Time Series Analysis I - Economy and Satisfaction With The United States

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Motivation

We are interested in identifying social issues trends and forecasting a given open topic. Referred by the project regulations, we selected data from public polls on the topic “Satisfaction With The United States” as advertised on Gallup.com. The polls are publicly available containing inquiries about Americans’ satisfaction with the way things are going in the U.S. Our data answers the question “In general, are you satisfied or dissatisfied with the way things are going in the United States at this time?” To explore the applications of advanced GARCH models, we selected relevant financially-related data on the field of the economy. The relationship helps illustrate associations between American’s satisfaction with the U.S. and its economy. Practical interest is given to the topic of confidence on current economic conditions answering the question “How would you rate economic conditions in this country today- as excellent, good, only fair, or poor?” It is chocking to see how America’s satisfaction ratings from its own citizens have been at strike since 2008 despite recent economic surpass.

Satisfaction With The U.S. Data

Data was entered from tables available on the web at news.gallup.com/poll. To assess seasonality, data was retrieved from the General-Mood-Country tab [^2]. The satisfaction was assessed in three levels namely ‘Satisfied,’ ‘Dissatisfied,’ and ‘No Opinion-’ measured in population percentage. The dissatisfaction can be easily calculated by subtracting (‘Satisfied’ + Dissatisfied’) from 100 %. Thus, it is more practical to focus on ‘Satisfied’ only. Timely data appears unsorted and containing duplicates which will be easily handled. The timeline ranges from February 1979 to March 2024 with irregular time intervals (!). The data-set contains 429 entries which is more than enough for our seasonal study. Satisfaction values range from as low as 7 % to 71 % with an average of about one third of the U.S. population. The fields needed proper formatting using the predetermined R tools. Character values were converted to numeric for time series analysis. The date has been formatted as YYYY/MM since daily data was virtually unavailable. Invalid or non-available entries appeared isolated and were hence filled by their previous observation, a common methodology in low-volatility seasonal data.

Satisfaction with the U.S. doesn’t exhibit any apparent linear trend (downward or upward) since 1979. It reaches its highest during the mid-late 80s to early 90s and around 2000, and it has been there about lower than average since 2008. The seasonal trends of the original series don’t appear obvious but we will find out that a seasonal model is a better fit than a non-seasonal one. The findings reveals underlying seasonal patterns of the undifferentiated series.

```
library(zoo) # To handle missing values
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
library(lubridate) # To format time data
```

```
##
## Attaching package: 'lubridate'
```

```
## The following objects are masked from 'package:base':
##
##     date, intersect, setdiff, union
```

```
library(xts) # To handle irregular time series data
library(forecast) # To aid seasonality detection
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
library(tseries) # Test for stationarity
library(TSA) # For extended autocorrelation function plots
```

```
## Registered S3 methods overwritten by 'TSA':
##   method       from
##   fitted.Arima forecast
##   plot.Arima   forecast
```

```
##
## Attaching package: 'TSA'
```

```
## The following objects are masked from 'package:stats':
##
##     acf, arima
```

```
## The following object is masked from 'package:utils':
##
##     tar
```

```
library(ggplot2) # Fancier ACF/ PACF plots
library(lmtest) # To detect heteroscedasticity

df <- read.csv("~/hbuchholz MA641 Manuscript/Satisfaction With The United States.cs
v")
View(df)
summary(df)
```

```
##      Date            Satisfied..   Dissatisfied..  No.opinion..
##  Length:429         Min.   : 7.0   Min.   :26.00   Length:429
##  Class :character   1st Qu.:23.0   1st Qu.:56.00   Class :character
##  Mode  :character   Median :30.0   Median :67.00   Mode  :character
##                     Mean   :33.3   Mean   :64.17
##                     3rd Qu.:42.0   3rd Qu.:75.00
##                     Max.   :71.0   Max.   :91.00
```

```
str(df) # Includes data type
```

```
## 'data.frame':    429 obs. of  4 variables:
##  $ Date          : chr  "2024 Mar 1-20" "2024 Feb 1-20" "2024 Jan 2-22" "2023 Dec
1-20" ...
##  $ Satisfied..   : int  23 19 20 22 19 19 20 20 19 18 ...
##  $ Dissatisfied..: int  75 79 78 77 80 80 80 79 80 81 ...
##  $ No.opinion..  : chr  "2" "2" "2" "1" ...
```

