

TVV 1

Chapter 02

Colorimetry

Basics of TV- and Video Technology

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1.1 Basics of spectrometry values

1.1 Basics of spectrometric values (Strahlungsphysikalische Grundlagen)

- A distinction is made between **spectrometric values** and **photometric values**.

- 1.1.1 Solid angle** (Raumwinkel)
- The solid angle Ω corresponds to the area bordered by the light radiation (*entspricht der durch die Lichtstrahlung begrenzte Fläche*).
- Unit of the solid angle Ω is **Steradian (sr)**.

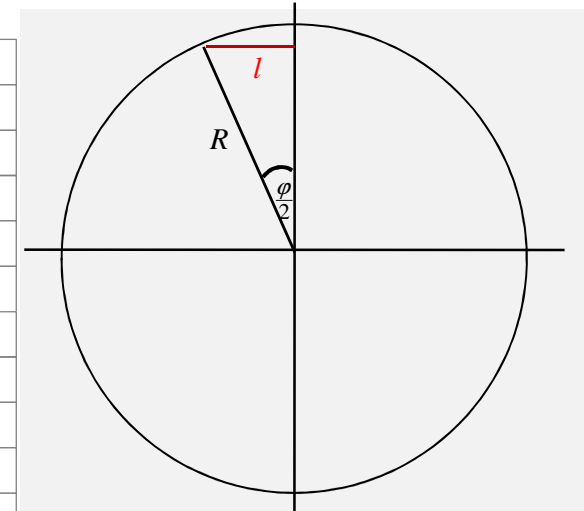
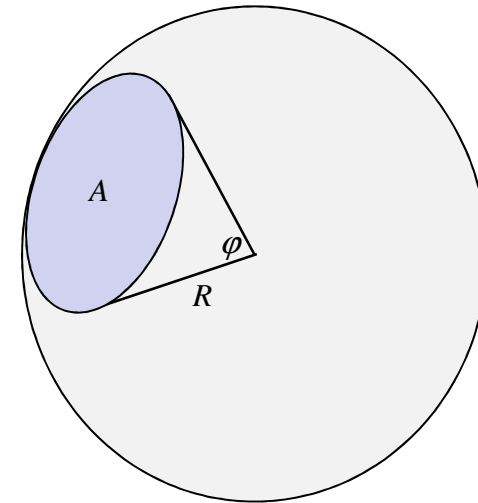
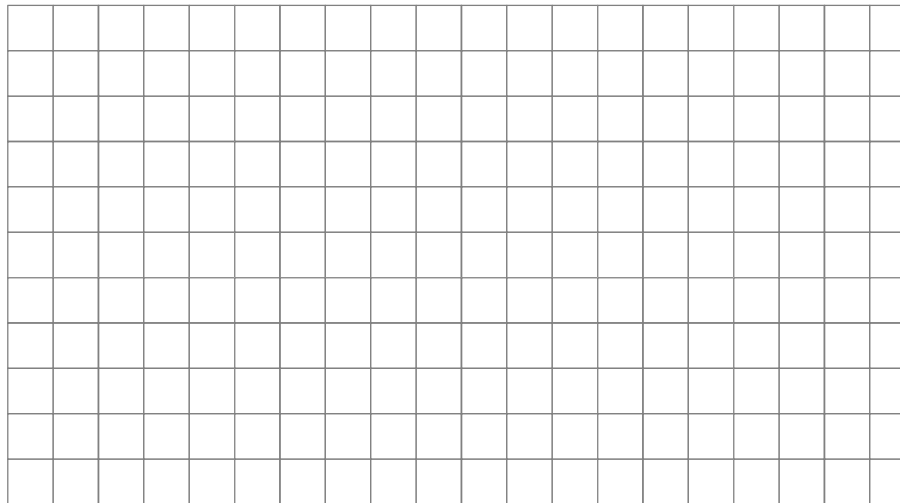
$$\Omega = \frac{A}{R^2} \text{ sr}$$

- A solid angle of 1 sr spans the area of 1m² on a ball with radius R=1m.
- The solid angle of the total ball is given by:

$$\Omega_{\text{ges}} = 4 \pi \text{ sr}$$

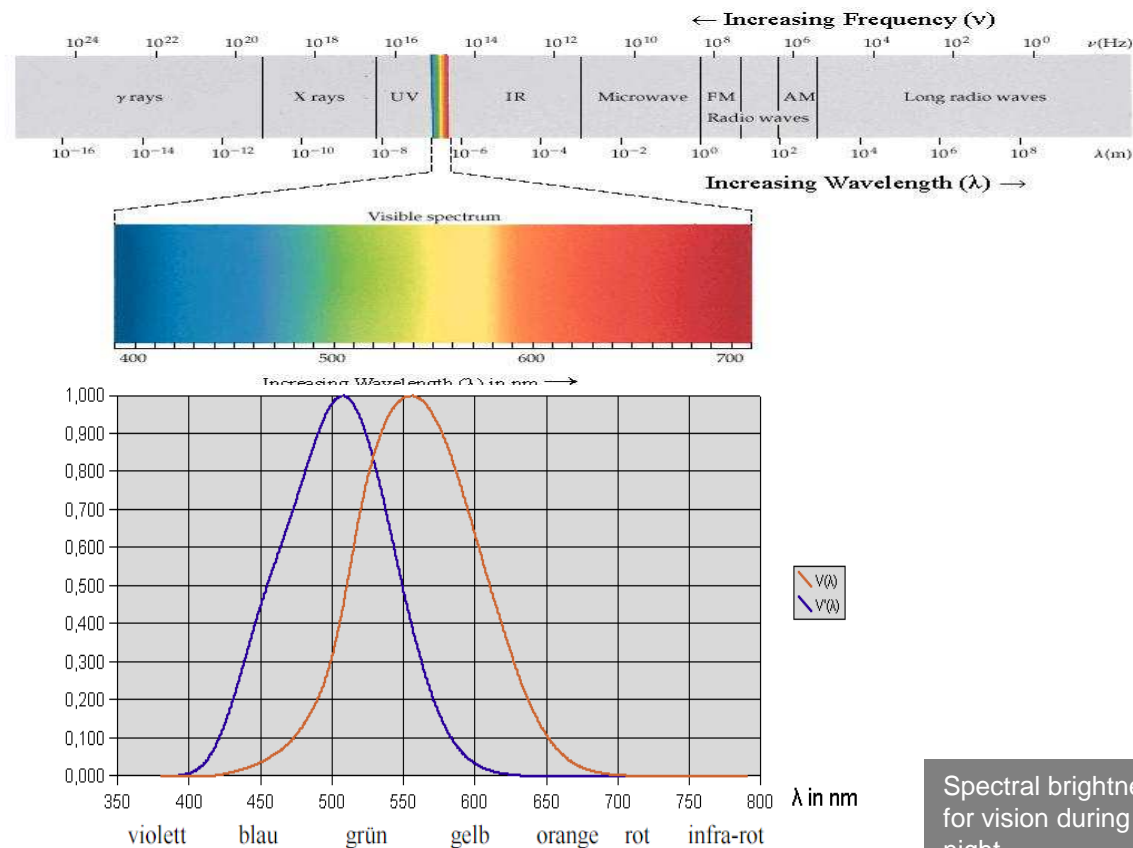
- Relationship between angle φ and solid angle Ω results in:

$$\Omega = 2\pi \left(1 - \cos \frac{\varphi}{2} \right)$$



1.2 Photometric values (Lichttechnische Größen)

- **1.2.1 Spectral brightness sensitivity** (*Spektrale Hellempfindung*)
- The spectrometric value rules are related to the complete frequency range of electromagnetic radiation.
- The sensitivity of the human eye is described by the so called spectral brightness sensitivity $V(\lambda)$.
- The brightness sensitivity $V(\lambda)$ during the day (photopic vision) differs from brightness sensitivity at night (scotopic vision)



Spectral brightness sensitivity $V(\lambda)$ for vision during day and $V'(\lambda)$ at night

- With help of the spectral brightness sensitivity $V(\lambda)$ it's possible now to convert the spectrometric values into photometric values., which are adapted to the human eye.
Here we talk about photometric values (*lichttechnischen Größen*).
- **1.2.2 Light flux** (*Lichtstrom*) Φ [Lumen]
- The relationship between the spectrometric value radiation flux (*Strahlungsflusses*) $\Phi_{e\lambda}$ and the photometric value light flux is given by the spectral brightness sensitivity $V(\lambda)$:

Spectrometric value		Photometric values
Radiation flux density	\rightarrow spectral brightness sensitivity \rightarrow	Light flux density
$\Phi_{e\lambda}$	$V(\lambda)$	$\Phi_{v\lambda} = k \cdot V(\lambda) \cdot \Phi_{e\lambda}$

- Light flux describes the complete in all directions emitted light energy within the visible wavelength range.
- The photometric value **light flux** Φ_v (unit Lumen (lm)) is given by:

$$\Phi_v = k \int \Phi_{e\lambda} \cdot V(\lambda) d\lambda$$

- For a wavelength $\lambda=555\text{nm}$:

$$\Phi_v = k \cdot \Phi_{e,555\text{nm}} \quad k = 683 \frac{\text{lm}}{\text{Watt}}$$

- \rightarrow Light with a wavelength of 555nm and a radiation flux of 1W results in a light flux of 683 lumens.
- A 1W radiation flux of $\lambda=675\text{nm}$ only generates a light flux of 70lm, by means that the brightness sensitivity of the human eye is considerably lower
- A 1W radiation flux of $\lambda=1000\text{nm}$ (infra red) or $\lambda=300\text{nm}$ (ultraviolet) generates no more visible light, since $V(\lambda)=0$ for those frequencies.

1.2 Photometric values

- The relationship between the spectrometric value radiation flux (*Strahlungsfluss*) Φ_{es} [W] and light flux Φ [Lumen] describes the luminous efficacy of a light source. Unit is Lumens/Watt.

– Spectral light ($\lambda=555\text{nm}$)	683 lm/W
– White light	200 lm/W
– fluorescent tube	95 lm/W
– Halogen lamp	25 lm/W
– LED	35 lm/W
– Incandescent lamp (<i>Glühbirne</i>) (2800K)	17 lm/W
– LCD display (backlight 80lm/W)	2 lm/W
– Plasma display	1 lm/W

- **1.2.3 Light intensity** (*Lichtstärke*) **I**
- Light sources normally do not radiate equally in all directions.
- The photometric value light intensity **I** describes the distribution of the light flux over the solid angle .

$$I = \frac{d\Phi}{d\Omega}$$

- Unit of light intensity is **candela (cd)**:

$$cd = \frac{lm}{sr}$$

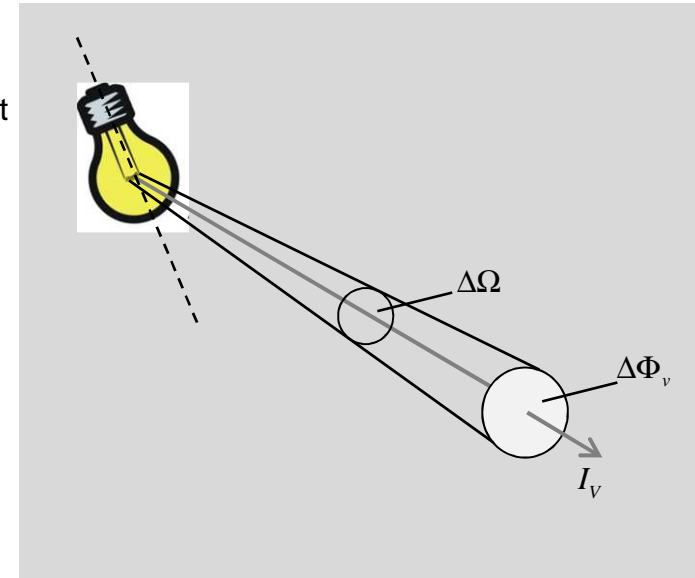
- The total radiated light flux is given by:

$$\Phi = \int_{\Omega} I(\Omega) d\Omega$$

- A spotlight with equal light intensity **I** in all directions results in a radiated light flux Φ :

$$\Phi = I \cdot 4 \cdot \pi \cdot sr$$

- Definition of light intensity only makes sense for spot lights, respectively when distance to the light source is minimum 5...10 times the lateral expansion of the light source.



1.2 Photometric values

- 1.2.4 Illumination (*Beleuchtungsstärke*) **E**

- The illumination **E** describes the light flux in relation to illuminated area:

$$E = \frac{d\Phi}{dA}$$

- Illumination corresponds to the spectrometric value irradiance (*Bestrahlungsstärke*)
- Unit of illumination is **lux**.

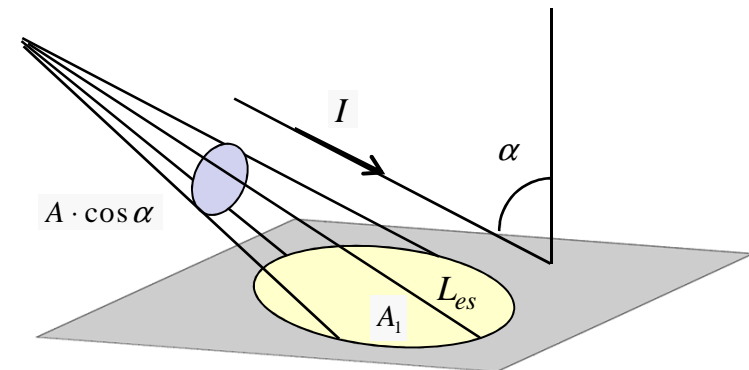
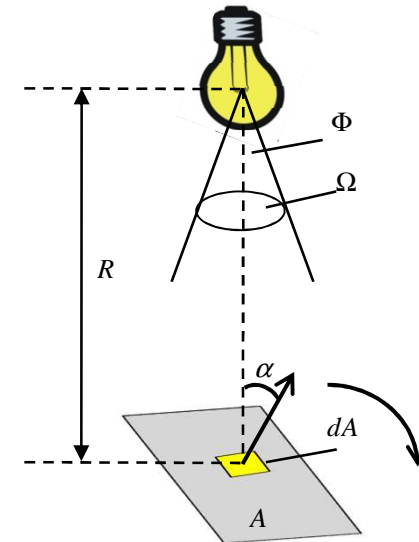
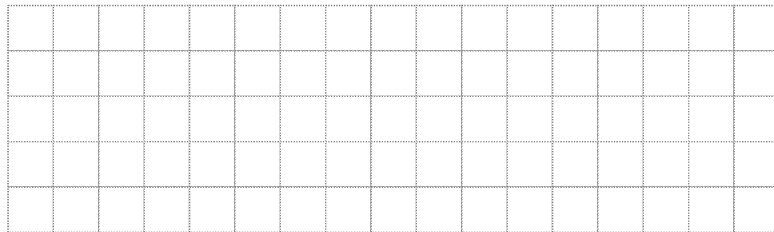
$$\text{lux} = \frac{\text{lm}}{\text{m}^2}$$

- For the illumination **E** we get:

$$E = \frac{I \cdot \cos \alpha}{R^2} \cdot sr$$

- In case of perpendicular incidence of light (*Lichteinfall*) ($\alpha=0^\circ$) the **Photometric Distance Law** is derived:

$$E = \frac{I}{R^2} \cdot sr$$



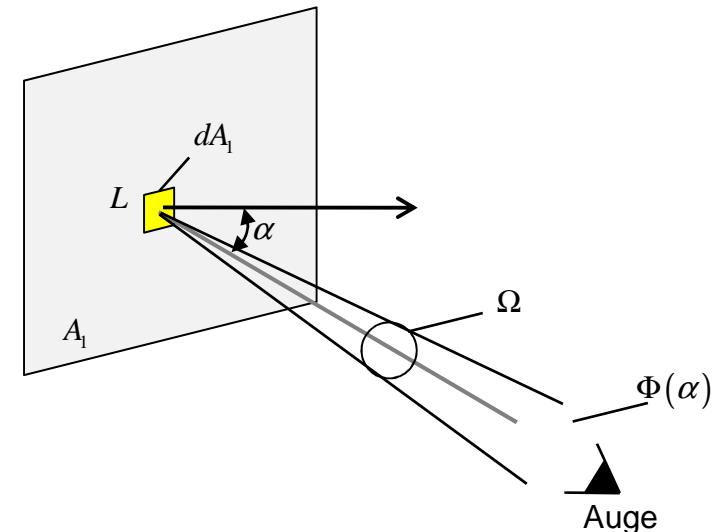
- **1.2.5 Luminance** (*Leuchtdichte*) **L**:

- The luminance **L** of a illuminating or illuminated **A** is responsible for the impression of brightness (*Helligkeitseindruck*).

$$L_v = \frac{dI_v}{dA} = \frac{d^2\Phi_v}{dA \cdot d\Omega}$$

- Luminance is the radiated light flux per solid angle and illuminating or illuminated surface.
- Unit of luminance is **Candela per square-meter**.
- As smaller the illuminating surface is, as larger is the luminance.
- Normally the luminance depends on the angle α of the illuminating surface

- | | |
|---------------------------------|------------------------------|
| • Street lighting | 2 cd/m ² |
| • White paper (400lux) | 100 cd/m ² |
| • LCD Monitor | 500...1000 cd/m ² |
| • Moon | 2500 cd/m ² |
| • Fluorescent lamp | 8000 cd/m ² |
| • Filament (<i>Glühfaden</i>) | 7 Mio cd/m ² |
| • Sun | 1650 Mio cd/m ² |



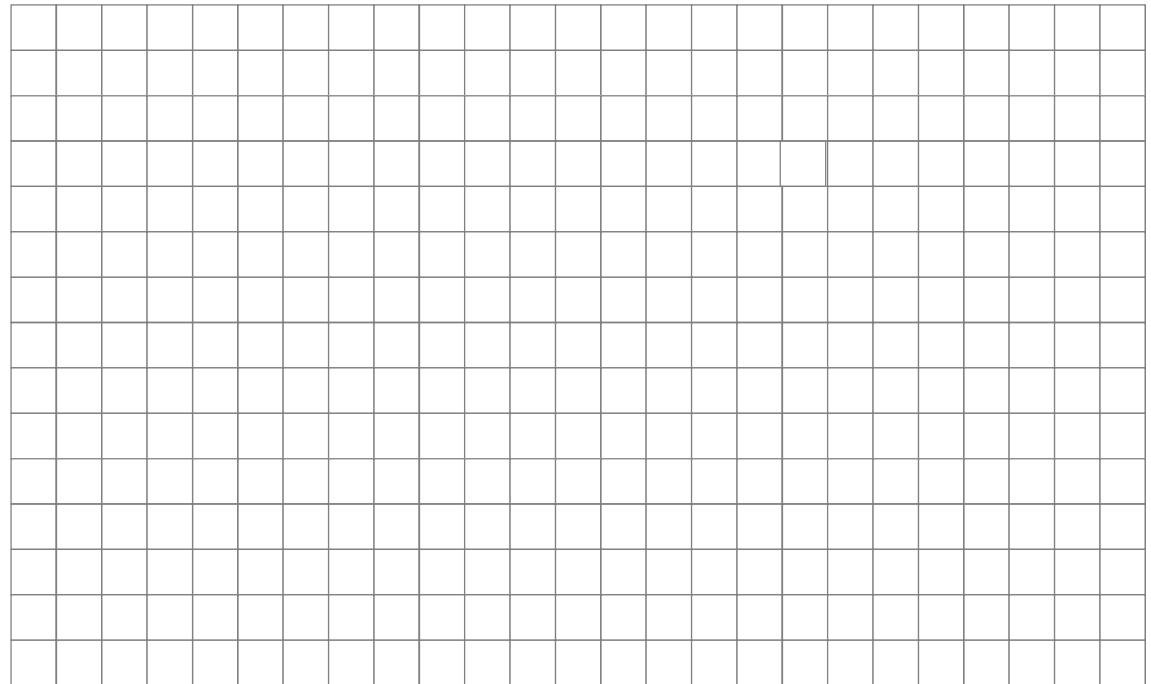
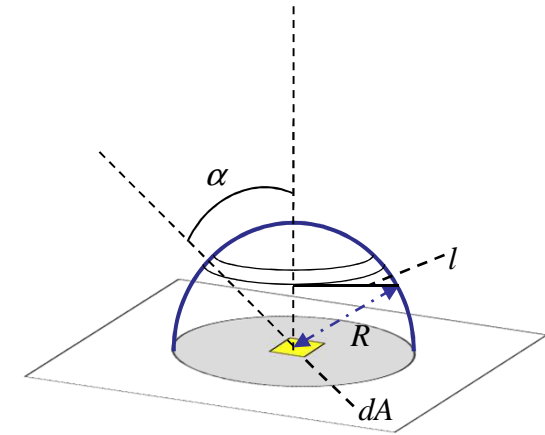
1.2 Photometric values

- **1.2.6 Lambertian radiator** (*Lambertstrahler*)
- A surface equally diffuse radiating in all directions is called **Lambertian radiator**.
- The luminance L of the Lambertian radiator is constant and does not angle depended.
- Assuming that the emitted light flux is lossless and diffuse reflected ($\rho=1$) (incident light flux is equal to reflected light flux)

