

# Signs of the Sine Illusion – why we need to care

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## Abstract

The sine illusion, which is part of a set of optical illusions which occur based on a conflict between a stimuli and the real-world situation which can cause similar stimuli, has a notable impact on perception of time-series data with a seasonal component. In this paper, we discuss the reasons for the illusion and various strategies which might be useful to break the illusion or reduce its' strength, demonstrating the presence of the illusion in real-world and theoretical situations. We also present data from user studies which demonstrate the dramatic effect the sine illusion can have on conclusions drawn from displayed data.

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## 1 Background and Introduction

this is where we need to introduce the sine-illusion  
and lay out the problem that we have with it from a statistical graphics side  
then we sketch out a remedy  
last paragraph is a sentence on the structure of the paper.

In the perception literature, the phenomenon is known as part of a group of geometrical optical misperceptions classified as Müller-Lyer illusions (Day and Stecher, 1991). Figure 1 shows a reproduction of Bach's applet (Bach, 2007) displaying the sine illusion. The sine illusion is closely related to the line width phenomenon – the line segments along the steepest slopes of the sine curve appear to be the shortest: "The illusion is explained in terms of a perceptual compromise between

the vertical extent and the greater overall dimensions of the section at the turn of the sine-wave figure and is thereby held to be the same in principle as the Müller-Lyer illusion.” (Day and Stecher, 1991). M Bach’s applet gives the option to compensate the line length manually for its perceived shortcoming. The amount of compensation chosen turns out to be highly dependent on both the length of the vertical line segments and the amplitude of the sine function. The amplitude directly affects the slope – the steeper the slope the more compensation is necessary, see section 3 for a more detailed discussion and some results from a user study .

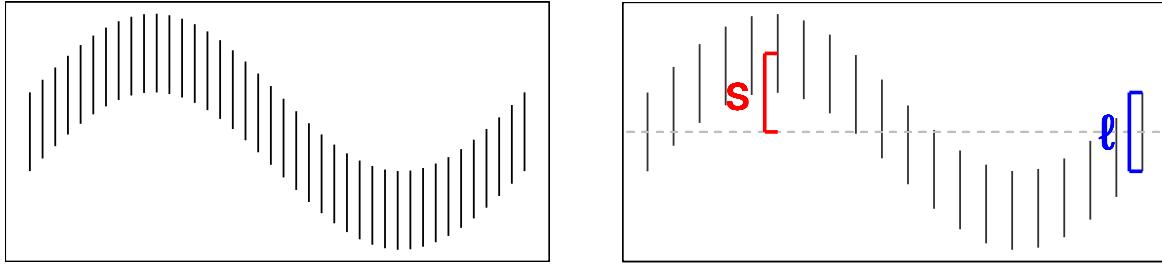


Figure 1: The original Sine Illusion. The regions of the graph in the peak and the valley appear to be composed of longer lines than the other regions.  $s$  denotes the amplitude of the function,  $\ell$  the length of the line segments.

could we start with our own data for the user study? that way we don't need IRB approval for now. data is in the data folder. I don't see any clear relationship, though/

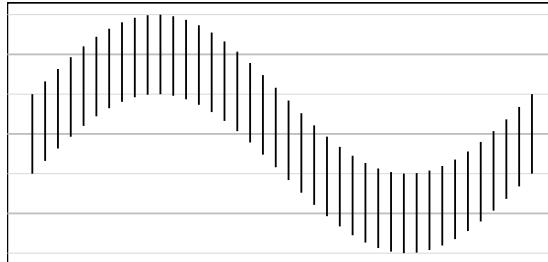


Figure 2: Horizontal grid lines do not help much in resolving the illusion, even though they provide a clear basis for comparison of line lengths.

## Discussion of Sine Illusion

Using the notation given in the applet, we assume a value  $s$  for the sine amplitude,  $\ell$  for the vertical length of line segments, and  $\ell_c$  for the amount of compensation. The sine curve is then given by function  $f$ , written as

$$f(x) = s/2 \sin(x)$$

for  $x$  in  $[-\pi, \pi]$ . The slope of the sine curve is the steepest in its reflection point at  $x = 0$ . We can calculate the slope as  $f'(x) = s/2 \cos(x)$ , which in  $x = 0$  yields a slope of  $s/2$ .

