Вариант 26

$\begin{cases} 2x+y=1 \\ 3x+2y=-3 \\ 4x+3y=-1 \end{cases}$ 1 Векторный вид: $\begin{pmatrix}2\\3\\4\end{pmatrix}x+\begin{pmatrix}1\\2\\3\end{pmatrix}y=\begin{pmatrix}1\\-3\\-1\end{pmatrix} \Longrightarrow \bar{a}_1=\begin{pmatrix}2\\3\\4\end{pmatrix}; \bar{a}_2=\begin{pmatrix}1\\2\\3\end{pmatrix}; \bar{a}_3=\begin{pmatrix}1\\-3\\-1\end{pmatrix}$

2 Нормальная система уравнений

$(\bar{a}_1,\bar{a}_1)x+(\bar{a}_1,\bar{a}_2)y=(\bar{a}_1,\bar{b})$

$(\bar{a}_1,\bar{a}_2)x+(\bar{a}_2,\bar{a}_2)y=(\bar{a}_2,\bar{b})$

$(\bar{a}_1,\bar{a}_1)=2\cdot2+3\cdot3+4\cdot4=4+9+16=29$

$(\bar{a}_1,\bar{a}_2)=2\cdot1+3\cdot2+4\cdot3=2+6+12=20$

$(\bar{a}_2,\bar{a}_2)=1\cdot1+2\cdot2+3\cdot3=1+4+9=14$

$(\bar{a}_1,\bar{b})=2\cdot1+3\cdot(-3)+4\cdot(-1)=2-9-4=-11$

$(\bar{a}_2,\bar{b})=1\cdot1+2(-3)+3\cdot(-1)=1-6-3=-8$

$\begin{cases} 29x+20y=-11 \\ 20x+14y=-8 \end{cases}$

3 Решение системы по формулам Крамера

$\Delta=\begin{vmatrix}29 & 20\\20 & 14\end{vmatrix}=29\cdot14-20\cdot20=6$ $x=\frac{\Delta x}{\Delta}=\frac{6}{6}=1$

$\Delta x=\begin{vmatrix}-11 & 20\\-8 & 14\end{vmatrix}=-11\cdot14+20\cdot8=6$ $y=\frac{\Delta y}{\Delta}=\frac{-12}{6}=-2$

$\Delta y=\begin{vmatrix}29 & -11\\20 & -8\end{vmatrix}=29(-8)+20\cdot11=-12$ $(1;-2)$

Вектор невязок: $\bar{\sigma}=\bar{b}-\bar{a}_1x-\bar{a}_2y=\begin{pmatrix}1\\-3\\-1\end{pmatrix}-\begin{pmatrix}2\\3\\4\end{pmatrix}+\begin{pmatrix}1\\2\\3\end{pmatrix}2=\begin{pmatrix}-1\\-6\\-5\end{pmatrix}+\begin{pmatrix}2\\4\\6\end{pmatrix}=$

$=\begin{pmatrix}1\\-2\\1\end{pmatrix}\Longrightarrow\bar{\sigma}=\cancel{(b-a_1x)}\;(\bar{\sigma},\bar{\sigma})=1+4+1=6$

Проверка

$\begin{pmatrix}2\\3\\4\end{pmatrix}\begin{pmatrix}1\\-2\\1\end{pmatrix}=2-6+4=0$

Ответ: $x=1; y=-2; \delta^2=6$

№2

$A=\begin{pmatrix}-4 & -2\\3 & 3\end{pmatrix}$ Вычислить: $2A^3-3A^2; e^{At}$

1 Собственные значение матрицы A

$|A-\lambda E|=0 \Longrightarrow |A-\lambda E|=\begin{vmatrix}-4-\lambda & -2\\3 & 3-\lambda\end{vmatrix}=(-4-\lambda)(3-\lambda)+6=$

$=-12-3\lambda+4\lambda+\lambda^2+6=0$

$\lambda^2+\lambda-6=0$ $\Longrightarrow D=\begin{pmatrix}-3 & 0\\0 & 2\end{pmatrix}$