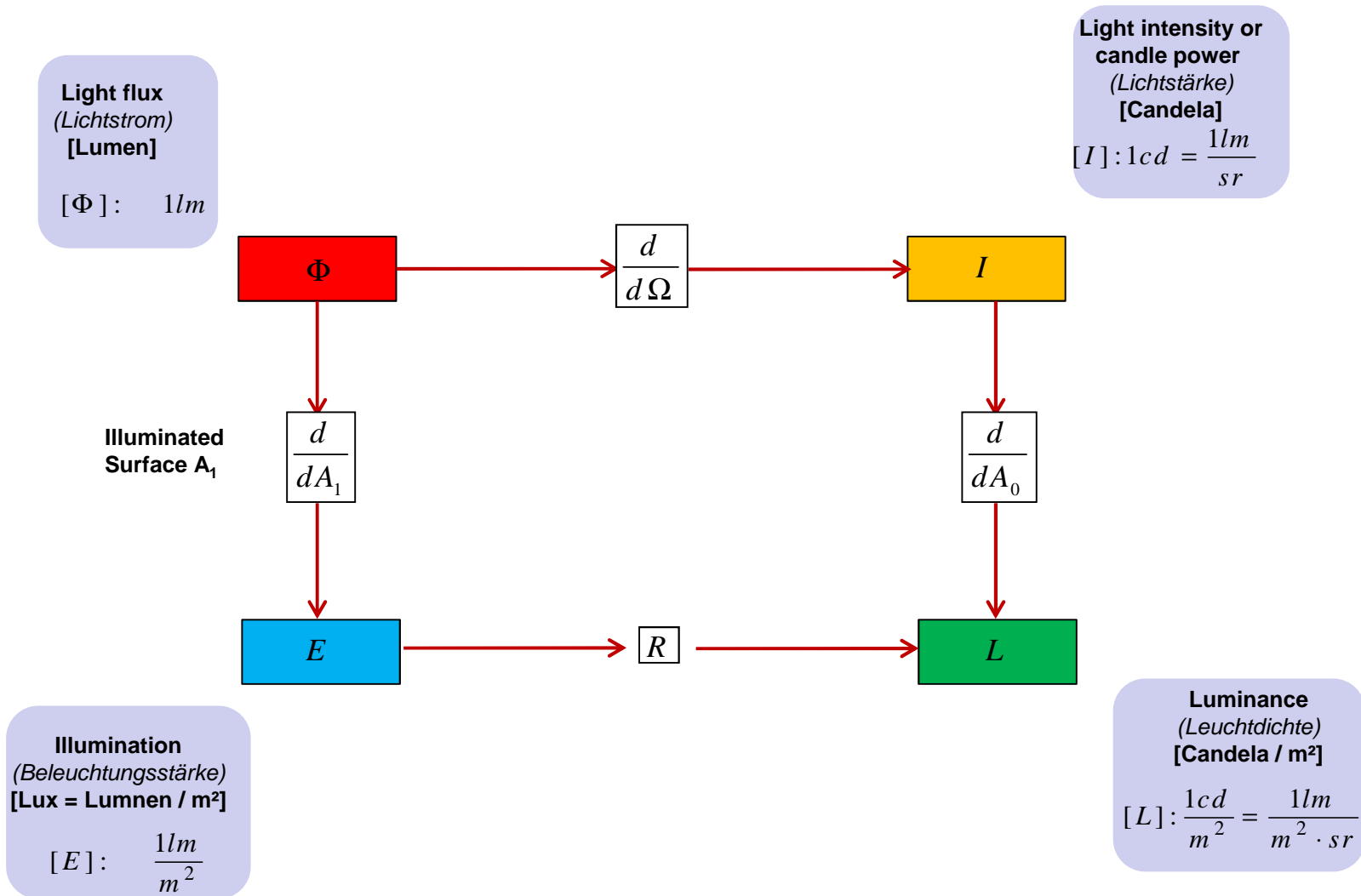
$$L = \frac{d\Phi}{\pi \cdot dA \cdot sr} = \frac{E}{\pi \cdot sr}$$

- A diffuse reflecting surface A , illuminated with the illumination $E = 1 \text{ lux} = 1 \text{ lm/m}^2$ results in a luminance L :

$$L = \frac{1 \text{ lm}}{\pi \cdot \text{m}^2 \cdot sr} = \frac{1}{\pi} \cdot \frac{cd}{\text{m}^2} = 1 \text{ Apostilb}$$



1.2 Photometric values

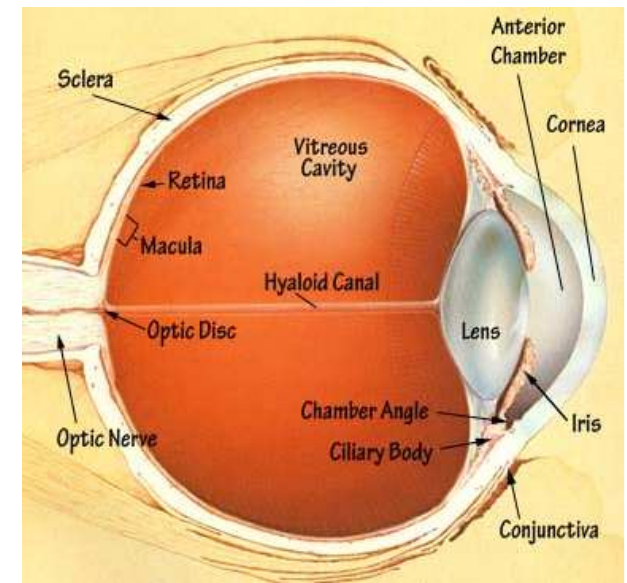


1.3 The human eye

1.3 The human eye

• 1.3.1 General properties

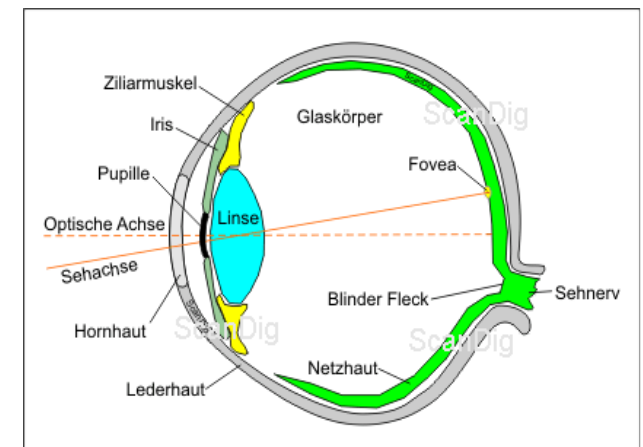
- The individual components of the eye work in a manner similar to a camera. Each part plays a vital role in providing clear vision. As the eye's main focusing element, the **cornea** (*Hornhaut*) takes widely diverging rays of light and bends them through the **pupil**, the dark, round opening in the center of the colored **iris**. The iris and pupil act like the aperture of a camera.
- Next is the lens which acts like a camera lens, helping to focus light to the back of the eye.
- The back of the eye is lined with a layer called the **retina** which acts very much like the film of the camera. The retina is a membrane containing photoreceptor nerve cells that lines the inside back wall of the eye. The photoreceptor nerve cells of the retina change the light rays into electrical impulses and send them through the optic nerve to the brain where an image is perceived.
- The center 10% of the retina is called the **macula**. This is responsible for your sharp vision, your reading vision.
- The peripheral retina is responsible for the peripheral vision.
- The human eye is able to accommodate to changing lighting conditions and focuses light rays originating from various distances from the eye.
- The accommodation capability decreases with the age..
 - The refractive power (*Brechkraft*) of the cornea with anterior eye chamber (vorderer Augenkammer) is about 43 dpt diopters (*Dioptrien*).
 - The refractive power of the lense, akkomodated in the far distance, is about 20dpt
 - In case of near vision (*Nahsehen*) the refractive power is about 33dpt.
 - The total refractive power can change between 59 dpt and 71dpt .



The human eye

Source

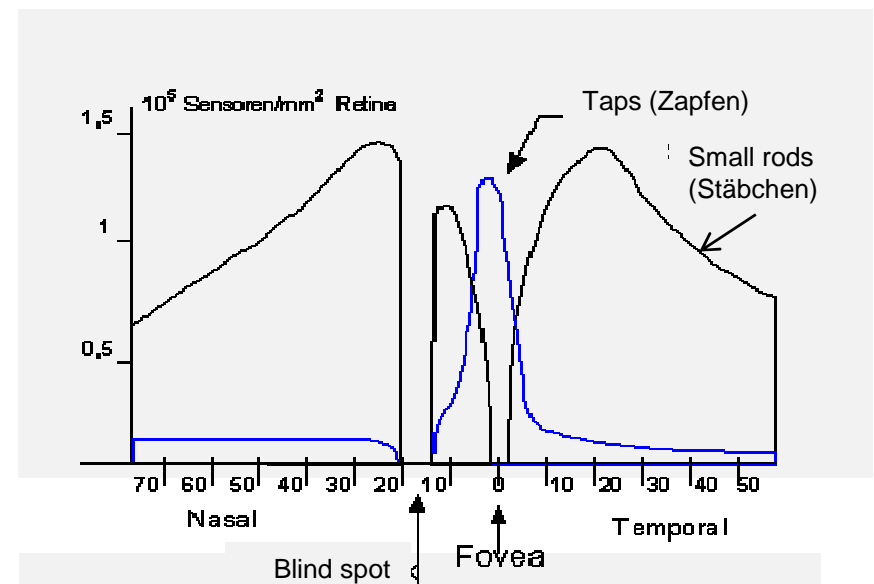
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Quelle: <http://www.filmscanner.info/BilderFM/Auge1.gif>

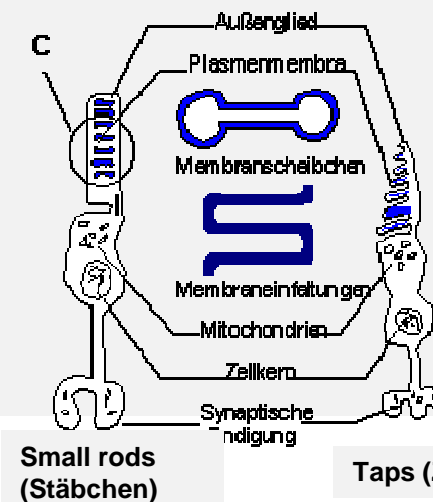
1.3 The human eye

- The **retina** is the photo sensitive organ of the eye. The photo-receptors subdivide in **taps** (Zapfen) and **small rods** (Stäbchen).
- The taps, all together 6,5 million are responsible for color vision $V(\lambda)$ and the all together 120 million **small rods** are responsible for black-and-white-vision $V'(\lambda)$.
- The taps are by far higher light sensitive than the rods
- The center of the retina, that so-called **makula** or "more yellow stains", the functionally most important share of the retina is. the makula is for the high one dissolution-is able, and this color-sees responsibly. here the receptor-dense one is the highest, she/it is regarded [..
- The Fovea Centralis has the highest detail sensitivity. It's diameter corresponds to an angle of 3° .
- The position where the optic nerve is connected to the eye we find the so called "blind spot" with a 6° diameter. The blind spot is about 15° shifted in direction of the nose.
- Outside the taps, which are also responsible for vision of details, the small rods (Stäbchen) are dominating.
- The temporal sensitivity of the eye is 10...20 pictures/s.
- With in total 120 millions of taps and small rods but only 1 million of nerve fibers, there must be a kind of pre-processing already in the retina (Netzhaut).



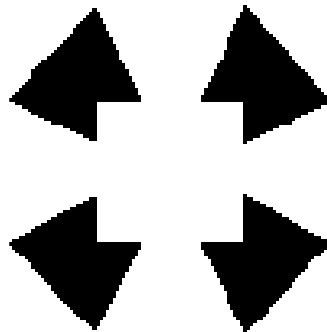
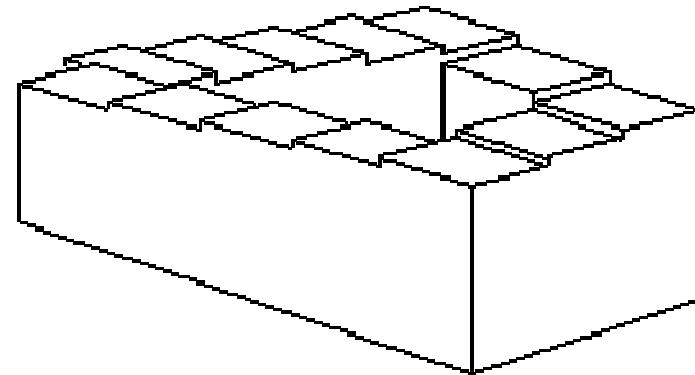
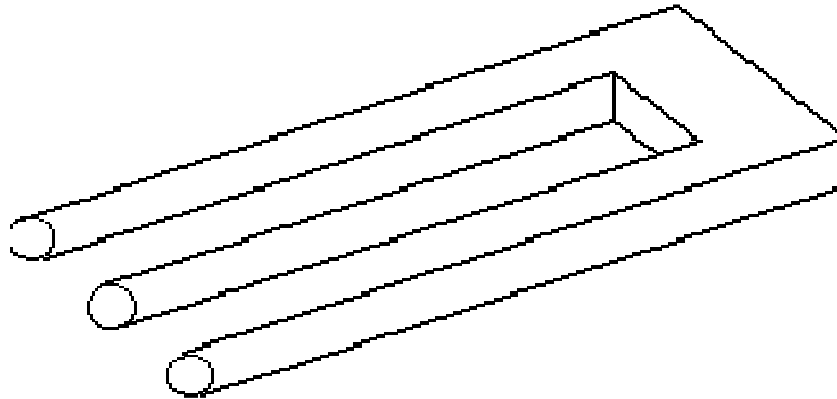
Taps (Zapfen) and small rods (Stäbchen).

http://www.klauslorenz.de/psychologie/phys_psy/images/auge.gif

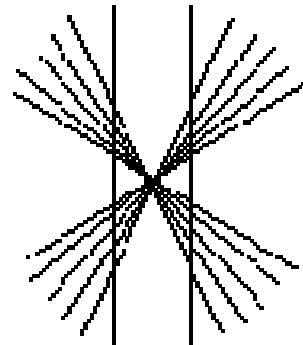


1.3 The human eye

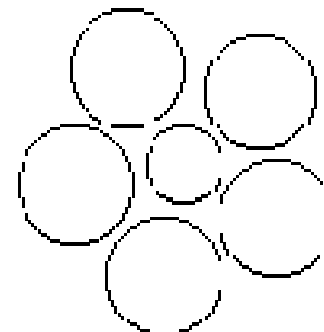
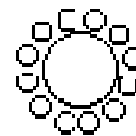
1.3.2 Optical illusion (*Optische Täuschungen*)



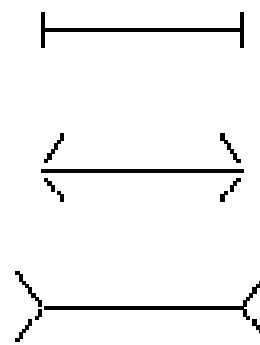
A



B



C

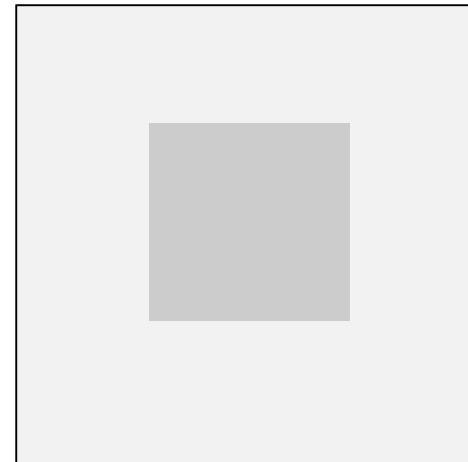
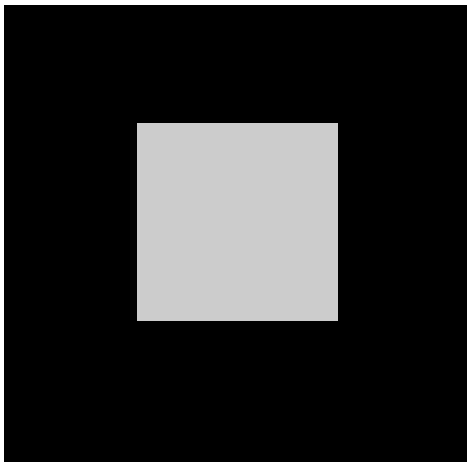
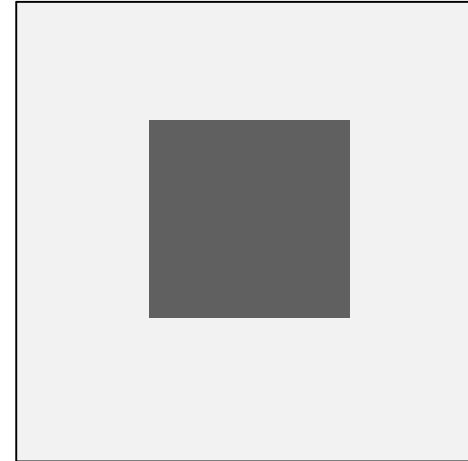


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Optische Täuschung des Auges.

1.3 The human eye

- 1.3.3 Brightness sensitivity and luminance



1.3 The human eye

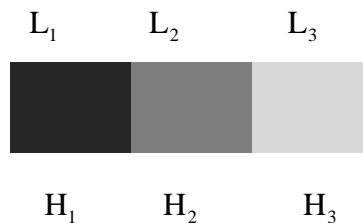
- The subjectively perceived brightness **H** of an object primarily depends on the luminance **L**.
- The perceived brightness also depends on the adaptation conditions (*Adaptionszustand*) of the eye.
 - A lamp which seems to be bright at darkness is very dark at daylight (instrument lighting of cars (*Instrumentenbeleuchtung*), night sky (*Sternenhimmel*)).
- It could be shown, that the subjective discrimination threshold (*Unterscheidungsschwelle*) dH depends on the ambient lighting (*Umfeldbeleuchtung*).

$$dH = k \cdot \frac{dL}{L}$$

- This results in the Weber-Fechner law.

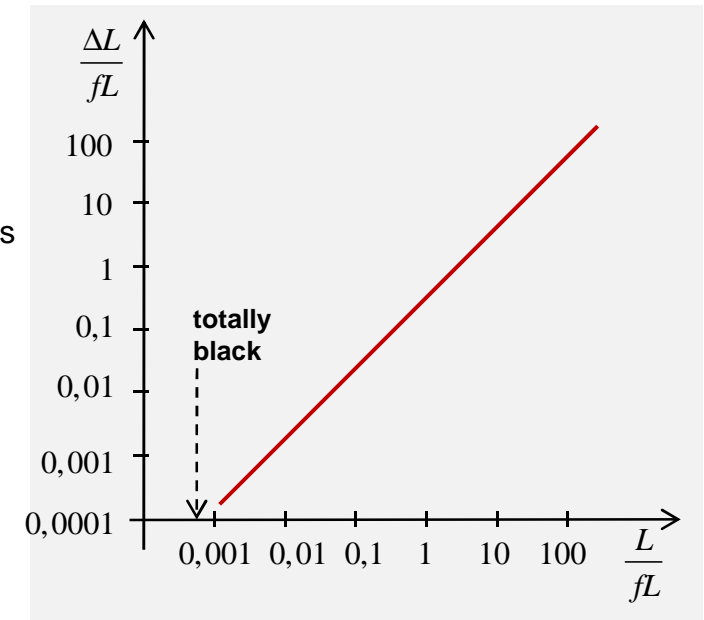
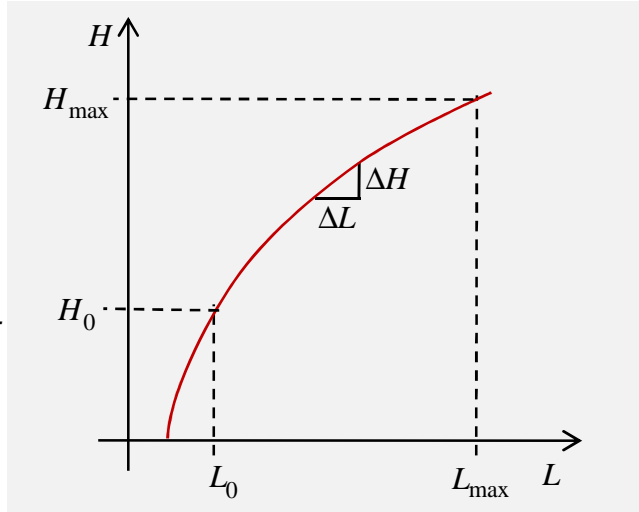
$$\int_{H_0}^{H_{\max}} dH = k \cdot \int_{L_0}^{L_{\max}} \frac{1}{L} dL \Leftrightarrow H_{\max} - H_0 = k \cdot \ln \left(\frac{L_{\max}}{L_0} \right)$$

- In case of low luminance, the subjectively perceived brightness-difference is higher than in case of high luminance.

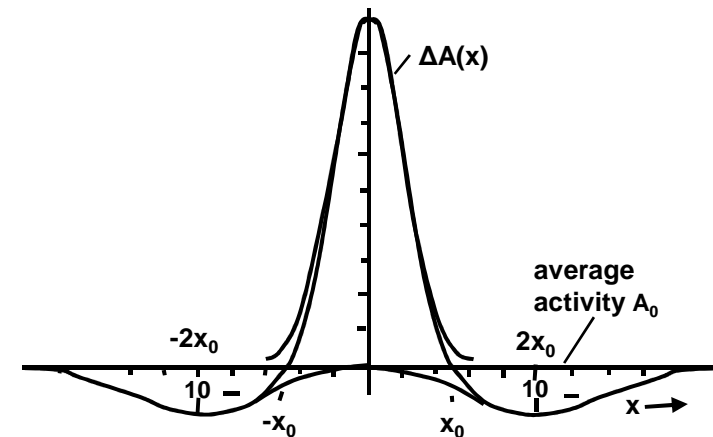
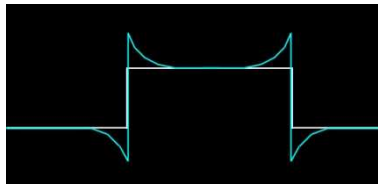


fL : Footlambert

$$1 fL = 3,43 \frac{cd}{m^2}$$

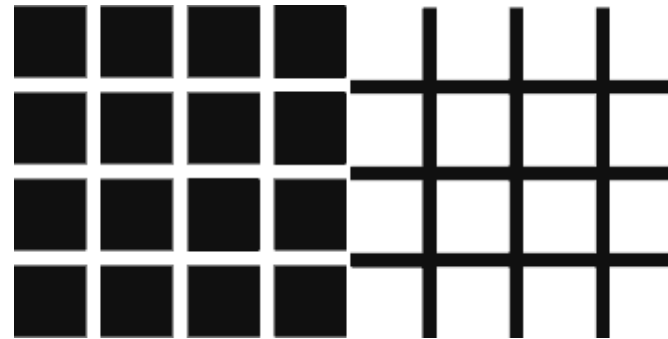


- **1.3.4 Lateral inhibition**
- The biophysicist Hartline could prove based on intensive examinations:
 - Excitations of luminous stimulus are transmitted to the brain by pulse sequences. The impulse rate (0...100/s) has a relation to the light intensity.
 - About 20% of the nerve fibers transmit impulses in case of a constant illumination (on cells) but 50% send impulses only when light is switched on or off.
 - Responsible for such a behavior is a relatively wide spread area of the illuminated retina.
- In case of an average illumination of the retina and an additional very fine illumination concentrated at a single point which is moved over the receptive area of a nerve, we can observe the following behavior:
 - Directly in the area of the nerve fiber there is a high activity of the nerve cells.
 - Quite next to center with increased activity we can observe an activity which is lower than the average one.
- This effect is called **lateral inhibition**. As already mentioned, this processing takes place in the retina and has the same effect as a high pass filter.



1.3 The human eye

- This kind of edge enhancement (*Kantenüberhöhung*) is also called Machian Stipes.