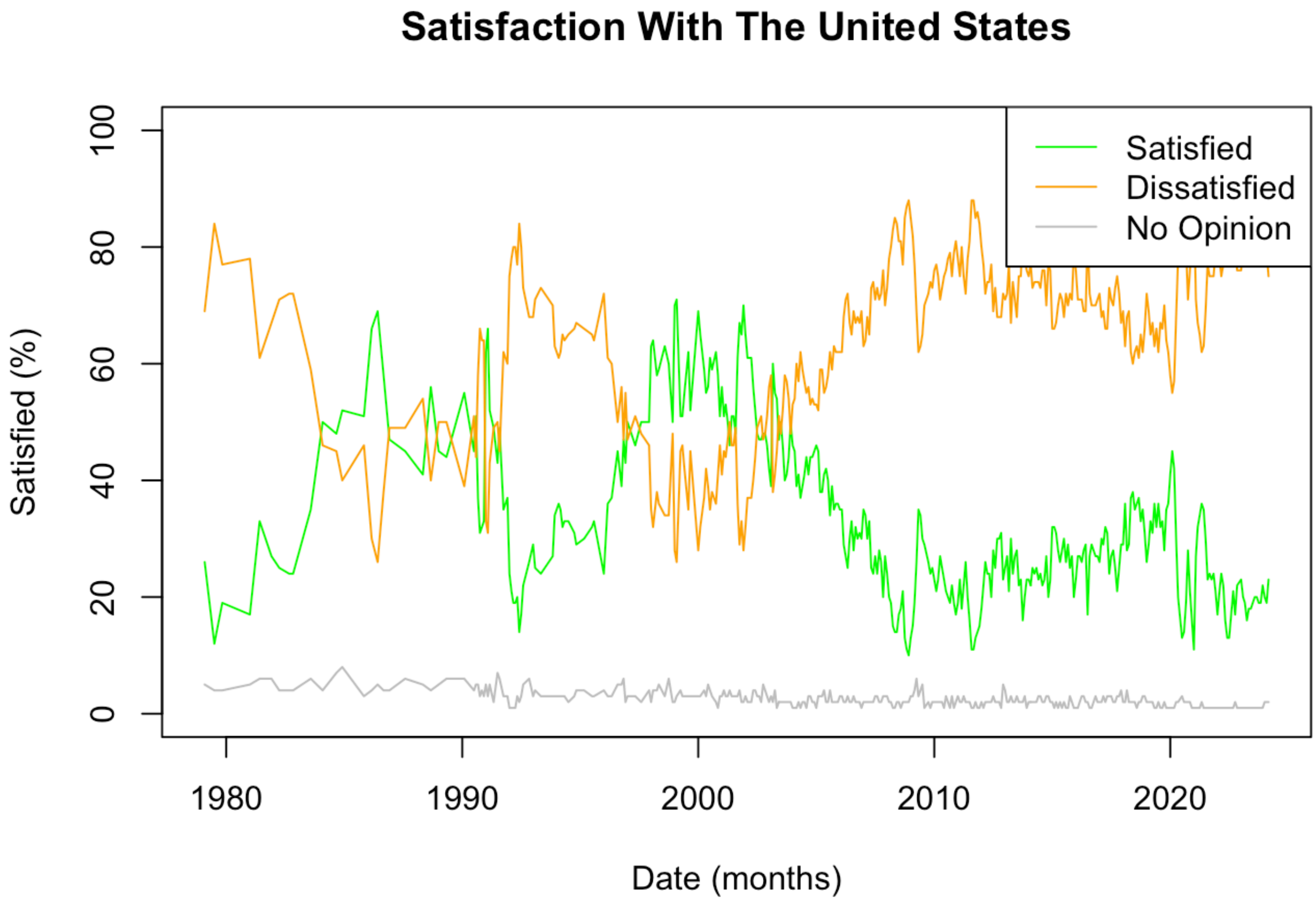
```
missing_values_per_column <- colSums(is.na(df)) # Include number of missing values
print(missing_values_per_column)
```

```
##           Date    Satisfied.. Dissatisfied..   No.opinion..
##              0              0              0              0
```

```
# Pre-processing
unique(df$No.opinion..) # Look at unique values that might not be numbers
```

```
## [1] "2" "1" "*" "3" "4" "5" "6" "7" "8"
```

```
df$No.opinion..[df$No.opinion.. %in% c("*")] <- NA
df$No.opinion.. <- na.locf(df$No.opinion..) # Replace missing values by-last-observed
-carried-forward
df$No.opinion.. <- as.numeric(df$No.opinion..) # Format "No Opinion" field as numeric
for TS analysis
df$Date <- substring(df$Date, 1, 8) # Trim the date
df$Date <- ym(df$Date) # format the date as YYYYMM
df <- df[order(df$Date), ] # Sort data by date
df <- df[!duplicated(df$Date), ] # Remove duplicates
X <- df$Date

# TS Plots
plot(df$Satisfied..~X, type = "l", col = "green", ylim = c(0, 100), main = "Satisfact
ion With The United States", xlab = "Date (months)", ylab = "Satisfied (%)")
lines(df$Dissatisfied..~X, type = "l", col = "orange")
lines(df$No.opinion..~X, type = "l", col = "grey")
legend("topright", legend = c("Satisfied", "Dissatisfied", "No Opinion"), col = c("gr
een", "orange", "grey"), lty = 1)
```




```
#Irregular TS
data_xts <- xts(df$Satisfied.., order.by = X)
print(head(data_xts))
```

```
##             [,1]
## 1979-02-01   26
## 1979-07-01   12
## 1979-11-01   19
## 1981-01-01   17
## 1981-06-01   33
## 1981-12-01   27
```

(!)

```
# Resampling to monthly data using interpolation
X <- seq(from = start(data_xts), to = end(data_xts), by = "month")
data_ts <- ts(na.approx(data_xts, xout = X), frequency = 12)

summary(data_ts)
```

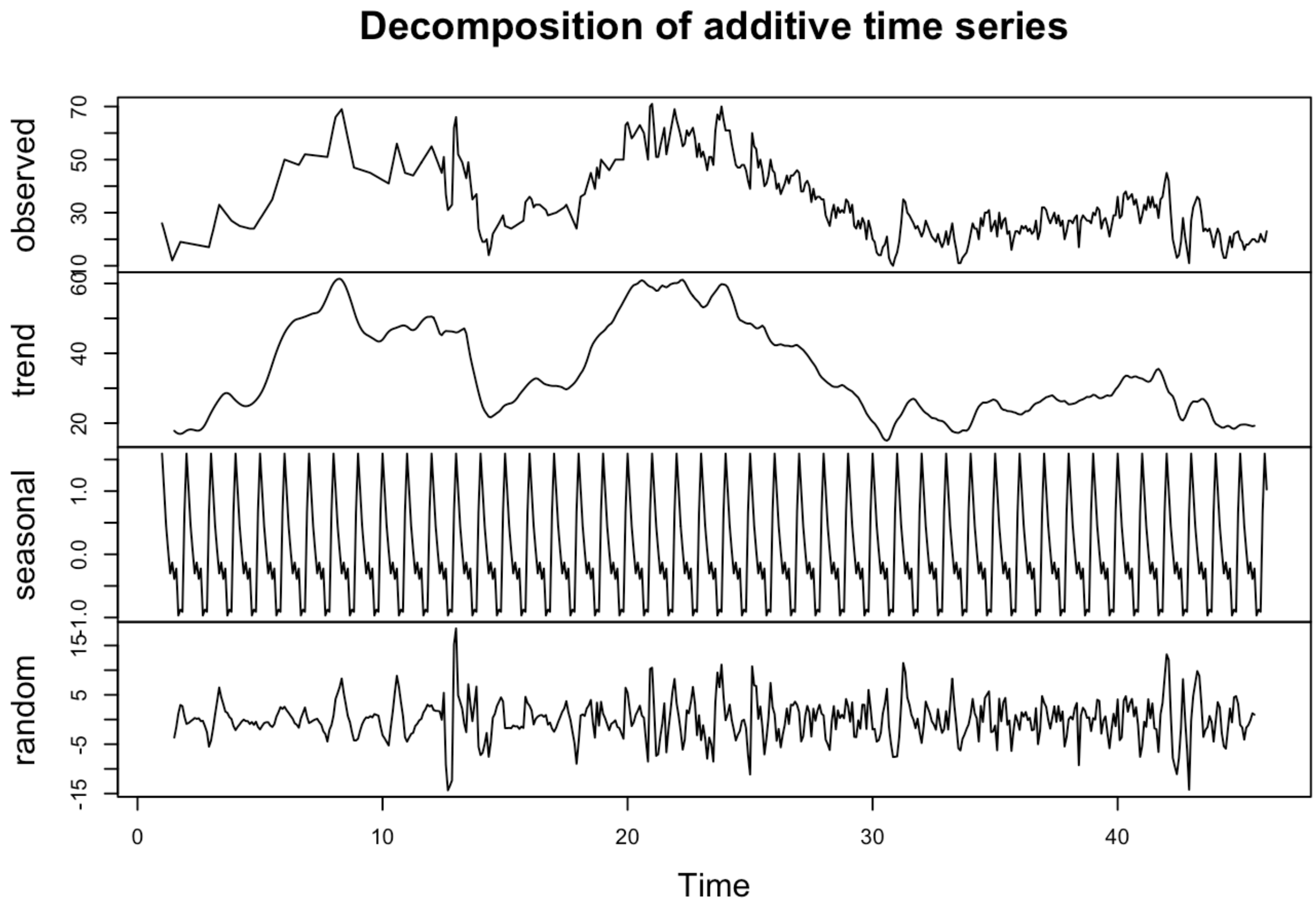
```
##        V1
##  Min.   :10.00
##  1st Qu.:24.00
##  Median :31.23
##  Mean   :35.03
##  3rd Qu.:46.94
##  Max.   :71.00
```

```
length(data_ts)
```

