Perception, 1991, volume 20, pages 49–55

# Sine of an illusion

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Received 5 September 1990, in revised form 17 November 1990

**Abstract.** In an informally observed sine-wave figure in which the vertical extent between contours was constant, apparent extent in the crest and trough (the 'turns') appeared greater than in the straight oblique sections of the figure. This observation was confirmed in two experiments in which the vertical extents were matched by two vertically arranged dots. It was found that in a turn the apparent extent was greater than the true extent, but in a straight section both extents were about equal. These outcomes were confirmed when the two sections were each separated from the figure and presented alone. The illusion is explained in terms of a perceptual compromise between the vertical extent and the greater overall dimensions of the section at the turn of the sine-wave figure and is thereby held to be the same in principle as the Müller-Lyer illusion.

## 1 Introduction

Although the vertical lines in figure 1 are the same length throughout, those at the crest and trough of the sine wave, ie in the 'turns', appear longer than those in the more or less straight oblique sections. This illusion was first noticed and drawn to the attention of the first author in the light grey computer-printout shown in figure 1.(1) Some informal observations showed that the illusion is also present in outline and solid forms of the figure (figure 2). The illusion persists when the figure is rotated

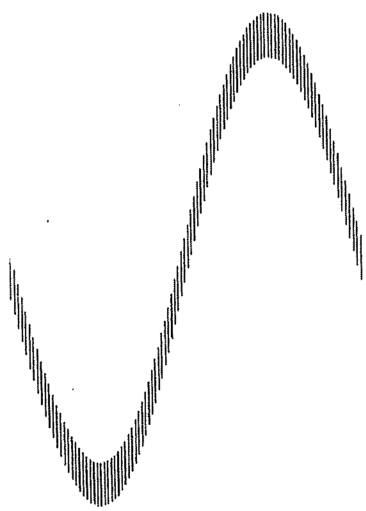


**Figure 1.** The Dickinson sine-wave figure in which the vertical bars are the same length throughout. The bars in the crest and trough ('turns') of the wave appear to be longer than those in the straight sections.

(1) The illusion was first drawn to the attention of the first author in a computer printout (figure 1) by the late Rod Dickinson who died young and unexpectedly in February 1990. Rod Dickinson was a valued colleague in earlier research projects and this paper is intended to commemorate his many collaborative contributions to experiments on perceptual illusions.

different orientations, and the effect tends to be weakened when the individual vertical bars are carefully scrutinised and compared.

Apart from not having been discussed before, the illusion is of interest because of what seems to be a basic difference between it and the well known Müller-Lyer illusion of extent. In the Müller-Lyer figure the spaces parallel to the interapical extents are either longer or shorter than them. This characteristic of the figure has often been invoked as a basis for the illusion; the equal extents are held to appear different because they assimilate to (or are comprised of) the longer or shorter extents that flank them (see Coren and Girgus 1978; Day 1989). This argument can not apply in figure 1 since all the parallel vertical lines and spaces are the same length throughout the sine wave.

The first of the two experiments reported here was designed to establish both the magnitude of the sine-wave illusion and the sections of the figure in which it occurs, ie whether it is an illusion of greater apparent extent in the turns, or smaller apparent extent in the oblique sections, or both. In the event, the illusion was found to be one of greater apparent extent in the turns. The second experiment was therefore concerned with whether the effect persists when a section about a turn is isolated from the sine-wave figure and presented alone (see figure 3).

## 2 Experiment 1

The purpose of the first experiment was to confirm and measure the sine-wave illusion and to establish where in the figure it is located—at the turns, in the straight oblique sections, or both. A matching procedure was used to measure the effect in the lower turn and in the central oblique section of both an outline and a solid figure. Subjects were required to adjust the separation between two vertically arranged dots so that it appeared to match the vertical distance between the lines or edges of the figure. For two reasons measurements were made in outline and solid forms rather than in the original figure consisting of vertical lines. First, as pointed out above, the illusion seems to be weakened when the individual lines in figure 1 are carefully inspected and directly compared. Thus it is possible that the vertical bars provide cues for true extent and thereby weaken the effect. These cues are not present in outline and solid forms of the figure. Second, there is evidence which indicates that at least one illusion is greater in a solid than in an outline figure. The Bourdon figure (see Day et al 1990) consists of two elongated triangular components joined at their apexes with two edges collinear and those opposite then forming a large obtuse angle. The Bourdon illusion is the apparent bentness of the straight edge in the same direction as the bent edge (obtuse angle) opposite. In a solid black figure this illusion is about double that in an outline figure of the same dimensions (Rozvany and Day 1980). It was therefore decided to establish whether any differences occur between the illusion of vertical extent in the solid and in the outline forms of the sine-wave figure.