1.4 Colorimetry (*Farbmetrik*)

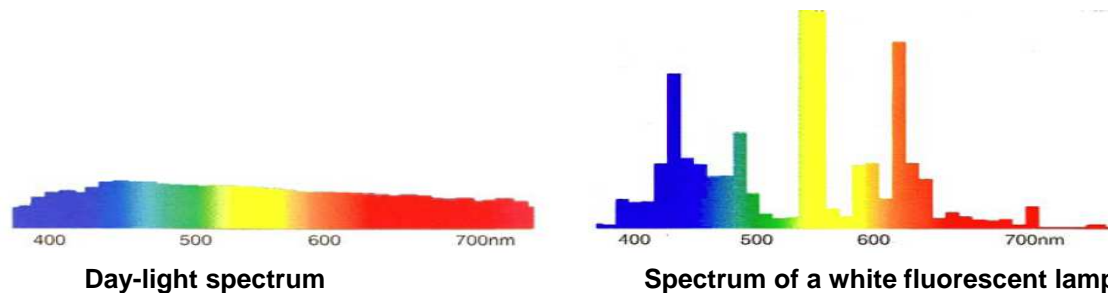
- Beside it's capabilities for luminance detection, the human eye is able to distinguish between different colors.
 - Color in this context means a subjective perception as result to a color stimulus by electromagnetic radiation in the range of visible light.
- Color sensitivity is directly not measurable.
 - Identical color stimulus results in different color perception depending on the colour scheme (*Farbstimmung*) of the eye.
 - Example white paper and red light.

1.4.1 Metameric colors

- Different color stimuli (different spectral composition) can generate the same color perception.
 - Metameric colors means conditional identical colors

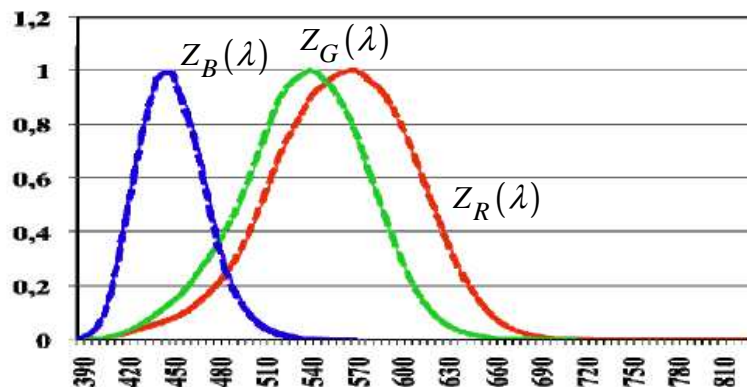
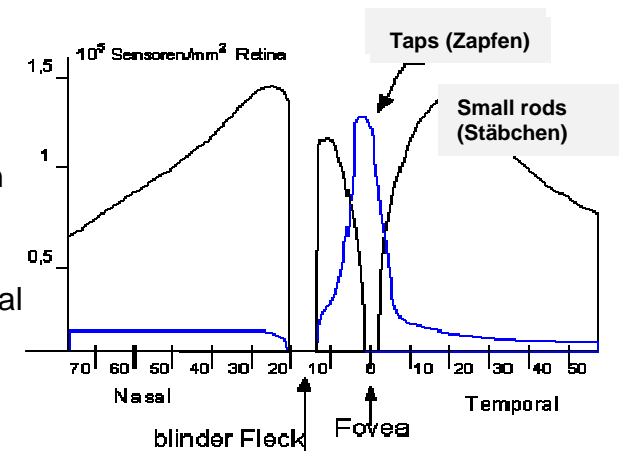
Metamerism: Different color stimuli can generate identical color perception.

- Reason for the behavior is the structure of the human eye, more precisely in the fact that the human eye is at all able to distinguish colors:
 - The human eye can only distinguish between colors which are mixed out of three suitable primary colors.



1.4 Colorimetry

- The ability to distinguish colors is linked to the taps (Zapfen).
- Taps are mainly located in the area of the fovea centralis.
 - → color sensitivity is limited to a small area of the visual field (Gesichtsfeld).
- The physicist H. von Helmholtz developed several theories about the color vision mid of the 19th century.
The retina has about 6 millions taps and about 120 millions small rods:
 - In the fovea centralis there are three different kind of taps with different spectral sensitivities:
 - S-taps (420 nm)
 - M-taps (530 nm)
 - L-taps (560 nm)



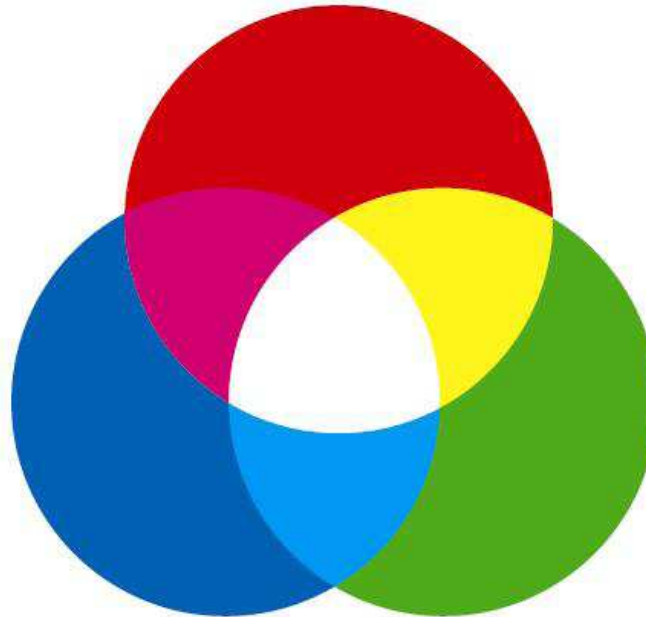
- With given weighting curves $Z_i(\lambda)$ (filters) of the taps ($i=R,G,B$) and a spectral distribution of the scene $\varphi(\lambda)$ the excitation of the eye is as follows:

$$E_i = \int_{380nm}^{780nm} \varphi(\lambda) \cdot Z_i(\lambda) d\lambda \quad i = R, G, B$$

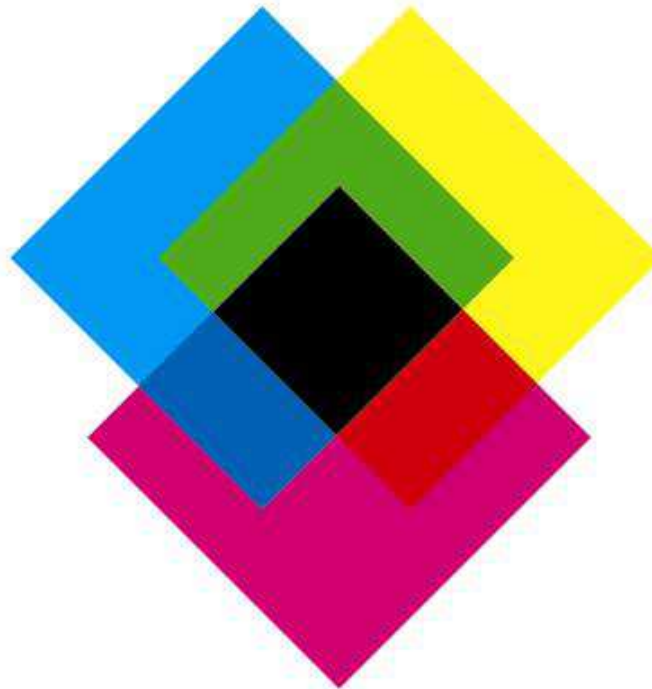
1.4.2 Color mixture

• 1.4.1.1 Additive color mixing

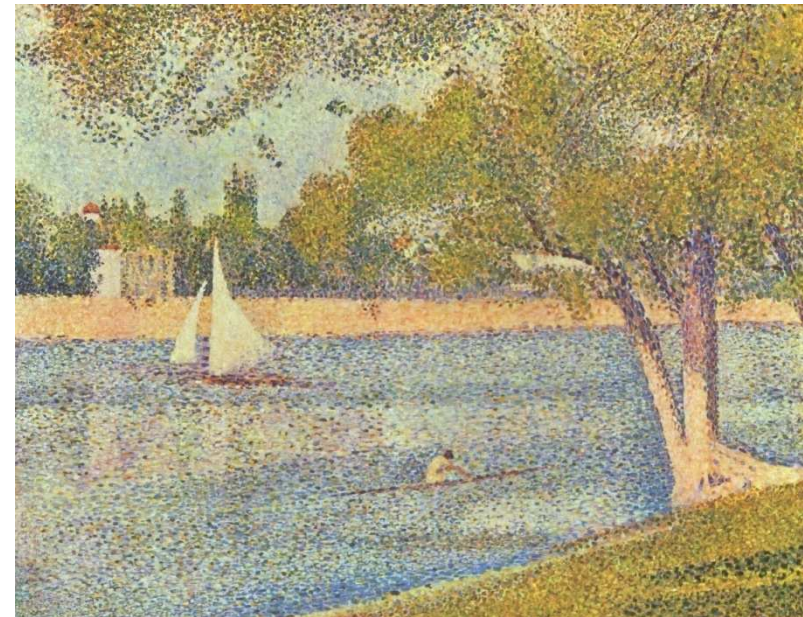
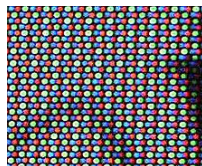
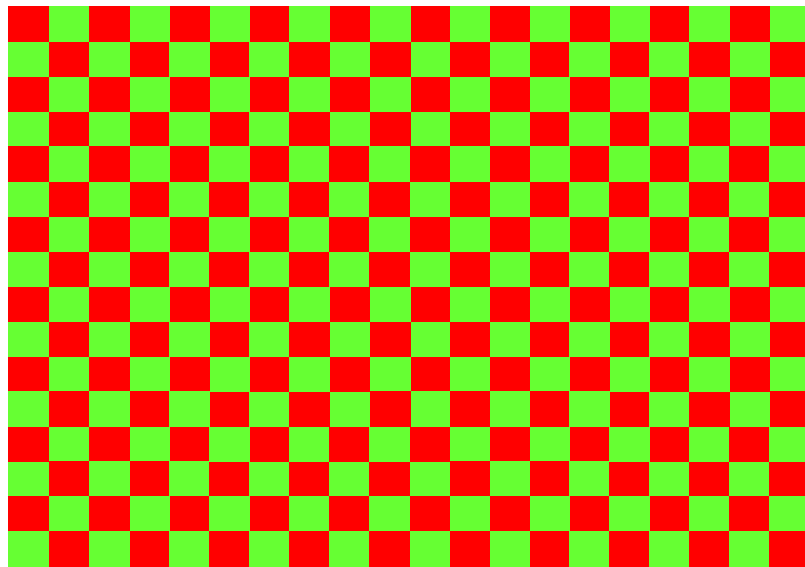
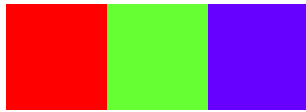
- Additive color mixing has the effect that out of two or more colored light sources a new light color is generated, which is brighter than the single colors, since the radiation flux is summed up.
 - For example: mixture of blue, green and red results cyan, yellow, magenta.
 - The additive mixture of the three primary colors results in white light.



- **1.4.1.2 Subtractive color mixing** (*Subtraktive Farbmischung*)
 - We talk about subtractive color mixing (in contrast to additive color mixing) when from a light source various parts of the light spectrum are absorbed (subtracted) by filters
 - That's the case as well as for mixture of filtered light as for mixture of elementary colors.
 - When we put a color filter (e.g. yellow and cyan) in front of a white light source the result is green light.
 - The same thing is valid for different colored paintings.



- **1.4.1.3 Optical color mixture** (*Optische Farbmischung*)
- In the field of industrial multicolor printing but also in case of pointillism painting the so called optical mixture is used which is basically an additive color mixture since the eye is not able to distinguish between small different colored point next to each other. From distance those points sum up to an additive color impression.
- That's the principle of multicolor printing where the single color points are very close together.
- That's also the principle of color TV!



- **1.4.3 Color valences / color stimulus specification** (*Farbvalenzen*)

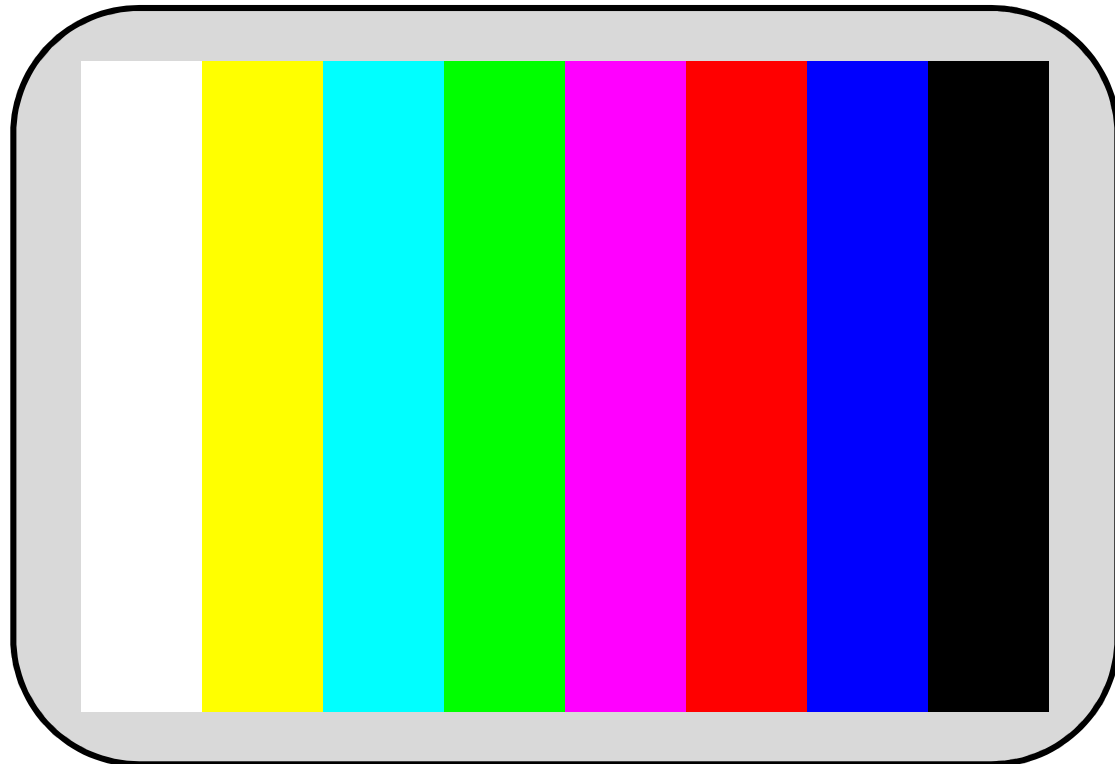
- Assume that mixture is made out of the three primaries red-, green- and blue-color values (*Farbwerten*)
- These three color values are scaled that:

$$R = G = B = 1$$

results in white (Color values are dimensionless)

- By additive mixture of these three primary valences we get the colors of the TV test picture:

	R	G	B
white	1	1	1
yellow	1	1	0
cyan	0	1	1
green	0	1	0
magenta	1	0	1
red	1	0	0
blue	0	0	1
black	0	0	0



1.4 Colorimetry

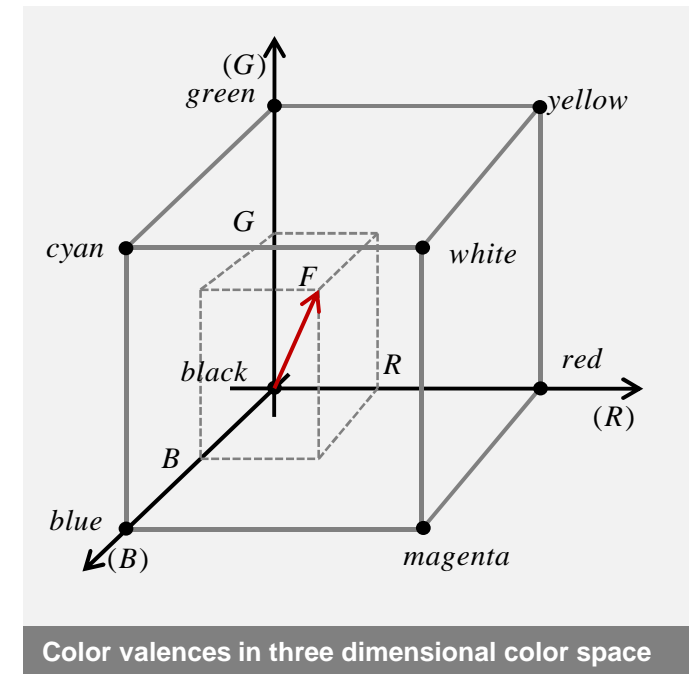
- The color dot (*Farbpunkt*) F in the color space (*Farbraum*) corresponds to the arrowhead of the vector with components R , G , and B .
- Color valences F with identical chromaticity (*Farbart*) only differ in the length of the vector (e.g. black to white), but not in the direction.
 - Hue and saturation remain unchanged
 - Only the brightness (vector length) is changed

Def: Chromaticity (*Farbart*) is defined by hue and color saturation

- The **chromaticity coordinates** (r, g, b *Farbwertanteile*) can be described by the color values or tri-stimulus values (*Farbwerte*) R, G, B normalized to the sum of the three color values:

$$r = \frac{R}{R+G+B}; \quad g = \frac{G}{R+G+B} \quad b = \frac{B}{R+G+B}$$

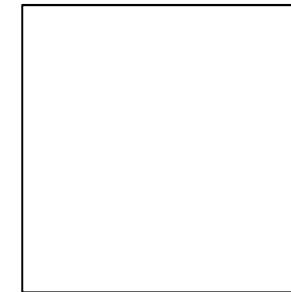
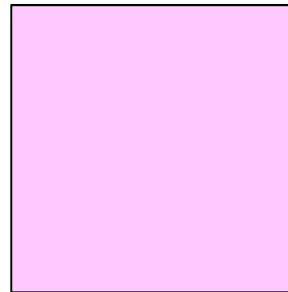
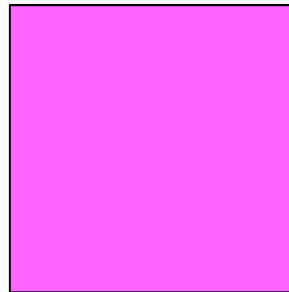
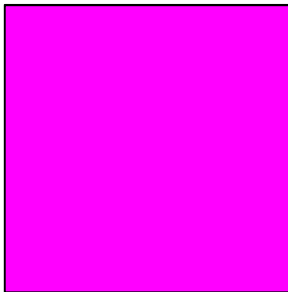
$$r + g + b = 1$$



R, G, B : Color values
(*Farbwerte*)

r, g, b Chromaticity coordinates
(*Farbwertanteile*)

- As can be seen for description of the chromaticity only two chromaticity coordinates are necessary (third one is difference to 1)



- **1.4.4 Hue (Farbton) and Color saturation (Farbsättigung)**
- Each color stimulus specification is specified by these three primary valences (R), (G), (B):

$$F = R \cdot (R) + G \cdot (G) + B \cdot (B)$$

- For white:

$$F_{white} = 1 \cdot (R) + 1 \cdot (G) + 1 \cdot (B)$$

- For black:

$$F_{black} = 0 \cdot (R) + 0 \cdot (G) + 0 \cdot (B)$$

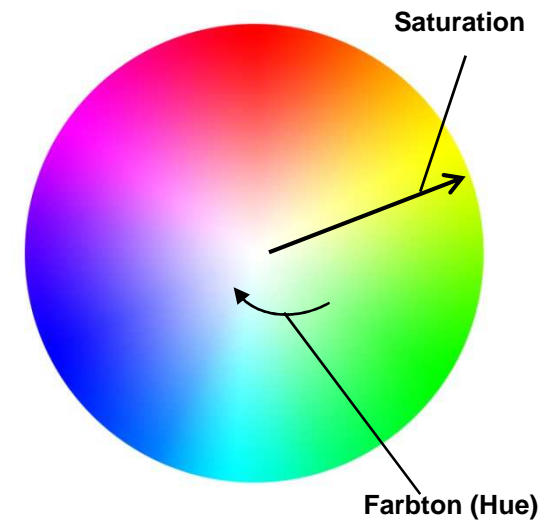
- For all grey shades (everything between black and white):

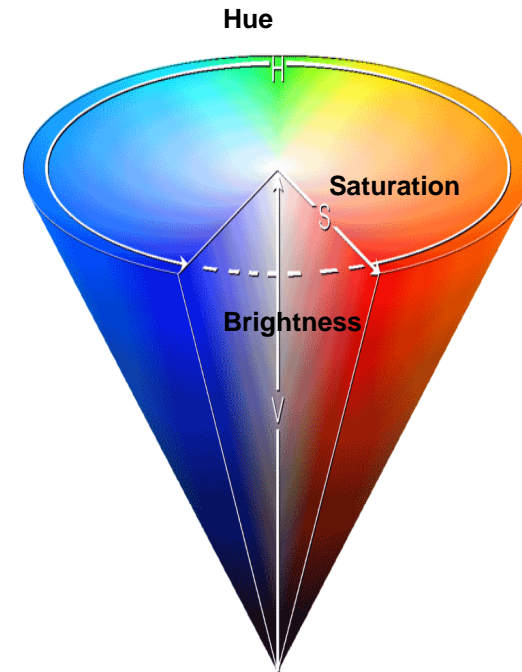
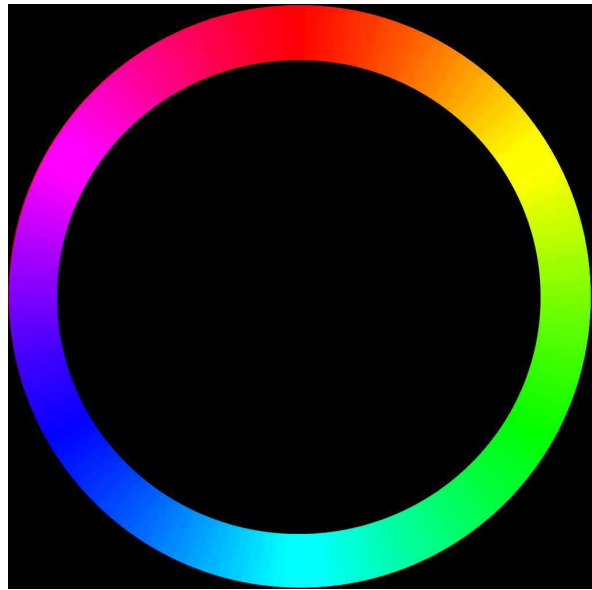
$$F_{grey} = K \cdot [1 \cdot (R) + 1 \cdot (G) + 1 \cdot (B)] \quad K \leq 1$$

- In case that one color valence (e.g. red) is reduced:

$$F_{grey} = K \cdot (R) + 1 \cdot (G) + 1 \cdot (B) = K \cdot [1 \cdot (R) + 1 \cdot (G) + 1 \cdot (B)] + (1 - K) \cdot [1 \cdot (G) + 1 \cdot (B)]$$

- This corresponds to the color cyan, but with reduced intensity, since grey (reduced white) is added to cyan)





Color-space with brightness, hue and saturation

1.4.5 Exterior color mixture (Äußere Farbmischung)

- We want to measure how the monitor colors can be generated out of monochromatic light
- 1.4.5.1 Measuring arrangement**
 - To the light of a monochromator (a device which generates pure spectral light e.g. laser) light from the monitor II (R,G,B) is added.
 - For white calibration $R_W=G_W=B_W=100$ (monochromator is switched off)
 - Now the monochromatic light is de-saturated by adding of light from the primary valences RGB II until the color of monitor I is identical to the color generated by the monochromator and monitor II
 - For the color valence F follows:

$$F_S + R_{II} \cdot (R) + G_{II} \cdot (G) + B_{II} \cdot (B) = R_I \cdot (R) + G_I \cdot (G) + B_I \cdot (B)$$

