization problem. For day-ahead and real-time problems of one day, $T$ and $\delta t$ are considered equal to 24 and $1[h]$, respectively. For the short-term planning problem, $T$ and $\delta t$ are considered equal to $24 \times 7$ and $1[h]$, respectively. For the long-term planning problem, $T$ and $\delta t$ are considered equal to 1 and $24 \times 7[h]$, respectively.

[5]The set $\mathcal{W}$ can be different for each optimization problem. For long-term planning of one year, $W$ is considered equal to 52. For short-term planning problem $\mathcal{W}$ only includes a value of respected week $w$.

[6]The suitable market products will be selected by studying the technical limitations of the generation and storage units of the VPP as well as potential modifications. As a result, $\mathcal{M} := \{\mathrm{DA}, \mathrm{BAL}\} \cup \mathcal{M}'$, where $\mathcal{M}'$ includes a number of flexibility market products that VPP is able to provide, i.e., $\mathcal{M}' \subset \{\mathrm{FCR}^{\uparrow}, \mathrm{FCR}^{\downarrow}, \mathrm{RR}^{\uparrow}, \mathrm{RR}^{\downarrow}, \mathrm{aFRR}^{\uparrow}, \mathrm{aFRR}^{\downarrow}, \mathrm{mFRR}^{\uparrow}, \mathrm{mFRR}^{\downarrow}\}$.

- The time series of day-ahead and hour-ahead prediction[7] of solar irradiation for all PV $r \in \mathcal{R}^{(\mathrm{PV})}$,

$$\mathbf{ir}_{\mathrm{r,s}} := (\mathrm{ir}_{\mathrm{r,w,t,z,s}})_{z\in\mathcal{Z},t\in\mathcal{T},w\in\mathcal{W}}, W = 52, T = 168,$$

- The day-ahead and hour-ahead prediction of water discharge for all HP unit $r \in \mathcal{R}^{(\mathrm{HP})}$,

$$\mathbf{wd}_{\mathrm{r,s}} := (\mathrm{wd}_{\mathrm{r,w,t,z,s}})_{z\in\mathcal{Z},t\in\mathcal{T},w\in\mathcal{W}}, W = 52, T = 168,$$

- The day-ahead and hour-ahead prediction of wind speed for all WT $r \in \mathcal{R}^{(\mathrm{WT})}$,

$$\mathbf{ws}_{\mathrm{r,s}} := (\mathrm{ws}_{\mathrm{r,w,t,z,s}})_{z\in\mathcal{Z},t\in\mathcal{T},w\in\mathcal{W}}, W = 52, T = 168,$$

- The day-ahead and hour-ahead prediction of market price for the market product $m \in \mathcal{M}$,

$$\boldsymbol{\pi}_{\mathrm{m,s}} := (\pi^{(\mathrm{m})}_{\mathrm{w,t,z,s}})_{z\in\{\mathrm{DA}\},t\in\mathcal{T},w\in\mathcal{W}}, W = 52, T = 168.$$

In the problem formulation, there are many variables that defined for combination of the indices $w$, $t$, $z$, $s$, and $l$. To present the equations simpler, we define an index $i := (w, t, z, s, l) \in \mathcal{I}$, where $\mathcal{I} := \mathcal{W} \times \mathcal{T} \times \mathcal{Z} \times \mathcal{S} \times \mathcal{L}$ represents the combination of week $w \in \mathcal{W}$, time step $t \in \mathcal{T}$, prediction horizon $z \in \mathcal{Z}$, scenario $s \in \mathcal{S}$, and design scheme $l \in \mathcal{L}$.

Other notations will be addressed further down in this section.

### 3.2 Objective Functions

We have four objective functions in problem formulation:
*First*, the real-time optimization objective is,

$$J^{(\mathrm{RT})}_{\mathrm{w,s,l}}\left(\tilde{\mathbf{x}}^{(\mathrm{RT})}_{\mathrm{w,s,l}}|\tilde{\mathbf{x}}^{(\mathrm{DA})}_{\mathrm{w,s,l}}, \tilde{\mathbf{x}}^{(\mathrm{Y})}_{\mathrm{w,l}}\right) := \max_{\mathbf{x}^{(\mathrm{RT})}_{\mathrm{w,s,l}}} \sum_{t\in\mathcal{T}} \delta t \cdot \sum_{m\in\{\mathrm{BAL}\}} p^{(\mathrm{m})}_{\mathrm{w,t,s,l}} \cdot \pi^{(\mathrm{m})}_{\mathrm{w,t,RT,s}}; \quad (1)$$

*Second*, the day-ahead optimization objective is,

$$J^{(\mathrm{DA})}_{\mathrm{w,s,l}}\left(\tilde{\mathbf{x}}^{(\mathrm{DA})}_{\mathrm{w,s,l}}|\tilde{\mathbf{x}}^{(\mathrm{Y})}_{\mathrm{s,l}}\right) := \max_{\mathbf{x}^{(\mathrm{DA})}_{\mathrm{w,s,l}}} \sum_{t\in\mathcal{T}} \delta t \cdot \sum_{m\in\mathcal{M}\setminus\{\mathrm{BAL}\}} p^{(\mathrm{m})}_{\mathrm{w,t,s,l}} \cdot \pi^{(\mathrm{m})}_{\mathrm{w,t,DA,s}}; \quad (2)$$

*Third*, the long-term optimization objective is,

$$J^{(\mathrm{Y})}_{\mathrm{s,l}}\left(\tilde{\mathbf{x}}^{(\mathrm{Y})}_{\mathrm{s,l}}\right) := \max_{\mathbf{x}^{(\mathrm{Y})}_{\mathrm{s,l}}} \sum_{w\in\mathcal{W}} \left[J^{(\mathrm{DA})}_{\mathrm{w,s,l}}\left(\tilde{\mathbf{x}}^{(\mathrm{DA})}_{\mathrm{w,s,l}}|\tilde{\mathbf{x}}^{(\mathrm{Y})}_{\mathrm{s,l}}\right) + J^{(\mathrm{RT})}_{\mathrm{w,s,l}}\left(\tilde{\mathbf{x}}^{(\mathrm{RT})}_{\mathrm{w,s,l}}|\tilde{\mathbf{x}}^{(\mathrm{DA})}_{\mathrm{w,s,l}}, \tilde{\mathbf{x}}^{(\mathrm{Y})}_{\mathrm{s,l}}\right)\right]; \quad (3)$$

[7] The difference between day-ahead and hour-ahead prediction data is denoted by the index $h$ in time series.

