

# 1. Exercise

## Dependable Systems

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### Task 1

Theoretical questions:

- What is an experiment?
- What is the event(sample) space?
- What is an event?
- What is an elementary/certain/impossible/random event?
- Define probability by Laplace.
- What is the difference between independent and mutually exclusive events?
- Combinatorial counting methods. The number of
  - sequences of length  $k$  with  $n$  symbols is ...
  - ordered subsets of  $k$  elements from a set with  $n$  elements is ...
  - subsets of  $k$  elements from a set with  $n$  elements denoted by  $\binom{n}{k}$  (read as "n choose k") is ...

### Task 2

Consider a random experiment with three consecutive coin tosses:

- What is the sample space  $S_1$  of the experiment outcome?
- What is the probability of every elementary event? Name two methods to compute them. What are the necessary conditions for each of the methods?
- Construct the event space  $S_2$  representing the number of tails in the three tosses.
- Give an example of a non-elementary event and find its probability.
- For the  $S_1$ , find the probability that there is at least one head, given that the event in the previous question has happened.
- For the  $S_2$ , name two mutually exclusive events.

### Task 3

There are  $n$  persons in a room

- a) What is the probability that at least two persons have the same birthday
- b) Calculate this probability for  $n = 50$ .
- c) How large need  $n$  be for this probability to be greater than 0.5?

#### **Task 4**

A committee of 5 persons is to be selected randomly from a group of 5 men and 10 women.

- a) Find the probability that the committee consists of 2 men and 3 women.
- b) Find the probability that the committee consists of all women.

#### **Task 5**

Consider four math books, five language books and two art books. Suppose that you put them on a shelf in a completely random order, what is the probability that the books are grouped per subject?

#### **Task 6**

Suppose there is a population of people and the probability that an individual has a certain disease is  $1/100$ . There is a test for this disease which is accurate for sick people in 90% cases. The probability that a healthy person is tested positively is 0.1. Now, one particular individual is tested positive. Usually, the way of thinking is "I have been tested positive by the 90% accurate test, hence the probability that I am sick is 0.9". And this is NOT correct. In the reality, it is asked for the conditional probability to have the disease, given that the test is positive, while given the conditional probability of a positive test, given the disease.  $P(A|B) \neq P(B|A)$  !!! What is the correct reasoning then?

## **Literatur**

- [1] Ronald Meester - A Natural Introduction to Probability Theory. 2008 Birkhäuser Verlag.
- [2] Hwei Hsu - Theory and Problems of Probability, Random Variables, and Random Processes. Schaum's Outline Series