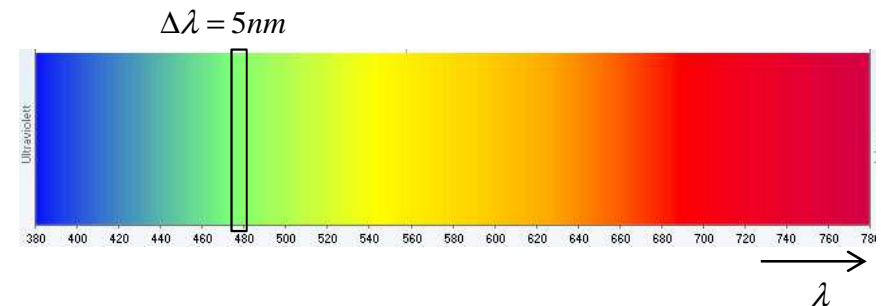
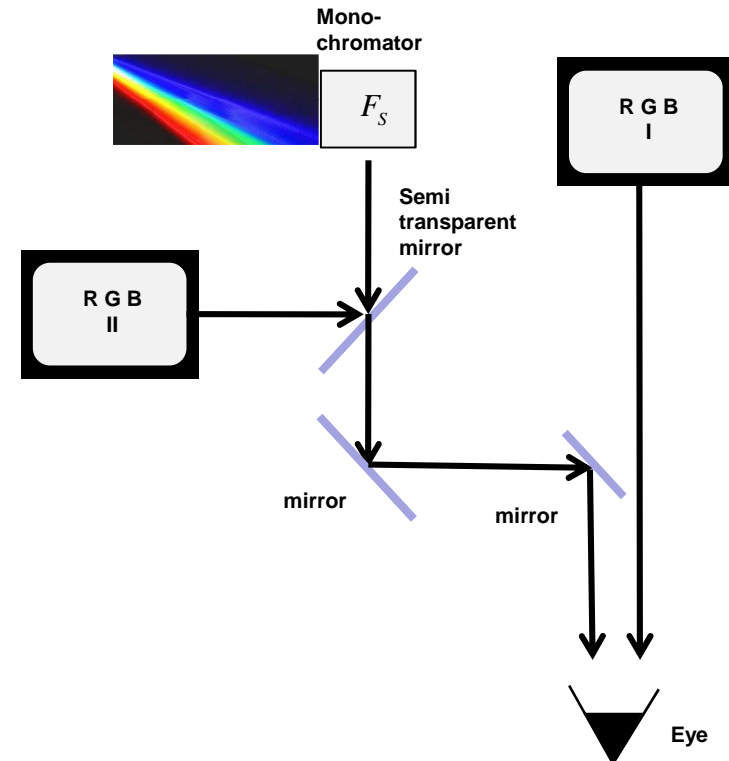
$$\Leftrightarrow F_S = (R_I - R_{II}) \cdot (R) + (G_I - G_{II}) \cdot (G) + (B_I - B_{II}) \cdot (B)$$

- Spectral color can **not** be mixed by additive mixture out of the three primary color valences
- To get the primary colors of monitor I (RGB I) pure monochromatic colors must be de-saturated (RGB II).
- Now we want determine the corresponding values for a spectral range of 5nm:

$$\bar{r}_i = R_I - R_{II} \quad \text{in range of } \lambda_1 \dots \lambda_2 \text{ mit } \lambda_1 + i \cdot \Delta\lambda; \quad \Delta\lambda = 5\text{nm}$$

$$\bar{g}_i = G_I - G_{II}$$

$$\bar{b}_i = B_I - B_{II}$$



1.4 Colorimetry

- **1.4.5.2 RGB Color matching function** (*Spektralwertkurven*)
- Since the radiation flux (Strahlungsfluss) of a light sources can be different, it is normalized to the average radiation flux :

$$\varphi(\lambda) = \frac{\Phi_{\lambda}(\lambda)}{\Phi_{\lambda,av}}$$

- With:

$$\Phi_{\lambda,av} = \frac{1}{\lambda_2 - \lambda_1} \cdot \int_{\lambda_1}^{\lambda_2} \Phi_{\lambda}(\lambda) d\lambda$$

- Accordingly the normalized radiation flux in the frequency range, $\Delta\lambda$ corresponds to:

$$\frac{\Delta\Phi_{\lambda}(\lambda)}{\Phi_{\lambda,av}} = \int_{\lambda_1+i\Delta\lambda}^{\lambda_1+(i+1)\Delta\lambda} \varphi_i(\lambda) d\lambda$$

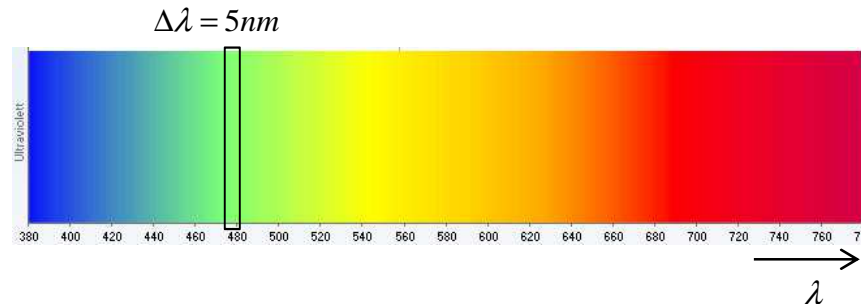


Table 1
Spectral values of display primaries

λ	$\bar{r}_i(\lambda)$	$\bar{g}_i(\lambda)$	$\bar{b}_i(\lambda)$	λ	$\bar{r}_i(\lambda)$	$\bar{g}_i(\lambda)$	$\bar{b}_i(\lambda)$
380	0.005	-0.005	0.036	580	6.246	3.671	-0.696
385	0.007	-0.008	0.059	585	7.287	2.876	-0.612
390	0.012	-0.015	0.112	590	8.184	2.099	-0.527
395	0.023	-0.027	0.202	595	8.888	1.379	-0.444
400	0.043	-0.051	0.379	600	9.304	0.761	-0.368
405	0.069	-0.082	0.614	605	9.455	0.246	-0.299
410	0.129	-0.154	1.156	610	9.288	-0.139	-0.241
415	0.227	-0.275	2.070	615	8.855	-0.404	-0.191
420	0.388	-0.473	3.598	620	8.175	-0.559	-0.150
425	0.601	-0.747	5.790	625	7.267	-0.622	-0.115
430	0.761	-0.966	7.718	630	6.264	-0.619	-0.088
435	0.825	-1.083	9.035	635	5.320	-0.583	-0.066
440	0.798	-1.095	9.718	640	4.421	-0.522	-0.050
445	0.692	-1.023	9.906	645	3.576	-0.446	-0.037
450	0.524	-0.893	9.834	650	2.819	-0.365	-0.027
455	0.312	-0.722	9.662	655	2.180	-0.291	-0.020
460	0.051	-0.493	9.225	660	1.646	-0.224	-0.014
465	-0.239	-0.204	8.418	665	1.211	-0.167	-0.010
470	-0.552	0.172	7.054	670	0.874	-0.122	-0.007
475	-0.852	0.576	5.656	675	0.637	-0.089	-0.005
480	-1.127	0.996	4.348	680	0.469	-0.066	-0.004
485	-1.377	1.416	3.215	685	0.330	-0.047	-0.003
490	-1.619	1.868	2.328	690	0.228	-0.033	-0.002
495	-1.894	2.396	1.646	695	0.159	-0.023	-0.001
500	-2.212	3.022	1.119	700	0.114	-0.017	-0.001
505	-2.591	3.802	0.690	705	0.081	-0.012	-0.001
510	-2.930	4.644	0.282	710	0.058	-0.008	0.000
515	-3.180	5.514	-0.092	715	0.041	-0.006	0.000
520	-3.263	6.286	-0.384	720	0.029	-0.005	0.000
525	-3.122	6.830	-0.581	725	0.020	-0.003	0.000
530	-2.799	7.196	-0.726	730	0.014	-0.002	0.000
535	-2.342	7.396	-0.835	735	0.010	-0.001	0.000
540	-1.761	7.446	-0.911	740	0.007	-0.001	0.000
545	-1.060	7.357	-0.956	745	0.005	0.000	0.000
550	-0.246	7.139	-0.973	750	0.003	0.000	0.000
555	0.675	6.811	-0.969	755	0.002	0.000	0.000
560	1.696	6.368	-0.944	760	0.002	0.000	0.000
565	2.795	5.815	-0.902	765	0.001	0.000	0.000
570	3.946	5.168	-0.844	770	0.001	0.000	0.000
575	5.112	4.445	-0.775	775	0.000	0.000	0.000
				Sum	100	100	100

1.4 Colorimetry

- Now related to the three primaries R,G,B:
 - Note $r(\lambda)$, $g(\lambda)$, $b(\lambda)$ are not known and should be determined!
 - And

$$R = R_I - R_{II}$$

$$G = G_I - G_{II}$$

$$B = B_I - B_{II}$$

- For the final primary valences:

$$R = \int_{\lambda_1}^{\lambda_2} \varphi(\lambda) \cdot r(\lambda) \cdot d\lambda = \int_{\lambda_1}^{\lambda_1 + \Delta\lambda} \varphi(\lambda) \cdot r(\lambda) \cdot d\lambda = \varphi(\lambda_1) \cdot \underbrace{r(\lambda_1) \cdot \Delta\lambda}_{\bar{r}(\lambda_1)} \Rightarrow R = \varphi(\lambda_1) \cdot \bar{r}(\lambda_1)$$

$$\Rightarrow \bar{r}(\lambda_1) = \frac{R}{\varphi(\lambda_1)}$$

$$G = \int_{\lambda_1}^{\lambda_2} \varphi(\lambda) \cdot g(\lambda) \cdot d\lambda = \int_{\lambda_1}^{\lambda_1 + \Delta\lambda} \varphi(\lambda) \cdot g(\lambda) \cdot d\lambda = \varphi(\lambda_1) \cdot \underbrace{g(\lambda_1) \cdot \Delta\lambda}_{\bar{g}(\lambda_1)} \Rightarrow G = \varphi(\lambda_1) \cdot \bar{g}(\lambda_1)$$

$$\Rightarrow \bar{g}(\lambda_1) = \frac{G}{\varphi(\lambda_1)}$$

$$B = \int_{\lambda_1}^{\lambda_2} \varphi(\lambda) \cdot b(\lambda) \cdot d\lambda = \int_{\lambda_1}^{\lambda_1 + \Delta\lambda} \varphi(\lambda) \cdot b(\lambda) \cdot d\lambda = \varphi(\lambda_1) \cdot \underbrace{b(\lambda_1) \cdot \Delta\lambda}_{\bar{b}(\lambda_1)} \Rightarrow B = \varphi(\lambda_1) \cdot \bar{b}(\lambda_1)$$

$$\Rightarrow \bar{b}(\lambda_1) = \frac{B}{\varphi(\lambda_1)}$$

λ	$\bar{r}_I(\lambda)$	$\bar{g}_I(\lambda)$	$\bar{b}_I(\lambda)$	λ	$\bar{r}_I(\lambda)$	$\bar{g}_I(\lambda)$	$\bar{b}_I(\lambda)$
380	0.005	-0.005	0.036	580	6.246	3.671	-0.696
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495	-1.894	2.396	1.646	695	0.159	-0.023	-0.001
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515	-3.180	5.514	-0.092	715	0.041	-0.006	0.000
520	-3.263	6.286	-0.384	720	0.029	-0.005	0.000
525	-3.122	6.830	-0.581	725	0.020	-0.003	0.000
530	-2.799	7.196	-0.726	730	0.014	-0.002	0.000
535	-2.342	7.396	-0.835	735	0.010	-0.001	0.000
540	-1.761	7.446	-0.911	740	0.007	-0.001	0.000
545	-1.060	7.357	-0.956	745	0.005	0.000	0.000
550	-0.246	7.139	-0.973	750	0.003	0.000	0.000
555	0.675	6.811	-0.969	755	0.002	0.000	0.000
560	1.696	6.368	-0.944	760	0.002	0.000	0.000
565	2.795	5.815	-0.902	765	0.001	0.000	0.000
570	3.946	5.168	-0.844	770	0.001	0.000	0.000
575	5.112	4.445	-0.775	775	0.000	0.000	0.000
				Sum	100	100	100

1.4 Colorimetry

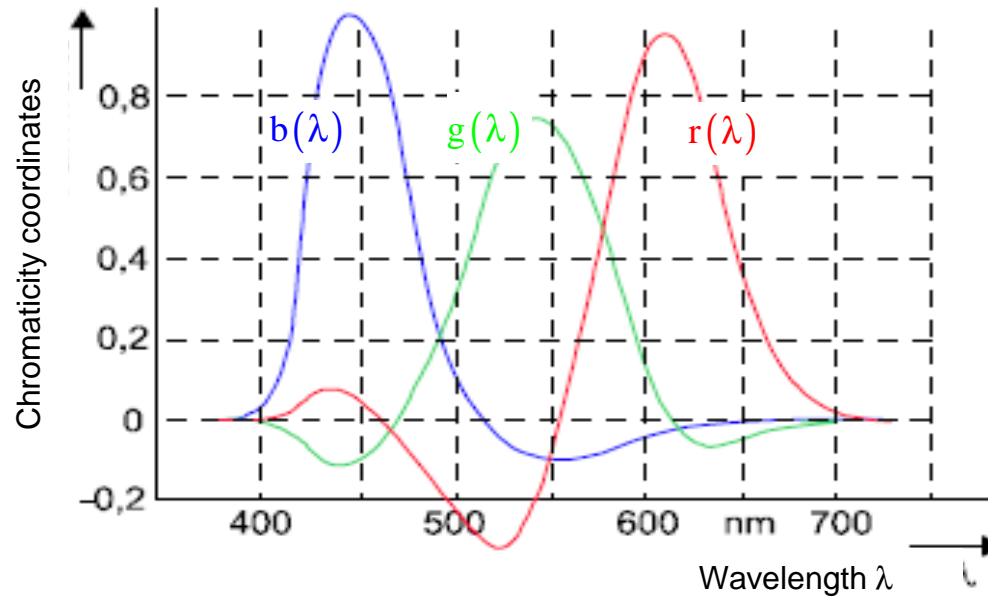
- For the equal-energy intervals $\Delta\lambda=5\text{nm}$ follows for the white point:

$$\sum_i \bar{r}_i(\lambda) = \sum_i \bar{g}_i(\lambda) = \sum_i \bar{b}_i(\lambda) = 100$$

$$\begin{aligned} \bar{r}_i &= R_I - R_{II} \quad \text{in range of } \lambda_1 \dots \lambda_2 \text{ mit } \lambda_i + i \cdot \Delta\lambda; & \Delta\lambda &= 5\text{nm} \\ \bar{g}_i &= G_I - G_{II} \\ \bar{b}_i &= B_I - B_{II} \end{aligned}$$

- R,G,B with the color stimulus function (*Farbreizfunktion*) $\varphi(\lambda)$ we get for the three color valences :

$$\begin{aligned} R &= \int_{\lambda_1}^{\lambda_2} \varphi(\lambda) \cdot r(\lambda) \cdot d\lambda \\ G &= \int_{\lambda_1}^{\lambda_2} \varphi(\lambda) \cdot g(\lambda) \cdot d\lambda \\ B &= \int_{\lambda_1}^{\lambda_2} \varphi(\lambda) \cdot b(\lambda) \cdot d\lambda \end{aligned}$$



- 1.4.5.3 Grassmann laws

- Due to definition of color values for monochromatic light it's possible now to determine for each color stimulus function (*Farbreizfunktion*) $\varphi(\lambda)$ the color valence **F** by three color values.
- Precondition is that every color of color stimulus function $\varphi(\lambda)$ can **additively** be mixed out of three suitable primary valences.

1. For the result of an additive mixture only the appearance but not the spectral composition is important.
2. Color mixture is linear.
3. For description of a color always three parts are necessary and sufficient.
(e.g. R,G,B or brightness, hue and saturation)

- **1.4.6 Spectrum locus** (*Spektralfarbenzug*)
- Chromaticity can be described by its chromaticity coordinates r, g, b , as already explained. Only two chromaticity coordinates are necessary:

$$r = \frac{R}{R+G+B}; \quad g = \frac{G}{R+G+B} \quad b = \frac{B}{R+G+B}$$

$$r + g + b = 1$$

- Starting with a **isosceles triangle** (*gleichschenkligen Dreieck*), where the edges represent the three primary colors (R), (G), (B)
- Accordingly the white point, or better the achromatic point, is in the middle:

$$R = G = B = 1$$

- For this achromatic point we get the following chromaticity coordinates:

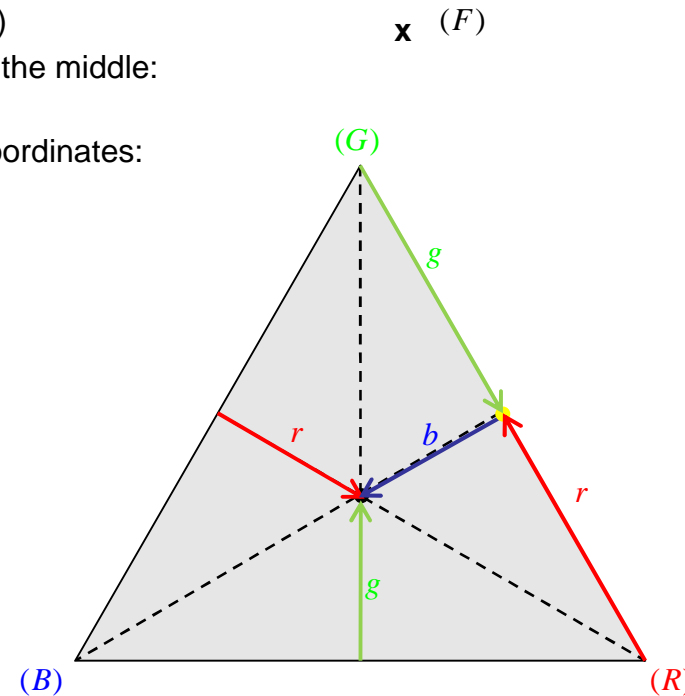
$$r = \frac{1}{3}; g = \frac{1}{3}; b = \frac{1}{3}$$

- Green for instance:

$$G = 1; R = B = 0 \Rightarrow r = 0; g = 1; b = 0$$

- Yellow for instance:

$$R = G = 1 \Rightarrow r = \frac{1}{2}; g = \frac{1}{2}; b = 0$$



1.4 Colorimetry

- For a color valence **F** with $\lambda=540\text{nm}$ we can see from table1:

$$\bar{r}_i(540\text{nm}) = -1,761; \quad \bar{g}_i(540\text{nm}) = 7,446; \quad \bar{b}_i(540\text{nm}) = -0,911$$

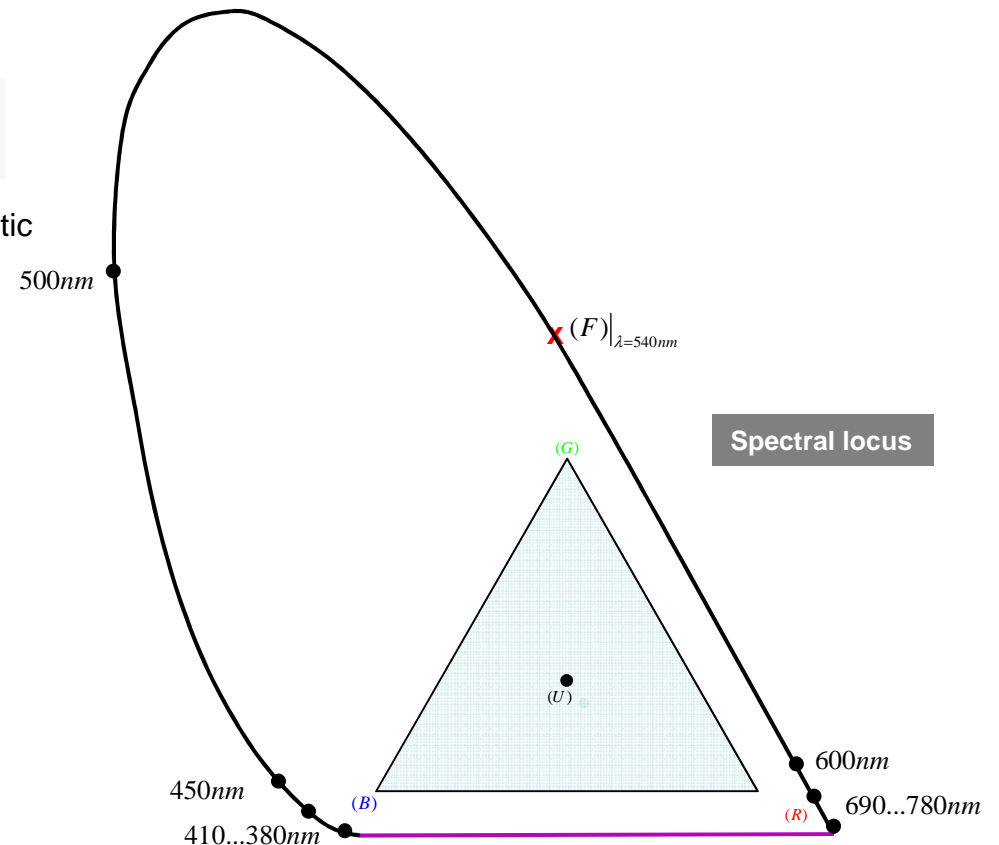
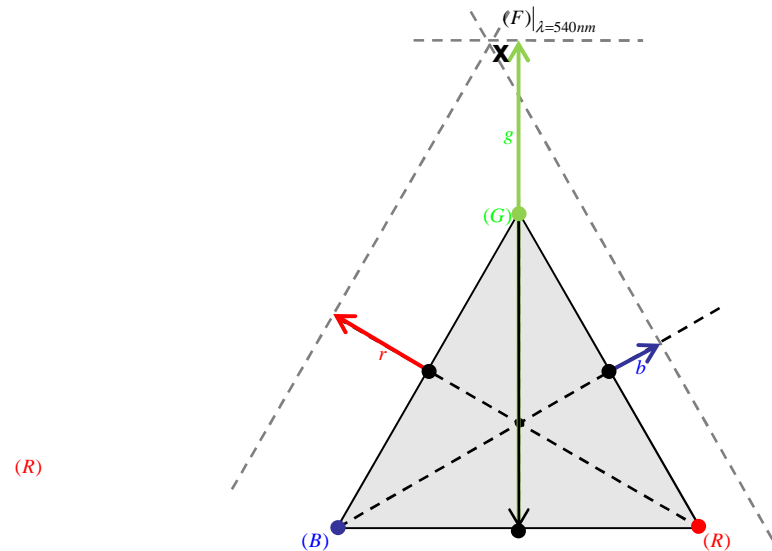
$$R = \bar{r}_i(540\text{nm}) = \int_{540\text{nm}-\Delta\lambda/2}^{540\text{nm}+\Delta\lambda/2} \varphi(\lambda) \cdot \bar{r}(\lambda) d\lambda = \varphi(\lambda) \cdot \bar{r}(540\text{nm}) = -1,761$$