*Fourth*, the profit of the VPP is,

$$J^* := \max_{l\in\mathcal{L}} \left[ \mathrm{NP}_{\mathrm{int,y}} \cdot \mathop{\mathbb{E}}_{s\in\mathcal{S}} \left( J^{(\mathrm{Y})}_{\mathrm{s,l}} \left( \tilde{\mathbf{x}}^{(\mathrm{Y})}_{\mathrm{s,l}} \right) \right) - \sum_{r\in\mathcal{R}_{\mathrm{l}}} \mathrm{capex}_{\mathrm{r}} \right], \tag{4}$$

where $p^{(\mathrm{m})}_{\mathrm{w,t,s,l}}$ is the sold power[8] exchanged by the VPP at time $t \in \mathcal{T}$, week $w \in \mathcal{W}$, scenario $s \in \mathcal{S}$ in the market $m \in \mathcal{M}$ for the design scheme $l \in \mathcal{L}$; $\mathbf{x}^{(\mathrm{RT})}_{\mathrm{w,s,l}}$, $\mathbf{x}^{(\mathrm{DA})}_{\mathrm{w,s,l}}$, and $\mathbf{x}^{(\mathrm{Y})}_{\mathrm{s,l}}$ are the decision variables of real-time, day-ahead, and long-term optimization problems[9]; the symbol $\tilde{(.)}$ denotes the optimal decision variables, $int \neq 0$ is the interest rate in [%/100], e.g., $int = 0.04$; $y$ is the expected number of year of functionality of the VPP; $\mathrm{NP}_{\mathrm{int,y}} := \frac{(1+int)^{y+1}-1}{int\cdot(1+int)^y}$ is the factor that multiplied by constant cash flow to determine the net present value (NPV) of the cash flow; and $\mathrm{capex}_{\mathrm{r}}$ is the capital expenditure per unit of capacity that is needed to install, modify, maintain, and expand unit $r$ to provide flexibility and will be considered in the portfolio of the VPP.

We must first solve the real-time and day-ahead optimization problems before we solve the long-term VPP planning (3). However, the real-time and day-ahead optimization problems ((1) and (2)) need the best annual decision, including the water discharge and reservoir volume of HPs in the VPP. Our approach to deal with the long-term optimization problem is to approximate it so that it is independent of day-ahead and real-time decision variables. To accomplish this, we use the weekly average prices $\overline{\pi}^{(\mathrm{m})}_{\mathrm{w,s}} := \frac{1}{T} \cdot \sum_{t\in\mathcal{T}} \pi^{(\mathrm{m})}_{\mathrm{w,t,z',s}}$[10] and then approximate (3) by the following:

$$\overline{J}^{(\mathrm{Y})}_{\mathrm{s,l}} \left( \tilde{\mathbf{x}}^{(\mathrm{Y})}_{\mathrm{s,l}} \right) := \max_{\mathbf{x}^{(\mathrm{Y})}_{\mathrm{s,l}}} \sum_{w\in\mathcal{W}} \delta t \cdot \sum_{m\in\mathcal{M}} p^{(\mathrm{m})}_{\mathrm{w,1,s,l}} \cdot \overline{\pi}^{(\mathrm{m})}_{\mathrm{w,s}}, \tag{5}$$

where $p^{(\mathrm{m})}_{\mathrm{w,1,s,l}}$ is the average exchanged power[11] in the market $m$ by the VPP in the week $w$, scenario $s$, and the design scheme $l$; and $\overline{J}^{(\mathrm{Y})}_{\mathrm{s,l}}(.)$ is the approximated annual objective of the VPP.

The day-ahead and real-time problems can be integrated into one weekly optimization problem. The objective of this problem is as follows:

$$\overline{J}^{(\mathrm{ST})}_{\mathrm{w,s,l}} \left( \tilde{\mathbf{x}}^{(\mathrm{DA})}_{\mathrm{w,s,l}}, \tilde{\mathbf{x}}^{(\mathrm{RT})}_{\mathrm{w,s,l}} | \tilde{\mathbf{x}}^{(\mathrm{Y})}_{\mathrm{s,l}} \right) := \max_{\mathbf{x}^{(\mathrm{DA})}_{\mathrm{w,s,l}}, \mathbf{x}^{(\mathrm{RT})}_{\mathrm{w,s,l}}} \sum_{t\in\mathcal{T}} \delta t \cdot \sum_{m\in\mathcal{M}} p^{(\mathrm{m})}_{\mathrm{w,t,s,l}} \cdot \pi^{(\mathrm{m})}_{\mathrm{w,t,z'(m),s}}, \tag{6}$$

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[8] It should be noted that the energy is exchanged conventionally in the day-ahead spot market and the balancing market, whereas the power capacity will be exchanged conventionally in the ancillary service market. Here, we use the notion of power for all markets, and the prices are adjusted accordingly.

[9] In the next sections, we will define the components of decision variable vectors.

[10] $h' = \mathrm{RT}$ for $m \in \{\mathrm{BAL}\}$ and $h' = \mathrm{DA}$ for the rest $m \in \mathcal{M} \setminus \{\mathrm{BAL}\}$.

[11] In problem (5), $T$ and $\delta t$ are considered equal to 1 and $24 \times 7[h]$, respectively. Therefore, $p^{(\mathrm{m})}_{\mathrm{w,1,s,l}}$ has the index of $t = 1$.

where $h'(m) = \mathrm{RT}$ for $m \in \{\mathrm{BAL}\}$ and $h'(m) = \mathrm{DA}$ for the rest $m \in \mathcal{M} \setminus \{\mathrm{BAL}\}$. Note that $\delta t$ in (6) is different from $\delta t$ in (5)[12].

We now have a better understanding of the algorithm presented in Figure 1. It gives us a near-optimal solution as we first solve the problem with objective (5) in step iii to find the annual optimal decision variables $\tilde{\mathbf{x}}_{\mathrm{s,l}}^{(\mathrm{Y})}$, including weekly water discharge and level of the reservoirs (step iv). Then, we solve the problem with the objective (6) to determine the optimal exchanged power in energy market and flexibility markets as well as the balancing power exchanged by the VPP (the vectors $\tilde{\mathbf{x}}_{\mathrm{w,s,l}}^{(\mathrm{DA})}$ and $\tilde{\mathbf{x}}_{\mathrm{w,s,l}}^{(\mathrm{RT})}$). As a result, we can calculate the precise annual revenue of the VPP defined in (3) in order to solve the problem with the objective (4). To reduce the financial risk of the VPP, a risk measure can be subtracted from the objective (4). Here, we define a risk measure called *conditional value at risk* (CVaR).

$$\begin{aligned}\mathrm{CVaR}(\alpha, l) = \mathrm{NP}_{\mathrm{int,y}} \cdot \max_{\eta^{(\mathrm{cv})}} \left( \eta^{(\mathrm{cv})} - \frac{1}{1-\alpha} \cdot \mathbb{E}\left( \max\left(0, \eta^{(\mathrm{cv})} - J_{\mathrm{s,l}}^{(\mathrm{Y})}(\tilde{\mathbf{x}}_{\mathrm{s,l}}^{(\mathrm{Y})})\right)\right)\right) \\ - \sum_{r \in \mathcal{R}_{\mathrm{r}}} \mathrm{capex}_{\mathrm{r}}, \qquad (7)\end{aligned}$$

where $\alpha \in (0,1)$ is the confidence level, $\eta$ is a variable used to determine the expected value of profit smaller than $(1-\alpha)$-quantile of the profit distribution.

### 3.3 Constraints

Each optimization problem has a unique set of constraints. These constraints can be classified into those related to component operation and capacity (HPs, PMs, PVs, WTs, BESs), those related to HP water discharge, and those related to market requirements, financial risk limits, and auxiliary constraints. The general version of the constraints will be described below. A number of these constraints over a specific set of indices can be included in each optimization problem. We will define which constraints must be considered for each optimization problem in Appendix B.