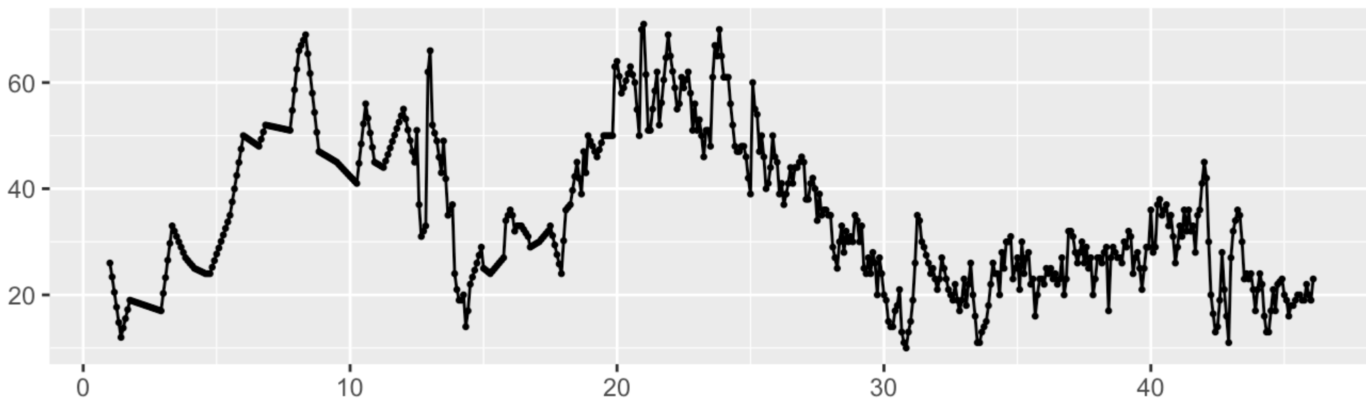
```
## [1] 542
```

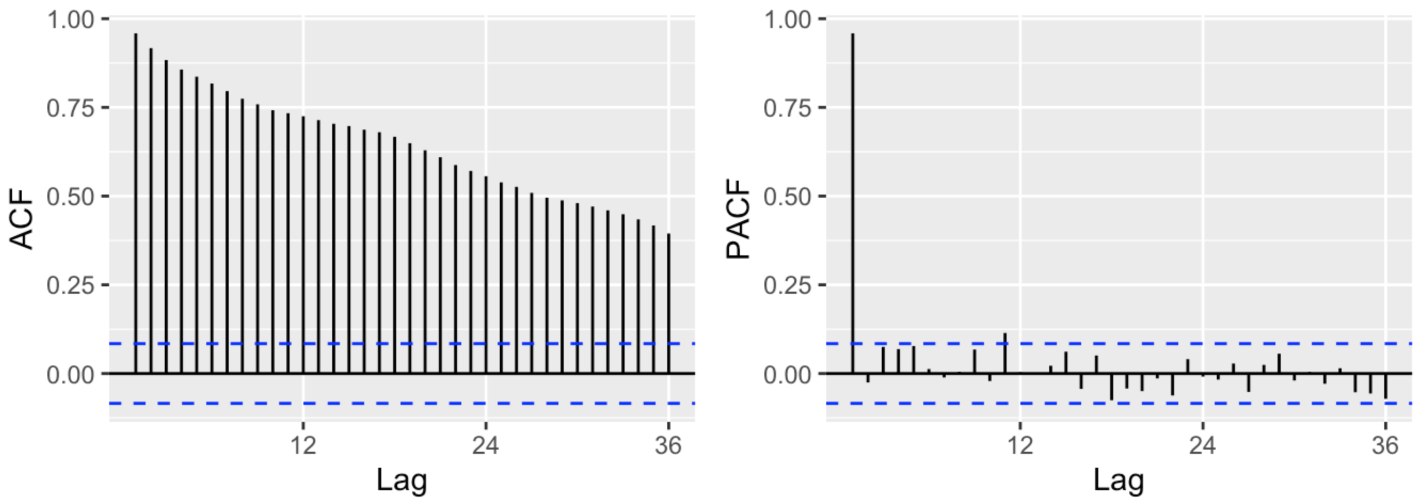
26% of monthly time instances were missing (limitation). The data has been re-sampled to monthly by filling the missing monthly values with their interpolated values according to the time series which introduces assumptions. Note that the time steps must be regular in order for the auto-correlations to be valid.

```
# Decomposition
plot(decompose(data_ts, type = "additive"))
```




```
# Regular and partial autocorrelation function plots
data_ts %>%
  ggtsdisplay()
```





ACF's slow exponential decay suggests non-stationarity of the monthly series. A seasonal trend can now be seen more clearly from the decomposition.

```
# Augmented Dickey-Fuller test
print(adf.test(data_ts, alternative = "stationary"))
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data_ts
## Dickey-Fuller = -2.9198, Lag order = 8, p-value = 0.189
## alternative hypothesis: stationary
```

Dickey-Fuller test's p-value > 0.01 fails to reject the null hypothesis of non-stationarity. Therefore, we must assume the monthly series is non-stationary. Since our series is monthly, we first consider a seasonal differencing at yearly lags to make the series stationary.

