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The Poggendorff Illusion: Consider All the Angles

Daniel J. Weintraub, David H. Krantz, and Thomas P. Olson
University of Michigan

In the Poggendorff display, which consists of parallel lines interrupting a transversal, one of the two transversal segments was replaced by a dot lying along the parallel. The angle between the remaining transversal segment and the parallels was varied in 15° increments, as was the orientation of the transversal with respect to the subject. Subjects set the dot to appear collinear with the transversal. Judgmental errors can be partitioned into additive components, one linearly related to the size of the obtuse angle between transversal and parallels and the other a sinusoidal function of transversal orientation (collinearity settings err toward the horizontal or vertical, whichever is closer), plus a meridional effect, an interaction term that magnifies the errors of a given obtuse angle as the transversal approaches an oblique orientation.

The Poggendorff display consists of parallel lines interrupting a transversal. In previous work (e.g., Weintraub & Krantz, 1971) one of the transversal segments was replaced with a dot lying on the parallel, as depicted in Figure 1. An important reason for the modification is that in the traditional configuration some subjects claim that the two transversal segments do not appear to be parallel. Thus, the achievement of perceived collinearity by translating one segment along a parallel cannot occur. The display modified as in Figure 1 was used in the experiment to be reported.

What requires explanation is the misperception of collinearity, an error in setting the dot along the parallel to lie on a visual extension of the transversal. The perceptual operation to be performed is no less important than the physical geometry of the stimulus. It is clear that adjusting a line to appear parallel to one of the sides of an angle (e.g., as in Blakemore, Carpenter, & Georgeson, 1970) produces outcomes different from the collinearity judgments necessary with the Poggendorff display (Weintraub & Tong, 1974). Emerson, Wenderoth, Curthoys, and Edmonds (1975) have carefully documented discrepancies among methods.

The goal is an adequate theory of the Poggendorff anomaly, and the experiment to be presented extends our previous efforts to determine the aspects of the configuration that lead to collinearity errors. The emphasis will be on gathering sufficient data to construct a response surface as a function of the physical variables. One of the problems is how best to describe the relationship between angular changes in the display and response errors. Major features of the experimental outcome turned out to be predictable from a relatively simple mathematical model. Some of the choices involved in developing the model were easy (e.g., between measuring errors in degrees or in millimeters) and others difficult (e.g., between characterizing the orientation of the display as the tilt of the transversal or as the tilt of the parallels).

Important features of the Poggendorff display are the width (W) between the parallels

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Requests for reprints should be sent to Daniel J. Weintraub, Human Performance Center, Perry Building, University of Michigan, 330 Packard Road, Ann Arbor, Michigan 48104.

and the acute angle (A) between the transversal and parallels. For the traditional display containing both transversal segments, the misjudgment (M) in linear units measured along the parallel is M = .162 W cot A (Weintraub & Krantz, 1971, corroborating Burmester, 1896). For a fixed acute angle A, the misjudgment is directly proportional to the width between the parallels. Therefore, the collinearity error measured in degrees does not depend on the width between the parallels. There are deviations from the proportionality of M to W, especially for a small W (Pressey & Sweeney, 1972; Tong & Weintraub, 1974), but the general finding is well established. (See also MacKay & Newbigging, 1977.) Errors will be measured in degrees; the width W between parallels will be fixed in the experiment to be reported.

Another critical variable is the orientation of the Poggendorff display in the subject's visual field (Weintraub & Krantz, 1971). For an acute angle A of 45°, the Poggendorff error approached zero whenever the transversal was oriented vertically or horizontally with respect to the subject, or, stated another way, whenever the parallels were oriented obliquely. No Poggendorff theory can ignore the angle between transversals and parallels, transversal orientation, and tilt of the parallels. Any two determine the third. How do they mutually determine collinearity errors? Displays containing various angles between transversal and parallels were presented in various orientations so that the three angle variables could be assessed every 15°.

Method

Subjects

Paid volunteers from the University of Michigan served, three groups of 48 subjects each.