$\lambda_1=-3; \lambda_2=2;$

2 Собственные вектора

$\lambda_1=-3\Longrightarrow(A-\lambda E)\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\Longrightarrow\begin{pmatrix}-4+3 & -2\\3 & 3+3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$$\begin{pmatrix}-1 & -2\\ 3 & 6\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix} \Longrightarrow -x-2y+3x+6y=0$$

$$2x+4y=0$$

$$2x=-4y$$

$$x=-2y \Longrightarrow \bar{S}_1=\begin{pmatrix}-2\\ 1\end{pmatrix}$$

$$\lambda_2=2 \Longrightarrow (A-\lambda E)\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix} \Longrightarrow \begin{pmatrix}-4-2 & -2\\ 3 & 3-2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix} \Longrightarrow$$

$$\begin{pmatrix}-6 & -2\\ 3 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix} \Longrightarrow -6x-2y+3x+y=0$$

$$-3x-y=0$$

$$x=-\frac{1}{3}y \Longrightarrow \bar{S}_2\begin{pmatrix}-\frac{1}{3}\\ 1\end{pmatrix}$$

3. $T_{B\to S}\begin{pmatrix}-2 & -\frac{1}{3}\\ 1 & 1\end{pmatrix}$;

$$T^{-1}_{B\to S}=\frac{1}{|T_{B\to S}|}\cdot\begin{pmatrix}-2 & -\frac{1}{3}\\ 1 & 1\end{pmatrix}=-\frac{3}{5}\begin{pmatrix}1 & \frac{1}{3}\\ -1 & -2\end{pmatrix}=\begin{pmatrix}-\frac{3}{5} & -\frac{1}{5}\\ \frac{3}{5} & \frac{6}{5}\end{pmatrix}$$

Проверка: $T^{-1}_{B\to S}\cdot T_{B\to S}=\begin{pmatrix}-\frac{3}{5} & -\frac{1}{5}\\ \frac{3}{5} & \frac{6}{5}\end{pmatrix}\begin{pmatrix}2 & -\frac{1}{3}\\ 1 & 1\end{pmatrix}=\begin{pmatrix}-2+\frac{1}{3} & -\frac{2}{3}+\frac{2}{3}\\ 1-1 & \frac{1}{3}-2\end{pmatrix}=$

$$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

4. $f(A)=2A^3-3A^2=\begin{pmatrix}-2 & -\frac{1}{3}\\ 1 & 1\end{pmatrix}\begin{pmatrix}f(-3) & 0\\ 0 & f(2)\end{pmatrix}\begin{pmatrix}1 & \frac{1}{3}\\ -1 & -2\end{pmatrix}\left(-\frac{3}{5}\right)=$

$$\cancel{=\begin{pmatrix}-2 & -\frac{1}{3}\\ 1 & 1\end{pmatrix}\begin{pmatrix}4 & 0\\ 0 & 27\end{pmatrix}\begin{pmatrix}1 & \frac{1}{3}\\ -1 & 2\end{pmatrix}\left(-\frac{3}{5}\right)=\begin{pmatrix}-8 & -9\\ 4 & 27\end{pmatrix}}\ \frac{1}{3}$$

$$=\begin{pmatrix}-2 & -\frac{1}{3}\\ 1 & 1\end{pmatrix}\begin{pmatrix}-81 & 0\\ 0 & 4\end{pmatrix}\begin{pmatrix}-\frac{3}{5} & -\frac{1}{5}\\ \frac{3}{5} & \frac{6}{5}\end{pmatrix}=\begin{pmatrix}162 & -\frac{4}{3}\\ -81 & 4\end{pmatrix}\begin{pmatrix}-\frac{3}{5} & -\frac{1}{5}\\ \frac{3}{5} & \frac{6}{5}\end{pmatrix}=\begin{pmatrix}-98 & -34\\ 51 & 21\end{pmatrix}$$

Проверка:

$$2\begin{pmatrix}-4 & -2\\ 3 & 3\end{pmatrix}^3-3\begin{pmatrix}-4 & -2\\ 3 & 3\end{pmatrix}^2=2\begin{pmatrix}10 & 2\\ -3 & 3\end{pmatrix}\begin{pmatrix}-4 & -2\\ 3 & 3\end{pmatrix}-3\begin{pmatrix}10 & 2\\ -3 & 3\end{pmatrix}=$$

$$=2\begin{pmatrix}-34 & -14\\ 21 & 15\end{pmatrix}-\begin{pmatrix}30 & 6\\ -9 & 9\end{pmatrix}=\begin{pmatrix}-68 & -28\\ 42 & 30\end{pmatrix}-\begin{pmatrix}30 & 6\\ -9 & 9\end{pmatrix}=\begin{pmatrix}-98 & -34\\ 51 & 21\end{pmatrix}$$