λ	$\bar{r}_i(\lambda)$	$\bar{g}_i(\lambda)$	$\bar{b}_i(\lambda)$
530	-1.799	7.196	-0.726
535	-1.342	7.396	-0.835
540	-1.761	7.446	-0.911
545	-1.060	7.357	-0.956
550	-0.246	7.139	-0.973

- For the chromaticity coordinates:

$$r = \frac{R}{R+G+B} = \frac{-1,761}{4,774} = -0,369; \quad g = 1,560; \quad b = -0,191$$

- Accordingly the color values of all the other monochromatic colors are determined and marked in á diagram:
→ We get the spectral locus curve

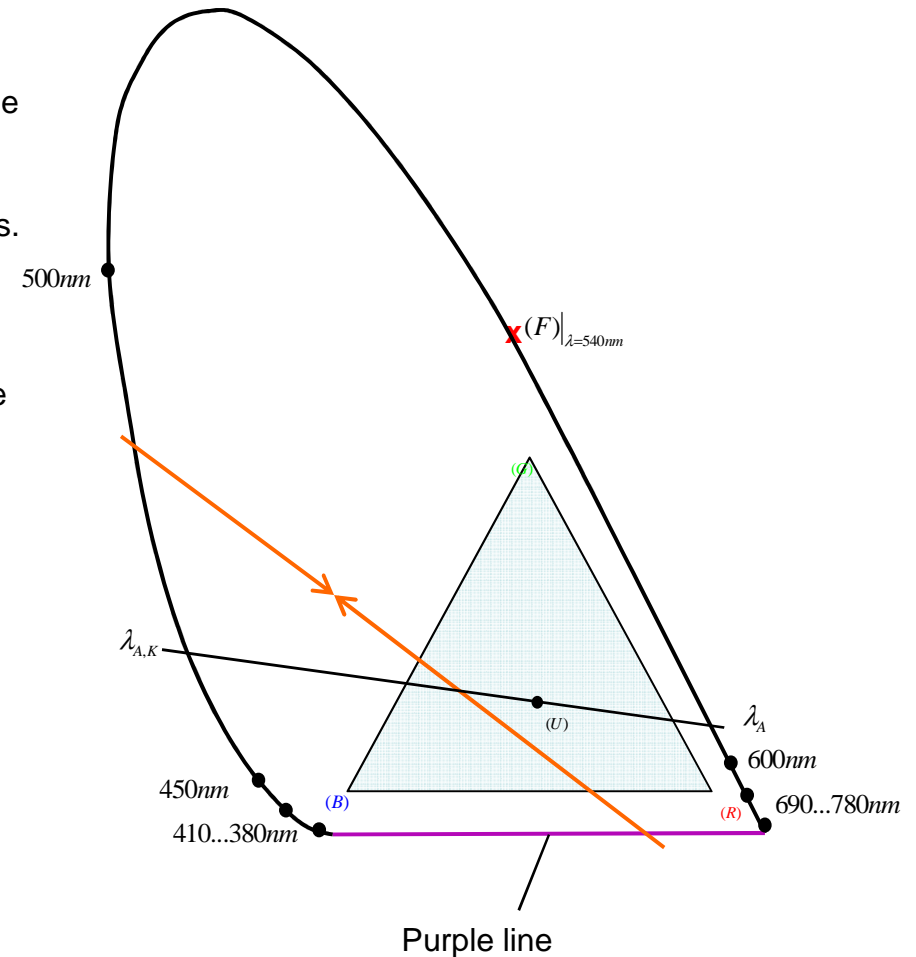


1.4 Colorimetry

- The spectral locus curve describes the colorimetric locus (*Farborte*) corresponding to monochromatic light.
 - Monochromatic colors are pure colors with maximum possible color saturation (e.g. laser light).
- All physically realizable colors are inside the color locus, by means that all colors can be mixed out of monochromatic colors.
- The purple line (*Purpurgerade*) connects the area of the monochromatic colors red and blue.
- As already indicated by the name that purple colors are no pure monochromatic colors but are mixtures out of red and blue.
- When two color components are mixed together the resulting color valence is located on the connecting line of these two.
 - In the range between 540 ... 690nm (but only in this range!) monochromatic colors can be mixed out of two other monochromatic colors.
 - The display primaries (R), (G), (B) form a isosceles triangle with the white point:

$$R_w = G_w = B_w = 1 \Rightarrow r = \frac{1}{3}; g = \frac{1}{3}; b = \frac{1}{3}$$

- A display or monitor can only reproduces colors which are located inside the triangle.



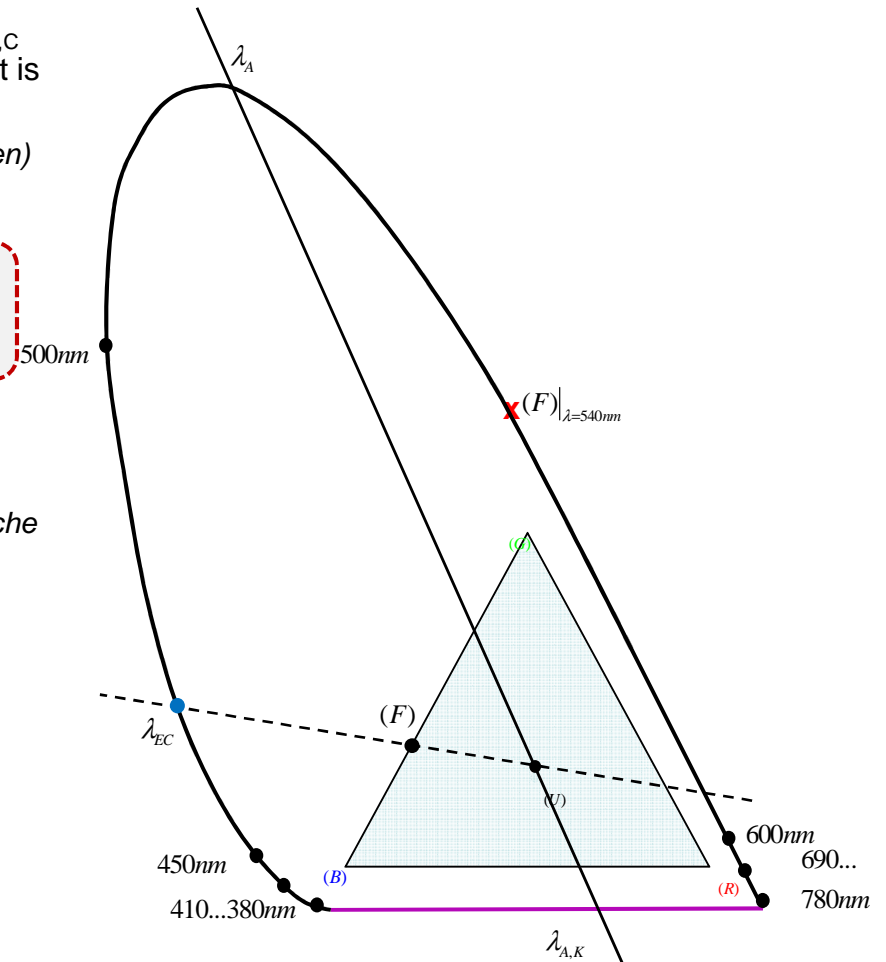
1.4 Colorimetry

- For a given color with wavelength λ_A the compensation color $\lambda_{A,C}$ is achieved by extension through the white point. The white point is obtained by mixture between color λ_A and color $\lambda_{A,C}$.
 - The compensation color of green prismatic colors (*Spektralfarben*) are located on the purple line.

Def: Compensative colores

Two colors where the connecting line goes through the white point are compensative to each other.

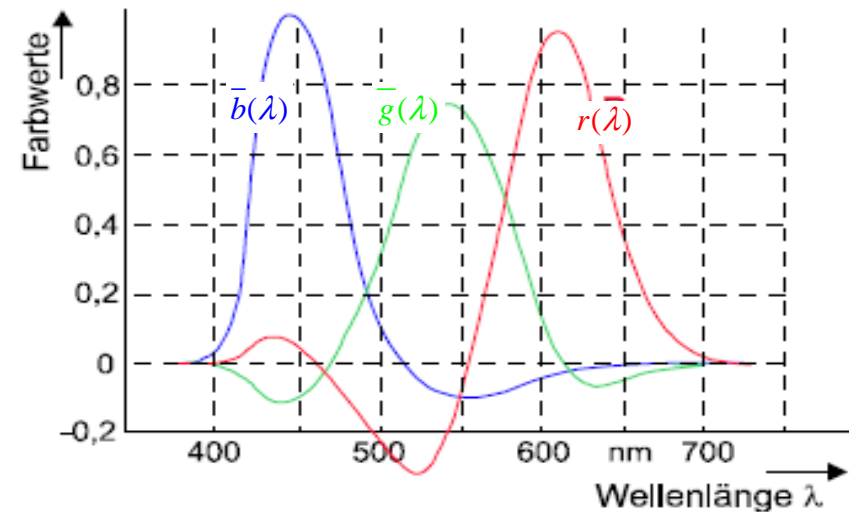
- The rear warded extension on the connecting line between color valence (F) and white point intersect the spectral locus at wavelength λ_{EC} . We talk about dominant wavelengths (*farbtongleiche Wellenlänge*).
 - For color valences on the purple line there are no dominant wavelengths. Here we indicate the compensative wavelength.



- For generation of the red color value for example, the camera would have to emulate exactly the filter curve $\bar{r}(\lambda)$.

$$R = \int_{380}^{760} \varphi(\lambda) \cdot \bar{r}(\lambda) \cdot d\lambda, \quad G = \int_{380}^{760} \varphi(\lambda) \cdot \bar{g}(\lambda) \cdot d\lambda, \quad B = \int_{380}^{760} \varphi(\lambda) \cdot \bar{b}(\lambda) \cdot d\lambda,$$

- This would mean that for at in some ranges (in case of red 460...550nm) the filter would have to generate negative values.
- Since no photo-electric cell is able to generate negative signals in certain frequency ranges and positive ones for other frequencies, such a filter function can not be realized directly.



- This problem is solved by matrixing (*Matrizierung*) in a way that only positive spectral value curves remain:

$$\begin{pmatrix} \bar{r}_0(\lambda) \\ \bar{g}_0(\lambda) \\ \bar{b}_0(\lambda) \end{pmatrix} = \begin{pmatrix} 0,36 & 0,60 & 0,04 \\ 0,13 & 0,77 & 0,10 \\ 0,03 & 0,12 & 0,85 \end{pmatrix} \cdot \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} \quad R_0 = \int_{380}^{760} \varphi(\lambda) \cdot \bar{r}_0(\lambda) \cdot d\lambda, \quad G_0 = \int_{380}^{760} \varphi(\lambda) \cdot \bar{g}_0(\lambda) \cdot d\lambda, \quad B_0 = \int_{380}^{760} \varphi(\lambda) \cdot \bar{b}_0(\lambda) \cdot d\lambda,$$

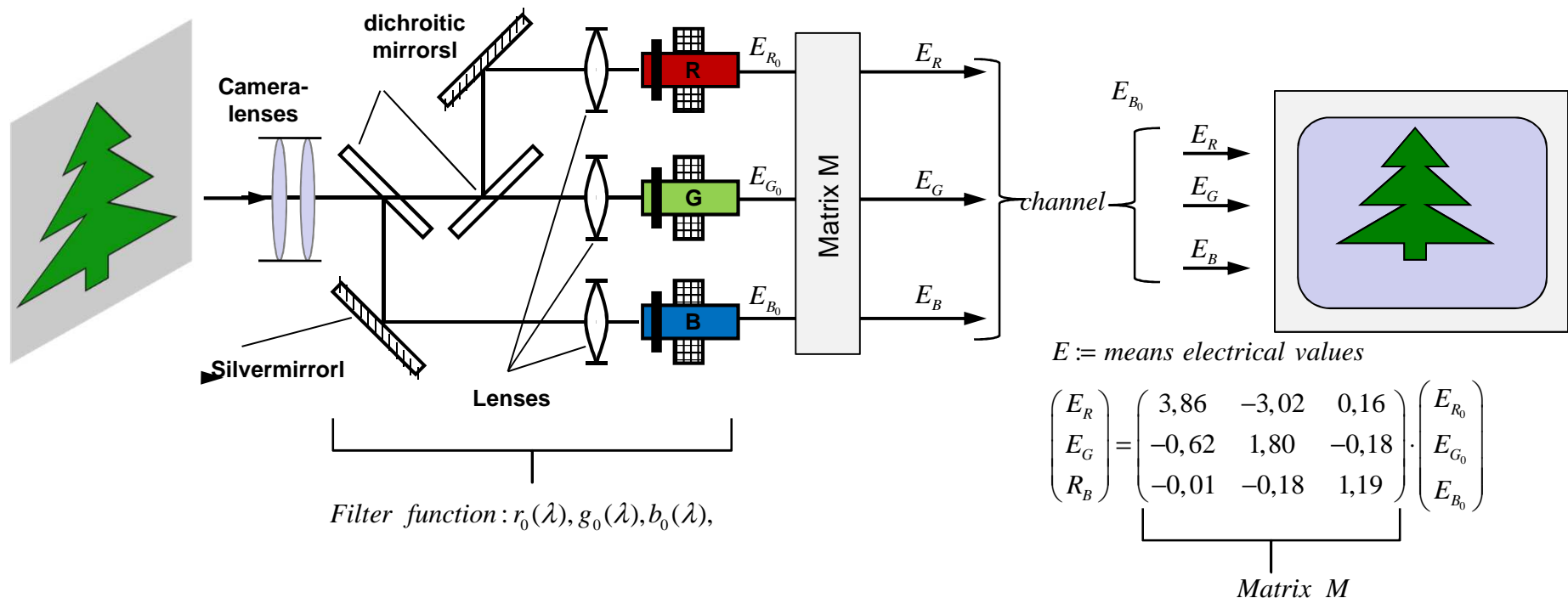
- By the inverse matrix the values $r(\lambda)$, $g(\lambda)$, $b(\lambda)$ can be calculated out of $r_0(\lambda)$, $g_0(\lambda)$, $b_0(\lambda)$:

$$\begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} = \begin{pmatrix} 3,86 & -3,02 & 0,16 \\ -0,62 & 1,80 & -0,18 \\ -0,01 & -0,18 & 1,19 \end{pmatrix} \cdot \begin{pmatrix} \bar{r}_0(\lambda) \\ \bar{g}_0(\lambda) \\ \bar{b}_0(\lambda) \end{pmatrix}$$

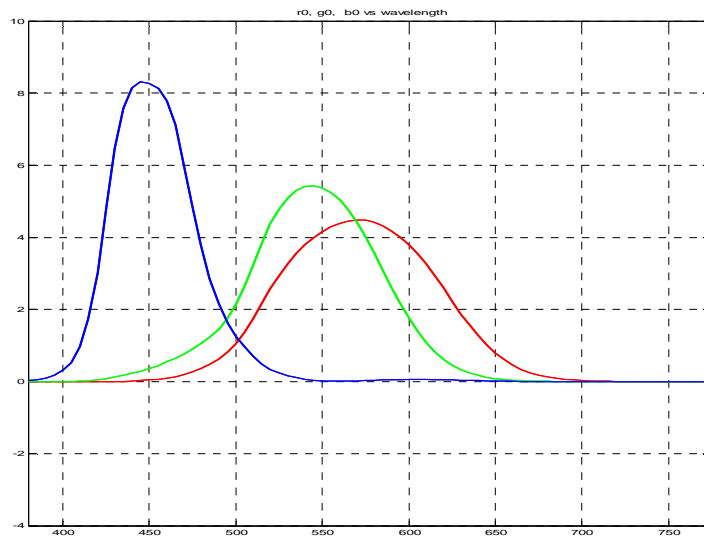
1.4.7 Primary valence systems (Primärvalenzsysteme)

1.4.7.1 Principle of video transmission scheme

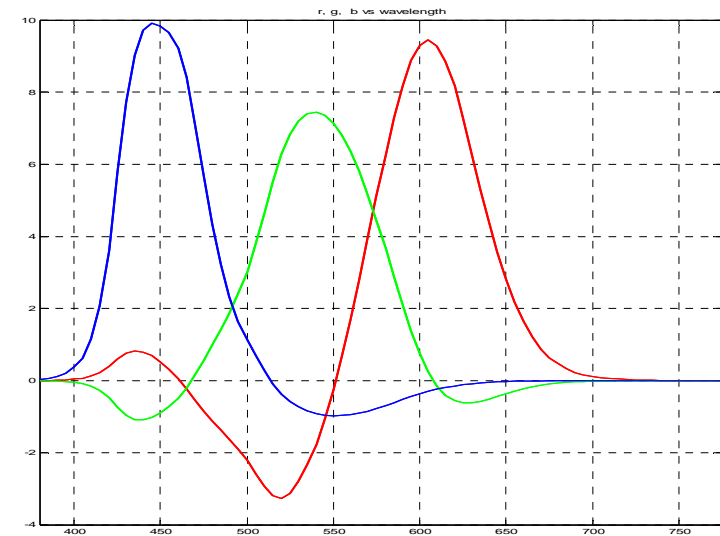
- As already explained in the previous chapter, not the original color stimulus function $\varphi(\lambda)$ (Farbreizfunktion) is transmitted to the receiver. It's sufficient that for every pixel point three color valences are determined and transmitted.
- At the receiver side, the same color valence is reconstructed again by additive color mixture out of the three color values R,G,B
- Due to the metamerism the human eye has exactly the same color impression as with the original scene.



1.4 Colorimetry



Filter curves in camera



Calculated filter curves behind Matrix

$$\begin{pmatrix} E_R \\ E_G \\ E_B \end{pmatrix} = \begin{pmatrix} 3,86 & -3,02 & 0,16 \\ -0,62 & 1,80 & -0,18 \\ -0,01 & -0,18 & 1,19 \end{pmatrix} \cdot \begin{pmatrix} E_{R_0} \\ E_{G_0} \\ E_{B_0} \end{pmatrix}$$

- **1.4.8 Standardized valence system** (*Normvalenzsystem*)

- **1.4.8.1 Definition**

- The primary valences of different displays are by far not identical (different phosphors, LCD, Plasma, OLED...)
- Question is how color values can be described in a standardized way.
- In 1931 the so called **CIE standardized valence system** with primaries **(X), (Y), (Z)** was defined.
- The corresponding curves $x(\lambda)$, $y(\lambda)$, $z(\lambda)$ are the standardized spectral value curves (*Normspektralwertkurven*).

The **standardized valence system** defines virtual primary valences, which can physically not be realized.

- Criteria:
 - All feasible colors (inside of the spectrum locus (*Spektralfarbenzug*)) should only deliver positive chromaticity coordinates.
 - The primary valences (X), (Y), (Z) must be located outside of the spectrum locus
 - The standardized spectral value curve (*Spektralwertkurve*) $y(\lambda)$ should correspond to the spectral brightness sensitivity curve of the human eye $V(\lambda)$.
 - This way the value Y directly delivers to the luminance value.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0,514 & 0,324 & 0,162 \\ 0,265 & 0,670 & 0,065 \\ 0,024 & 0,123 & 0,853 \end{pmatrix} \cdot \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

- For the reverse transformation.

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 2,565 & -1,167 & -0,398 \\ -1,022 & 1,978 & 0,044 \\ 0,075 & -0,252 & 1,177 \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

1.4 Colorimetry

- For achromatic (unbunt):

$$X = Y = Z = 1$$

- For representing all feasible colors (everything within the spectrum locus) there are no negative values for X, Y or Z.
- Sum of the spectral values of all primaries is 100.

$$\sum_i \bar{r}_i = \sum_i \bar{g}_i = \sum_i \bar{b}_i = 100$$

- The standardized spectral value curve (Normspektralwertkurve) $y(\lambda)$ should correspond to $V(\lambda)$ as already mentioned:

$$Y = \begin{pmatrix} 0,265 & 0,670 & 0,065 \end{pmatrix} \cdot \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

λ	$\bar{r}_i(\lambda)$	$\bar{g}_i(\lambda)$	$\bar{b}_i(\lambda)$
550	-0.246	7.139	-0.973
555	0.675	6.811	-0.969
560	1.696	6.368	-0.944

$$V(\lambda) = 1|_{\lambda=555nm} \Rightarrow$$

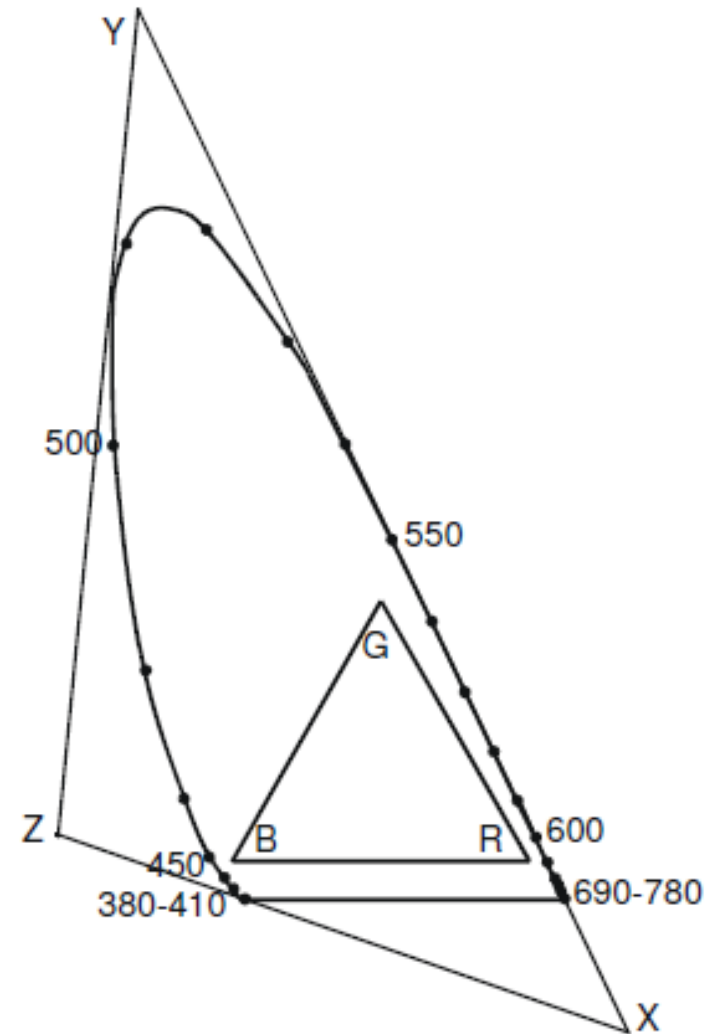
$$\bar{y}_i(555nm) = 1 = k \cdot \begin{pmatrix} 0,265 & 0,675 & 0,065 \end{pmatrix} \cdot \begin{pmatrix} 0,675 \\ 6,811 \\ -0,969 \end{pmatrix} = k \cdot 4,67926$$

$$\Rightarrow k = \frac{1}{4,67926} = 0,213714$$

- We get:

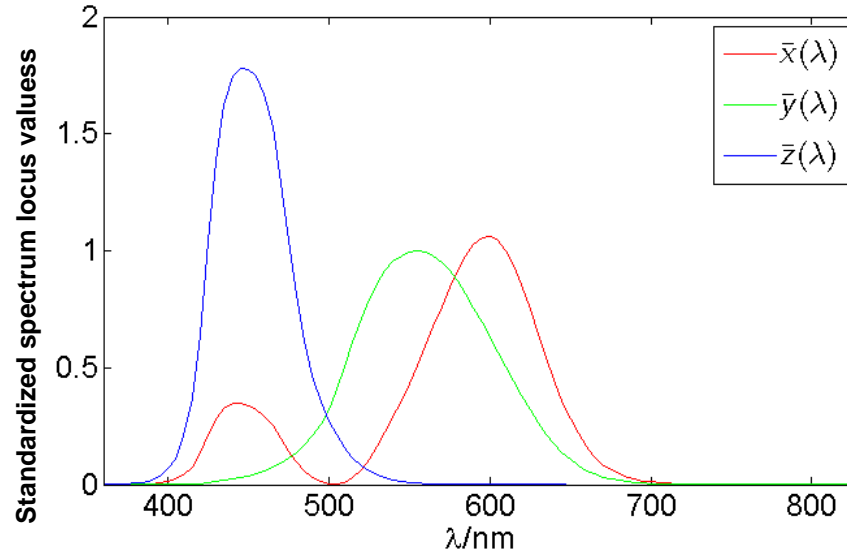
$$\sum_i \bar{y}_i = k \cdot \underbrace{\left(0,265 \cdot \sum_i \bar{r}_i + 0,675 \cdot \sum_i \bar{g}_i + 0,065 \cdot \sum_i \bar{b}_i \right)}_{100} = k \cdot 100$$

$$\sum_i \bar{y}_i = \sum_i \bar{x}_i = \sum_i \bar{z}_i = 21,3714$$



1.4 Colorimetry

1.4.8.2 Standardized spectrum locus curves (Normspektralwertkurven)



- The standardized spectrum locus curves in the figure above are always positive.
- The standardized spectrum locus curve $y(\lambda)$ corresponds to the brightness sensitivity of the human eye.
- Thus the color value Y contains the complete luminance information.
- The values (X) and (Z) do not contain any brightness information.