#### 3.3.1 Components: Small Hydro Power Plants

The first constraint of a small HP $r \in \mathcal{R}^{(\mathrm{HP})}$ is the relation between its flow and its power production.

$$p_{\mathrm{r,i}}^{(\mathrm{HP})} = \rho^{(\mathrm{HP})} \cdot h_{\mathrm{r}}(v_{\mathrm{r,i}}) \cdot q_{\mathrm{r,i}} \cdot \frac{\eta_{\mathrm{r}}^{(\mathrm{HP})}(h_{\mathrm{r}}(v_{\mathrm{r,i}}), q_{\mathrm{r,i}})}{100}, \qquad (8)$$

where $p_{\mathrm{r,i}}^{(\mathrm{HP})}$ is the power production for unit $r \in \mathcal{R}^{(\mathrm{HP})}$ in $[kW]$; $\rho^{(\mathrm{HP})} = 9.80665 \times 998$ $[kg/(m^2 \cdot s^2)]$ is the multiplication of gravitational acceleration

[12] $\delta t$ is considered equal to $1[h]$ in problem (6).

and water density; $h_{\mathrm{r}(.)}$ is the head function of unit $r \in \mathcal{R}^{(\mathrm{HP})}$ $[m]$, $v_{\mathrm{r,i}}$ is the volume of unit $r \in \mathcal{R}^{(\mathrm{HP})}$ in $[m^3]$, $q_{\mathrm{r,i}}$ is the water discharge of unit $r \in \mathcal{R}^{(\mathrm{HP})}$ in $[m^3/s]$, and $\eta_{\mathrm{r}}^{(\mathrm{HP})}(.,.)$ is the efficiency function of unit $r \in \mathcal{R}^{(\mathrm{HP})}$ in $[\%]$.

The equation (8) is non-linear because of multiplications of variables and existence of the functions $h_{\mathrm{r}}(.)$ and $\eta_{\mathrm{r}}^{(\mathrm{HP})}(.,.)$. This equation must be linearized, as explained in Appendix A.

Other constraints of small HPs are presented in the following. These constraints refer to the minimum and maximum flow of HPs, the volume equation, and minimum and maximum volume of HPs:

$$q_{\mathrm{r,i}}^{(\mathrm{min})} \leq q_{\mathrm{r,i}} \leq q_{\mathrm{r,i}}^{(\mathrm{max})}, \tag{9}$$

$$v_{\mathrm{r,l(i)},1}^{(\mathrm{min})} \leq v_{\mathrm{r,i}} + (q_{\mathrm{r,i}}^{(\mathrm{in})} - q_{\mathrm{r,i}}) \cdot \delta t \leq v_{\mathrm{r,l(i)},K_{\mathrm{r}}}^{(\mathrm{max})}, \tag{10}$$

$$v_{\mathrm{r,n}(i)} = v_{\mathrm{r,i}} + (q_{\mathrm{r,i}}^{(\mathrm{in})} - q_{\mathrm{r,i}}) \cdot \delta t, \tag{11}$$

$$\Delta q_{\mathrm{r,w,z,s,l}}^{(\mathrm{pos})} - \Delta q_{\mathrm{r,w,z,s,l}}^{(\mathrm{neg})} = q_{\mathrm{r,w,s,l}}^{(\mathrm{avg})} - \frac{1}{\sum_{t \in \mathcal{T}} \delta t} \cdot \sum_{i \in \{(w,t,z,s,l)|t \in \mathcal{T}\}} \delta t \cdot q_{\mathrm{r,i}}, \tag{12}$$

$$\Delta q_{\mathrm{r,w,s,l}}^{(\mathrm{pos})} \geq 0, \tag{13}$$

$$\Delta q_{\mathrm{r,w,s,l}}^{(\mathrm{neg})} \geq 0, \tag{14}$$

$$v_{\mathrm{r,i}} = \hat{v}_{\mathrm{r,i}}, \tag{15}$$

where $q_{\mathrm{r,i}}^{(\mathrm{min})}$ and $q_{\mathrm{r,i}}^{(\mathrm{max})}$ are the minimum and maximum permissible flow[13] of the unit $r$ at index $i$; $q_{\mathrm{r,i}}^{(\mathrm{in})}$ is the input water discharge[14] to unit $r$ in index $i$; the function $n(.)$ determines the next time step[15] depending on the problem we are solving; $l(i)$ is the respected design scheme for index $i$; $q_{\mathrm{r},\mathbf{w},\mathbf{s},\mathrm{l}}^{(\mathrm{avg})}$ is the average water discharge determined by the long-term planning for week $w$, scenario $s$, and design scheme $l$; $\Delta q_{\mathrm{r},\mathbf{w},\mathrm{z},\mathbf{s},\mathrm{l}}^{(\mathrm{pos})}$ and $\Delta q_{\mathrm{r},\mathbf{w},\mathrm{z},\mathbf{s},\mathrm{l}}^{(\mathrm{neg})}$ are positive and negative variations from planned flow, which will be minimized in short-term planning; $\hat{v}_{\mathrm{r,i}}$ is the initial volume of unit $r$, which is determined after solving the optimization problem for time $w-1$. Note that the constraint (15) must be only considered for $i \in \{(w,t,z,s,l)|t=0, z \in \mathcal{Z}\}$.

### 3.3.2 Components: Pumps

We considered that each pump $r \in \mathcal{R}^{(\mathrm{PM})}$ would be installed over a particular dam $r'$, with $r' \in \mathcal{R}^{(\mathrm{HP})}$ being an HP unit existing over that dam[16].

[13] They are calculated taking into account the environmental constraints and using the associated time $t$, week $w$, and scenario $s$ for the index $i$.

[14] It differs from $\mathrm{wd}_{\mathrm{r,w,t,z,s}}$ and will be defined later in this section as a variable.

[15] For long-term planning $n(w,t,z,s,l) = (w+1,t,z,s,l)$ and for short-term planning $n(w,t,z,s,l) = (w,t+1,z,s,l)$.

[16] Even though a dam does not have a turbine, we assume in our model that there is an HP unit $r'$ with a capacity equal to zero.

As a result, the operation in a combination of a pump and a turbine is considered. Each pump has only two operational states, namely idle and full consumption. However, they can consume power in a duration less than $\delta t$. Thus, in each index $i$ the pump $r \in \mathcal{R}^{(\mathrm{PM})}$ is modeled as follows:

$$p^{(\mathrm{PM})}_{\mathrm{r,i}} = p^{(\mathrm{PM,max})}_{\mathrm{r}} \cdot \phi_{\mathrm{r,i}}/\delta t, \tag{16}$$

$$q_{\mathrm{r,i}} = q^{(\mathrm{PM,max})}_{\mathrm{r}} \cdot \phi_{\mathrm{r,i}}/\delta t, \tag{17}$$

$$0 \leq \phi_{\mathrm{r,i}} \leq \delta t, \tag{18}$$

where $p^{(\mathrm{PM})}_{\mathrm{r,i}}$ is the power consumption of the pump $r$ in index $i \in \mathcal{I}$; $q_{\mathrm{r,i}}$ is the flow rate of the pump $r$ in index $i \in \mathcal{I}$; $\phi_{\mathrm{r,i}}$ is the duration that the pump $r$ is working; $p^{(\mathrm{PM,max})}_{\mathrm{r}}$ is the maximum power consumption of the pump, $q^{(\mathrm{PM,max})}_{\mathrm{r}}$ is the maximum flow rate of the pump $r$.

### 3.3.3 Water Discharge

In the current version of the code, only pumps are considered without any travel time.