5. e^{At}; $D=\begin{pmatrix}e^{-3t} & 0\\ 0 & e^{2t}\end{pmatrix}$

$$e^{At}=\begin{pmatrix}-2 & -\frac{1}{3}\\ 1 & 1\end{pmatrix}\begin{pmatrix}e^{-3t} & 0\\ 0 & e^{2t}\end{pmatrix}\begin{pmatrix}-\frac{3}{5} & -\frac{1}{5}\\ \frac{3}{5} & \frac{6}{5}\end{pmatrix}=\begin{pmatrix}-\frac{2}{e^{3t}} & -\frac{e^{2t}}{3}\\ \frac{1}{e^{3t}} & e^{2t}\end{pmatrix}\times$$

$$\times\begin{pmatrix}-\frac{3}{5} & -\frac{1}{5}\\ \frac{3}{5} & \frac{6}{5}\end{pmatrix}=\begin{pmatrix}\frac{-e^{5t}+6}{5e^{3t}} & \frac{-2e^{5t}+2}{5e^{3t}}\\ \frac{3e^{5t}-3}{5e^{3t}} & \frac{6e^{5t}-1}{5e^{3t}}\end{pmatrix}$$

Ответ: $2A^3-3A^2=\begin{pmatrix}-98 & -34\\ 51 & 21\end{pmatrix}$; $e^{At}=\begin{pmatrix}\frac{-e^{5t}+6}{5e^{3t}} & \frac{-2e^{5t}+2}{5e^{3t}}\\ \frac{3e^{5t}-3}{5e^{3t}} & \frac{6e^{5t}-1}{5e^{3t}}\end{pmatrix}$

$\sqrt{3}$

$F(x_1,x_2,x_3)=x_1^2+5x_2^2+x_3^2-4x_1x_2-4x_1x_3+4x_2x_3$

$A=\begin{pmatrix}1 & -2 & -2\\ -2 & 5 & 2\\ -2 & 2 & 1\end{pmatrix}$; $|A-\lambda E|=0$; $\begin{vmatrix}1-\lambda & -2 & -2\\ -2 & 5-\lambda & 2\\ -2 & 2 & 1-\lambda\end{vmatrix}=-\lambda^3+7\lambda^2+\lambda-7=0$

$\lambda_1=1$

$$\begin{array}{r|l} -\lambda^3+7\lambda^2+\lambda-7 & \lambda-1 \\ \hline -\lambda^3+\lambda^2 & -\lambda^2+6\lambda+7 \\ \hline 6\lambda^2+\lambda-7 & \\ 6\lambda^2-6\lambda & \\ \hline 7\lambda-7 & \\ 7\lambda-7 & \\ \hline 0 & \end{array}$$

$-\lambda^2+6\lambda+7=0$

$\lambda^2-6\lambda-7=0$

$\lambda_2=-1;\ \lambda_3=7$

$\lambda_1=1$

$\begin{cases}-2x_2-2x_3=0\\ -2x_1+4x_2+2x_3=0\\ -2x_1+2x_2=0\end{cases}$ $\begin{pmatrix}0 & -2 & -2\\ -2 & 4 & 2\\ -2 & 2 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=0$ $\sim\begin{pmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 0 & 1\end{pmatrix}$

$\begin{pmatrix}0 & -2 & -2\\ -2 & 4 & 2\\ -2 & 2 & 0\end{pmatrix}$ 2, 1 $\sim\begin{pmatrix}0 & -2 & -2\\ -2 & 0 & -2\\ -2 & 0 & -2\end{pmatrix}\sim$ $\begin{cases}x_2+x_3=0\\ x_1+x_3=0\end{cases}$ $\begin{cases}x_2=-x_3\\ x_1=-x_3\end{cases}$