Apparatus

A standing subject looked directly downward through a rigid opaque viewing mask (Lone Ranger mask) at the display that was lying horizontally on a table covered with a white cloth. The viewing mask set eye to screen distance at 57 cm. Narrow eye slits in the mask ensured the proper head position; with a misaligned head the subject could not view the display with both eyes. Side lighting to the left and to the right provided illumination. An identical display booth, separated by a partition, was located adjacent to the first.

Stimuli

The stimulus configuration was the single-transversal Poggendorff display depicted in Figure 1, drawn in black India ink (3% reflectance) on a white disk of drafting paper (85% reflectance), 45 cm in diameter. Line thickness was .3 mm. The parallels, 40 mm apart measured from the center of each line, were centered on the disk and proceeded all the way across. The transversal segment coincided with a radius of the disk and extended from a parallel to the nearest edge of the disk. A display was drawn for each angle (A) between transversal and parallel. All Poggendorff dimensions were verified under 25-power magnification. The disk, properly oriented for a given trial, was centered directly below the subject's hypothetical cyclopean eye. None of the aids for centering or orienting the disk on the table was ever visible to the subject.

Measuring from the transversal toward the parallel and following the convention of counterclockwise (CCW) angles as positive and clockwise (CW) angles as negative, 11 displays were drawn, 1 every 15° of Angle A as recorded in Figure 1, from −15° through +15°, including ±90, a display with the tranversal perpendicular to the parallels. Transversal orientation, Angle B, was achieved by rotating a given display in 15° increments through the left half of the subject's visual field (Mathematical Quadrants 2 and 3), from 90°, free end of the transversal pointing directly upward, through 180°, transversal pointing horizontally to the left, to and including 270°, transversal pointing directly downward, a total of 13 orientations.

The disk constituted an underlay. A 46-cm diameter matte-surface thin plastic (Mylar) sheet with a gently scalloped edge (scissored freehand) served as a transparent overlay. A black dot, 1.5 mm in diameter, on the underside of the Mylar overlay was located 10 cm from the edge. Thus, centering the overlay on the underlay or using the edge as an aid were meant to appear as obviously inappropriate strategies for positioning the dot. Illuminance at the location of the stimulus displays was 600 lux.

Procedure

The subject viewed the stimulus display binocularly through the mask and was instructed to adjust the overlay by hand, sliding the dot along the parallel *opposite the transversal, with the stipulation that the* overlay cover the entire transversal segment and most of the parallels. The task was to set the dot to lie on a visual extension of the transversal, that is, to produce a collinearity setting. The Poggendorff anomaly is a collinearity error toward the acute-angle side of the transversal-parallel intersection. Thus, the dot is set too low in the display depicted in Figure 1. Measurements were recorded as errors in millimeters.

Having produced a collinearity setting, the subject moved to the adjacent booth for the next judgment.

The experimenter, on the opposite side of the table from the subject in the booth just vacated, measured the response and positioned the next display.

The experiment was conducted in three parts. One group of subjects judged all Angle Bs from 180° through 270° for all +As and for half of the −As. The second group judged all Bs from 90° through 180° for all −As and for half of the +As. The third group judged the remaining −A and +A angles as well as six stimuli judged previously, three by each preceding group, as a check. The order of the 62 stimulus conditions was chosen randomly for each subject in a group (36 randomized conditions for the third group). Each condition was judged once by each subject.

Results

Measurements were first transformed from errors in millimeters to errors in degrees; CCW collinearity errors are designated as positive and CW collinearity errors as negative. The outcome of the experiment, mean error in degrees as a function of the acute angle subtended between transversal and parallel (Angle A) and transversal orientation (Angle B), is shown in Figure 1. The 143 data points systematically cover B/A combinations in which the single transversal lies toward the observer's left. For the 17 points that were judged by two groups of subjects (the following B/A pairs: 105/75, 135/45, 165/15, 180/allA, 195/−15, 225/−45, 255/−75), mean errors are based on 96 judgments.

Note first that for any transversal orientation, B, there is indeed the usual Poggendorff anomaly. The dot is displaced from true collinearity toward the acute-angle side of the display. Furthermore, the strong influence of the size of the subtended angle is evident. For example, at B = 225°, as the acute Angle A approaches 90°, the judgmental error declines toward zero. The ordering of errors as a function of A tends to be preserved at every transversal orientation. Second, the major impact of B occurs via an AB interaction. In words, there is always an A effect, but it is minimized when the collinearity path is horizontal or vertical, giving a wasp-waisted appearance to Figure 1. Finally, there are undulations that seem to be superimposed on this pinching effect. These might be characterized as the smaller independent effect of B.