These differences in the slope are the reason we perceive the lines to have different length, as we have an innate tendency to evaluate line or band widths orthogonally rather than vertically, as would be the mathematically correct way.

Orthogonal  $w_o$  and vertical  $w_v$  line widths are related – the orthogonal line width depends on the angle (or, equivalently, the slope) of the line:

$$w_o = w_v \cos \theta, \quad (1)$$

where  $\theta$  is the angle of the line with respect to the horizontal line.

Should we make the  $w_o$  vs. b and  $w_v$  vs. a terminology consistent?

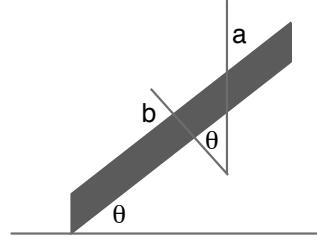


Figure 3: Sketch of line width assessments: (a) is showing vertical width, (b) shows width orthogonal to the slope. From previous surveys (we could cite Marie's survey here) we know that observers associate line width more with orthogonal width (b) than vertical width (a).

One way to correct for the sine illusion is to make all slopes the same (by transforming the  $x$  axis), another approach is to make the lines appear to be of the same height, by adjusting their lengths to maintain a constant orthogonal width. This is the approach that we pursue when transforming the  $Y$  values.

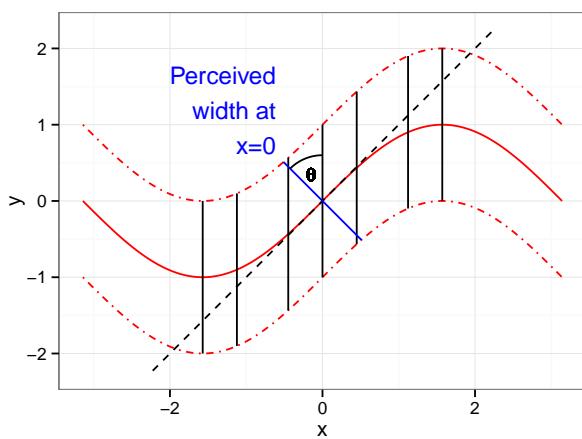


Figure 4: The perceived width orthogonal to the slope differs from the length of the lines.

## 2 Correction Approaches to the Sine Illusion

Web Link: [http://www.michaelbach.de/ot/sze\\_sineIllusion/index.html](http://www.michaelbach.de/ot/sze_sineIllusion/index.html)

The original authors (Day and Stecher, 1991) write in their abstract: The illusion is explained in terms of a perceptual compromise between the vertical extent and the greater overall dimensions of the section at the turn of the sine-wave figure and is thereby held to be the same in principle as the Müller-Lyer illusion.

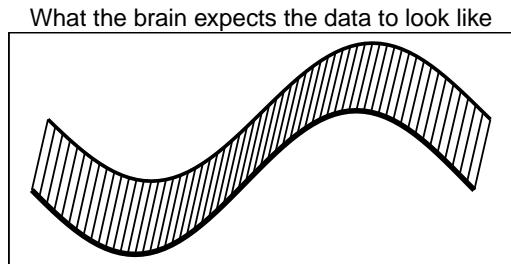


Figure 5: Experience with real-world objects suggests that the data are most similar to a slightly angled top view of a 3-dimensional strip such as a ribbon. Even though the sine illusion lines are not angled, the brain often recruits paradigms from one situation when processing novel stimuli. Optical illusions result from the mismatch between the applied heuristic and the stimulus. (Goldstein, 2010)

## 2.1 Data Setup

Describe the setup of the data in words and leave out the code from the paper.

We begin with lines of length 2 evenly spaced along the x-axis, where the center of the vertical line is at  $y = \sin(x)$ . For this example, the amplitude  $s = 1$  and the line length  $\ell = 2$ . Our goal is to break the sine illusion by transforming the line length,  $f(\ell) = \ell + \ell_c$  so that the perceived width is constant.