λ	$\bar{x}_i(\lambda)$	$\bar{y}_i(\lambda)$	$\bar{z}_i(\lambda)$	λ	$\bar{x}_i(\lambda)$	$\bar{y}_i(\lambda)$	$\bar{z}_i(\lambda)$
380	0,0014	0,0000	0,0065	580	0,9163	0,8700	0,0017
385	0,0022	0,0001	0,0105	585	0,9786	0,8163	0,0014
390	0,0042	0,0001	0,0201	590	1,0263	0,7570	0,011
395	0,0076	0,0002	0,0362	595	1,0567	0,6949	0,0010
400	0,0143	0,0004	0,0679	600	1,0622	0,6310	0,0008
405	0,0232	0,0006	0,1102	605	1,0456	0,5668	0,0006
410	0,0435	0,0012	0,2074	610	1,0026	0,5030	0,0003
415	0,0778	0,0022	0,3713	615	0,9384	0,4412	0,0002
420	0,1344	0,0040	0,6456	620	0,8544	0,3810	0,0002
425	0,2148	0,0073	1,0391	625	0,7514	0,3210	0,0001
430	0,2839	0,0116	1,3856	630	0,6424	0,2650	0,0000
435	0,3285	0,0168	1,6230	635	0,5419	0,2170	0,0000
440	0,3483	0,0230	1,7471	640	0,4479	0,1750	0,0000
445	0,3481	0,0298	1,7826	645	0,3608	0,1382	0,0000
450	0,3362	0,0380	1,7721	650	0,2835	0,1070	0,0000
455	0,3187	0,0480	1,7441	655	0,2187	0,0816	0,0000
460	0,2908	0,0600	1,6692	660	0,1649	0,0610	0,0000
465	0,2511	0,0739	1,5281	665	0,1212	0,0446	0,0000
470	0,1954	0,0910	1,2876	670	0,0874	0,0320	0,0000
475	0,1421	0,1126	1,0419	675	0,0636	0,0232	0,0000
480	0,0956	0,1390	0,8130	680	0,0468	0,0170	0,0000
485	0,0580	0,1693	0,6162	685	0,0329	0,0119	0,0000
490	0,0320	0,2080	0,4652	690	0,0227	0,0082	0,0000
495	0,0147	0,2586	0,3533	695	0,0158	0,0057	0,0000
500	0,0049	0,3230	0,2720	700	0,0114	0,0041	0,0000
505	0,0024	0,4073	0,2123	705	0,0081	0,0029	0,0000
510	0,0093	0,5030	0,1582	710	0,0058	0,0021	0,0000
515	0,0291	0,6082	0,1117	715	0,0041	0,0015	0,0000
520	0,0633	0,7100	0,0782	720	0,0029	0,0010	0,0000
525	0,1096	0,7932	0,0573	725	0,0020	0,0007	0,0000
530	0,1655	0,8620	0,0422	730	0,0014	0,0005	0,0000
535	0,2257	0,9149	0,0298	735	0,0010	0,0004	0,0000
540	0,2904	0,9540	0,0203	740	0,0007	0,0002	0,0000
545	0,3597	0,9803	0,0134	745	0,0005	0,0002	0,0000
550	0,4334	0,9950	0,0087	750	0,0003	0,0001	0,0000
555	0,5121	1,0000	0,0057	755	0,0002	0,0001	0,0000
560	0,5945	0,9950	0,0039	760	0,0002	0,0001	0,0000
565	0,6784	0,9786	0,0027	765	0,0001	0,0000	0,0000
570	0,7621	0,9520	0,0021	770	0,0001	0,0000	0,0000
575	0,8425	0,9154	0,0018	775	0,0001	0,0000	0,0000
				Sum	21,3714	21,3714	21,3714

• 1.4.8.3 Standardized chromaticity diagram (Normfarbtafel)

- For the standardized chromaticity coordinates:

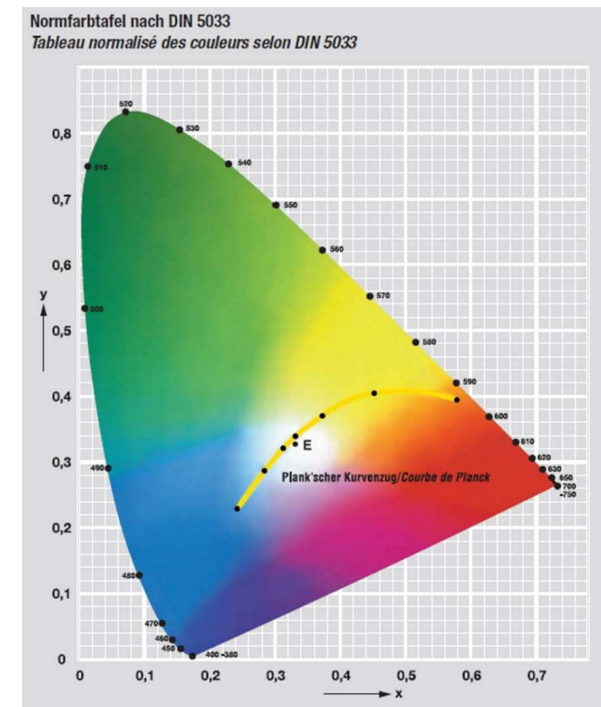
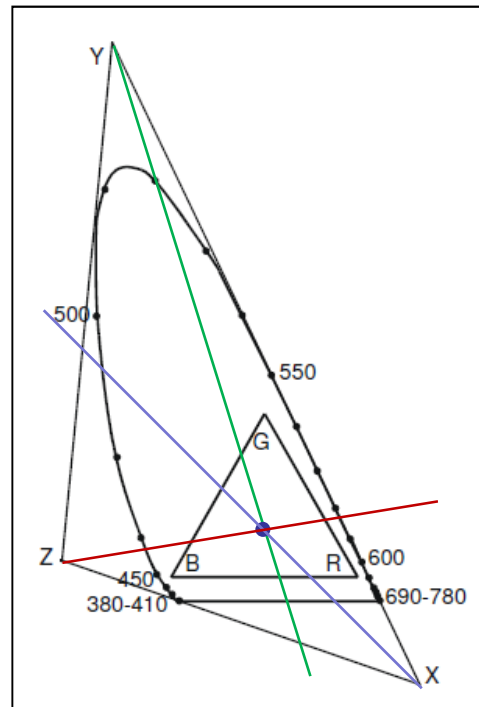
$$x = \frac{X}{X+Y+Z}; \quad y = \frac{Y}{X+Y+Z} \quad z = \frac{Z}{X+Y+Z}$$

$$x + y + z = 1$$

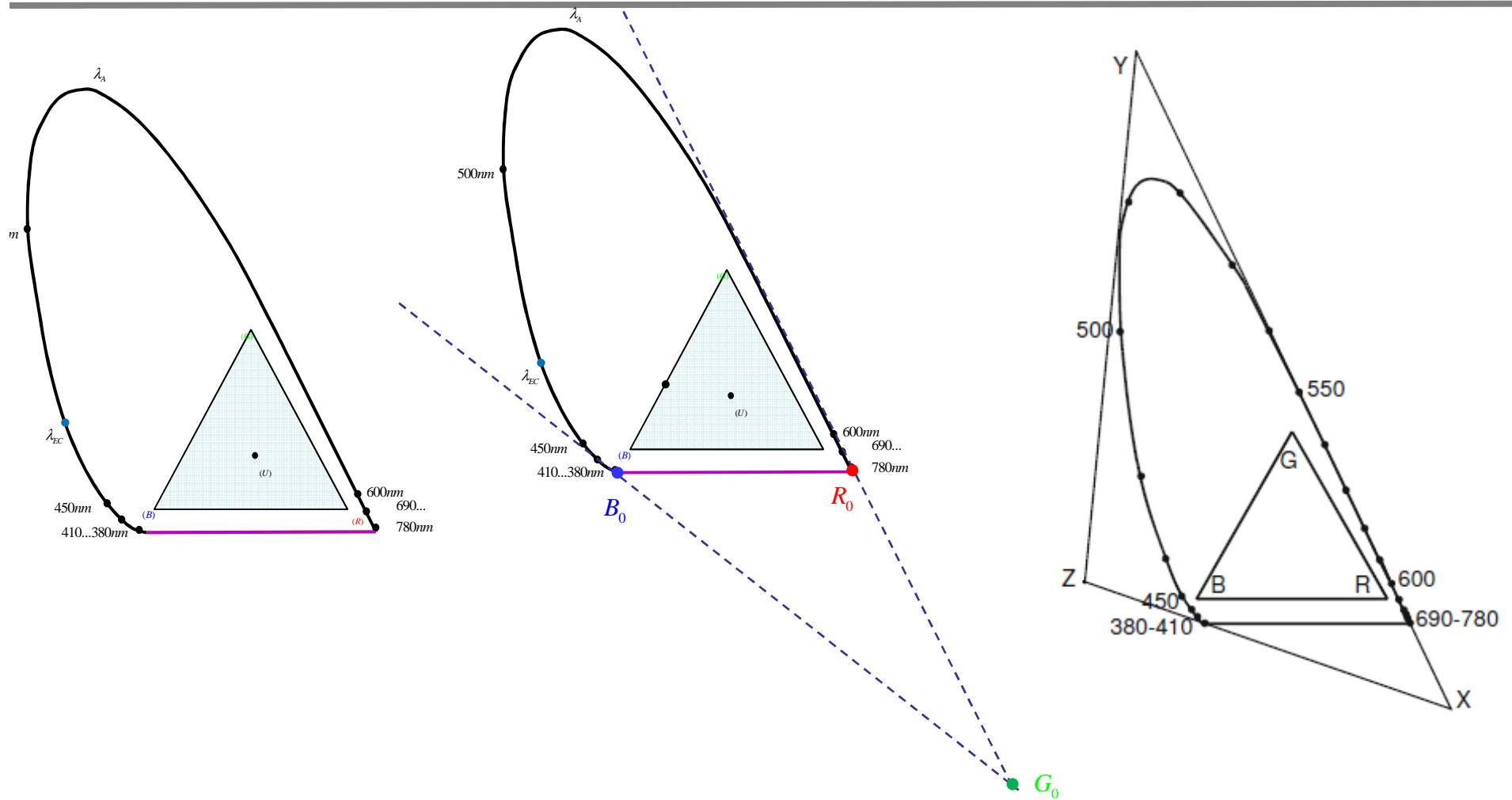
- Now the representation of the standardized spectrum locus curve is replaced by a rectangular coordinate system.
 - Therefore all color valences (F) together with their standardized chromaticity coordinates x and y are transferred into the new rectangular coordinate system
- The white point is located at:

$$x_U = y_U = 0,33$$

- A color valence is completely described by
 - Standardized color values X, Y, Z or
 - Standardized color value Y and chromaticity coordinates x and y.



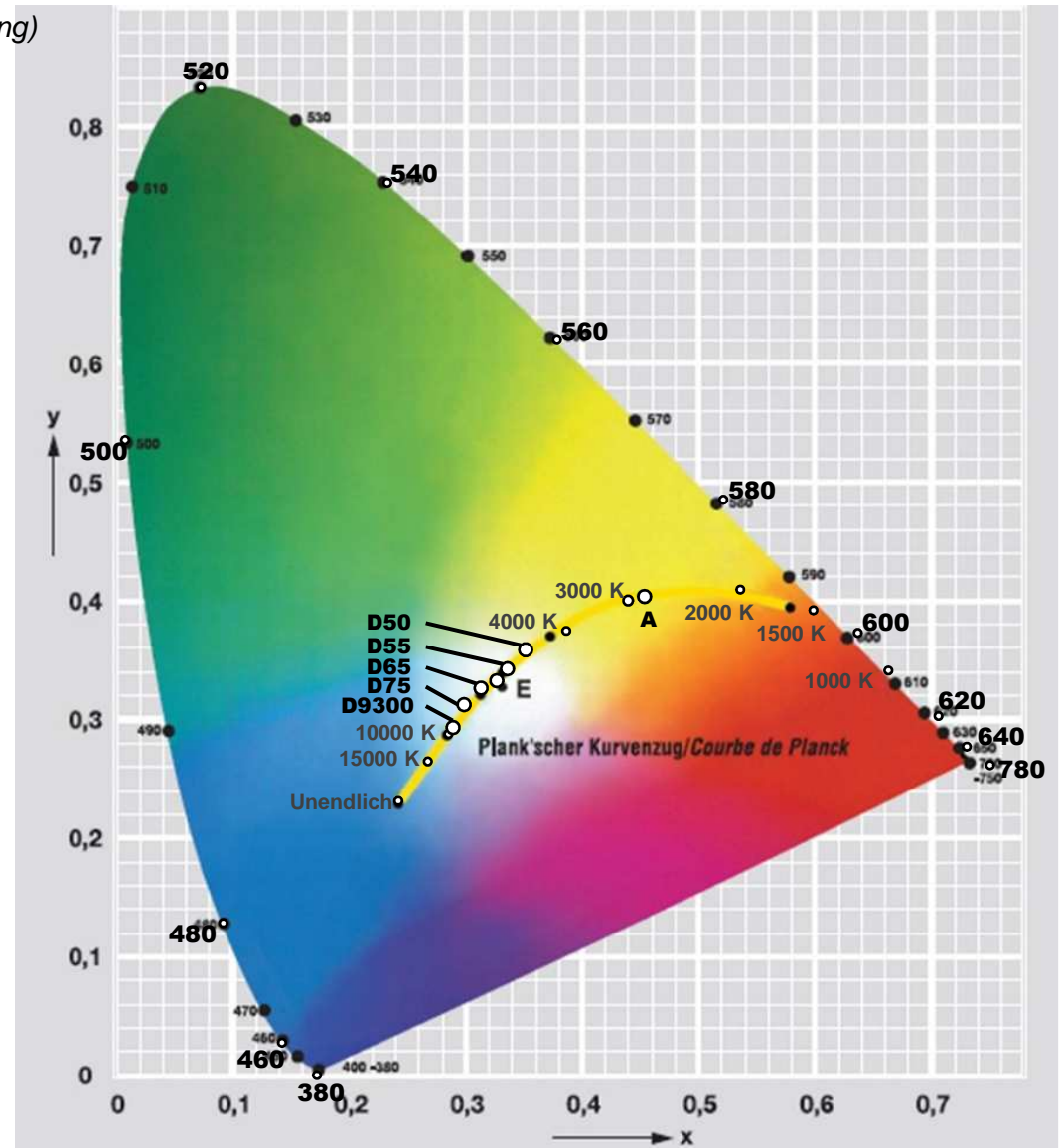
1.4 Colorimetry



1.4 Colorimetry

- 1.4.8.4 CIE Standard illumination (Normbeleuchtung)

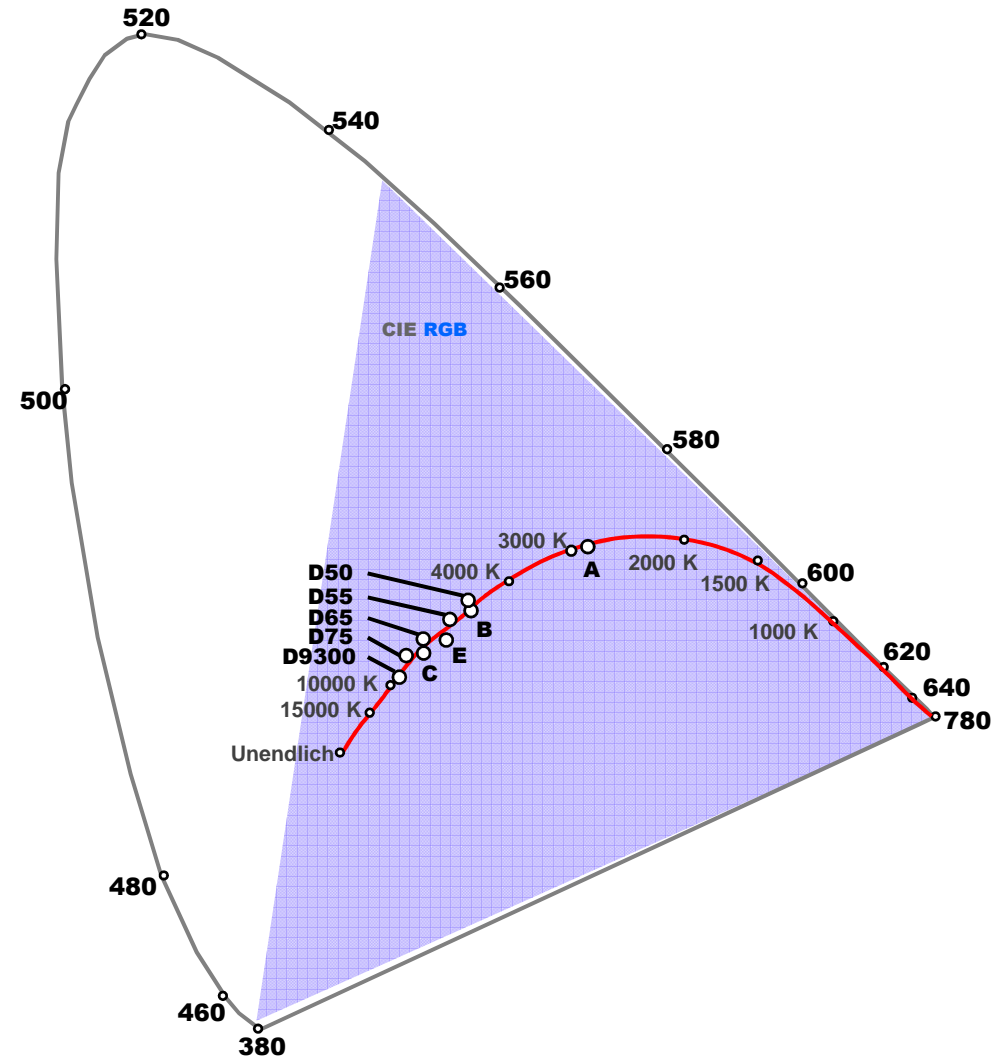
CIE standard illumination	x	y	
CIE standard illumination A	0,4476	0,4074	Standardized incandescent lamp (Glühlampe)
CIE standard illumination B	0,3484	0,3516	Old standard for daylight
CIE standard illumination C	0,3101	0,3162	Whitepoint NTSC - TV
CIE standard illumination E	0,3333	0,3333	Whitepoint X=Y=Z
D50=5000K	0,3457	0,3585	Whitepoint Wide Gammut RGB
D55=5500K	0,3324	0,3474	Daylight similar radiation of a black body (schwarzer Strahler) at 5500K
D65=6500K	0,3127	0,3290	Corresp. To average daylight Whitepoint of PAL/Secam, sRGB and Adobe-RGB
D75=7500K	0,2990	0,3149	Black body at 7500K



- 1.4.9 Color spaces

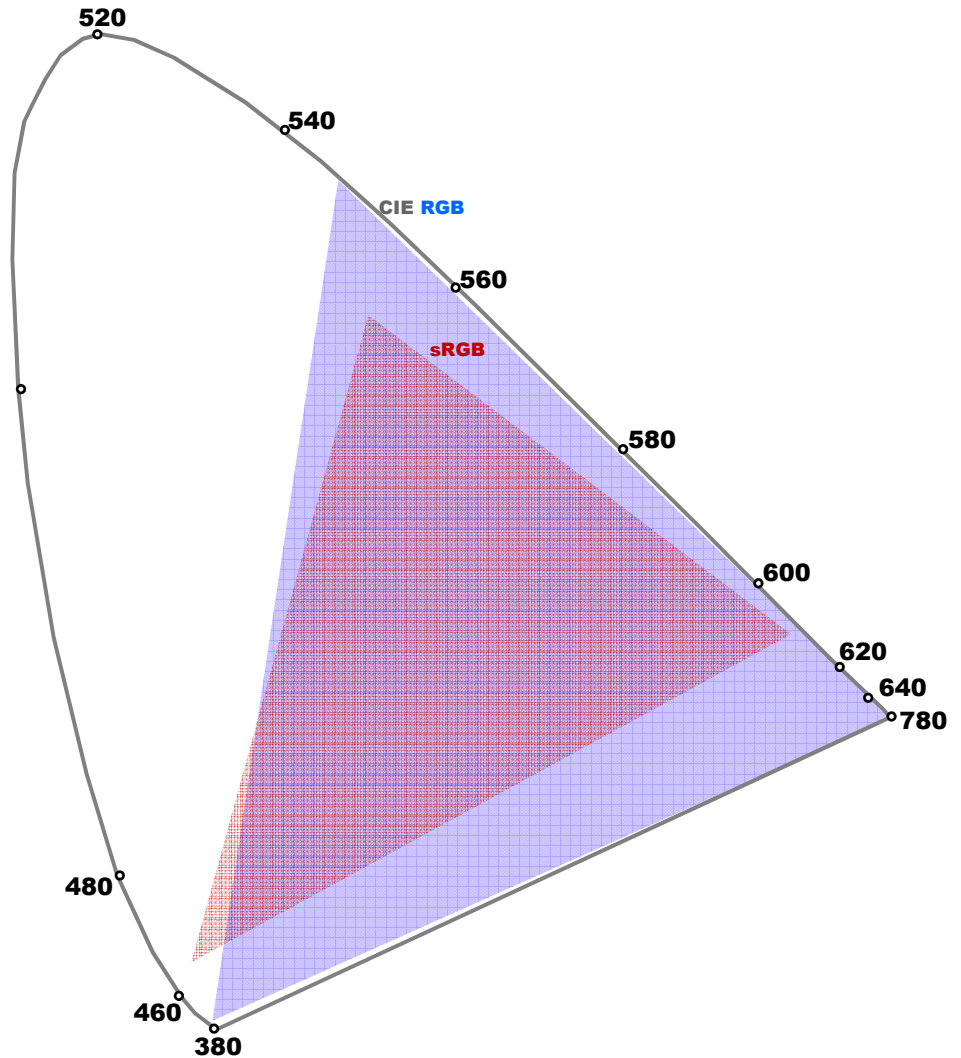
- 1.4.9.1 CIE-RGB - color space

- The real CIE-RGB color space contains the normally reproducible spectrum locus points
 - Red: in practice all wavelengths above 650 nm are identical for the human eye. Wavelengths above 650 nm do not contribute to the color impression (e.g. 690,1-nm-Hg-line)
 - This way a nearly perfect representation of red, orange, yellow, blue and the purple areas is achieved.
 - Noticeable weaknesses are in the area of green and turquoise (*türkis*).