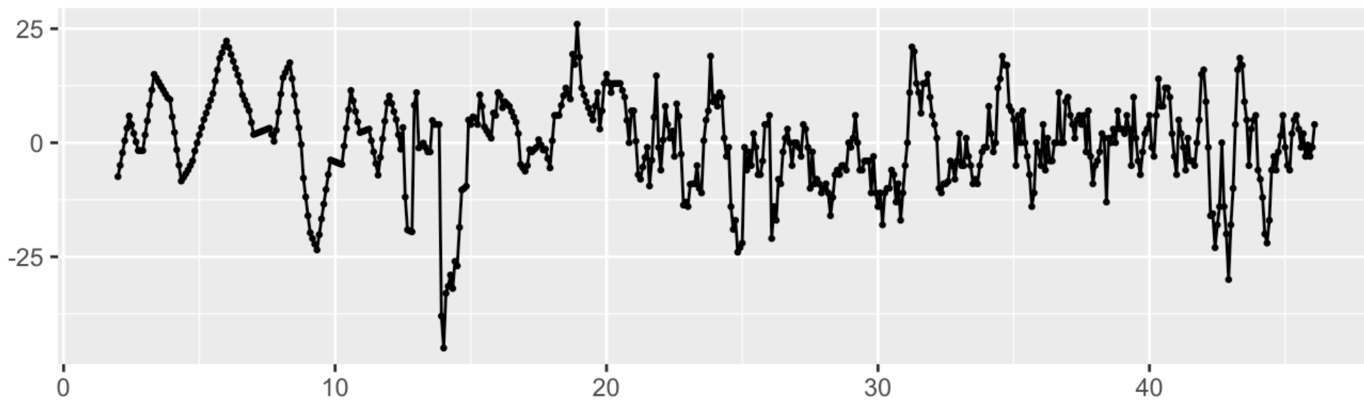
```
# Seasonal Differencing
data_ts_Diff <- diff(data_ts, lag = 12)

# Augmented Dickey-Fuller test on the differenced series
print(adf.test(data_ts_Diff, alternative = "stationary"))
```

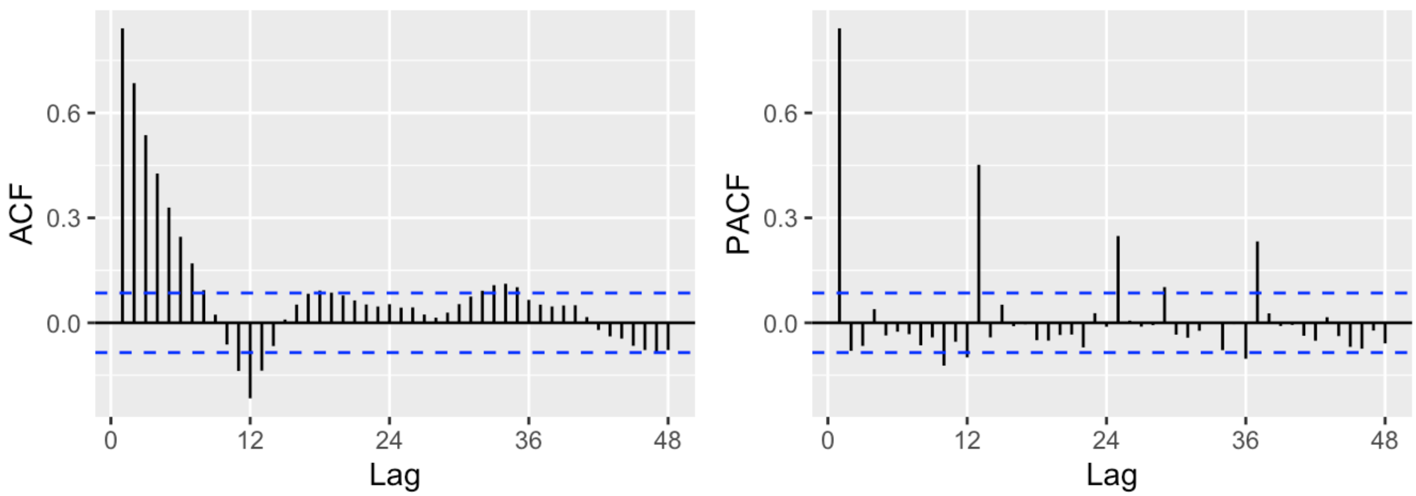
```
## Warning in adf.test(data_ts_Diff, alternative = "stationary"): p-value smaller
## than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data_ts_Diff
## Dickey-Fuller = -6.6048, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
```

Dickey-Fuller test's p-value is now much less than 0.01 rejecting the null hypothesis of non-stationarity. Thus, the seasonally differenced series is stationary at 1% significance.

```
data_ts_Diff %>%
  ggtsdisplay(lag.max = 48)
```





PACF’s exponential decay of the seasons and ACF’s 1 repeated season suggests a seasonal MA(1) part. ACF’s exponential decay and PACF’s 1 signifficant non-seasonal lag suggests a non-seasonal AR(1) part. Including the seasonal difference order D = 1, that is definitely a multiplicative SARIMA(1, 0, 0) X (0, 1, 1)12 model.

```
# ~ SARIMA(1, 0, 0) X (0, 1, 1)12
model = Arima(data_ts, order = c(1, 0, 0), seasonal = list(order = c(0, 1, 1), period
= 12), method = 'ML') # Maximum likelihood
model
```

```
## Series: data_ts
## ARIMA(1,0,0)(0,1,1)[12]
##
## Coefficients:
##          ar1     sma1
##       0.9612  -0.9651
## s.e.  0.0130   0.0378
##
## sigma^2 = 16.33:  log likelihood = -1507.36
## AIC=3020.72   AICc=3020.76   BIC=3033.53
```

```
# Automatically fitting an ARIMA model
model_auto <- auto.arima(data_ts, D = 1, stepwise = TRUE, approximation = FALSE, trac
e = TRUE)
```

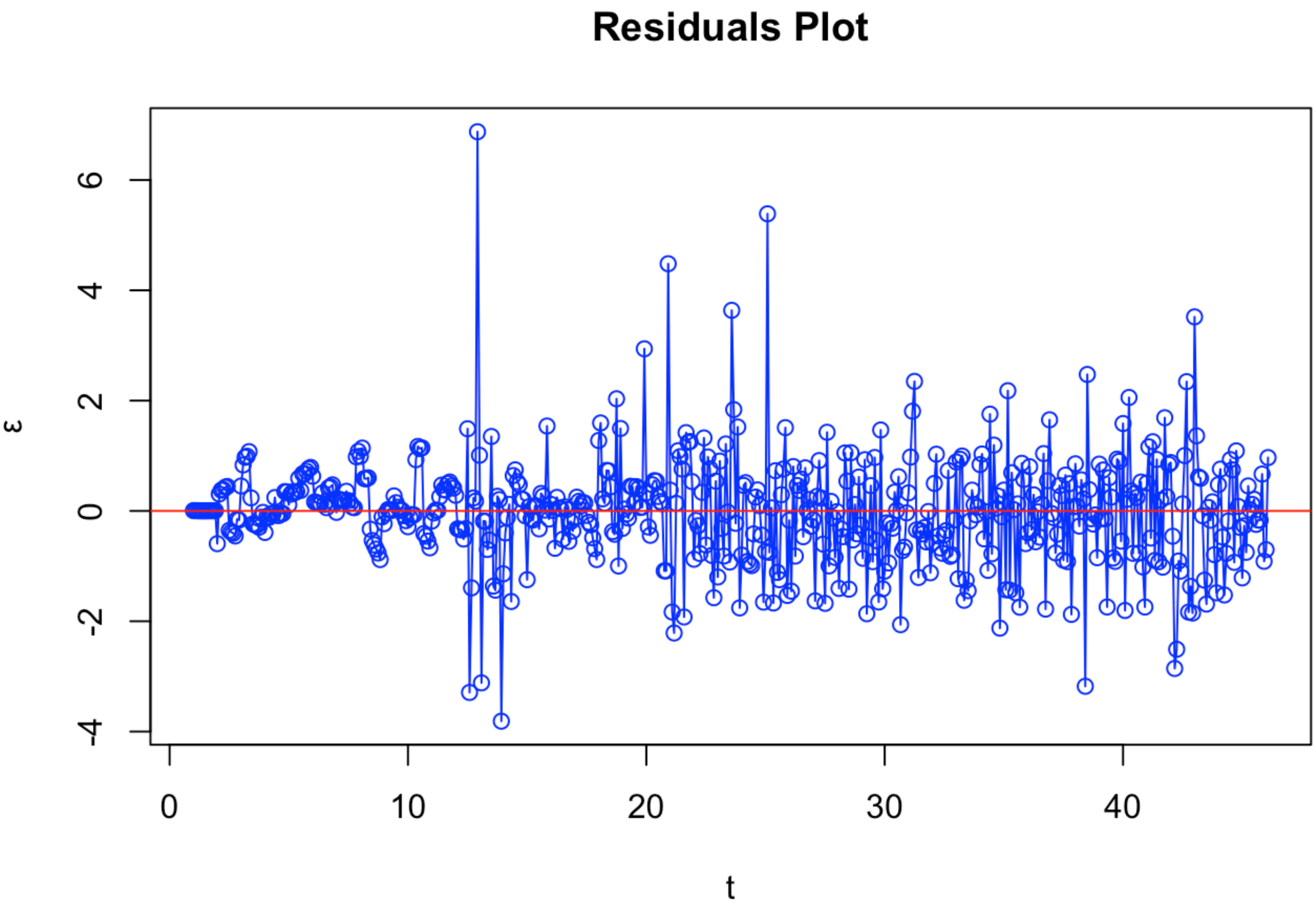
```
##
##  ARIMA(2,0,2)(1,1,1)[12] with drift         : Inf
##  ARIMA(0,0,0)(0,1,0)[12] with drift         : 3952.79
##  ARIMA(1,0,0)(1,1,0)[12] with drift         : 3164.603
##  ARIMA(0,0,1)(0,1,1)[12] with drift         : 3534.429
##  ARIMA(0,0,0)(0,1,0)[12]                    : 3950.779
##  ARIMA(1,0,0)(0,1,0)[12] with drift         : 3300.881
##  ARIMA(1,0,0)(2,1,0)[12] with drift         : 3133.556
##  ARIMA(1,0,0)(2,1,1)[12] with drift         : Inf
##  ARIMA(1,0,0)(1,1,1)[12] with drift         : Inf
##  ARIMA(0,0,0)(2,1,0)[12] with drift         : 3931.897
##  ARIMA(2,0,0)(2,1,0)[12] with drift         : Inf
##  ARIMA(1,0,1)(2,1,0)[12] with drift         : 3133.691
##  ARIMA(0,0,1)(2,1,0)[12] with drift         : 3532.888
##  ARIMA(2,0,1)(2,1,0)[12] with drift         : 3135.548
##  ARIMA(1,0,0)(2,1,0)[12]                    : 3131.519
##  ARIMA(1,0,0)(1,1,0)[12]                    : 3162.573
##  ARIMA(1,0,0)(2,1,1)[12]                    : Inf
##  ARIMA(1,0,0)(1,1,1)[12]                    : Inf
##  ARIMA(0,0,0)(2,1,0)[12]                    : 3929.873
##  ARIMA(2,0,0)(2,1,0)[12]                    : 3131.807
##  ARIMA(1,0,1)(2,1,0)[12]                    : 3131.646
##  ARIMA(0,0,1)(2,1,0)[12]                    : 3530.854
##  ARIMA(2,0,1)(2,1,0)[12]                    : 3133.498
##
##  Best model: ARIMA(1,0,0)(2,1,0)[12]
```

```
summary(model_auto)
```

```
## Series: data_ts
## ARIMA(1,0,0)(2,1,0)[12]
##
## Coefficients:
##          ar1     sar1     sar2
##       0.9021  -0.6090  -0.2482
## s.e.  0.0189   0.0426   0.0422
##
## sigma^2 = 21.11:  log likelihood = -1561.72
## AIC=3131.44   AICc=3131.52   BIC=3148.53
##
## Training set error measures:
##                       ME    RMSE     MAE       MPE    MAPE      MASE       ACF1
## Training set 0.007688578 4.53012 3.18585 -1.260178 10.6207 0.4154167 0.05265958
```