In this section, we define the relation between the variables $q^{(\mathrm{in})}_{\mathrm{r,i}}$ and the day-ahead and hour-ahead prediction $\mathrm{wd}_{\mathrm{r,w(i),t(i),h(i),s(i)}}$ for unit $r \in \mathcal{R}^{(\mathrm{HP})}$, as well as other flows over the turbines and pumps. Figure 4 shows the schematic of an exemplary unit $r \in \mathcal{R}^{(\mathrm{HP})}$, in which the HPs and PMs $r' \in \mathcal{O}_{\mathrm{r}}$ discharge water towards this unit and the PMs $r' \in \mathcal{W}_{\mathrm{r}}$ discharge water from this unit.

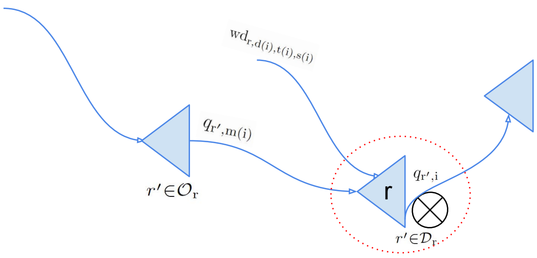


Figure 4: Water discharge model.

The sets $\mathcal{O}_{\mathrm{r}}$ and $\mathcal{W}_{\mathrm{r}}$ are known and predetermined. As a result, the net water discharge into unit $r$ (the water injected to the red circle area in Figure 4) is as follows:

$$q^{(\mathrm{in})}_{\mathrm{r,i}} = \mathrm{wd}_{\mathrm{r,w(i),t(i),h(i),s(i)}} - \sum_{r' \in \mathcal{W}_{\mathrm{r}}} q_{\mathrm{r}',\mathrm{i}} + \sum_{r' \in \mathcal{O}_{\mathrm{r}}} q_{\mathrm{r}',\mathrm{m}_{\mathrm{r}'}(\mathrm{i})}, \tag{19}$$

where, $w(i)$, $t(i)$, $h(i)$, and $s(i)$ are the respected week, time, horizon, and scenario for index $i$, and $m_{\mathrm{r}}(i)$ is a function to model the travel time between the unit $r$ and the following unit. If the unit travel times are less than the 1/10 of the time resolution of the optimization problem, e.g., long-term planning problem, then $m_{\mathrm{r}}(i) = i$. Otherwise, it would be $m_{\mathrm{r}}(i) = (w, t - \lceil \frac{\tau_{\mathrm{r}}}{\delta t} \rceil, z, s, l)$ such that $\tau_{\mathrm{r}}$ is the travel time in $[h]$. Note that if $t - \lceil \frac{\tau_{\mathrm{r}}}{\delta t} \rceil$ is less than zero, then the water discharge of previous day $q_{\mathrm{r,m_r(i)}}$ is considered if it exists; otherwise, $q_{\mathrm{r,m_r(i)}}$ is considered equal to zero.

### 3.3.4 Components: PVs

The PV production of unit $r \in \mathcal{R}^{(\mathrm{PV})}$ is mathematically modeled considering a number of factors, including the global horizontal irradiation $\mathrm{irr}_{\mathrm{r,w,t,z,s}}$, the surface area of the PV system $\mathrm{area}_{\mathrm{r}}$, and the efficiency of the PV system $\eta_{\mathrm{r}}^{(\mathrm{PV})}$. The model is typically expressed as a function of these factors, as follows:

$$p_{\mathrm{r,i}}^{(\mathrm{PV})} = \mathrm{irr}_{\mathrm{r,w}(i),\mathrm{t}(i),\mathrm{z}(i),\mathrm{s}(i)} \cdot \mathrm{area}_{\mathrm{r}} \cdot \eta_{\mathrm{r}}^{(\mathrm{PV})}. \tag{20}$$

### 3.3.5 Components: Wind Turbines

The wind production model is a mathematical model that predicts the power output of a WT $r \in \mathcal{R}^{(\mathrm{WT})}$. The model takes into account a number of factors, including the air density, the rotor swept area, the power curve, the efficiency factor, and the cut-in and cut-off wind speeds. The model is typically expressed as a function of these factors, as follows:

$$p_{\mathrm{r,i}}^{(\mathrm{WT})} = \frac{\rho^{(\mathrm{a})} \cdot \mathrm{A}_{\mathrm{r}}^{(\mathrm{WT})}}{2} \cdot \mathrm{cp}_{\mathrm{r}}(\mathrm{ws}_{\mathrm{r,w}(i),\mathrm{t}(i),\mathrm{z}(i),\mathrm{s}(i)}) \cdot \mathrm{ef}_{\mathrm{r}} \cdot (\mathrm{ws}_{\mathrm{r,w}(i),\mathrm{z}(i),\mathrm{t}(i),\mathrm{s}(i)})^3 \cdot k_{\mathrm{r,i}}, \tag{21}$$

$$k_{\mathrm{r,i}} = \mathbb{1}(\mathrm{ws}_{\mathrm{r,w}(i),\mathrm{t}(i),\mathrm{z}(i),\mathrm{s}(i)} \leq \mathrm{ws}_{\mathrm{r}}^{(\mathrm{c-off})}) \cdot \mathbb{1}(\mathrm{ws}_{\mathrm{r,w}(i),\mathrm{t}(i),\mathrm{z}(i),\mathrm{s}(i)} \geq \mathrm{ws}_{\mathrm{r}}^{(\mathrm{c-in})}), \tag{22}$$

where $p_{\mathrm{r,i}}^{(\mathrm{WT})}$ is the predicted power output of the wind turbine $r \in \mathcal{R}^{(\mathrm{WT})}$ at a given index $i$; $\rho^{(\mathrm{a})}$ is the air density, which is typically 1.079 $[kg/m^3]$; $\mathrm{A}_{\mathrm{r}}^{(\mathrm{WT})}$ is the rotor swept area of the wind turbine, in $[m^2]$; the function $\mathrm{cp}_{\mathrm{r}}(.)$ s the power curve, which is a function of the wind speed and it determines the amount of power that can be extracted from the wind in [%/100]; $\mathrm{ef}_{\mathrm{r}}$ is the efficiency factor in [%/100], which is typically 0.8, and it accounts for losses in the wind turbine; $k_{\mathrm{r,i}}$ is a variable that determines whether the WT is producing power. The variable $k_{\mathrm{r,i}}$ is equal to 1 if the wind speed is greater than or equal to the cut-in wind speed $\mathrm{ws}_{\mathrm{r}}^{(\mathrm{c-in})}$ and less than or equal to the cut-off wind speed $\mathrm{ws}_{\mathrm{r}}^{(\mathrm{c-off})}$. Otherwise, the function is equal to 0. The operator $\mathbb{1}(.)$ returns 1 if its input is true, otherwise it returns 0.

Equations (21)-(22) make the optimization problem non-linear due the non-necessary multiplications of variables. To resolve this issue, we compute the multiplication of $\gamma_{\mathrm{r,i}} = \mathrm{cp}_{\mathrm{r}}(\mathrm{ws}_{\mathrm{r,w}(i),\mathrm{t}(i),\mathrm{z}(i),\mathrm{s}(i)}) \cdot \mathrm{ef}_{\mathrm{r}} \cdot (\mathrm{ws}_{\mathrm{r,w}(i),\mathrm{t}(i),\mathrm{z}(i),\mathrm{s}(i)})^3 \cdot k_{\mathrm{r,i}}$ before running the optimization problem. Then, the production of WTs can be calculated in an optimization problem using $\gamma_{\mathrm{r,i}}$.

### 3.3.6 Components: Battery Energy Storage Systems

In the primary version of implementation, the battery storage is not considered.