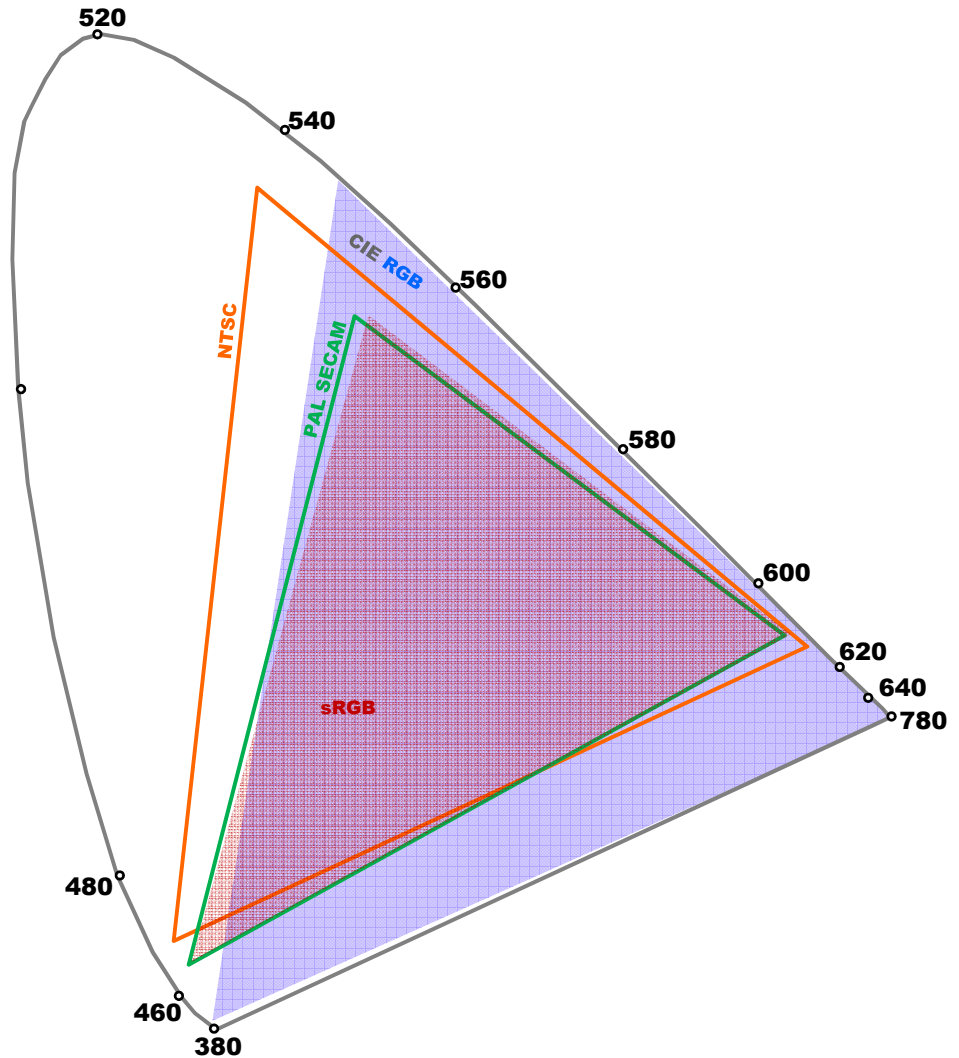


- **1.4.9.2 sRGB - color space**

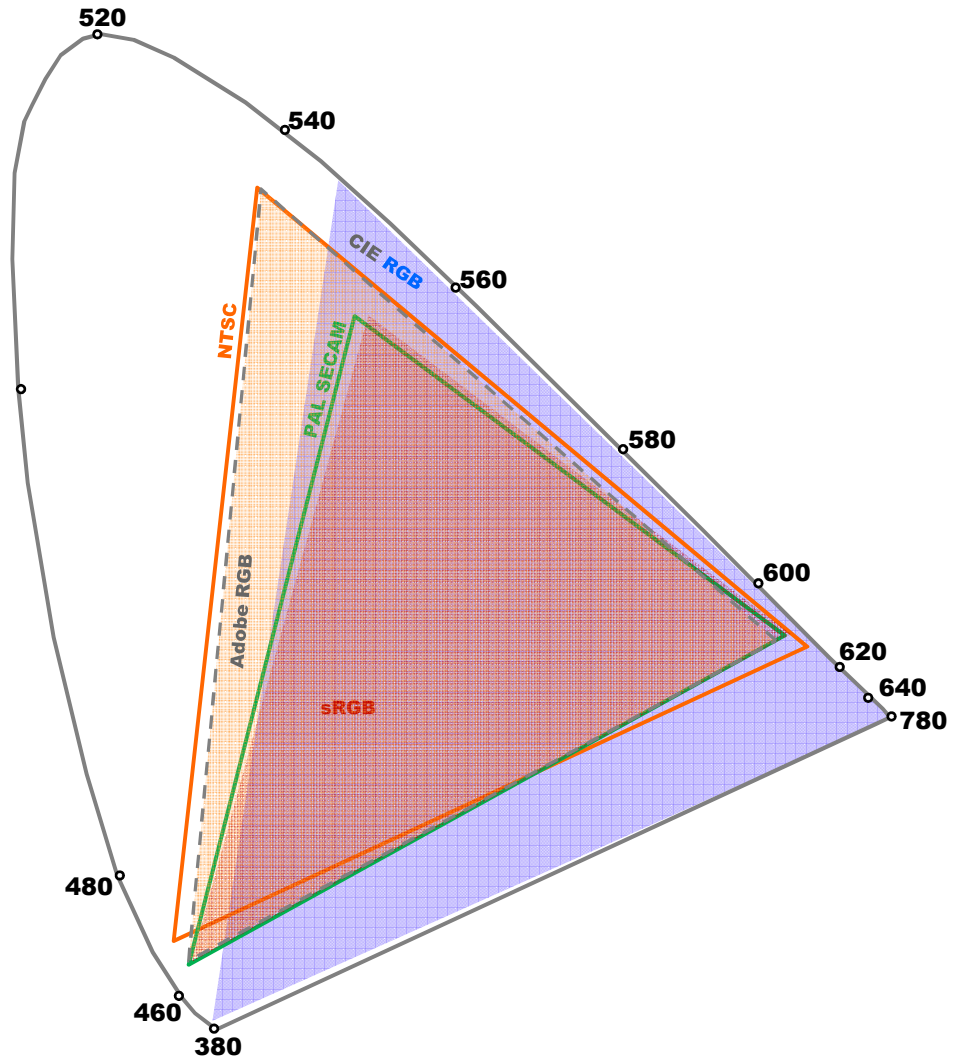
- The sRGB – color space was defined in 1966 by Hewlett-Packard and Microsoft..
- Intention was that in case of direct representation of the color triplets (Farbtripel) a good color reproduction should be achieved.
- This color model was oriented on the available phosphors (Leuchtstoffen) and has weaknesses with saturated colors red, green and bluish color shades.
- Not all in the CMYK printing process printable colors can be reproduced.
- Especially in the green to turquoise area (480 nm to 510 nm) , there are some weaknesses.



- **1.4.9.3 Color space of PAL and SECAM as well as later NTSC (EBU 3213, ITU-R BT.470-2, SMPTE-C)**
- Parallel to the standardization of color representation for computer monitors with sRGB also the color television standards were revised and adapted.
- Since for both technical systems the same electronic constraints are valid, the possibilities are also nearly identical for computer- and TV applications.
- Same as with sRGB the color representation in the green area is restricted compared with that of the red and blue area.
- Due to parallel running standardization activities beside the EBU/ITU-R- color space also a SMPTE-C color space was defined.
- With introduction of HDTV the sRGB color space is prevailed also for TV applications.

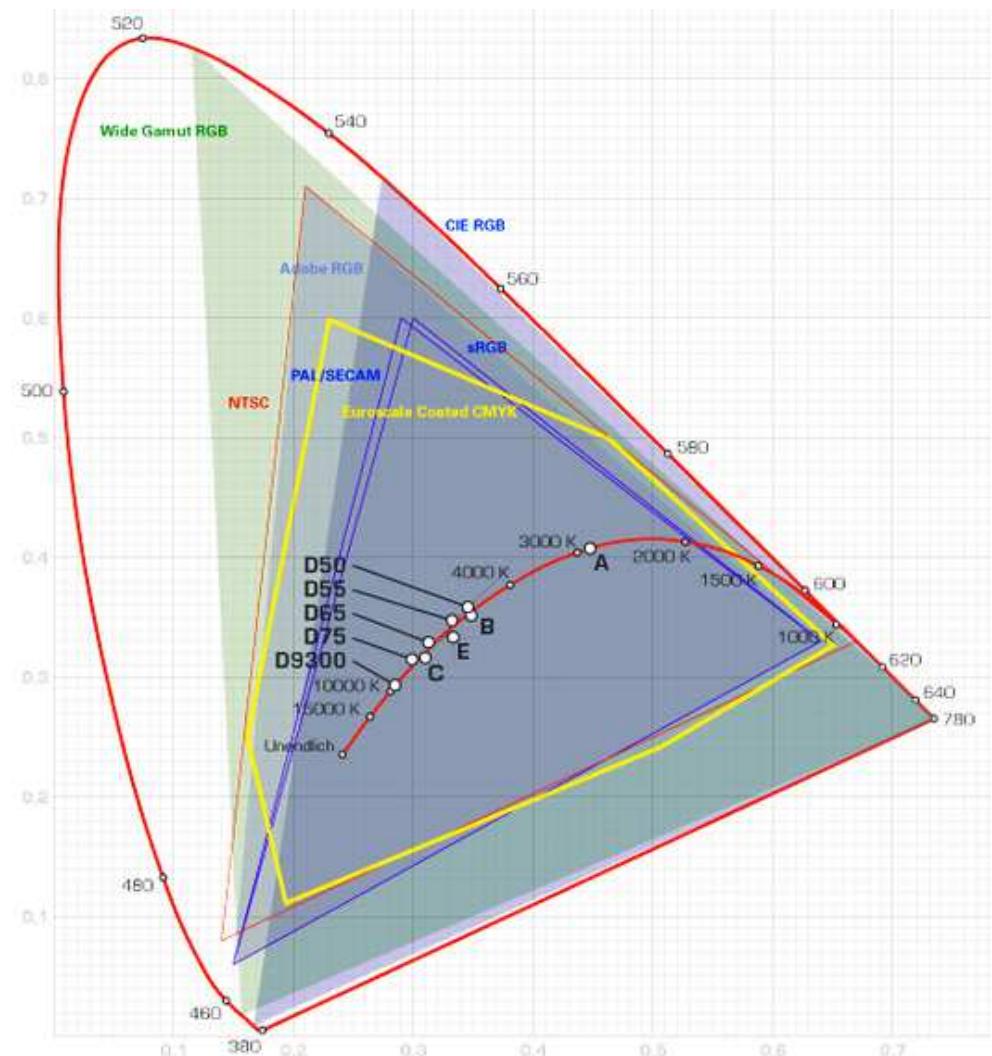


- **1.4.9.4 Adobe-RGB-(1998) – color space**
- In 1998 Adobe implemented a new concept which should make it possible to represent all relevant colors of the CMYK color space (Adobe-RGB-(1998)-Gamut)
- In comparison with sRGB there are clearly improvements in the green and turquoise area. However the reproduction of red is just barely improved whereby blue is even a bit worse.
- Advantage of Adobe-RGB-(1998) is that nearly all colors of the CMYK screen printing (*Siebdruck-verfahren*) can be reproduced in the RGB color space.

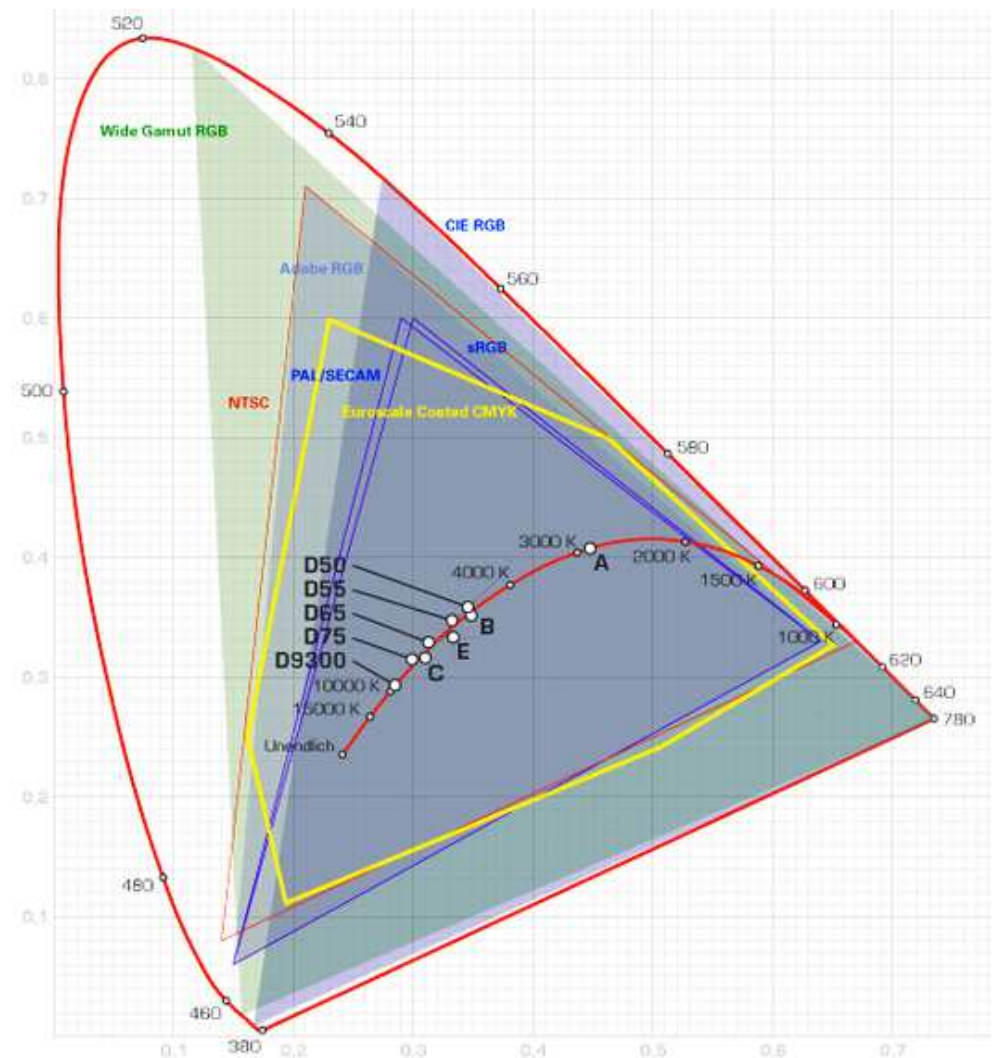


- **1.4.9.5 Adobe-Wide-Gamut-RGB - color space**

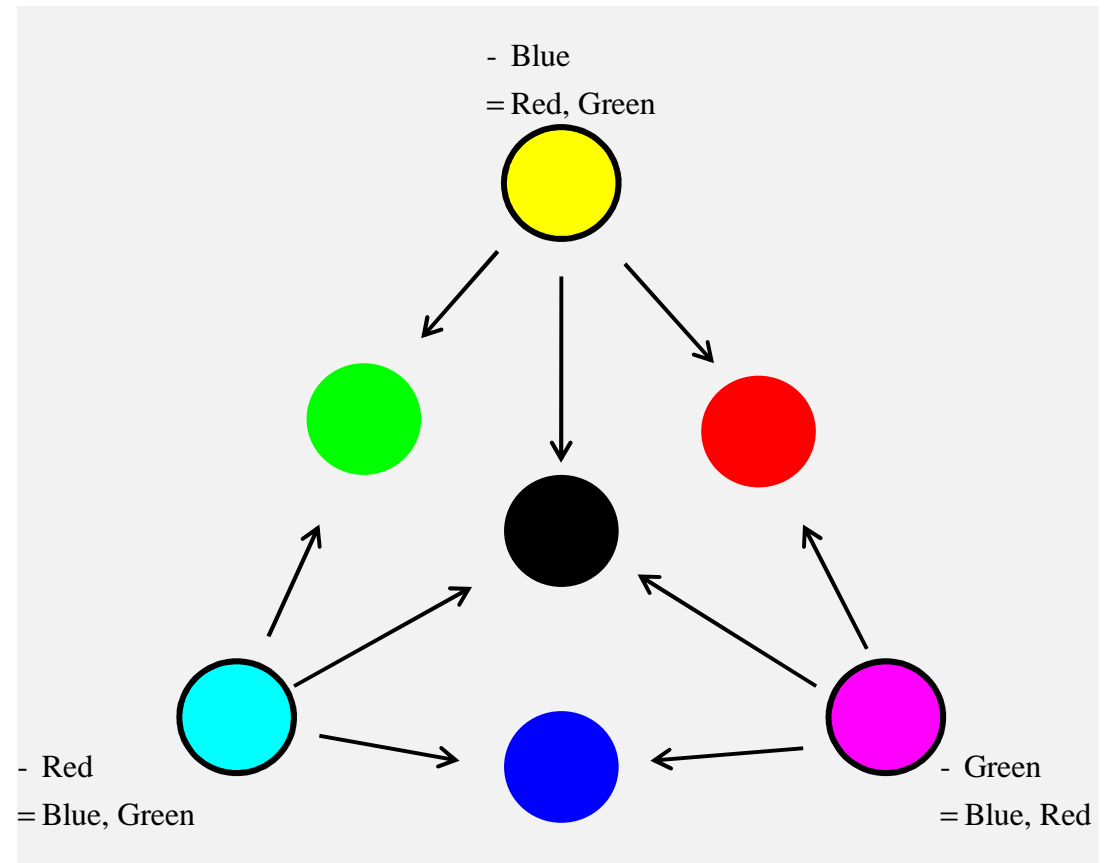
- Although Adobe-RGB is a further development, it doesn't fulfill all the increasing requirements from praxis. As example colored company logos could not continuously forwarded within one workflow from one device to the next one.
- As possible solution the Wide Gammut color space was defined, again under Adope leadership,
- The Wide-Gamut-RGB uses primary colors: 700 nm, 525 nm und 450 nm and higher color saturation near the limits of feasibility (*Machbarkeitsgrenze*).
- This way a perfect coverage of red and a nearly perfect coverage of purple and blue as well as a good greed coverage are achieved.
- Small errors in the green-turquoise range (470 nm to 520 nm) are accepted for the benefit of a system easy to manage in practical environments.
- All colors reproducible in CMYK-2 color printing are covered by the Adobe-Wide-Gamut- color space.



- **1.4.9.6 RGBY color space**
- In 2010 Sharp moved into a totally new direction with their LCD TV: They implemented a fourth primary color.
- Additive color mixture is realized with additionally yellow color points next to red green and blue ones (RGBY).
- Target is an improved color reproduction in the so far critical yellow, gold and brownish color shades. Also the reproduction of skin tones is improved.
- Meanwhile there are multicolor LEDs available, which in addition to RGB also a yellow LED inside.
- In the CE market these types of displays are also indicated as „RGYB“



- 1.4.9.7 CMYK- color space
- Used for color printing
- Primary colors are: **Cyan, Magenta, Yellow, Black**
- Subtractive color missing of ink or toner (covering)
- The principle is to take colors away from white.
 - When the paper is printed with **Cyan**, then no more red light is reflected (Cyan subtracts red from the reflected white light)
→ Blue and Green remain.
 - When the paper is printed with **Magenta**, no more green light is reflected
→ Red and Blue remain (but red is already eliminated by cyan)
 - When the paper is printed with **Yellow**, no more blue light is reflected
→ Red and Green remain.
- A white paper printed with Cyan, Magenta and Yellow results in Black (in principle)



1.4 Colorimetry

- Reproduction of black is only insufficiently possible out of Cyan, Magenta and Yellow .

- → Additional black ink necessary.

- Conversion CMY → CMYK

- Basic CMY colors:

$$\{C_o; M_o; Y_o\}$$

- Calculation minimum of C_o , M_o , Y_o : that's Black (K)

$$K_1 = \min \{C_o; M_o; Y_o\}$$

- For the new colors Magenta, Cyan and Yellow

$$C_1 = \frac{C_o - K_1}{1 - K_1}$$

$$M_1 = \frac{M_o - K_1}{1 - K_1}$$

$$Y_1 = \frac{Y_o - K_1}{1 - K_1}$$

Subtract the minimum (K_1)
Normalize to the remaining ($1-K_1$)

Cyan



Magenta



Gelb



Black



red
suppr.

green
suppr.

Red
and
blue
suppr.

