Prof. Dr.-Ing. Gangolf Hirtz
Dipl.-Ing. Julia Richter

WS 2015/2016

Digital Systems

Exercise E01

Exercise 1 Sampling

A 12 bit analog to digital converter has a nominal input range of [-10V..10V]. The digital value is interpreted as a positive integer value.

(a) What is the integer code for the incoming voltage of -3V?

(b) What is the binary code for the incoming voltage of -3V?

(c) What is the incoming voltage calculated from that integer code?

(d) What is the range of incoming voltages that would receive the same code?

Exercise 2 Discrete Fourier Transform: Real and imaginary part

Figure 1 shows a complex signal consisting of cosine and sine components.

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j \cdot 2\pi \cdot n \cdot \frac{k}{N}} = \sum_{n=0}^{N-1} x[n] \cdot cos(2\pi \cdot n \cdot \frac{k}{N}) - j \sum_{n=0}^{N-1} x[n] \cdot sin(2\pi \cdot n \cdot \frac{k}{N}) \quad \text{with } k = 0, 1, \ldots N-1 \tag{1}$$

The following statements are valid:

- The similarity of the real part of x[n] with a cosine is represented by the DFT as real-part of X[k] (axis symmetry).
- The similarity of the real part of x[n] with a sine is represented by the DFT as imaginary part of X[k] (point symmetry).
- The similarity of the imaginary part of x[n] with a cosine is represented by the DFT as imaginary part of X[k] (axis symmetry).
- The similarity of the imaginary part of x[n] with a sine is represented by the DFT as real part of X[k] (point symmetry).

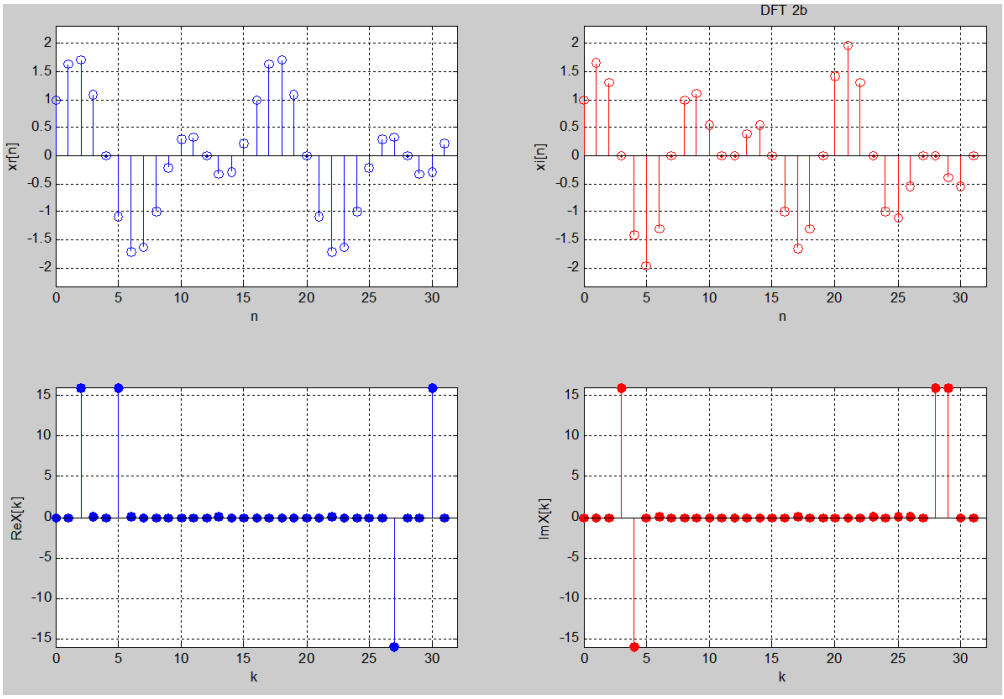

Figure 1: Signal x[n] and corresponding DFT spectrum

Consider the DFT coefficients in the bottom images of Figure 1 and answer the following questions:

(a) Which of the peak pairs represent a cosine component, which a sine component? Do they belong to the real or the imaginary part of the signal?

(b) Determine the frequencies the respective cosine and sine functions.

(c) In how many DFT coefficients does the DFT transformation of the given signal x[n] result?

Exercise 3 Discrete Fourier Transform

The Discrete Fourier Transform (DFT) $X[k]$ of a discrete signal $x[n]$ is given by:

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j \cdot 2\pi \cdot n \cdot \frac{k}{N}} \text{ with } k = 0, 1, \ldots N-1 \tag{2}$$

(a) What is the difference between FFT and DFT?

(b) Given the following input signal, which was sampled with a sampling frequency $f_s = 8$ kHz for a period of 1 ms.

$$x(t) = 0.5 + \cos(2\pi \cdot 1 \text{ kHz} \cdot t) + j \cdot \cos(2\pi \cdot 3 \text{ kHz} \cdot t) \tag{3}$$

Calculate the coefficients of the DFT (real and imaginary part). Draw the spectrum.