The production or consumption of ESS $r \in \mathcal{R}^{(\mathrm{ES})}$ can be regulated by taking into account the constraints over its state of energy and state. The conventional equations used for BESs are presented below.

$$p^{(\mathrm{ES})}_{\mathrm{r,i}} = p^{(\mathrm{ES,d})}_{\mathrm{r,i}} - p^{(\mathrm{ES,c})}_{\mathrm{r,i}}, \tag{23}$$

$$0 \leq p^{(\mathrm{ES,c})}_{\mathrm{r,i}} \leq \alpha^{(\mathrm{ES,c})}_{\mathrm{r,i}} \cdot p^{(\mathrm{ES,c,max})}_{\mathrm{r}}, \tag{24}$$

$$0 \leq p^{(\mathrm{ES,d})}_{\mathrm{r,i}} \leq \alpha^{(\mathrm{ES,d})}_{\mathrm{r,i}} \cdot p^{(\mathrm{ES,d,max})}_{\mathrm{r}}, \tag{25}$$

$$\alpha^{(\mathrm{ES,c})}_{\mathrm{r,i}} + \alpha^{(\mathrm{ES,d})}_{\mathrm{r,i}} \leq 1, \tag{26}$$

$$\mathrm{SOE}^{(\mathrm{min})}_{\mathrm{r}} \leq \mathrm{SOE}_{\mathrm{r,i}} + \delta t \cdot \left( p^{(\mathrm{ES,c})}_{\mathrm{r,i}} \cdot \eta^{(\mathrm{ES,c})}_{\mathrm{r}} - p^{(\mathrm{ES,d})}_{\mathrm{r,i}} \cdot \frac{1}{\eta^{(\mathrm{ES,w})}_{\mathrm{r}}} \right) \leq \mathrm{SOE}^{(\mathrm{max})}_{\mathrm{r}}, \tag{27}$$

$$\mathrm{SOE}_{\mathrm{r,n(i)}} = \mathrm{SOE}_{\mathrm{r,i}} + \delta t \cdot \left( p^{(\mathrm{ES,c})}_{\mathrm{r,i}} \cdot \eta^{(\mathrm{ES,c})}_{\mathrm{r}} - p^{(\mathrm{ES,d})}_{\mathrm{r,i}} \cdot \frac{1}{\eta^{(\mathrm{ES,w})}_{\mathrm{r}}} \right), \tag{28}$$

where $p^{(\mathrm{ES})}_{\mathrm{r,i}}$ is the power generated or consumed by ESS $r$ in $[kW]$; $p^{(\mathrm{ES,d})}_{\mathrm{r,i}}$ is the discharging power in $[kW]$; $p^{(\mathrm{ES,c})}_{\mathrm{r,i}}$ is the charging power in $[kW]$; $\mathrm{SOE}_{\mathrm{r,i}}$ is the state of energy of the ESS in $[kWh]$; $\eta^{(\mathrm{ES,c})}_{\mathrm{r}}$ and $\eta^{(\mathrm{ES,w})}_{\mathrm{r}}$ are charging and discharging efficiencies in $[\%/100]$; $\alpha^{(\mathrm{ES,c})}_{\mathrm{r,i}}$ and $\alpha^{(\mathrm{ES,d})}_{\mathrm{r,i}}$ are binary variables determining whether the unit is in charging state or discharging state; $p^{(\mathrm{ES,c,max})}_{\mathrm{r}}$ and $p^{(\mathrm{ES,d,max})}_{\mathrm{r}}$ are the maximum charging and discharging power; $\mathrm{SOE}^{(\mathrm{min})}_{\mathrm{r}}$ and $\mathrm{SOE}^{(\mathrm{max})}_{\mathrm{r}}$ are the minimum and maximum permissible state of energy in $[kWh]$; and the function $n(.)$ determines the next time step depending on the problem we are solving, i.e., for long-term planning $n(w,t,z,s,l) = (w+1,t,z,s,l)$ and for short-term planning $n(w,t,z,s,l) = (w,t+1,z,s,l)$.

### 3.3.7 Market

The day-ahead and balancing exchange power will be the sum of production and consumption of distributed resources considering the associated prediction horizon. The following constraints determine the power exchanged in day-ahead and balancing markets.

$$p^{(\mathrm{DA})}_{\mathrm{w,t,s,l}} = \sum_{g\in\{\mathrm{HP,PV,WT}\}} \sum_{r\in\mathcal{R}^{(\mathrm{g})}_{1}} p^{(\mathrm{g})}_{\mathrm{r,w,t,DA,s,l}} - \sum_{g\in\{\mathrm{PM,ES}\}} \sum_{r\in\mathcal{R}^{(\mathrm{g})}_{1}} p^{(\mathrm{g})}_{\mathrm{r,w,t,DA,s,l}}, \tag{29}$$

$$\begin{aligned} p^{(\mathrm{BAL})}_{\mathrm{w,t,s,l}} &= -p^{(\mathrm{DA})}_{\mathrm{w,t,s,l}} + \sum_{g\in\{\mathrm{HP,PV,WT}\}} \sum_{r\in\mathcal{R}^{(\mathrm{g})}_{1}} p^{(\mathrm{g})}_{\mathrm{r,w,t,RT,s,l}} \\ &\quad - \sum_{g\in\{\mathrm{PM,ES}\}} \sum_{r\in\mathcal{R}^{(\mathrm{g})}_{1}} p^{(\mathrm{g})}_{\mathrm{r,w,t,RT,s,l}}. \end{aligned} \tag{30}$$

We assume that HPs, PMs, and BESs only provide flexibility. We divide the flexibility products into two sets $\mathcal{M}^{\uparrow}$ and $\mathcal{M}^{\downarrow}$ for upward and downward flexibility, respectively[17]. We define the set $\mathcal{FL} := \{\mathrm{FL}^{\uparrow}, \mathrm{FL}^{\downarrow}\}$ to use as an index of horizon. To calculate the exchanged flexibility, we must add the constraints $(47), (49)-(10), \forall r \in \mathcal{R}^{(\mathrm{HP})}_{1}, \forall z \in \mathcal{FL}$, $(16)-(18), \forall r \in \mathcal{R}^{(\mathrm{PM})}_{1}, \forall z \in \mathcal{FL}$, $(19), \forall r \in \mathcal{R}^{(\mathrm{HP})}_{1}, \forall z \in \mathcal{FL}$, and $(23)-(27), \forall r \in \mathcal{R}^{(\mathrm{ES})}_{1}, \forall z \in \mathcal{FL}$. In addition, we must add the following constraints $\forall h(i) \in \mathcal{FL}$ to ensure that there is enough capacity to provide flexibility for a given time period.

$$v_{\mathrm{r,i}} = v_{\mathrm{r,w(i),t(i),DA,s(i),l(i)}}, \forall r \in \mathcal{R}^{(\mathrm{HP})}_{1}, \tag{31}$$

$$\mathrm{SOE}_{\mathrm{r,i}} = \mathrm{SOE}_{\mathrm{r,w(i),t(i),DA,s(i),l(i)}}, \forall r \in \mathcal{R}^{(\mathrm{ES})}_{1}. \tag{32}$$