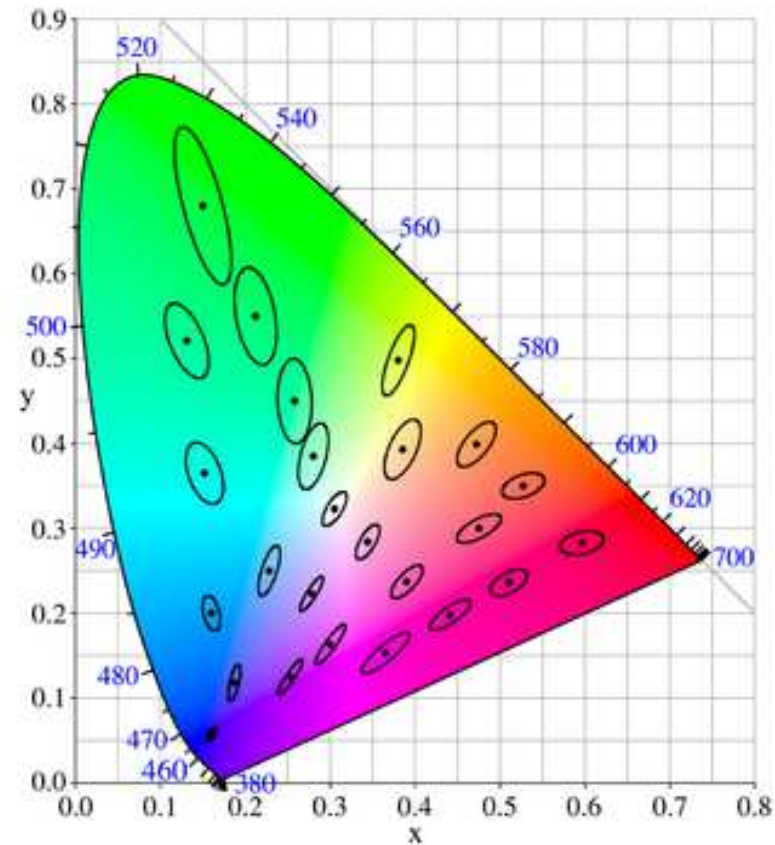


• 1.4.10 Mac Adam ellipses

- In 1942 MacAdam published its investigations he made with a lot of test persons:
 - Mac Adam investigated dependent on the color locus (Farbort) whether the human eye is even able to distinguish color differences.
 - Visual photometer with 2° diameter
 - Environmental luminance 25 cd/m²
 - Photometer luminance 50 cd/m².
 - He came to the conclusion that the standard deviation over all test persons (sd) where a color can not be distinguished from that of the center point can be described as ellipse.
 - Mac Adam introduced the “Just Noticeable Difference” (jnd). That’s the area where a color deviation is just noticeable.

$$1jnd \approx 3sd$$

- He could also show, that the eye is quite sensitive to distinguish colors in the blue and red area. The sensitivity to distinguish colors in the green area is considerably reduced.



Mac-Adam ellipses shown with 10x enlarged standard deviation.

1.5.1 Bibliography

- [1] Theile Richard: Fernsehtechnik, Band 1 Grundlagen. Springer Verlag Berlin, 1973
- [2] Wendland Broder: Fernsehtechnik, Band I: Grundlagen. Hüthig Verlag, 1988
- [3] Wendland Broder: Fernsehtechnik, Band II: Systeme und Komponenten. Hüthig Verlag, 1988
- [4] http://www.pasadenaeye.com/faq/faq15/faq15_text.html

1.5.2 Questionnaire

- 1.1 Why are spectrometric and photometric values distinguished?
Warum unterscheidet man strahlungsphysikalische und lichttechnische Größen?
- 1.2 From a punctiform light source the illumination E of 100lx is measured in a 3 meters distance
Von einer punktförmigen Lichtquelle wird im Abstand von 3m eine Beleuchtungsstärke E von 100lx gemessen.
- a) Calculate the light intensity I of the lamp.
Berechnen Sie die Lichtstärke I der Lampe
 - b) Calculate the complete light flux of the light source.
Berechnen Sie den gesamten Lichtstrom der Lichtquelle
- 1.3 Explain the spectral brightness sensitivity curve, what is the difference between day-time and night-time vision?
Beschreiben Sie die spektrale Hellempfindlichkeitskurve, was versteht man unter Tages und Nachtsehen?
- 1.4 A Lambertian radiator, used as projection screen is illuminated from the backside
Eine Lambertstrahler als Projektionsfläche wird von hinten beleuchtet.
- a) Determine the light intensity dependant on the angle α ?
Berechnen Sie die Lichtstärke in Abhängigkeit vom Winkel α ?
 - b) The luminance is 400cd/m². The projection area is 1m². What is the amount of the light flux necessary, when only 50% of the light flux reaches the projection screen and the grade of transmission of the projection screen is 75%.
Die Leuchtdichte soll 400cd/m² betragen. Die Projektionsfläche sei 1m², Wie groß muss der Lichtstrom der Leuchtquelle sein, wenn lediglich 50% auf den Schirm gelangt. und die Projektionswand einen Transmissionsgrad der Projektionwand 75% beträgt?
 - c) Now the projection is done with the three color primaries Red, Green and Blue. Each of these three projection systems should generate a luminance of 400cd/m². $V(\text{green}) = 1$, $V(\text{red}) = 0,1$, $V(\text{blue}) = 0,05$. calculate the necessary light power each of these 3 projection systems must have.
Die Projektion soll nun durch die drei Primärfarben Rot Grün und Blau erfolgen. Jede Strahlkanone für sich soll die Leuchtdichte von 400cd/m² erzeugen. $V(\text{grün}) = 1$, $V(\text{rot}) = 0,1$, $V(\text{blau}) = 0,05$. Berechnen Sie jeweils die erforderliche Lichtleistung.

- 1.5 A projection screen has a luminance distribution $L(\alpha) = L_0 \cos \alpha$ in the range from $-90^\circ < \alpha < 90^\circ$

Eine Projektionsfläche habe eine Leuchtdichtevertellung $L(\alpha) = L_0 \cos \alpha$ im Bereich $-90^\circ < \alpha < 90^\circ$

- a) Calculate the light flux Φ necessary.

Berechnen Sie den erforderlichen Lichtstrom Φ .

- 1.6 Explain the tasks of taps and small rods in the human eye.

Beschreiben Sie die Aufgaben von Zapfen und Stäbchen im menschlichen Auge.

- 1.7 Explain adaptation of the human eye

Was versteht man unter Adaptation des Auges?

- 1.8 a) What does the Weber Fechner law say?

Was besagt das Weber Fechnersche Gesetz?

- b) The the human eye more sensitive to noise in case of dark or light pictures, explanation.

Ist das Auge für Rauschen empfindlicher bei hellem oder dunklem Bildschirm?

- 1.9 Explain lateral inhibition.

Was versteht man unter lateraler Hemmung?

- 1.10 The modulation transfer function of a camera lens can nearly be described as Gaussian function.

Die Modulationsübertragungsfunktion eines Objektivs kann in guter Näherung als Gaußfunktion betrachtet werden.

$$M(f^x) = \exp \left\{ -\pi \cdot \left(\frac{f^x}{f_g^x} \right)^2 \right\}$$

Calculate the cutoff frequency, in case that the template has a rectangular luminance distribution

Bestimmen Sie die Grenzfrequenz, wenn als Vorlage eine rechteckförmige Leuchtdichtevertellung verwendwird.

1.11 Explain additive and subtractive color mixing

Was versteht man unter additiver und subtraktiver Farbmischung?

1.12 What does the term “metameric colors” mean?

Erläutern Sie den Begriff metamere Farben.

1.13 Based on the three display primary valences (R), (G),(B). Determine for following wavelengths $\lambda=470\text{nm}$, $\lambda=510\text{nm}$, $\lambda=560\text{nm}$; zusammen $\lambda=500\text{nm}$, $\lambda=600\text{nm}$, the chromaticity coordinates r,g,b .

Auszugehen von den Bildschirmprimärvalenzen (R), (G),(B). Für folgende Wellenlängen sind die Farbwertanteile r,g,b zu bestimmen: $\lambda=470\text{nm}$, $\lambda=510\text{nm}$, $\lambda=560\text{nm}$; zusammen $\lambda=500\text{nm}$, $\lambda=600\text{nm}$.

1.14 Assumed a color stimulus function $\phi(\lambda)$ normalized to 1

Gegeben ist eine auf 1 normierte Farbreizfunktion $\phi(\lambda)$,

a) Define the color values for the visible wavelengths

Bestimmen Sie die Farbwerte für: :

$$\phi[\lambda] = 1 \quad \text{im sichtbaren Bereich}$$

b) Define the color values for

Bestimmen Sie die Farbwerte für:

$$\phi[\lambda] = \begin{cases} 1 & 580\text{nm} < \lambda < 775\text{nm} \\ 0 & \text{sonst} \end{cases}$$

1.15 Explain why the color values R,G,B, which come from photoelectric sensors can't directly be used for a TV transmission?

Erläutern Sie, warum die Farbwerte R,G,B die von photoelektrischen Empfängern geliefert werden nicht unmittelbar zur TV-Übertragung geeignet?

1.16 Explain the difference between the spectral value curves:

Erläutern Sie den Unterschied zwischen den Spektralwertkurven:

$$\bar{r}_0(\lambda), \bar{g}_0(\lambda), \bar{b}_0(\lambda) \quad \text{und} \quad \bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$$

1.17 Why isn't it possible to add constant values to the spectral curves $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ in order to obtain only positive values?

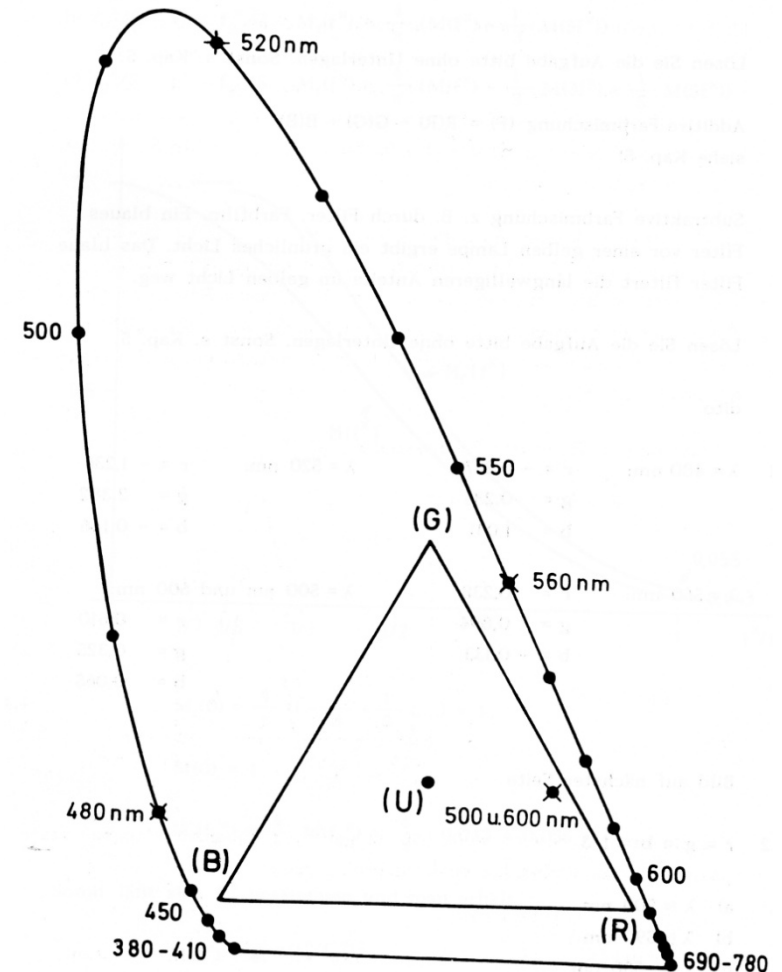
Warum kann man nicht einfach den Spektralwertkurven $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ konstante Werte hinzufügen, damit diese keine negativen Anteile mehr aufweisen?

1.18 Determine for the spectral values $\lambda=500\text{nm}$ and $\lambda=570\text{nm}$ the chromaticity coordinates. What happens, when the negative parts are simply ignored? How does the color valence change in this case?

Bestimmen Sie für die Spektralwerte $\lambda=500\text{nm}$ und $\lambda=570\text{nm}$ die Farbwertanteile. Was geschieht, wenn die negativen Anteile weggelassen werden. Wie verändert sich dann die Farbvalenz (Darstellung im Spektralfarbenzug rechts).

1.19 Assume a color valence composed of two monochromatic colors at $\lambda=500\text{nm}$ and $\lambda=600\text{nm}$. How does the chromaticity change when the negative parts in the camera (Color values) are ignored? Gegeben ist eine Farbvalenz, die aus zwei monochromen Strahlquellen bei $\lambda=500\text{nm}$ und $\lambda=600\text{nm}$ besteht.

Wie ändert sich die Farbart, wenn ebenfalls die negativen Anteile in der Kamera (Farbwerte) unberücksichtigt bleiben.



End



1.5.3 Eratum

- Ver 1.1 → ver 1.2 (27.04.2011)
 - Bild mit $ro(\lambda)$, $go(\lambda)$, und $bo(\lambda)$, eingefügt S46;
 - Rechtschreibkorrekturen
 - S24 Kurven a, b
 - Inhaltsverzeichnis update
 - S37 Formel für äußere Farbmischung korrigiert
 - Aufgabenblatt und Eratum eingefügt
 - Augenmodell ersetzt
 - Animationen eingefügt

1.5.4 Lösungen

@ 1.14 siehe Tabelle Seite 38 (Aufaddieren der Werte im entsprechenden Frequenzbereich)

a) $R=G=B=100$

b) $R=113,5$; $G=5,65$; $B=-2,8$

@1.15 Photoelektrische Empfänger können keine neg. Werte liefern, die zur Nachbildung der Spektralwertkurven erforderlich wären.

@1.16 Siehe Skript

@1.17

Die Farbvalenz Rot ergibt sich zu:

$$R = \int \varphi(\lambda) \cdot \bar{r}(\lambda) \cdot d\lambda$$

Durch Hinzufügen eines additiven Anteils ergäbe sich:

$$\begin{aligned} R^* &= \int \varphi(\lambda) \cdot (\bar{r}(\lambda) + \bar{r}_0) \cdot d\lambda \\ &= \int \varphi(\lambda) \cdot \bar{r}(\lambda) \cdot d\lambda + \int \varphi(\lambda) \cdot \bar{r}_0 \cdot d\lambda \\ &= R + \bar{r}_0 \int \varphi(\lambda) d\lambda \end{aligned}$$

d.H der Anteil der bei der Farbvalenz hinzugefügt würde hängt vom Spektralen Verlauf ab und ist somit nicht mehr konstant → es wäre unklar, was von der Farbvalenz wieder zu subtrahieren sei.

@ 1.18

$\lambda=500\text{nm}$: Farbvalenz F mit Farbwertanteilen:

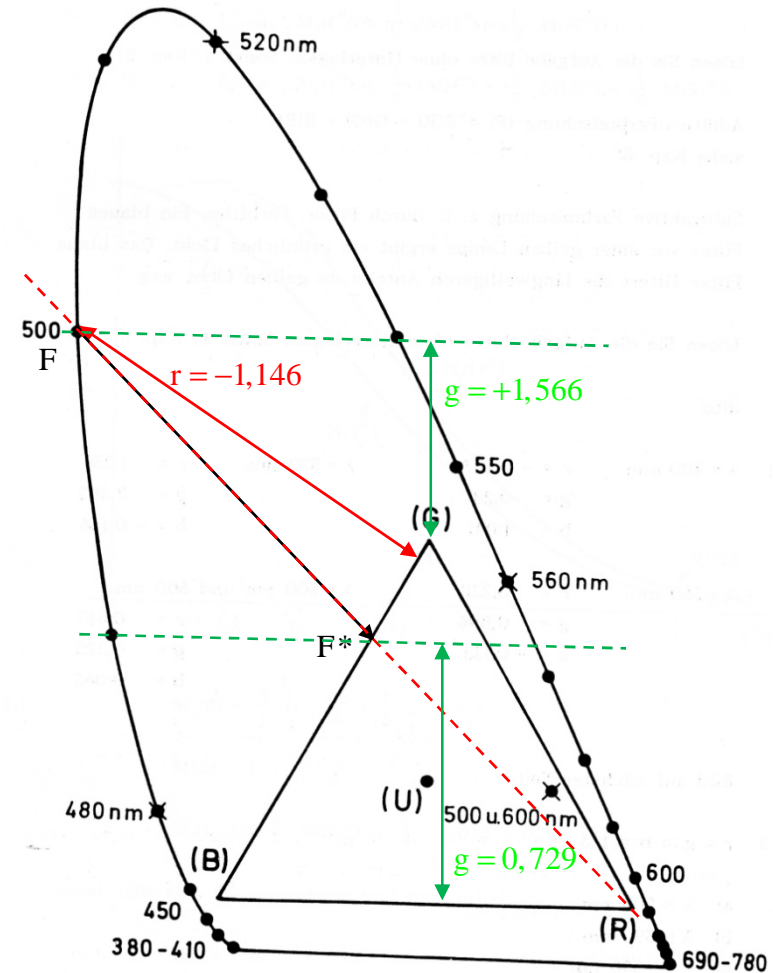
$$\left. \begin{array}{l} \bar{r}(\lambda) = -2,212 \\ \bar{g}(\lambda) = +3,022 \\ \bar{b}(\lambda) = +1,119 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} r = \frac{-2,212}{1,929} = -1,146 \\ g = \frac{3,022}{1,929} = +1,566 \\ b = \frac{1,119}{1,929} = 0,580 \end{array} \right.$$

$$\left. \begin{array}{l} \bar{r}(\lambda) = 0 \\ \bar{g}(\lambda) = +3,022 \\ \bar{b}(\lambda) = +1,119 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} r = 0 \\ g = \frac{3,022}{4,141} = +0,729 \\ b = \frac{1,119}{4,141} = +0,270 \end{array} \right.$$

Hierdurch erfolgt eine Verschiebung der Farbvalenz von F nach F*

Für den Spektralwert $\lambda=570\text{nm}$ ergibt sich:

$$\left\{ \begin{array}{l} r = 0,4330 \\ g = 0,567 \\ b = 0 \end{array} \right.$$



@ 1.19 Für $\lambda=500\text{nm}$ und $\lambda=600\text{nm}$:

$$\left. \begin{array}{l} \bar{r}(500\text{nm}) = -2,212 \\ \bar{g}(500\text{nm}) = +3,022 \\ \bar{b}(500\text{nm}) = +1,119 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} r = \frac{-2,212}{1,929} = -1,146 \\ g = \frac{3,022}{1,929} = +1,566 \\ b = \frac{1,119}{1,929} = 0,580 \end{array} \right.$$

$$\left. \begin{array}{l} \bar{r}(600\text{nm}) = +9,304 \\ \bar{g}(600\text{nm}) = +0,761 \\ \bar{b}(600\text{nm}) = -0,368 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} r = \frac{9,304}{9,697} = 0,959 \\ g = \frac{0,761}{9,697} = 0,0785 \\ b = \frac{-0,368}{9,697} = -0,0379 \end{array} \right.$$

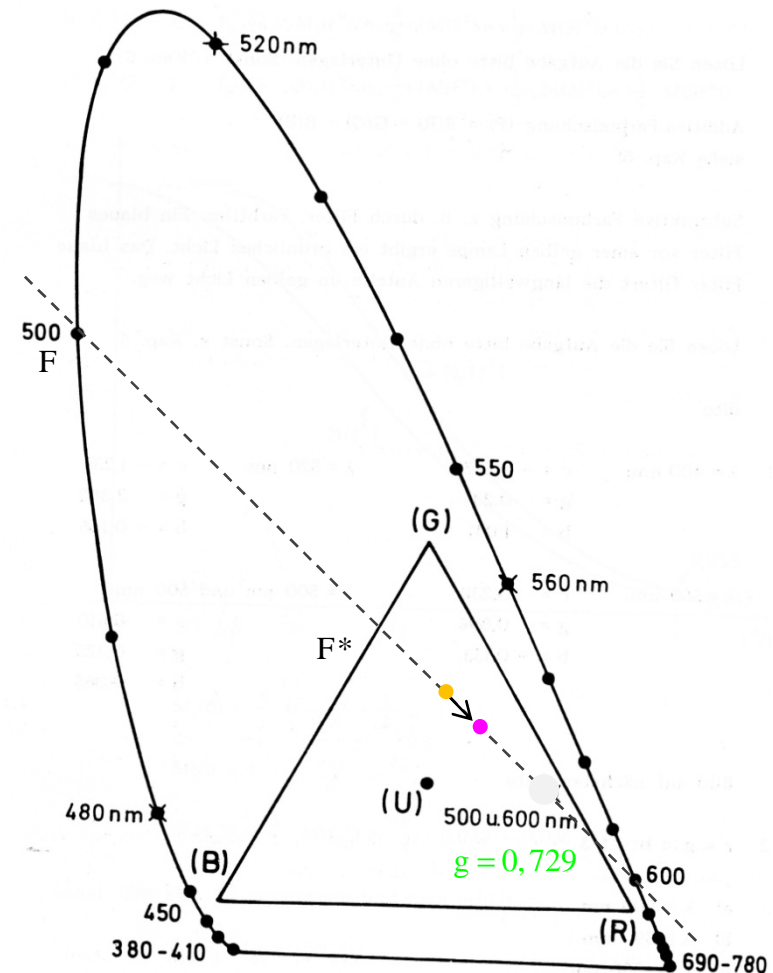
Für die zusammengesetzte Farbvalenz F ergibt sich:

$$\bar{F}(500\text{nm} + 600\text{nm}) = \bar{F}(500\text{nm}) + \bar{F}(600\text{nm})$$

$$\left. \begin{array}{l} R = -2,212 + 9,304 = 7,092 \\ G = 3,022 + 0,761 = 3,783 \\ B = 1,119 - 0,368 = 0,751 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} r = \frac{7,092}{11,626} = 0,610 \\ g = \frac{3,783}{11,626} = 0,325 \\ b = \frac{0,751}{11,626} = 0,0646 \end{array} \right.$$

Bei Nichtbeachten der negative'n Anteile:

$$\left. \begin{array}{l} R = 0 + 9,304 = 9,304 \\ G = 3,022 + 0,761 = 3,783 \\ B = 1,119 - 0 = 1,119 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} r = \frac{9,304}{14,206} = 0,654 \\ g = \frac{3,783}{14,206} = 0,266 \\ b = \frac{1,119}{14,206} = 0,0084 \end{array} \right.$$



@ 1.19 Für $\lambda=500\text{nm}$ und $\lambda=600\text{nm}$:

Für die zusammengesetzte Farbvalenz F ergibt sich:

$$\vec{F}(500\text{nm} + 600\text{nm}) = \vec{F}(500\text{nm}) + \vec{F}(600\text{nm})$$

Bei Nichtbeachten der negative'n Anteile:

