

I. ILLEGITIMATE TRANSFORMATIONS

In this section we will consider the transformation from Gibbons form of GR solution to the Horava-type solution. We start with the Gibbons solution given in [1] (with $\lambda = 0$)

$$ds^2 = \left(-1 + \left(\frac{2Mr}{a^2 + r^2 - a^2\mu^2} \right) \right) dt^2 - \frac{4aMr\mu^2}{a^2 + r^2 - a^2\mu^2} dt d\varphi + \frac{a^2 + r^2 - a^2\mu^2}{a^2 - 2Mr + r^2} dr^2 \\ + \left(a^2 - \frac{r^2}{\mu^2 - 1} \right) d\theta^2 + \frac{a^2(a^2 + r(r - 2M))\mu^4 - (r^2 + a^2)^2\mu^2}{a^2(\mu^2 - 1) - r^2} d\varphi^2 \quad (1)$$

Then we consider the following transformation with the unknown function $H(r, \mu)$

$$d\tau = dt, \quad dR = dr, \quad d\tilde{\mu} = d\mu, \quad d\tilde{\varphi} - H(r, \mu)d\tau = d\varphi, \quad (2)$$

where $\tau, R, \tilde{\mu}, \tilde{\varphi}$ are the new coordinates. The new components of the metric function can be found solving the following set

$$g_{tt} - 2g_{t\varphi}H(r, \varphi) + g_{\varphi\varphi}H(r, \varphi)^2 - g_{\tau\tau} = 0, \quad (3)$$

$$g_{t\varphi} - g_{\varphi\varphi} - g_{\tau\tilde{\varphi}} = 0, \quad (4)$$

in which the solution is given by (note that the rest of the components does not transform)

$$g_{\tau\tau} = \frac{\mu^2 H \left(H \left((a^2 + r^2)^2 - a^2\mu^2 (a^2 + r(r - 2M)) \right) + 4aMr \right) + a^2 (\mu^2 - 1) + r(2M - r)}{r^2 - a^2 (\mu^2 - 1)} \quad (5)$$

$$g_{\tau\tilde{\varphi}} = \frac{\mu^2 \left(H \left((a^2 + r^2)^2 - a^2\mu^2 (a^2 + r(r - 2M)) \right) + 2aMr \right)}{a^2 (\mu^2 - 1) - r^2}. \quad (6)$$

In terms of ADM variables this only introduces a change in shift function

$$N_\phi = \frac{\mu^2 (2aMr + ((a^2 + r^2)^2 - a^2(a^2 + (r - 2M))\mu^2)H)}{a^2(\mu^2 - 1)r^2}. \quad (7)$$

Using the new metric components in Horava's equations we will have PDEs for the function $H(r, \mu)$ which are discussed in mathematica files.

[1] G. W. Gibbons *et al.*, J. Geom. Phys. **53**, 49 (2005), Appendix **E**.