$x_1=\begin{pmatrix}-k\\ -k\\ k\end{pmatrix}$; $||x_1||=\sqrt{k^2+k^2+k^2}=k\sqrt{3}$

$\lambda_2=-1$

$\begin{cases}2x_1-2x_2-2x_3=0\\ -2x_1+6x_2+2x_3=0\\ -2x_1+2x_2+2x_3=0\end{cases}$ $\begin{pmatrix}2 & -2 & -2\\ -2 & 6 & 2\\ -2 & 2 & 2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=0$

$\begin{pmatrix}2 & -2 & -2\\ -2 & 6 & +2\\ -2 & 2 & 2\end{pmatrix}\sim\begin{pmatrix}2 & -2 & -2\\ 0 & 4 & 0\\ 0 & 0 & 0\end{pmatrix}\sim\begin{pmatrix}1 & -1 & -1\\ 0 & 2 & 0\\ 0 & 0 & 0\end{pmatrix}$ $\begin{cases}x_1-x_2-x_3=0\\ 2x_2=0\end{cases}$ $\begin{cases}x_1=x_3\\ x_2=0\end{cases}$

$x_2 = \begin{pmatrix} k \\ 0 \\ k \end{pmatrix} \quad \|x_2\| = \sqrt{k^2+0+k^2} = k\sqrt{2}$

$\lambda_3 = 7$

$\begin{cases} -6x_1 - 2x_2 - 2x_3 = 0 \\ -2x_1 - 2x_2 + 2x_3 = 0 \\ -2x_1 + 2x_2 - 6x_3 = 0 \end{cases} \quad \begin{pmatrix} -6 & -2 & -2 \\ -2 & -2 & 2 \\ -2 & 2 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$

$\begin{pmatrix} 3 & 1 & 1 \\ -1 & -1 & 1 \\ -1 & 1 & -3 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 & 1 \\ 2 & 0 & 2 \\ +4 & 0 & -4 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & -2 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad x_3 = \begin{pmatrix} -k \\ 2k \\ k \end{pmatrix}$

$\begin{cases} x_2 - 2x_3 = 0 \\ x_1 + x_3 = 0 \end{cases} \quad \begin{cases} x_2 = 2x_3 \\ x_1 = -x_3 \end{cases} \quad \|x_3\| = \sqrt{k^2+4k^2+k^2} = k\sqrt{6}$

$x_1' = \frac{x_1}{\|x_1\|} = \frac{1}{k\sqrt{3}} \begin{pmatrix} -k \\ -k \\ k \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}; \quad x_2' = \frac{x_2}{\|x_2\|} = \frac{1}{k\sqrt{2}} \begin{pmatrix} k \\ 0 \\ k \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

$x_3' = \frac{x_3}{\|x_3\|} = \frac{1}{k\sqrt{6}} \begin{pmatrix} -k \\ 2k \\ k \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}; \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = S \cdot \begin{pmatrix} x_1'' \\ x_2'' \\ x_3'' \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{3}} x_1'' + \frac{1}{\sqrt{2}} x_2'' - \frac{1}{\sqrt{6}} x_3'' \\ -\frac{1}{\sqrt{3}} x_1'' \cdot \frac{2}{\sqrt{6}} x_3'' \\ \frac{1}{\sqrt{3}} x_1'' + \frac{1}{\sqrt{2}} x_2'' + \frac{1}{\sqrt{6}} x_3'' \end{pmatrix}$

Ответ: $F(x_1'', x_2'', x_3'') = x_1''^2 - x_2''^2 + 7x_3''^2$

$x_1 = -\frac{1}{\sqrt{3}} x_1'' + \frac{1}{\sqrt{2}} x_2'' - \frac{1}{\sqrt{6}} x_3''$

$x_2 = -\frac{1}{\sqrt{3}} x_1'' + \frac{2}{\sqrt{6}} x_3''$

$x_3 = \frac{1}{\sqrt{3}} x_1'' + \frac{1}{\sqrt{2}} x_2'' + \frac{1}{\sqrt{6}} x_3''$

№4

$7x^2-12xy-2y^2+10x+20y-35=0$

$A=\begin{pmatrix}7 & -6\\ -6 & -2\end{pmatrix}$ $|A-\lambda E|=\begin{vmatrix}7-\lambda & -6\\ -6 & -2-\lambda\end{vmatrix}=0$ $(7-\lambda)(-2-\lambda)-36=0$