As a result, the flexibility provided by VPP is calculated as follows:

$$\sum_{m\in\mathcal{M}^{\downarrow}} p^{(\mathrm{m})}_{\mathrm{w,t,s,l}} = p^{(\mathrm{DA})}_{\mathrm{w,t,s,l}} - \sum_{r\in\mathcal{R}^{(\mathrm{HP})}} p^{(\mathrm{HP})}_{\mathrm{r,w,t,FL^{\downarrow},s,l}} + \sum_{g\in\{\mathrm{PM,ES}\}} \sum_{r\in\mathcal{R}^{(\mathrm{g})}_{1}} p^{(\mathrm{g})}_{\mathrm{r,w,t,FL^{\downarrow},s,l}}, \tag{33}$$

$$\sum_{m\in\mathcal{M}^{\uparrow}} p^{(\mathrm{m})}_{\mathrm{w,t,s,l}} = -p^{(\mathrm{DA})}_{\mathrm{w,t,s,l}} + \sum_{r\in\mathcal{R}^{(\mathrm{HP})}} p^{(\mathrm{HP})}_{\mathrm{r,w,t,FL^{\uparrow},s,l}} - \sum_{g\in\{\mathrm{PM,ES}\}} \sum_{r\in\mathcal{R}^{(\mathrm{g})}_{1}} p^{(\mathrm{g})}_{\mathrm{r,w,t,FL^{\uparrow},s,l}}, \tag{34}$$

$$p^{(\mathrm{m})}_{\mathrm{w,t,s,l}} \geq 0, \forall m \in \mathcal{M} \setminus \{\mathrm{DA}, \mathrm{BAL}\}. \tag{35}$$

We also need to consider the ramp rate of each unit as follows:

$$-\mathrm{RR}_{\mathrm{r}} \leq \frac{p^{(\mathrm{g})}_{\mathrm{r,w,t,FL^{\uparrow},s,l}} - p^{(\mathrm{g})}_{\mathrm{r,w,t,DA,s,l}}}{\delta t} \leq \mathrm{RR}_{\mathrm{r}}, \forall g \in \{\mathrm{HP,PM,ES}\}, \forall r \in \mathcal{R}^{(\mathrm{g})}, \tag{36}$$

$$-\mathrm{RR}_{\mathrm{r}} \leq \frac{p^{(\mathrm{g})}_{\mathrm{r,w,t,FL^{\downarrow},s,l}} - p^{(\mathrm{g})}_{\mathrm{r,w,t,DA,s,l}}}{\delta t} \leq \mathrm{RR}_{\mathrm{r}}, \forall g \in \{\mathrm{HP,PM,ES}\}, \forall r \in \mathcal{R}^{(\mathrm{g})}, \tag{37}$$

[17] $\mathcal{M}^{\uparrow} \subset \{\mathrm{FCR}^{\uparrow}, \mathrm{RR}^{\uparrow}, \mathrm{aFRR}^{\uparrow}, \mathrm{mFRR}^{\uparrow}\}$ and $\mathcal{M}^{\downarrow} \subset \{\mathrm{FCR}^{\downarrow}, \mathrm{RR}^{\downarrow}, \mathrm{aFRR}^{\downarrow}, \mathrm{mFRR}^{\downarrow}\}$.

where $\mathrm{RR_r}$ is the ramp-rate of unit $r$.

If flexibility products other than primary reserves are chosen, additional constraints must be imposed. In that case, we must determine whether we have sufficient energy storage to provide flexibility for more than an hour.

### 3.4 Decision Variables

Let define $\mathcal{I}_{\mathrm{w,z,s,l}} := \{w\} \times \mathcal{T} \times \{z\} \times \{s\} \times \{l\}$. The operational and auxiliary variables of small HP units are summarized in the following vector:

$$\begin{aligned}\mathbf{u}^{(\mathrm{HP})}_{\mathrm{w,z,s,l}} := [&(p^{(\mathrm{HP})}_{\mathrm{r,i}}, q_{\mathrm{r,i}}, v_{\mathrm{r,i}})_{r\in\mathcal{R}^{(\mathrm{HP})}, i\in\mathcal{I}_{\mathrm{w,z,s,l}}}, (\hat{p}_{\mathrm{r,i,k}}, \zeta_{\mathrm{r,i,k}})_{r\in\mathcal{R}^{(\mathrm{HP})}, k\in\mathcal{K}_{\mathrm{r}}, i\in\mathcal{I}_{\mathrm{w,z,s,l}}},\\ &(\Delta q^{(\mathrm{pos})}_{\mathrm{r,w,s,l}}, \Delta q^{(\mathrm{neg})}_{\mathrm{r,w,s,l}})_{r\in\mathcal{R}^{(\mathrm{HP})}}].\end{aligned} \tag{38}$$

The operational and auxiliary variables of pumps are summarized in the following vector:

$$\mathbf{u}^{(\mathrm{PM})}_{\mathrm{w,z,s,l}} := (p^{(\mathrm{PM})}_{\mathrm{r,i}}, q_{\mathrm{r,i}}, \phi_{\mathrm{r,i}})_{r\in\mathcal{R}^{(\mathrm{PM})}, i\in\mathcal{I}_{\mathrm{w,z,s,l}}}. \tag{39}$$

The operational variables of PVs are summarized in the following vector:

$$\mathbf{u}^{(\mathrm{PV})}_{\mathrm{w,z,s,l}} := (p^{(\mathrm{PV})}_{\mathrm{r,i}})_{r\in\mathcal{R}^{(\mathrm{PV})}, i\in\mathcal{I}_{\mathrm{w,z,s,l}}}. \tag{40}$$

The operational variables of WTs are summarized in the following vector:

$$\mathbf{u}^{(\mathrm{WT})}_{\mathrm{w,z,s,l}} := (p^{(\mathrm{WT})}_{\mathrm{r,i}})_{r\in\mathcal{R}^{(\mathrm{WT})}, i\in\mathcal{I}_{\mathrm{w,z,s,l}}}. \tag{41}$$

The operational variables of BESs are summarized in the following vector:

$$\mathbf{u}^{(\mathrm{ES})}_{\mathrm{w,z,s,l}} := (p^{(\mathrm{ES})}_{\mathrm{r,i}}, p^{(\mathrm{ES,d})}_{\mathrm{r,i}}, p^{(\mathrm{ES,c})}_{\mathrm{r,i}}, \alpha^{(\mathrm{ES,d})}_{\mathrm{r,i}}, \alpha^{(\mathrm{ES,c})}_{\mathrm{r,i}}, \mathrm{SOE}_{\mathrm{r,i}})_{r\in\mathcal{R}^{(\mathrm{WT})}, i\in\mathcal{I}_{\mathrm{w,z,s,l}}}. \tag{42}$$

The market variables are summarized in the following vector:

$$\mathbf{u}^{(\mathrm{m})}_{\mathrm{w,s,l}} := (p^{(\mathrm{m})}_{\mathrm{w,t,s,l}})_{t\in\mathcal{T}}. \tag{43}$$