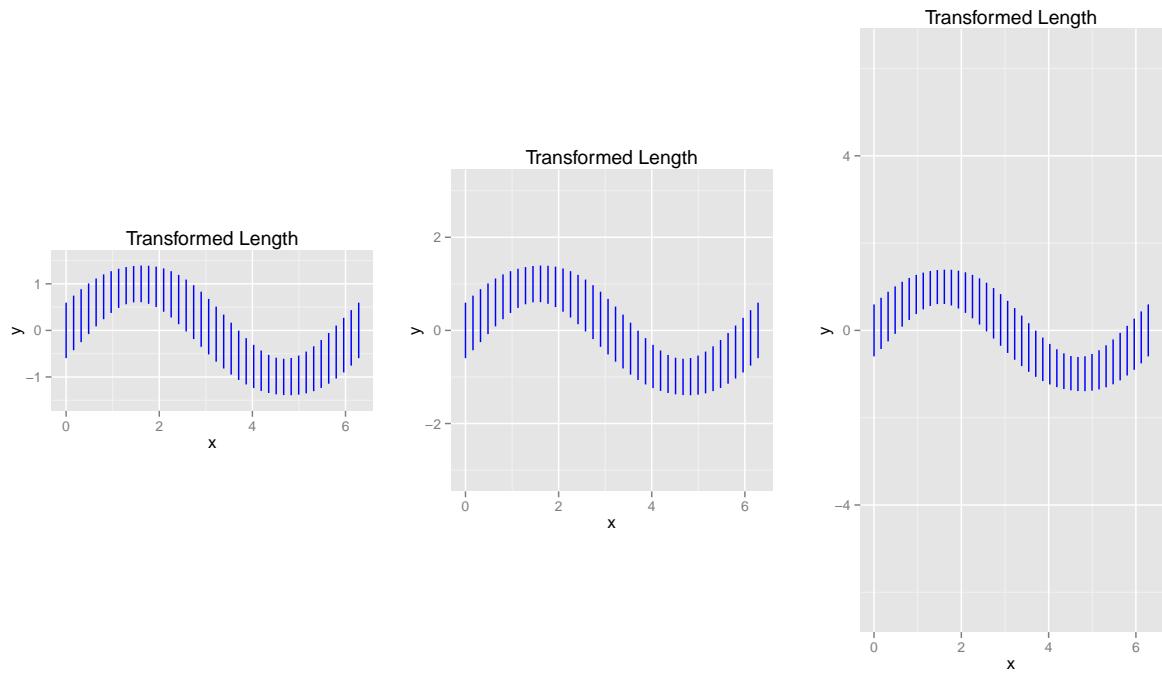
## 2.2 Transformation of Y values

This approach aims to extend the orthogonal width, which onlookers generally evaluate, to the amount of the vertical width, which we want to represent. For that, we multiply the vertical width by a factor of  $1/\cos \theta$ , where  $\theta$  is the angle under which the line is drawn. This angle is given by the slope of the function, mathematically we know that in position  $x$  the angle  $\theta(x)$  of function  $f(x)$  is given as:

$$\tan \theta(x) = f'(x).$$

We only distinguish the size, not the sign of the angle, which gives us an overall correction factor for the line width in vertical direction of

$$\text{atan} (\cos |f'(x)|)^{-1} \quad (2)$$



(a) Aspect Ratio of .5

(b) Aspect ratio of 1

(c) Aspect ratio of 2

Figure 6: Changing aspect ratios changes the strength of the illusion and the effectiveness of the transformation

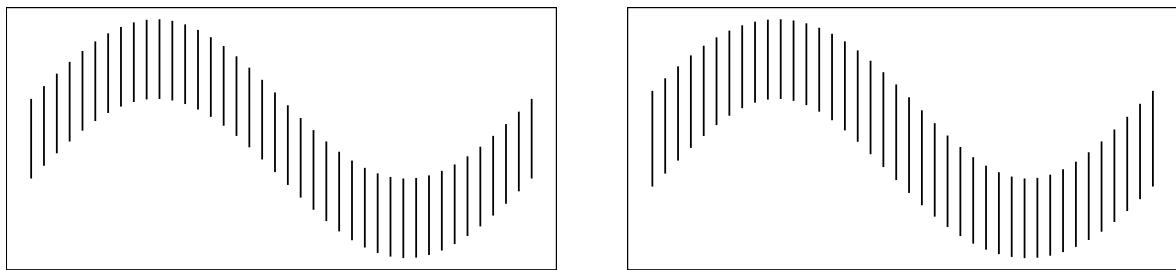


Figure 7: First fix of the sine illusion by using the factor of  $1/\cos(\text{atan}(|f'(x)|))$  for adjusting the vertical extent of the lines.

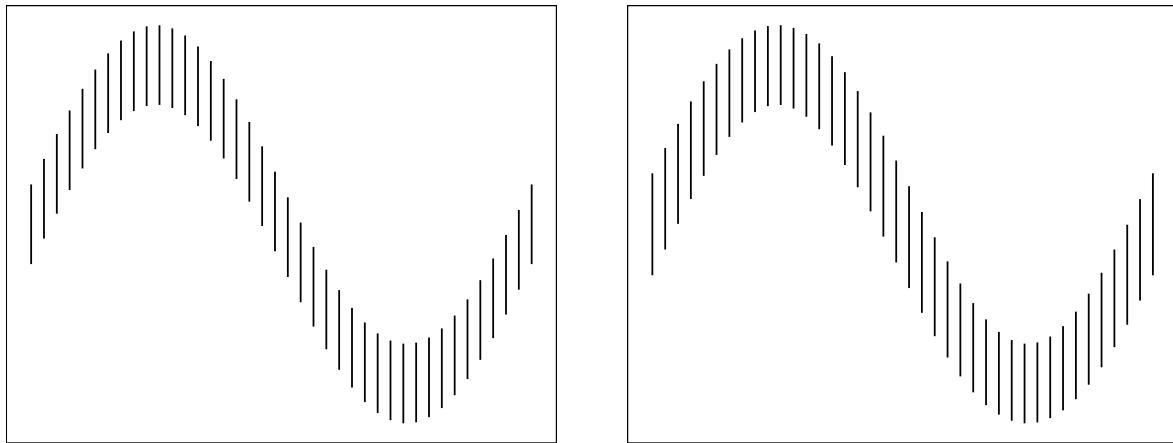


Figure 8: Aspect ratio seems to be a scalar in the correction factor.

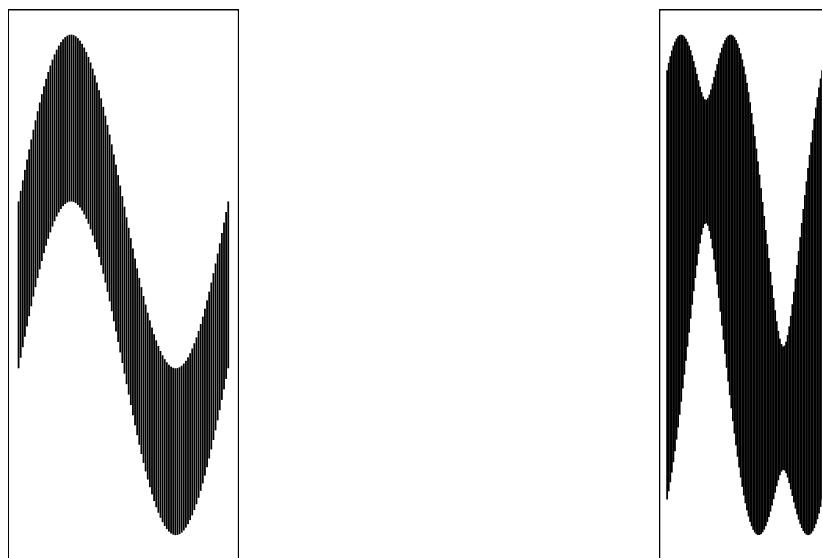
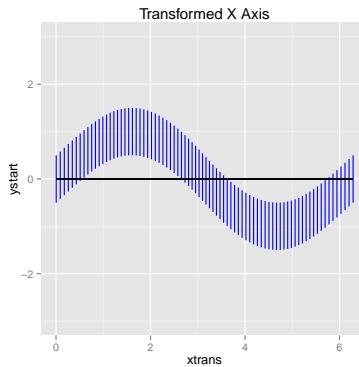


