TECHNISCHE UNIVERSITÄT
CHEMNITZ

Professur
Betriebssysteme

2. Excercise

Dependable Systems

Jafar Akhundov

Task 1

Theoretical Questions:

a) What is a random variable? Define its domain and range.

b) What is a (cumulative) distribution function?

c) What is a probability mass/density function? Define its properties.

d) Define the mean (expected value) of a random variable.

e) Define the variance of a random variable.

f) Give examples for an exponential, Poisson and a Gaussian process/event.

Task 2

Consider the previous experiment with tossing a coin three times. Let X be a random variable giving the number of heads obtained. Assume that the tosses are independent and the probability of a head is p

a) What is the range of X?

b) Find the probabilities of $P(X = 0), P(X = 1), P(X = 2), P(X = 3)$.

c) Sketch the $F_X(x)$.

Task 3

Consider a sequence of Bernoulli trials with probability p of success. This sequence is observed until the first success occurs. Let the random variable X denote the trial number on which this first success occurs.

a) Find the pmf of X $p_X(x)$?

b) Show that $p_X(x)$ satisfies the property $\sum_k p_X(x_k) = 1$.

c) Find the $F_X(x)$ of X.

Task 4

Find the mean and the variance of X representing the outcome of throwing a fair die.

Task 5

The number of telephone calls arriving at a switchboard during any 10-minute period is known to be Poisson random variable X with $\lambda = 2$.

a) Find the probability that more than three calls arrive during any 10-minute period.

b) Find the probability that no calls will arrive during any 10-minute period.

Task 6

Assume that the length of a phone call in minutes is an exponential random variable X with parameter $\lambda = 0.1$. If someone arrives at a phone booth just before you arrive, find the probability that you will have to wait

a) less than 5 minutes.

b) between 5 and 10 minutes.

Task 7

A production line manufactures 1000-ohm resistors that have 10 percent tolerance. Let X denote the resistance of a resistor. Assuming that X is normal random variable, find the probability that a resistor picked at random will be rejected.

Literatur

[1] Ronald Meester - A Natural Introduction to Probability Theory. 2008 Birkhäuser Verlag.

[2] Hwei Hsu - Theory and Problems of Probability, Random Variables, and Random Processes. Schaum's Outline Series