Considering other candidates with the help of the auto ARIMA function, our best model remains SARIMA(1,0,0)(2,1,0)[12] in terms of Akaike information criterion (AIC) and parsimony. That is, our model AIC is 3020.72, much smaller than auto ARIMA's lowest AIC (3131.44). Moreover, our model has only 2 parameters and auto ARIMA's best model has 3. Because we are interested in prediction, we only use AIC for model comparison.

```
ε <- rstandard(model) # Standardized residuals
plot(ε, type = 'o', col = 'blue', main = 'Residuals Plot', xlab = "t")
abline(h = 0, col = "red")
```



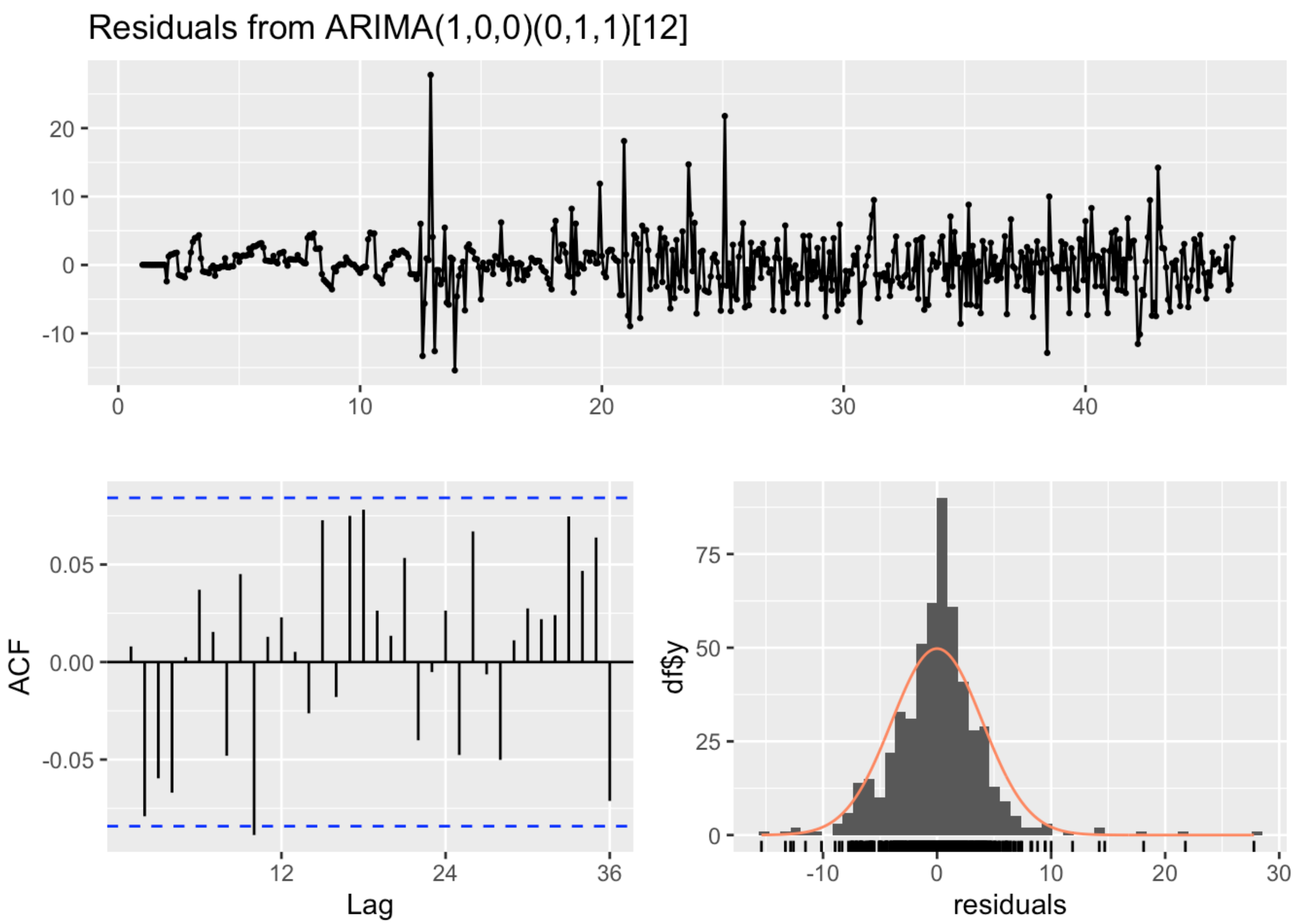
No apparent shape indicating non-constant variance of the standardized residuals with a few potential outliers.

```
# Fit a linear model
Y <- fitted(model)

data_lm <- data.frame(residuals = ε, fitted = Y)
model_lm <- lm(ε ~ Y + I(Y^2), data = data_lm)

# Breusch-Pagan Test
print(bptest(model_lm))
```

```
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 1.4229, df = 2, p-value = 0.4909
```

Breusch-Pagan's p-value greater than 0.01 fails to reject the null hypothesis of constant variance. Thus, the constancy of variance assumption has been satisfied.

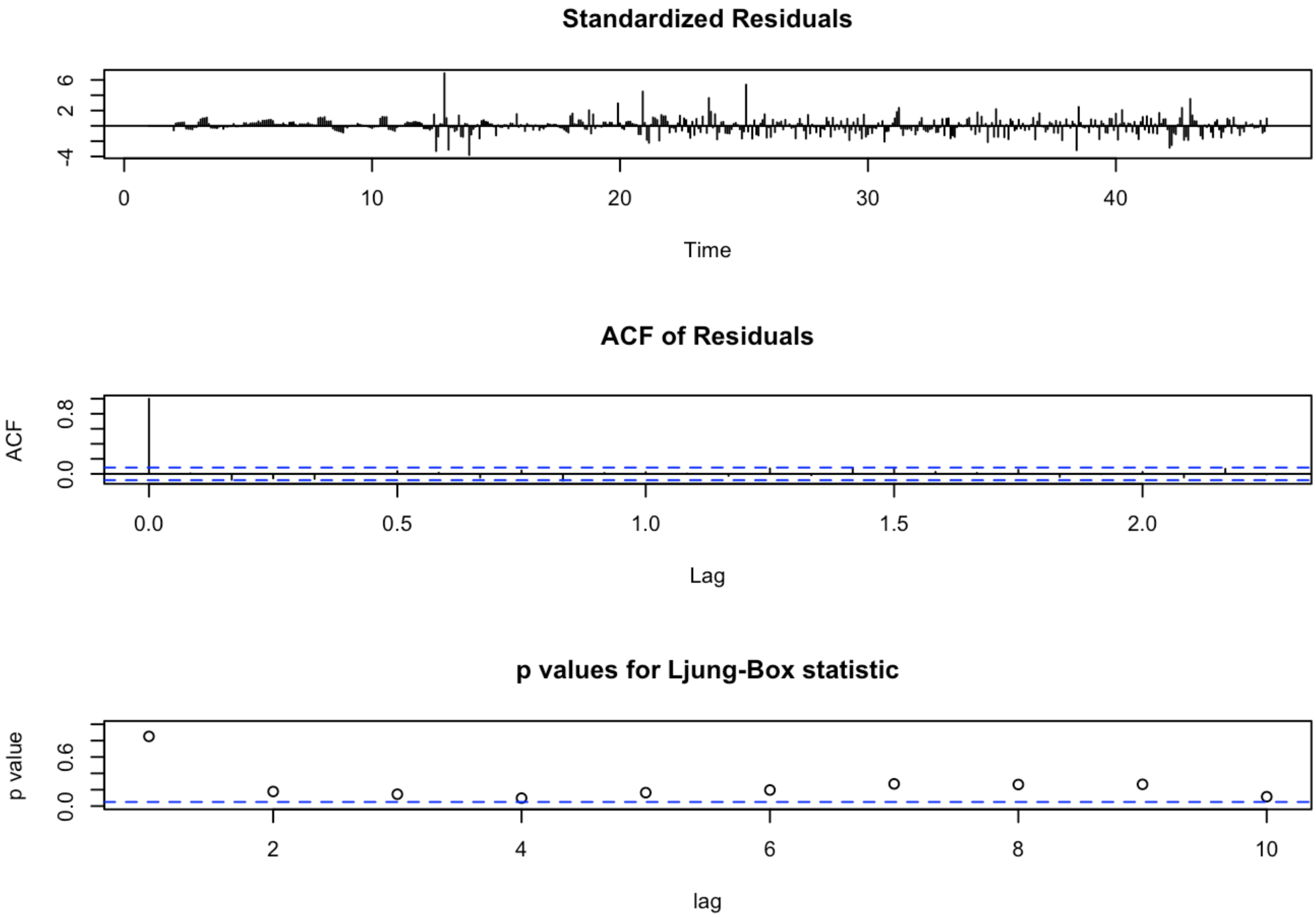
```
# Residual Diagnostics
checkresiduals(model)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(1,0,0)(0,1,1)[12]
## Q* = 29.46, df = 22, p-value = 0.1322
##
## Model df: 2.   Total lags used: 24
```

The standardized residuals fall significantly outside the normal distribution indicating non-normality of the residuals. On the other hand, Ljung-Box's p-value greater than 0.01 fails to reject the null hypothesis of uncorrelated residuals so the independence assumption has been satisfied.

```
tsdiag(model)
```




```
# Ljung-Box test for individual lags 2, 3, 4, ...
Ljung_Box_p <- sapply(2: 12, function(ε_lag) {
    Ljung_Box <- Box.test(ε, lag = ε_lag, type = "Ljung-Box")
    return(Ljung_Box$p.value)
})
# P-values for each lag
names(Ljung_Box_p) <- 2: 12
print(Ljung_Box_p)
```

```
##          2          3          4          5          6          7          8
## 0.17810392 0.14495103 0.09701211 0.16417293 0.19636212 0.27130177 0.26341292
##          9         10         11         12
## 0.26566113 0.11453432 0.15650681 0.19603339
```

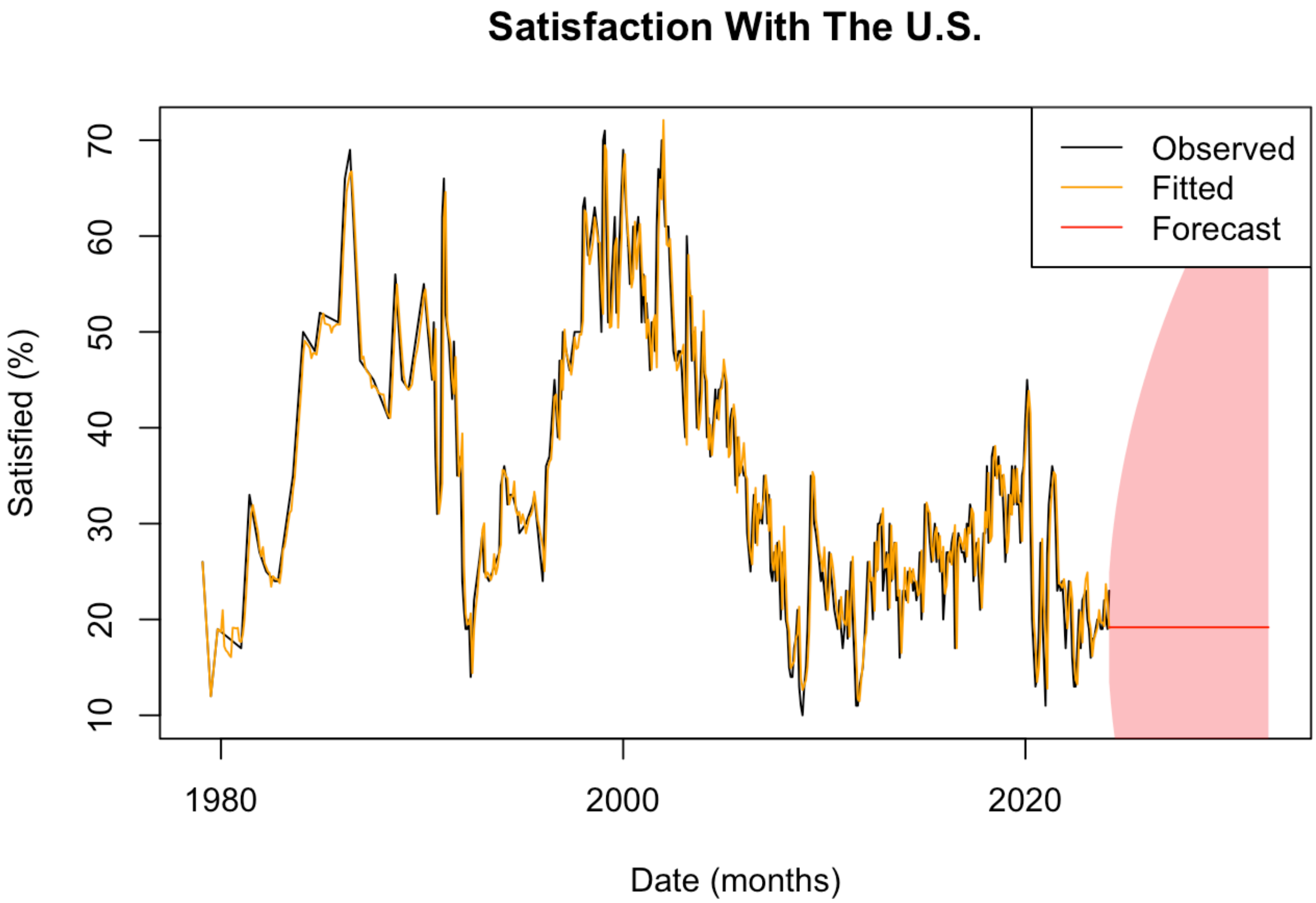
Standardized residuals mostly within [-2, +2] with no apparent patterns is indicative of the model fitting the data well. ACF of residuals shows all bars within the confidence bounds so the residuals appear as white noise. Ljung-Box p-values > 0.01 for all considered lags implies no significant autocorrelation/ no patterns left unexplained.

```
# Normality Test
shapiro.test(ε)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  ε
## W = 0.92047, p-value = 2.642e-16
```