The decision to present Poggendorff errors in degrees as a function of A and B is the culmination of our attempts to describe and understand the nature of the anomaly. The orientation of the parallels

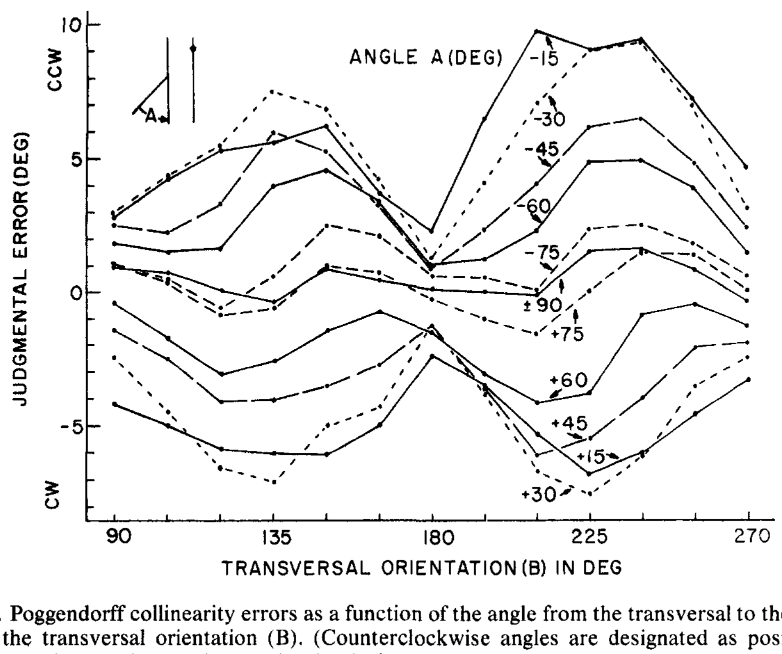

Figure 1. Poggendorff collinearity errors as a function of the angle from the transversal to the parallels (A) and the transversal orientation (B). (Counterclockwise angles are designated as positive. The transversal points straight up in the visual field for B = 90, horizontally to the left for B = 180, and down for B = 270. DEG = degrees.)

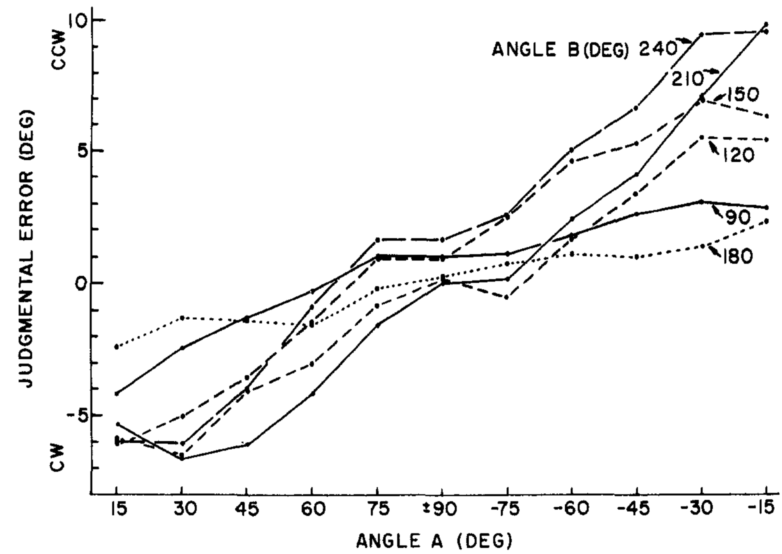

Figure 2. Data replotted from Figure 1 to show, for a given B, the linear trend between Angle A and collinearity errors. (DEG = degrees.)

was treated initially as a salient variable because, introspectively, the "uprightness" of the configuration seemed to depend on it. Nevertheless, no insights were gained by considering the data as a function of orientation of the parallels, and attempts at modeling were hindered.

Consider Figure 1 to represent a response surface in three dimensions by imagining Variable A plotted along a third axis extending backward from the plane of the figure. The response surface climbs as it recedes; valleys rise to become peaks. Mentally rotate the three-dimensional representation to view it from the side. The side view is equivalent to presenting A along the abscissa and connecting points of equal B, which is the representation in Figure 2. For clarity, only six B functions are shown, 90° through 240°, in increments of 30°, but all functions show strong linear trends. A least-squares best-fit line to each of the 13 sets of B data gives a small standard deviation about the line; the median deviation is .74°, the largest is 1.24°. Each line represents a contour of the response surface at a particular transversal orientation. The lines differ in slope, being shallow for vertical and horizontal transversals, another way of representing the pinching of Figure 1. The lines also differ in intercept if A = ± 90 is taken as the origin, another way of representing the undulations of Figure 1.

Given the linear trends, linearity can be assumed and the slopes and intercepts used to characterize each straight line. The slopes and intercepts derived from the least-squares straight line for each B are plotted in Figure 3. The functions connecting the points were fitted by eye.

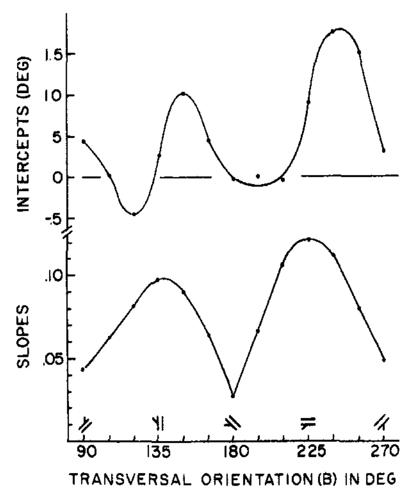

Figure 3. Intercepts and slopes from least-squares straight lines, a line fitted to the As at each transversal orientation, B. (DEG = degrees.)

What has been lost by describing the data as the slopes and intercepts of straight lines? The main effect of A can be determined by averaging the Bs at each A. The resulting function based on these averages is relatively straight, of course. But, plotted in the coordinates of Figure 2, the lower limb (15°–90°) is slightly S shaped, as is the upper limb (−90° to −15°). Using the average function, the "main effect of A" described earlier, in place of a straight line for fitting the individual Bs resulted in little improvement in variance accounted for. There would be good reason to investigate the Poggendorff anomaly in detail for $|A|$ less than 15°, because this region is most likely to reveal large deviations from linearity, if any exist.

Discussion

Let us construct a model, explaining and defending as we proceed. The ultimate theory of the Poggendorff must take into consideration both the subtended angle between transversal and parallels (A) and the orientation of the transversal (B). Although the A effect has been designated by the associated acute angle, the evidence is very strong that the major geometrical determinant is the obtuse angle rather than the acute angle. A convincing type of evidence is obtained by removing line segments from the configuration. Removing any element that destroys the obtuse angle greatly attenuates the anomaly, sometimes reversing it; destroying the acute angle has a minor effect (e.g., Houck & Mefferd, 1973; Krantz & Weintraub, 1973; Weintraub & Krantz, 1971).

For a given transversal orientation, the obtuse-angle (A) effect has a strong linear component, an outcome that was not anticipated. Data have not previously been available to assess the obtuse-angle effect while holding *transversal* orientation constant.

The main effect of B can be described by the intercepts function of Figure 3. A single line (e.g., Bouma & Andriessen, 1968) or lines that do not intersect (Weintraub & Virsu, 1972) or lines that do (Weintraub & Krantz, 1971; Hotopf, Ollerearnshaw, & Brown, 1974) conform to what Hotopf et al. have called the horizontal-vertical alignment tendency: A collinearity judgment is more horizontal or vertical than it should be depending on whether the line to be judged is oriented in the visual field nearer the horizontal or the vertical axis. The postulate states that the tendency will be zero at horizontal, at vertical, and at the 45° obliques where opposing tendencies cancel; it has been modeled by a sinusoidal function. The intercepts data of Figure 3 tend to oscillate as expected except that they do not oscillate about zero. A term of the form $t + v \sin 4B$ will model the intercept data reasonably well.

The AB interaction is represented in the slopes data of Figure 3. Judgmental errors are constrained as the transversal approaches horizontal or vertical in the visual field. This is a version of the meridional effect (Sekuler, 1974) that manifests itself as poorer performance (reduced acuity, greater errors, etc.) along the obliques. (See a comprehensive review by Appelle, 1972.) The slopes data can be interpreted as a susceptibility function. For each A, judgmental error is smallest at the cardinal visual axes. Errors become magnified (multiplied by the larger slope parameter) as the obliques are approached. A function of the form $h + k|\sin 2B|$ captures the effect; h represents minimum error susceptibility at horizontal and vertical.

In summary, consider the data of the experiment as plotted in Figure 2. The action of Variable A, the angle between transversal and parallels, is assumed to be linear. The slope is the susceptibility factor, a function of Variable B (the transversal orientation) that interacts with A. The intercept associated with a given B, considering the origin at $A = \pm 90°$, is the noninteractive component of B.

A Poggendorff Model

Let collinearity errors, E, be measured in degrees. To translate the origin to $A = \pm 90°$ and change the sign, let the acute angle A between transversal and parallel be transformed to the corresponding angle of incidence, A_I, which is, like an incident light ray, the number of degrees from the transversal to a perpendicular erected on the parallel, that is, $A_I = A \pm 90$, +90 for negative A, −90 for positive A. To repeat, B is the orientation of the transversal. With all counterclockwise angles designated as positive, $E = f(B)A_I + g(B)$, with $f(B) = h + k|\sin 2B|$, $g(B = t + v \sin 4B$. For each B, the slope, $f(B)$, and intercept, $g(B)$, were estimated by least squares (estimates plotted in Figure 3). Then, the least-squares line predicting $f(B)$ from $|\sin 2B|$ yielded estimates of h and k. Similarly, the line predicting $g(B)$ from sin 4B yielded estimates of t and v. Based on this two-level least-squares approach, the model for predicting Poggendorff collinearity errors (in degrees) is $E = (.0380 + .0684|\sin 2B|)A_I + (.481 - .761 \sin 4B)$.

The plotted data contain complex regularities that entertain the eye, whereas the model seems rather simple. Nevertheless, predicted E accounts for 93.8% of the variance among the observed values of E plotted in Figure 1.

Although good marks might be awarded for ingenuity (and persistence) in curve fitting, the mettle of the model is tested by predicting data other than those from which it was derived. Table 1 exhibits the correlations of our predicted E with the observed data of five earlier studies.

Before we comment in detail on these studies, it should be noted that a linear regression model relating observed to predicted E in effect allows two degrees of freedom worth of parameter change in the basic model; all of the parameters, h, k, t, and v, are multiplied by the regression slope, and then the constant, t, is modified further by the intercept. Given the differences in display configurations, procedures, and so forth, these parameter changes are not unreasonable. In the two best cases, however (rows 3 and 5 of Table 1), less change in parameters was needed for a good fit: The intercept could be omitted with not too much loss for the Hotopf et al. data, whereas no parameter change at all was needed to give a good prediction of our own study (Weintraub & Krantz, 1971, Figure 5).

Weintraub and Krantz (1971, Figure 1) varied A while holding the parallels vertical, so that B covaried with A. Linear errors were converted to degrees and averaged across the different widths between parallels. These data were initially fit by the cotangent equation discussed in the introduction. However, the cotangent rule predicts poorly when the parallels are held constant at an orientation other than horizontal or vertical.

Velinsky's (1925) data (see also MacKay & Newbigging, 1977) were gathered from displays like those discussed in the previous paragraph. Linear errors were first converted to degrees error. Once again, parameter changes were needed.

Weintraub and Krantz (1971, Figure 5) held A, the angle between transversal and parallel, constant at 45°, and rotated the entire display (varied B) through 360°. These data (converted from error in millimeters) are predicted beautifully. In addition, the raw predictions from the model, that is those made with no parameter changes, accounted for .90 of the variance.

