I. ILLEGITIMATE TRANSFORMATIONS

In this section we will consider the transformation from Gibbons form of GR solution to the Horava-type solution. We start with the Gibbons solution given in [1] (with $\lambda = 0$)

$$\begin{aligned} ds^2 &= \left(-1 + \left(\frac{2Mr}{a^2+r^2-a^2\mu^2}\right)\right) dt^2 - \frac{4aMr\mu^2}{a^2+r^2-a^2\mu^2} dt d\varphi + \frac{a^2+r^2-a^2\mu^2}{a^2-2Mr+r^2} dr^2 \\ &+ \left(a^2 - \frac{r^2}{\mu^2-1}\right) d\theta^2 + \frac{a^2(a^2+r(r-2M))\mu^4-(r^2+a^2)^2\mu^2}{a^2(\mu^2-1)-r^2} d\varphi^2 \end{aligned} \tag{1}$$

Then we consider the following transformation with the unknown function $H(r,\mu)$

$$d\tau = dt, \quad dR = dr, \quad d\tilde{\mu} = d\mu, \quad d\tilde{\varphi} - H(r,\mu) d\tau = d\varphi, \tag{2}$$

where $\tau, R, \tilde{\mu}, \tilde{\varphi}$ are the new coordinates. The new components of the metric function can be found solving the following set

$$g_{tt} - 2g_{t\varphi} H(r,\varphi) + g_{\varphi\varphi} H(r,\varphi)^2 - g_{\tau\tau} = 0, \tag{3}$$

$$g_{t\varphi} - g_{\varphi\varphi} - g_{\tau\tilde{\varphi}} = 0, \tag{4}$$

in which the solution is given by (note that the rest of the components does not transform)

$$g_{\tau\tau} = \frac{\mu^2 H\left(H\left(\left(a^2+r^2\right)^2 - a^2\mu^2\left(a^2+r(r-2M)\right)\right) + 4aMr\right) + a^2\left(\mu^2-1\right) + r(2M-r)}{r^2 - a^2\left(\mu^2-1\right)} \tag{5}$$

$$g_{\tau\tilde{\varphi}} = \frac{\mu^2\left(H\left(\left(a^2+r^2\right)^2 - a^2\mu^2\left(a^2+r(r-2M)\right)\right) + 2aMr\right)}{a^2\left(\mu^2-1\right)-r^2}. \tag{6}$$

In terms of ADM variables this only introduces a change in shift function

$$N_\phi = \frac{\mu^2(2aMr + ((a^2+r^2)^2 - a^2(a^2+(r-2M))\mu^2)H)}{a^2(\mu^2-1)r^2}. \tag{7}$$

Using the new metric components in Horava's equations we will have PDEs for the function $H(r,\mu)$ which are discussed in mathematica files.

[1] G. W. Gibbons *et al.*, "The General Kerr-de Sitter metrics in all dimensions," J. Geom. Phys. **53**, 49 (2005), Appendix **E**.