The decision variables used in Section 3.2 are defined in the following:

$$\begin{aligned}\mathbf{x}^{(\mathrm{Y})}_{\mathrm{s,l}} &= [(\mathbf{u}^{(\mathrm{g})}_{\mathrm{w,z,s,l}})_{g\in\{\mathrm{HP,PM,ESS}\}, w\in\mathcal{W}, z\in\mathcal{Z}\cup\mathcal{FL}},\\ &\quad(\mathbf{u}^{(\mathrm{g})}_{\mathrm{w,z,s,l}})_{g\in\{\mathrm{PV,WT}\}, w\in\mathcal{W}, z\in\mathcal{Z}}, (\mathbf{u}^{(\mathrm{m})}_{\mathrm{w,s,l}})_{m\in\mathcal{M}, w\in\mathcal{W}}], &(44)\\ \mathbf{x}^{(\mathrm{DA})}_{\mathrm{w,s,l}} &= [(\mathbf{u}^{(\mathrm{g})}_{\mathrm{w,z,s,l}})_{g\in\{\mathrm{HP,PM,ESS}\}, z\in\{\mathrm{DA}\}\cup\mathcal{FL}}, (\mathbf{u}^{(\mathrm{g})}_{\mathrm{w,z,s,l}})_{g\in\{\mathrm{PV,WT}\}, z\in\{\mathrm{DA}\}},\\ &\quad(\mathbf{u}^{(\mathrm{m})}_{\mathrm{w,s,l}})_{m\in\mathcal{M}\setminus\{\mathrm{BAL}\}}], &(45)\\ \mathbf{x}^{(\mathrm{RT})}_{\mathrm{w,s,l}} &= [(\mathbf{u}^{(\mathrm{g})}_{\mathrm{w,z,s,l}})_{g\in\{\mathrm{HP,PM,PV,WT,ESS}\}, z\in\{\mathrm{RT}\}}, (\mathbf{u}^{(\mathrm{m})}_{\mathrm{w,s,l}})_{m\in\{\mathrm{BAL}\}}]. &(46)\end{aligned}$$

# 4 Next Steps

The following steps must be taken to implement the proposed algorithm numerically:

- Fine-tuning the mathematical formulation: This step will involve making adjustments to the mathematical formulation of the algorithm in order based on the feedback from other project partners to improve its performance. This may involve changing the parameters of the algorithm, adding a new set of constraints (e.g., financial risk of the VPP), or the way that the data is processed.

- Collecting data: This step will involve collecting data on the candidate installation or renovation projects, as well as existing assets. This data will be used to train the algorithm and to evaluate its performance.

- Generating hydro-meteorological time-series scenarios: This step will involve generating a set of hydro-meteorological time-series scenarios. These scenarios will be used to test the robustness of the algorithm to different weather conditions. We will use the data available in "`https://gletsch.slf.ch/`" and "`meteonorm`".

- Generating forecasting time-series: This step will involve using a short-term forecasting method to generate day-ahead and real-time forecasting time-series. These time-series will be used to evaluate the performance of the algorithm.

- Collecting electricity market prices data: This step will involve collecting data on electricity market prices. This data will be used to evaluate the economic performance of the algorithm. The average positive or negative secondary control energy prices and average positive or negative tertiary control energy prices for 2014-2023 are available in the website of Swiss-grid. Furthermore, the day-ahead price of FCR, RR, a-FRR, and m-FRR for 2015-2023 are available in "`https://transparency.entsoe.eu`".

- Running the proposed algorithm numerically: This step will involve running the proposed algorithm numerically with Python and GUROBI on selected use cases. This will allow the algorithm to be evaluated on real-world data.

# Appendix

# A Linearization Technique

We use the step-wise linearization technique demonstrated in Figure 5.

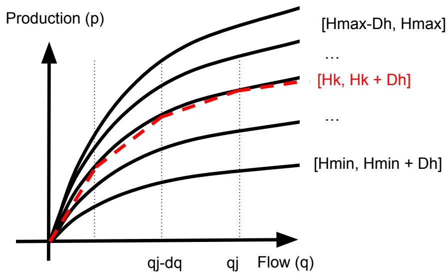


Figure 5: Linearization technique.

Let consider $k \in \mathcal{K}_{\rm r}$ and $j \in \mathcal{J}_{\rm r}$ be the indices of step-wise linearization, $\mathcal{K}_{\rm r} := \{1, 2, \ldots, K_{\rm r}\}$ and $\mathcal{J}_{\rm r} := \{1, 2, \ldots, J_{\rm r}\}$ include a number of possible head ranges and flow rates for unit $r \in \mathcal{R}^{\rm (HP)}$, respectively. Each $k \in \mathcal{K}_{\rm r}$ and $j \in \mathcal{J}_{\rm r}$ represent the head range $[h_{\rm k} - \Delta h_{\rm k}, h_{\rm k} + \Delta h_{\rm k}]$ and flow range $[q_{\rm j} - \Delta q_{\rm j}, q_{\rm j} + \Delta q_{\rm j}]$. All $k \in \mathcal{K}_r$ and $j \in \mathcal{J}_{\rm r}$ cover all possible heads and flow rates. For the head range $k$, we define an auxiliary variable called $\hat{p}^{\rm (HP)}_{\rm r,i,k}$, in which it is equal to the production of the unit $r$ (the linearized dashed red line in Figure 5), if the head is inside the range of $[h_{\rm k} - \Delta h_{\rm k}, h_{\rm k} + \Delta h_{\rm k}]$; otherwise, it is equal to zero. As a result, the production of unit $r$ would be,

$$p^{\rm (HP)}_{\rm r,i} = \sum_{k \in \mathcal{K}_{\rm r}} \hat{p}^{\rm (HP)}_{\rm r,i,k}. \tag{47}$$

If the head is inside the range of $[h_{\rm k} - \Delta h_{\rm k}, h_{\rm k} + \Delta h_{\rm k}]$, $\hat{p}^{\rm (HP)}_{\rm r,i,k}$ is approximated as follows:

$$\hat{p}^{\rm (HP)}_{\rm r,i,k} = \min_{j \in \mathcal{J}_{\rm r}} \left[ \rho^{\rm (HP)} \cdot (\hat{h}_{\rm r,k,j} \cdot q_{\rm r,i} + \beta_{\rm r,k,j}) \right], \tag{48}$$

where $\hat{h}_{\rm r,k,j}$ and $\beta_{\rm r,k,j}$ are the net head and y-intercept in $[m \cdot \%/100]$ and $[kW]$, approximating the hill chart of unit $r$ using step-wise linear curves. They are calculated beforehand by calculating the production at $q_{\rm j} - \Delta q_{\rm j}$ and $q_{\rm j} + \Delta q_{\rm j}$ using (8).

The equation (48) is not linear. By replacing it with the following constraints, and because we maximize the VPP profit, which is directly proportional to unit $r$ production, the output would be the maximum possible value[18], i.e., the minimum of estimated line curves over all $j \in \mathcal{J}_{\rm r}$.

$$0 \leq \hat{p}^{\rm (HP)}_{\rm r,i,k} \leq \rho^{\rm (HP)} \cdot (\hat{h}_{\rm r,k,j} \cdot q_{\rm r,i} + \beta_{\rm r,k,j}), \forall j \in \mathcal{J}_{\rm r}. \tag{49}$$

The constraints (47) and (49) are not enough. We have to add a number of auxiliary constraints to check which head range $k \in \mathcal{K}_{\rm r}$ is active. To this

[18]We must test this after solving the problem to avoid any bug.

end, we first calculate the associated volume range $k$ equal to $[v^{(\min)}_{\mathrm{r,l,k}}, v^{(\max)}_{\mathrm{r,l,k}}]$ using the "*volume-versus-head*" curve of the unit $r$. Then, the following constraints must be imposed.

$$0 \leq \hat{p}^{(\mathrm{HP})}_{\mathrm{r,i,k}} \leq p^{(\mathrm{HP,max})}_{\mathrm{r}} \cdot \zeta_{\mathrm{r,i,k}}, \forall k \in \mathcal{K}_{\mathrm{r}}, \tag{50}$$

$$v^{(\min)}_{\mathrm{r,l}} \cdot (1-\zeta_{\mathrm{r,i,k}}) + v^{(\min)}_{\mathrm{r,l,k}} \cdot \zeta_{\mathrm{r,i,k}} \leq v_{\mathrm{r,i}} \leq v^{(\max)}_{\mathrm{r,l}} \cdot (1-\zeta_{\mathrm{r,i,k}}) + v^{(\max)}_{\mathrm{r,l,k}} \cdot \zeta_{\mathrm{r,i,k}}, \quad \forall k \in \mathcal{K}_{\mathrm{r}}, \tag{51}$$

$$\sum_{k \in \mathcal{K}_{\mathrm{r}}} \zeta_{\mathrm{r,i,k}} = 1, \tag{52}$$

where $\zeta_{\mathrm{r,i,k}}$ is a binary number indicating whether the volume is within the range of the $k$ piece.

# B Summary of Optimization Problems

The optimization problem of long-term planning for scenario $s$ and design scheme $l$ is to maximize (5) subject to the following constraints:

$$(47), (49)-(52), (9)-(10), (19),$$

$$\forall r \in \mathcal{R}^{(\mathrm{HP})}_{\mathrm{l}}, \forall i \in \mathcal{W} \times \{1\} \times (\mathcal{Z} \cup \mathcal{FL}) \times \{s\} \times \{l\}, \tag{53}$$

$$(11), \forall r \in \mathcal{R}^{(\mathrm{HP})}_{\mathrm{l}}, \forall i \in \mathcal{W} \setminus \{W\} \times \{1\} \times \mathcal{Z} \times \{s\} \times \{l\}, \tag{54}$$

$$(15), \forall r \in \mathcal{R}^{(\mathrm{HP})}_{\mathrm{l}}, \forall i \in \{1\} \times \{1\} \times \mathcal{Z} \times \{s\} \times \{l\}, \tag{55}$$

$$(16)-(18), \forall r \in \mathcal{R}^{(\mathrm{PM})}_{\mathrm{l}}, \forall i \in \mathcal{W} \times \{1\} \times (\mathcal{Z} \cup \mathcal{FL}) \times \{s\} \times \{l\}, \tag{56}$$

$$(20), \forall r \in \mathcal{R}^{(\mathrm{PV})}, \forall i \in \mathcal{W} \times \{1\} \times \mathcal{Z} \times \{s\} \times \{l\}, \tag{57}$$

$$(21)-(22), \forall r \in \mathcal{R}^{(\mathrm{WT})}_{\mathrm{l}}, \forall i \in \mathcal{W} \times \{1\} \times \mathcal{Z} \times \{s\} \times \{l\}, \tag{58}$$

$$(23)-(27), \forall r \in \mathcal{R}^{(\mathrm{ES})}_{\mathrm{l}}, \forall i \in \mathcal{W} \times \{1\} \times (\mathcal{Z} \cup \mathcal{FL}) \times \{s\} \times \{l\}, \tag{59}$$

$$(28), \forall r \in \mathcal{R}^{(\mathrm{ES})}_{\mathrm{l}}, \forall i \in \mathcal{W} \setminus \{W\} \times \{1\} \times \mathcal{Z} \times \{s\} \times \{l\}, \tag{60}$$

$$(29)-(30), (33)-(37), \forall w \in \mathcal{W}, \forall t \in \{1\}, \tag{61}$$

$$(31)-(32), \forall i \in \mathcal{W} \times \{1\} \times \mathcal{FL} \times \{s\} \times \{l\}, \tag{62}$$

where $\mathcal{W} := \{1, 2, \ldots, 52\}$, $\mathcal{T} := \{1\}$, $\delta t = 24 \times 7[h]$, and decision variables are defined in (44).

The optimization problem of short-term planning is to maximize the objective of day-ahead and real-time together (presented in (6)) minus a penalty term equal to,

$$\psi_{\mathrm{w,s,l}} = \mathrm{BM} \cdot \sum_{r \in \mathcal{R}^{(\mathrm{HP})}} (\Delta q^{(\mathrm{pos})}_{\mathrm{r,w,s,l}} + \Delta q^{(\mathrm{neg})}_{\mathrm{r,w,s,l}}). \tag{63}$$

This penalty term is subtracted from the objective function (6) to minimize the deviation from short-term planning. Normally, this penalty term is zero; however, to avoid non-feasibility, we use this penalty function as a soft constraint over $\Delta q^{(\mathrm{pos})}_{\mathrm{r,w,s,l}}$ and $\Delta q^{(\mathrm{neg})}_{\mathrm{r,w,s,l}}$. The constraints of short-term planning problem of week $w$, scenario $s$, and design scheme $l$ are as follows:

$$(47), (49) - (52), (9) - (10), (19),$$

$$\forall r \in \mathcal{R}_1^{(\mathrm{HP})}, \forall i \in \{w\} \times \mathcal{T} \times (\mathcal{Z} \cup \mathcal{FL}) \times \{s\} \times \{l\}, \tag{64}$$

$$(11), \forall r \in \mathcal{R}_1^{(\mathrm{HP})}, \forall i \in \{w\} \times \mathcal{T} \setminus \{T\} \times \mathcal{Z} \times \{s\} \times \{l\}, \tag{65}$$

$$(12) - (14), \forall r \in \mathcal{R}_1^{(\mathrm{HP})}, \tag{66}$$

$$(15), \forall r \in \mathcal{R}_1^{(\mathrm{HP})}, \forall i \in \{w\} \times \{1\} \times \mathcal{Z} \times \{s\} \times \{l\}, \tag{67}$$

$$(16) - (18), \forall r \in \mathcal{R}_1^{(\mathrm{PM})}, \forall i \in \{w\} \times \mathcal{T} \times (\mathcal{Z} \cup \mathcal{FL}) \times \{s\} \times \{l\}, \tag{68}$$

$$(20), \forall r \in \mathcal{R}_1^{(\mathrm{PV})}, \forall i \in \{w\} \times \mathcal{T} \times \mathcal{Z} \times \{s\} \times \{l\}, \tag{69}$$

$$(21) - (22), \forall r \in \mathcal{R}_1^{(\mathrm{WT})}, \forall i \in \{w\} \times \mathcal{T} \times \mathcal{Z} \times \{s\} \times \{l\}, \tag{70}$$

$$(23) - (27), \forall r \in \mathcal{R}_1^{(\mathrm{ES})}, \forall i \in \{w\} \times \mathcal{T} \times (\mathcal{Z} \cup \mathcal{FL}) \times \{s\} \times \{l\}, \tag{71}$$

$$(28), \forall r \in \mathcal{R}_1^{(\mathrm{ES})}, \forall i \in \{w\} \times \mathcal{T} \setminus \{T\} \times \mathcal{Z} \times \{s\} \times \{l\}, \tag{72}$$

$$(29) - (30), (33) - (37), \forall t \in \mathcal{T}, \tag{73}$$

$$(31) - (32), \forall i \in \{w\} \times \mathcal{T} \times \mathcal{FL} \times \{s\} \times \{l\}, \tag{74}$$

where $\mathcal{W} := \{w\}$, $\mathcal{T} := \{1, 2, \ldots, 24{\times}7\}$, $\delta t = 1[h]$, and the decision variables of short-term planning are defined in (45) and (46).