### 2.1 *Method*

2.1.1 *Subjects.* There were twelve subjects, seven women and five men, recruited from among the undergraduates, graduate students, and research assistants from the Department of Psychology.

2.1.2 *Apparatus.* The sine-wave patterns and the two dots were presented on the screen of a personal computer (NEC Multisyne 2A) fitted with a video graphics adaptor (VGA). The screen was 27.5 cm wide and 20.4 cm high. Subjects were seated at a viewing distance of about 62 cm, at which 1 cm on the screen subtends a visual angle of about 56 min. The computer was programmed to generate the patterns and the dots, to present the patterns in a particular order, to control the initial separation between the dots, and to score in millimetres the difference between the final separation of the dots and the vertical extent between the upper and the lower boundaries of the pattern. The vertical separation between the dots was varied by pressing one of two keys on a keyboard on the table at which the subject was seated. One key was marked with an arrow directed away from the subject, and the other was marked with an arrow directed towards the subject. The first moved the upper dot upwards on the screen, and the second moved it downwards. The lower dot was fixed. Both keys when tapped moved the dot by 0.5 mm and moved it continuously when held down. Subjects used either or both modes to adjust the distance between the dots to match the apparent vertical separation between the boundaries of the sine wave. The next figure in a sequence was brought into view by pressing a third key.

2.1.3 *Stimulus figures.* The outline and solid sine-wave figures and the two vertically arranged dots are shown in figure 2 with their main dimensions. So that the vertical distance between the ends of the lines or edges in figure 2 could not provide a cue for extent they each were not themselves vertical. The ends of the outline figure fell on an oblique axis, and those of the solid figure formed a right-angled edge. The two dots were about 2 mm in diameter and were positioned to the lower left of the figure as shown in figure 2.

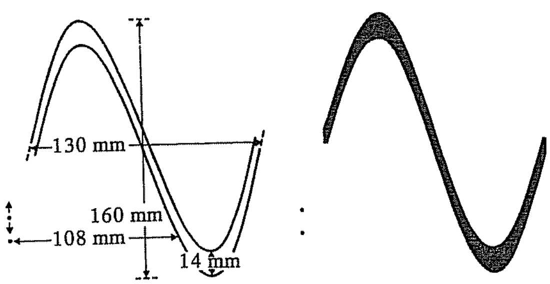



**Figure 2.** The sine-wave stimulus figures for experiment 1 including the positions of the two dots used for matching vertical extent.

2.1.4 *Procedure.* The task of a subject was to move the upper dot up or down so that its distance from the lower dot matched that of one of the two identical vertical extents, that at the midpoint of the upper turn (the crest of the wave), or that in about the middle of the straight oblique section at the centre of the figure. With both figures, four matches were made for each of these two positions, two from an initial dot separation of 7 mm, and two from a separation of 22 mm. The initial separations and the nominated extents—turn or middle section—alternated over the eight matches for each figure; half the subjects matched extents first in the outline figure and then in the solid figure, and the other half did so in the reverse order. Before each match the experimenter nominated which of the two extents was to be matched. The score calculated in all cases was the mean difference in millimetres between the true extent (14 mm) and the apparent extent derived from the mean of the four matches. When the matched extent was greater than 14 mm the difference was scored as being positive, and when it was less the score was negative.

2.2 *Results*

The mean matching scores and their standard deviations are shown for the turn and middle oblique sections of the two figures in table 1. It can be seen that the scores for the turns were positive and markedly greater than those for the middle sections.

In the case of the latter, the mean score for the outline figure was small and negative and that for the solid figure small and positive. It can also be seen that the mean scores for both locations in the solid figure were greater than those in the outline figure. Independent $t$ tests showed that whereas the mean scores for the turns in the outline (O) and the solid (S) figures were significantly greater than 0 ($t_{11}^{\mathrm{O}} = 5.92$, $p < 0.001$; $t_{11}^{\mathrm{S}} = 5.91$, $p < 0.001$), those for the oblique sections were not ($t_{11}^{\mathrm{O}} = 0.65$, $p > 0.05$; $t_{11}^{\mathrm{S}} = 1.10$; $p > 0.05$). A two-way ANOVA showed that both the form (F) of the figure and the location (L) of the extent were significant factors: $F_{\mathrm{F}1,\,11} = 9.77$, $p < 0.01$; $F_{\mathrm{L}1,\,11} = 28.24$, $p < 0.005$. The interaction between these factors was not significant: $F_{1,\,11} = 3.92$, $p > 0.05$. Thus although the illusion for the solid figure was only about 0.5 mm greater than that for the outline figure, this difference was significant. However, the difference could be due to a statistical anomaly. Of the twelve position mean scores for the turn, six were greater for the solid figure than for the outline figure, five were smaller, and one was the same. Of the seven positive scores and five negative scores for the middle section of the solid figure, eleven were greater (ie more positive or less negative) and one was the same as for the outline figure. However, the interaction between the form of the figure (solid or outline) and the position at which matches to vertical extent were made (turn or middle) just failed to reach significance ($p > 0.07$). It seems reasonable to conclude, therefore, that the difference between the two positions is attributable to the scores for the middle section only, the means of which did not reach significance.