Anton (1976) collected Poggendorff evidence suitable for prediction. He kindly supplied an enlarged copy of Figure 4 from his article from which errors in centimeters were reconstituted. Errors were then converted to degrees and plotted for com-

Table 1
Assessing Predictions of Other Data via Linear Regression

Experiment	Points predicted	r	Slope	Intercept
Weintraub and Krantz (1971) Figure 1	4	.81	.65	−1.99
Velinsky (1925)	8	.70	1.22	−4.73
Weintraub and Krantz (1971) Figure 5	9	.99	1.31	.62
Anton (1976)	40	.48	2.15	9.53
Hotopf, Ollerearnshaw, and Brown (1974)	35	.90	.55	−.65

parison with Figure 1 of this article. The data are similar but lack the separation of Angle A effects to be found in Figure 1. We did not discern discrepant trends; rather, we judged the Anton data to be noisy. As Table 1 reveals, unimpressive (but statistically significant at the .01 level) proportions of variance can be predicted.

The Hotopf et al. (1974) data speak for themselves. Displays contained a variety of Angles A and B; errors were measured in degrees and listed in a table in the article. Data excepting the control condition were plotted for comparison with Figure 1. Similarities were evident by eye, and the proportion of variance accounted for (Table 1) corroborated the visual impression. The additive-components model developed by Hotopf et al. from their data contains a sin 4B term to account for the horizontal–vertical alignment tendency. The main effect of A, regression to right angles in Hotopf et al.'s terminology, and the AB interaction were handled differently, requiring, in all, six free parameters. Their own formulation accounted for .928 of the variance among judgmental errors. Modifying our raw predictions by only a multiplier (slope .50) accounted for .69 of the variance of the Hotopf et al. judgmental errors.

Table 1 can serve as a summary. Our model can predict evidence other than that from whence it sprang, and in the case of Hotopf et al. and one set of our own data, very well indeed. It is reasonable to conclude that the model has captured regularities existing across experiments.

There is evidence that tracking upward has an effect different from tracking downward. Weintraub and Virsu (1972) decided that subjects exhibit a bias toward placing their judgments too high in their visual fields. For an A of 45°, a greater Poggendorff error can be obtained when tracking downward compared to tracking upward, which can also be interpreted as an upward judgmental bias (Tong & Weintraub, 1974, Experiment VI; Weintraub & Krantz, 1971, Figure 5). The same result can be found in the present data. The error for the downward tracking A/B stimulus, −45/135, was 5.94°; for the corresponding upward tracking stimulus, 45/225, it was −5.47, a difference of .47 favoring the downward track. However, the finding does not hold for all pairs of A. From Figure 3 it can be noted that both slopes and intercepts were greater when B > 180°, that is when tracking upward. The increase in slope has the effect of magnifying judgmental errors. The raised intercepts, however, increase the positive (CCW) errors associated with negative A but bring negative (CW) judgmental errors associated with positive A closer to zero (see Figure 2). The bias, then, is not as Weintraub and Virsu (1972) would have it, an elevator factor (i.e., a simple bias for elevating every judgment slightly). The perturbations are not likely to be chance outcomes. Yet, they are relatively small, and we have little by way of theory to offer. Therefore, they are not accounted for in the model.

Summary

A model containing four free parameters was used to estimate 143 data points gathered by varying the orientation of the Poggendorff transversal and the angle between transversal and parallels. The model nicely describes the data on which it is based. It nicely predicts data gathered by Hotopf et al. without modification of the empirical constants. More complicated models can be better, but we have managed to approximate the main characteristics of such data with a relatively modest assortment of additive terms.

The terms of the model have, in some sense, been explained via theoretical statements. Judgments are made as though (a) obtuse angles are underestimated, (b) a horizontal–vertical alignment tendency exists, and (c) a meridional effect akin to increased error susceptibility at the visual obliques exists. We are inclined to accept such statements as theoretically meaningful. If so, then the Poggendorff anomaly does not have a simple single-factor explanation. There are obtuse-angle effects, orientation effects, and susceptibility effects.

A different approach to theory construction might partition our observed effects in quite a different way, but it does seem that the time is past for single-factor qualitative models. Any theory should be pitted against these quantitative data.

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