Figure 9: The ratio of the length of the line to amplitude of the function needs to be included in the correction. Here, the amplitude is still 1, but the length of the line is 5. The correction factor leads to a massive over correction that makes the function almost unrecognizable. The plot on the left is uncorrected, the plot on the right is ‘corrected’.

## 2.3 Transformation of X Values

Could you try to write the correction factor out in terms of a formula? - I still think that all of the lines should appear under the same slope, which means that the result would need to be an up/down triangle, like we had before. I'm a bit worried about the 'fudge' factor  $a$  – let's first use a correction factor that we can derive mathematically from counteracting the line illusion and then allow some fudging.

In order to counteract the illusion of changing line length, we can slightly transform the  $X$  values, narrowing the distance between values at the peak and trough of the curve and widening the distance between values at the inflection point. If we alter the  $X$  axis such that  $\Delta X^* = c * a(\cos(2x) + 1)$ , where  $\Delta X = c$  is the distance between two evenly spaced  $X$  points, the illusion's effect is broken and all  $y$  values appear to be approximately the same length. Consistent with the hypothesis that the apparent line length is dependent on the slope of the tangent line, changing  $\Delta x$  reduces the apparent length discrepancy  $\Delta y$  significantly.

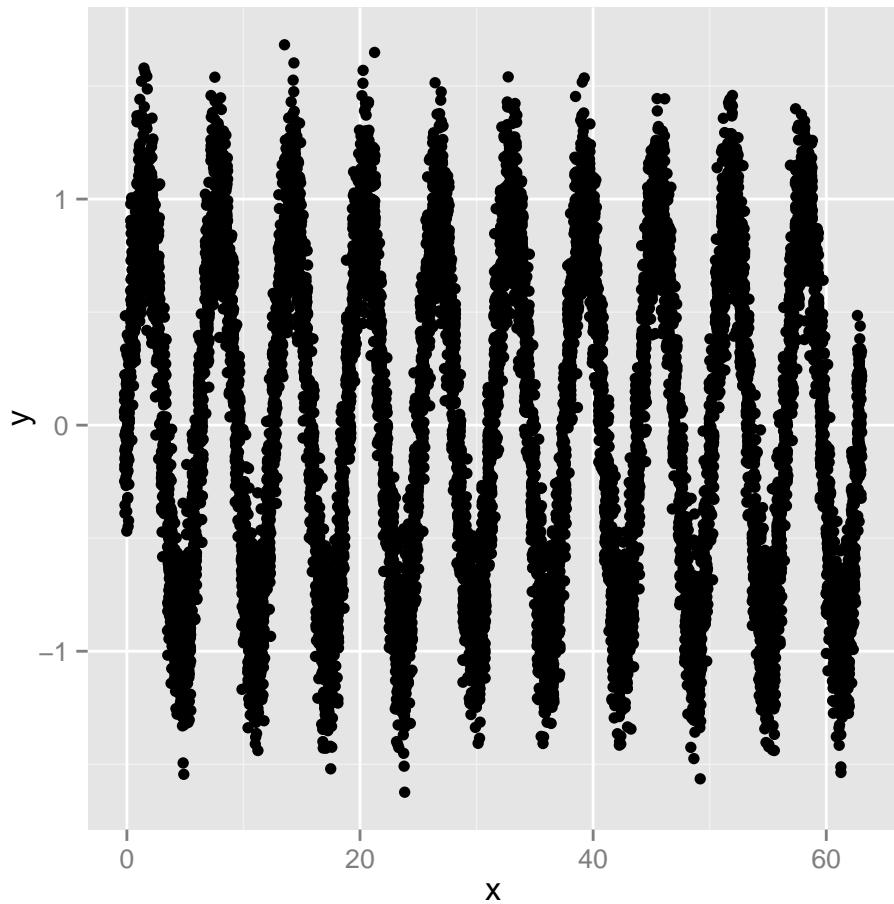


## 3 Distortion

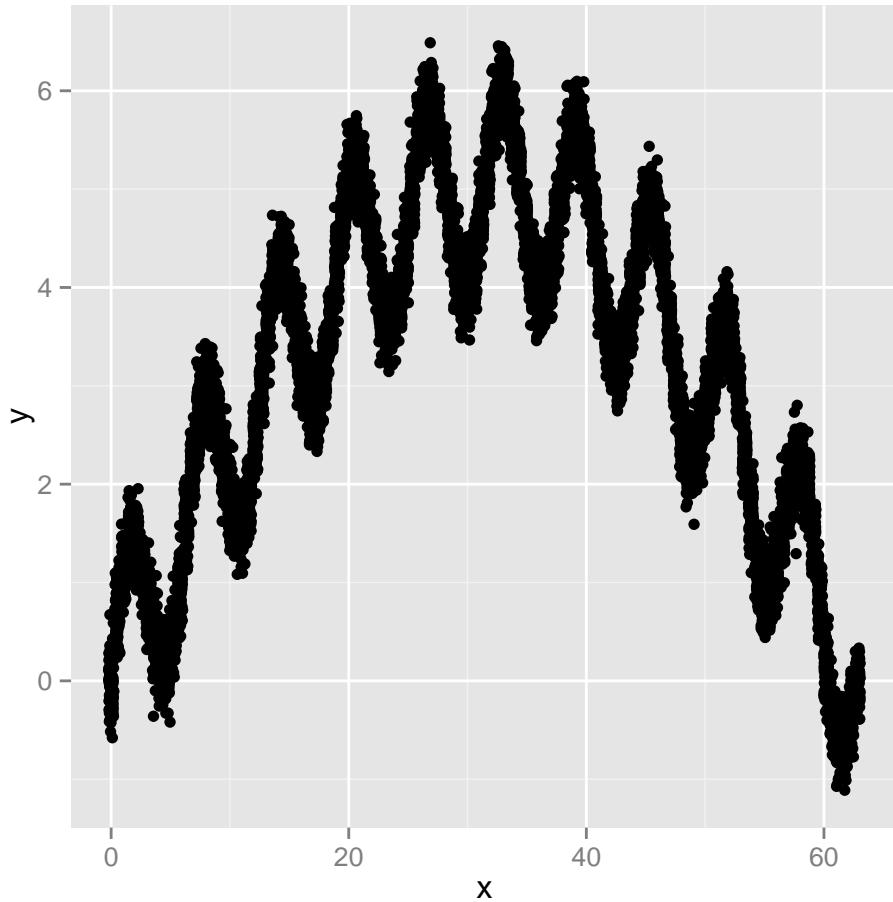
place holder

## 4 Application: Simulation

```
x <- rep(seq(0, 2 * pi * 10, length = 120), each = 100)
y <- rnorm(n = length(x), mean = sin(x), sd = 0.2)
qplot(x, y, geom = "jitter")
```



```
x <- rep(seq(0, 2 * pi * 10, length = 120), each = 100)
y <- rnorm(n = length(x), mean = 5 * sin(x/20) + sin(x), sd = 0.2)
qplot(x, y, geom = "jitter")
```



## 5 Application: USHCN Data

Figures 10 and 12 show temperature data from 1218 stations across the US. The first plot shows raw temperatures, the second residuals from a ‘baseline’ model. The caption for the raw temperature plot suggests that the baseline model is not suitable to capture all of the variability in summer and winter peaks. However, the model used is actually based on averages in each time point. This model is not susceptible to shrinking values, and it is our perception of the plot rather than the data that makes us see any higher variability in the peaks. After removing the averages in figure 12 the impression of higher variability in the peaks is gone with the exception of a few outliers.