In summary, the illusion in the sine-wave figure is one of increased apparent extent in the turns, not one of decreased extent in the oblique sections. This outcome immediately posed the question as to whether the sine-wave form is necessary for the occurrence of the illusion. With a strong illusion in the turns and none in the oblique sections, it could reasonably be expected that the same outcome would be obtained if the two sections were separated from the figure and each presented alone. This expectation was tested in the second experiment.

**Table 1.** Means and standard deviations (in mm) of matching scores for the turn and oblique sections of the stimulus figures.

| | Outline | | Solid | |
|---|---|---|---|---|
| | turn | oblique section | turn | oblique section |
| Experiment 1 | $3.46 \pm 2.0$ | $-0.05 \pm 2.9$ | $3.95 \pm 2.3$ | $0.77 \pm 2.4$ |
| Experiment 2 | | | $3.62 \pm 1.7$ | $2.35 \pm 4.6$ |

## 3 Experiment 2

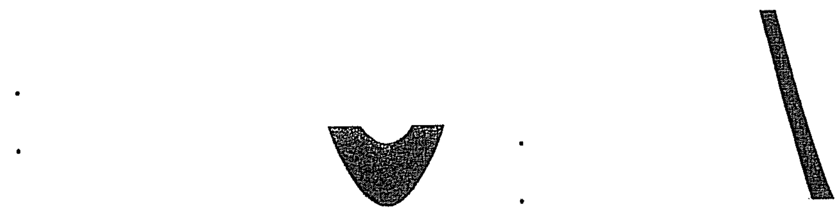
A section of the figure at a turn, this time about the trough rather than the crest, and the middle section as before were each presented alone. The trough was selected in the interest of establishing that the illusion occurs there as well as at the crest, as found in experiment 1.

### 3.1 *Method*

3.1.1. *Subjects.* There were again twelve subjects, seven women and five men, recruited from among the same groups as before but consisting mainly of undergraduates.

3.1.2 *Apparatus.* The apparatus was the same as that used in experiment 1.

3.1.3 *Stimulus figures.* The figures taken from the turn and the middle section of the solid waveform (figure 2) are shown in figures 3a and 3b respectively. The maximum vertical height of figure 3a was 28 mm and the maximum horizontal width across the top was 36 mm. The vertical extents from edge to edge within the figure were the same as before. Figure 3b was 65 mm along its axis and all vertical extents from edge to edge within it were also 14 mm. The two vertically arranged dots were positioned as before and are shown in figure 3.



(a) (b)

**Figure 3.** Sections of the sine-wave figure presented above in experiment 2. The positions of the matching dots are shown.

3.1.4 *Procedure.* The procedure and scoring were essentially the same as before with subjects matching the distance between the dots to the vertical extent at the centre of figures 3a and 3b, four times each. The initial distance between the dots was also varied as before. Half the subjects made the matches first with figure 3a and then with figure 3b, and the other half did it in the reverse order.

3.2 *Results*

The two mean matching scores and their standard deviations are shown in table 1. Both mean scores were positive with that for the turn section being greater than that for the middle oblique section. The latter was considerably greater than the mean scores for the same section in experiment 1. However, whereas for figure 3a all twelve scores were positive, for figure 3b eight were positive and four were negative. Two $t$ tests showed that the mean score for figure 3a was significantly greater than zero ($t_{11} = 7.27$, $p < 0.001$) but the score for figure 3b was not ($t_{11} = 1.78$, $p > 0.05$).

These results confirm those of the first experiment. There is a strong illusion of increased vertical extent in a turn of the sine-wave figure but none in the oblique middle section between turns. The data also show that this illusion of increased vertical extent persists when the curved section is completely separated from the sine-wave figure. In other words, the whole figure is not necessary for the occurrence of the illusion. Rather, the illusion is generated by some characteristic of the section about the crest and trough of the sine wave.

## 4 Discussion

The data for the two experiments show that the sine-wave illusion occurs in the regions of the turns in the waveform, ie about the crest and trough, and that it persists when these regions are separated from the figure and presented alone.

How then is the illusion to be explained? Day (in press) has proposed a general theory of veridical and illusory perception. He has argued that apparent properties of objects, situations, and events, eg their size, orientation, and distance, are commonly correlated with various physical properties of the stimulus array. Illusions occur when these properties, called stimulus correlates or 'cues', are manipulated in certain ways. The type of manipulation determines the particular class of perceptual illusions. One class derives from arranging the cues for the properties of an object so that they are in conflict. The outcome in perception is a compromise between the conflicting cues.