$-14-7\lambda+2\lambda+\lambda^2-36=0$

$\lambda^2-5\lambda-50=0$

$\lambda_1=-5;\ \lambda_2=10$

$\Lambda=\begin{pmatrix}-5 & 0\\ 0 & 10\end{pmatrix}$

$\lambda_1=-5$

$(A-\lambda E)\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}7+5 & -6\\ -6 & -2+5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}12 & -6\\ -6 & 3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\Rightarrow$

$\begin{cases}12x-6y=0\\ -6x+3y=0\end{cases}\Rightarrow\begin{cases}x=1\\ y=2\end{cases}\Rightarrow \bar{s}_1=\begin{pmatrix}1\\2\end{pmatrix}$

$\bar{f}_1=\frac{1}{\sqrt{4+1}}\cdot\begin{pmatrix}1\\2\end{pmatrix}=\frac{1}{\sqrt5}\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt5}\\ \frac{2}{\sqrt5}\end{pmatrix}$

$\lambda_2=10$

$(A-\lambda_2E)\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}7-10 & -6\\ -6 & -2-10\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}-3 & -6\\ -6 & -12\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\Rightarrow$

$\begin{cases}-3x-6y=0\\ -6x-12y=0\end{cases}\Rightarrow\begin{cases}x=-2\\ y=1\end{cases}\Rightarrow\bar{s}_2=\begin{pmatrix}-2\\1\end{pmatrix}$

$\bar{f}_2=\frac{1}{\sqrt5}\begin{pmatrix}-2\\1\end{pmatrix}=\begin{pmatrix}-\frac{2}{\sqrt5}\\ \frac{1}{\sqrt5}\end{pmatrix}$

Матрица перехода: $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt5} & -\frac{2}{\sqrt5}\\ \frac{2}{\sqrt5} & \frac{1}{\sqrt5}\end{pmatrix}\begin{pmatrix}x_1\\y_2\end{pmatrix}$

$\begin{cases}x=\frac{1}{\sqrt5}x'-\frac{2}{\sqrt5}y'\\ y=\frac{2}{\sqrt5}x'+\frac{1}{\sqrt5}y'\end{cases}\Rightarrow\begin{cases}x+2y=\frac{5}{\sqrt5}x'\\ y-2x=\frac{5}{\sqrt5}y'\end{cases}\Rightarrow\begin{cases}x'=\frac{1}{\sqrt5}(x+2y)\\ y'=\frac{1}{\sqrt5}(y-2x)\end{cases}$

$-5x'^2+10y'^2+10\cdot\left(\frac{1}{\sqrt5}x'-\frac{2}{\sqrt5}y'\right)+20\left(\frac{2}{\sqrt5}x'+\frac{1}{\sqrt5}y'\right)-35=0$

$-5\left(x'^2-2x'\frac{5}{\sqrt5}+\left(\frac{5}{\sqrt5}\right)^2\right)+10y'^2-35=\left(\frac{5}{\sqrt5}\right)^2\cdot(-5)$

$-5\left(x'-\frac{5}{\sqrt5}\right)^2+10y'^2=35-25;$ $x''=x'-\frac{5}{\sqrt5}$

$-5x''^2+10y''^2=10\ |:10$ $y''=y'$

$-\frac{x''^2}{(\sqrt2)^2}+y''^2=1$ – гипербола

$a=\sqrt2$ $y''=\pm\frac{b}{a}x_2\Rightarrow y''=\pm\frac{1}{\sqrt2}x''$

$b=1,$

$$\begin{cases} x'' = \frac{1}{\sqrt{5}}(x+2y) - \frac{5}{\sqrt{5}} \\ y'' = \frac{1}{\sqrt{5}}(y-2x) \end{cases} \Longrightarrow \begin{cases} \sqrt{5}\,x_2 = x+2y-5 \\ \sqrt{5}\,y_2 = y-2x \end{cases} \Longrightarrow \text{оси} \begin{cases} x+2y=5 \\ y=2x \end{cases}$$

центр $x+4x=5$ $\begin{cases} x=1 \\ y=2 \end{cases}$