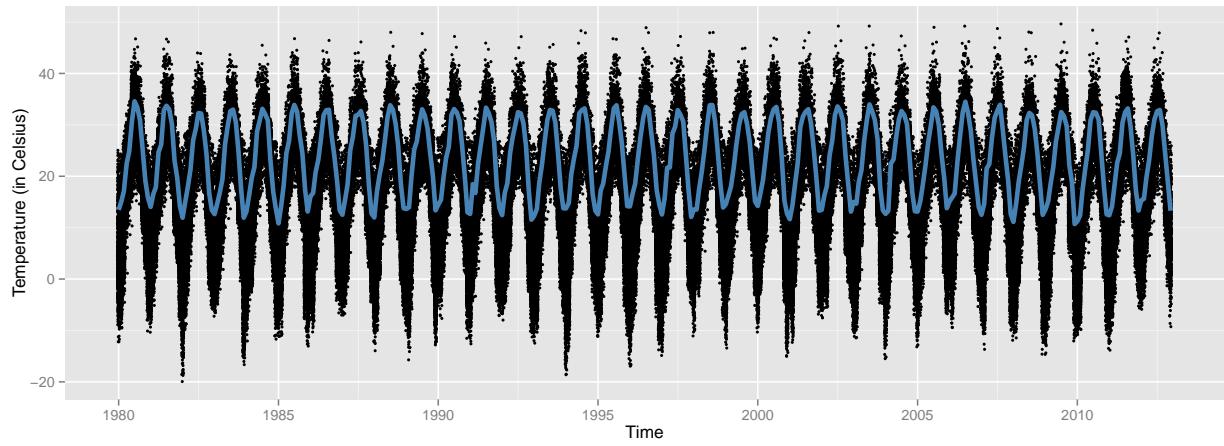


Figure 10: Raw monthly maximum temperatures (in Celsius) between 1980 and 2012 at 1218 stations across the US. The blue line shows fitted temperatures from a baseline model. The plot suggests that the model fails to capture the more extreme values in summer and winter.

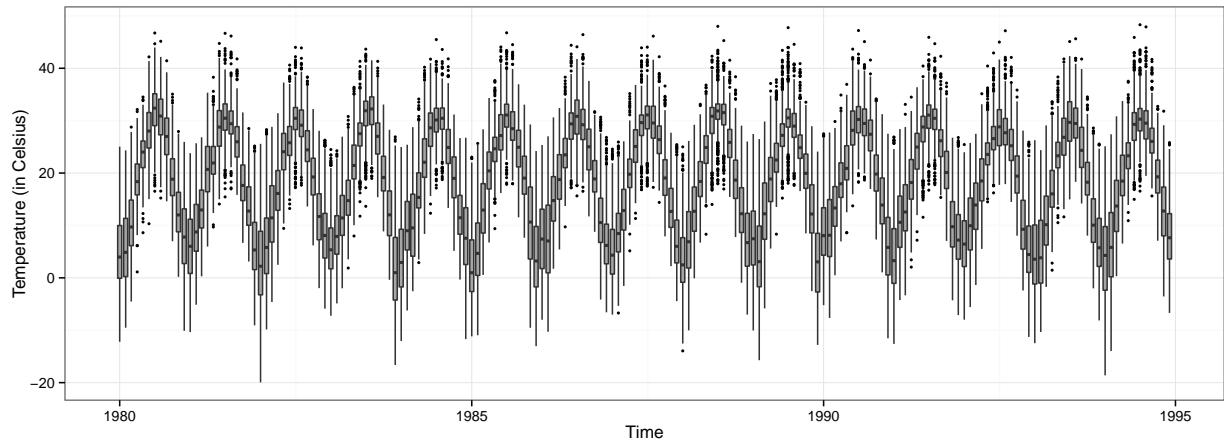


Figure 11: Monthly boxplot of maximum temperatures (in Celsius) between 1980 and 1995 at 1218 stations across the US. The sine illusion is less powerful but still visible in three locations: the box area of the boxes, the whiskers, and in the outliers.