The Müller-Lyer illusion is a case in point. Among the cues for extent are the lengths of the figures themselves and the distances between their apexes. The latter can be called the target extent. A figure with the outward-directed angles is longer overall than this extent. In consequence, all end-to-end dimensions in the figure are greater than the target extent. The opposite state of affairs obtains in a figure with inward-directed angles; all end-to-end dimensions in the figure are smaller than the target extent. It is proposed that the differences between target and figure lead to a perceptual compromise so that the former appears either greater or less than its true extent.

The occurrence of an illusion of extent in the turns of the sine-wave figure and the absence of an illusion in the straight oblique sections can be accounted for in essentially the same terms. As shown in figure 4, the section of the figure at the turn is greater than the vertical target extent in all directions obliquely through the centre and also horizontally above about the lower third of the section. Thus a compromise between target extent and the overall size of the section can be expected to result in the former appearing greater than it is. The situation in the straight sections of the figure is different. Whereas the figure is greater than the target extent along the axis of the section and obliquely in one direction, transversely through the section and obliquely in the other direction the figure is less. In other words, the average extent of the figure approximates to the vertical extent. Therefore, it can be expected that the apparent extent will approximate to the true vertical extent, which it does.

Finally, in an earlier series of experiments on the Poggendorff illusion, Day et al (1987) found that the vertical extent between oblique (45°) parallels 15 mm apart was reproducibly underestimated by about 2–4 mm. In experiments 1 and 2 reported above there was no such vertical underestimation between the oblique (18°) parallel lines or edges about 14 mm apart. However, the stimulus figures and the procedures were both different in the earlier experiments. The parallels were orientated at a greater angle relative to the vertical, the whole display was tilted back from the subject at an angle of about 60°, and subjects matched the vertical extent by making dots on a sheet of paper placed flat on the table. One or more of these differences could have contributed to the different outcomes. This issue warrants further enquiry.

In conclusion, two informal observations are worth following up. First, the illusion also occurs in a V-shaped figure in which the vertical and oblique dimensions are similar to those in the turns of a sine-wave figure. This indicates that the illusion is not confined to a particular sine-wave element like that in figure 3. Second, as in

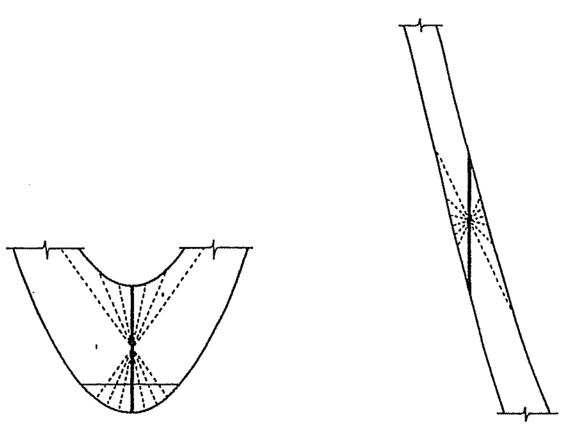


**Figure 4.** Relative vertical, oblique, and horizontal dimensions in the trough and middle section of the sine-wave figure. In the trough horizontal extents above the horizontal line are greater than vertical extents, and those below it are less.

numerous other figures exhibiting visual illusions, this sine-wave effect seems to occur in the haptic mode when the fingertips are moved vertically across the turns and straight sections of a flat object with the same shape as those in figure 2.

**Acknowledgments.** This research was supported by a grant to the first author by the Australian Research Committee. We gratefully acknowledge the assistance of Mike Durham with computer programming, Rosemary Williams with the illustrations, and Vladimir Kohout with photography.

**Addendum.** Following submission of this paper one reviewer drew our attention to two papers in press, the content of which bears closely on the illusion of vertical extent reported here. In the last of their four experiments Morgan et al (1991) found that a vertical line between horizontal parallels (ie an H figure rotated through 90°) appeared longer than that between 45° oblique parallels. This confirms by means of an alternative procedure the results reported earlier by Day et al (1987). However, it is to be noted that in this study the apparent extent at right-angles to the parallels (ie in the H figure) was compared directly with the apparent extent between the 45° tilted parallels. Therefore, it is not possible to deduce whether the vertical between the horizontal parallels (right-angled extent) appears longer than a vertical line alone, and/or whether the vertical between tilted parallels (oblique extent) appears shorter than the vertical alone.

Two experiments by Mather et al (1991) were also concerned with apparent extent between parallels, in this case the horizontal extent between vertical parallels (0°) and oblique parallels tilted by up to 80°. The apparent extent was found to decrease as the angle of tilt of the parallels increased up to about 50 – 55° and then to increase again (see their figure 2).

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