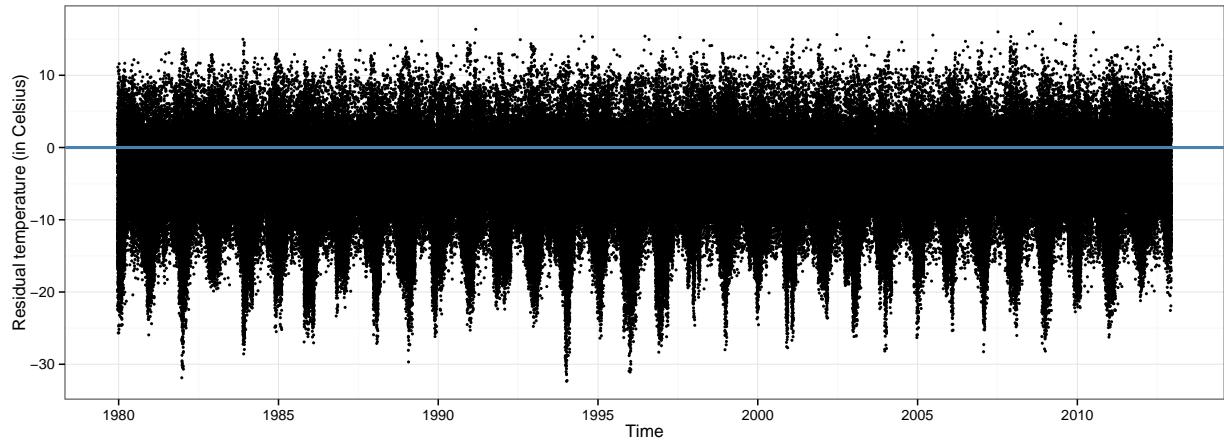
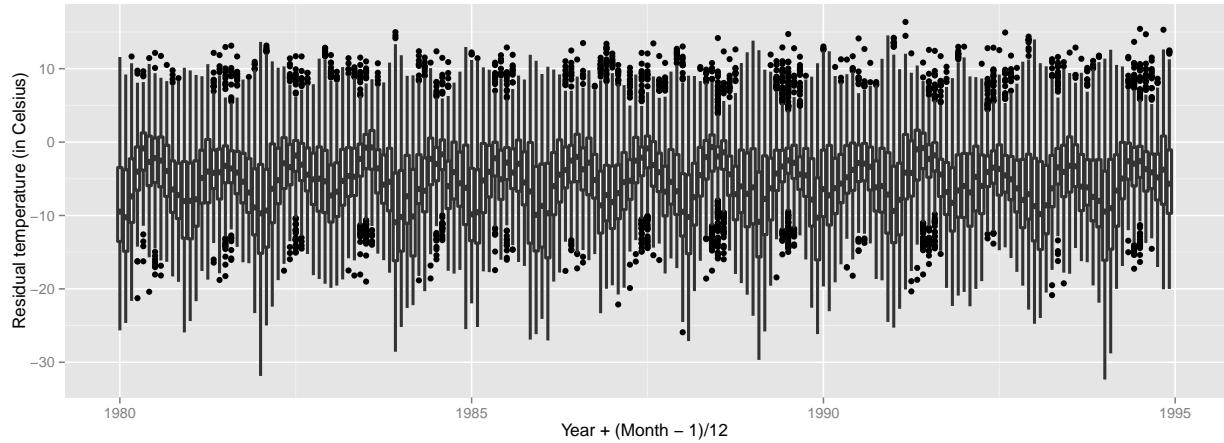


Figure 12: Residual monthly temperatures (in Celsius) from the baseline model fit. The original impression of smaller deviations during Spring and Fall is now almost gone: Winters have larger negative residuals, while Summers have larger positive ones.



## References

- Bach, M. (2007), “Sine Illusion from Visual Phenomena & Optical Illusions,” .
- Day, R. H. and Stecher, E. J. (1991), “Sine of an illusion,” *Perception*, 20, 49–55.
- Goldstein, E. B. (2010), *Sensation & Perception*, Belmont, CA: Thomason Wadsworth.