Shapiro-Wilk test p-value much less than 0.01 rejects the null hypothesis of normally distributed residuals so the normality assumption has been violated at 1% significance.

```
# Forecast
Y_h <- forecast(Y, h = 96, level = c(85)) # Eight years into the future
H <- seq(from = end(data_xts), length.out = 96, by = "month")

plot(X, data_ts, type = "l", main = "Satisfaction With The U.S.", xlab = "Date (month
s)", xlim = range(c(X, H)), ylab = "Satisfied (%)")
lines(X, Y, col = "orange", type = "l")
x_poly <- c(H, rev(H))
y_poly <- c(Y_h$lower[, '85%'], rev(Y_h$upper[, '85%']))
polygon(x_poly, y_poly, col = rgb(1, 0, 0, 0.3), border = NA)
lines(H, Y_h$mean, col = "red", type = "l")
legend("topright", legend = c("Observed", "Fitted", "Forecast"), col = c("black", "or
ange", "red"), lty = 1)
```



Although the model fits the data well, the forecast shows an absence of trend, that is, the overall rate of satisfaction with the United States is not expected to change in the long run. The seasonal differencing possibly sets the mean of the series to 0. Future directions are needed.

Economy Data

Similarly, data was retrieved from Consumer-Views-Economy at Gallup.com [1]. The economic conditions were rated in five different levels namely 'Poor,' 'Only Fair,' 'Good,' 'Excellent,' and 'No Opinion' measured in population percentage. 'Excellent' has a lot of 0-values that may add multi-collinearity issues for our model so we rather focus on 'Good.' The timeline ranges from January 1992 to March 2024. The data-set contains 270 entries. 'Good' values range from 4 % to 57 % with an average of 28.36 %. Economy ratings don't exhibit any apparent linear or seasonal trend since 1992. They reach their highest during the early 90s and around 2008 with times of high volatility (explosive increase/ decrease) around 2008 and 2020, proper behavior of financial data.

```
library(zoo) # To handle missing values
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
library(lubridate) # To format time data
```

```
##
## Attaching package: 'lubridate'
```

```
## The following objects are masked from 'package:base':
##
##     date, intersect, setdiff, union
```

```
library(xts) # To handle irregular time series data
library(forecast) # To aid seasonality detection
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
library(tseries) # Test for stationarity
library(TSA) # Extended autocorrelation function plots
```

```
## Registered S3 methods overwritten by 'TSA':
##   method       from
##   fitted.Arima forecast
##   plot.Arima   forecast
```

```
##
## Attaching package: 'TSA'
```

```
## The following objects are masked from 'package:stats':
##
##     acf, arima
```

```
## The following object is masked from 'package:utils':
##
##     tar
```

```
library(lmtest) # To detect heteroscedasticity
library(rugarch) # To incorporate GARCH model
```

```
## Loading required package: parallel
```

```
##
## Attaching package: 'rugarch'
```

```
## The following object is masked from 'package:stats':
##
##     sigma
```

```
library(ggplot2) # Fancier ACF/ PACF plots

df <- read.csv("~/hbuchholz MA641 Manuscript/Economy.csv")
View(df)
summary(df)
```

```
##      Date           Excellent..           Good..       Only.fair..
##  Length:270         Length:270         Min.   : 4.00   Min.   :21.00
##  Class :character   Class :character   1st Qu.:20.00   1st Qu.:36.00
##  Mode  :character   Mode  :character   Median :28.00   Median :42.00
##                                        Mean   :28.36   Mean   :40.84
##                                        3rd Qu.:36.00   3rd Qu.:46.00
##                                        Max.   :57.00   Max.   :57.00
##      Poor..      No.Opinion..
##  Min.   : 4.00   Length:270
##  1st Qu.:15.00   Class :character
##  Median :22.00   Mode  :character
##  Mean   :25.51
##  3rd Qu.:33.00
##  Max.   :73.00
```

```
str(df) # Include data type
```

```
## 'data.frame':    270 obs. of  6 variables:
##  $ Date        : chr  "2024 Mar 1-20" "2024 Feb 1-20" "2024 Jan 2-22" "2023 Dec 1-20" ...
##  $ Excellent.. : chr  "4" "4" "5" "3" ...
##  $ Good..      : int  26 22 22 19 17 17 17 21 20 18 ...
##  $ Only.fair.. : int  30 32 29 33 31 33 32 35 35 37 ...
##  $ Poor..      : int  39 41 45 45 50 47 48 42 42 44 ...
##  $ No.Opinion..: chr  "1" "*" "*" "*" ...
```

```
missing_values_per_column <- colSums(is.na(df)) # Include number of missing values
print(missing_values_per_column)
```

```
##         Date  Excellent..       Good..  Only.fair..       Poor.. No.Opinion..
##            0            0            0            0            0            0
```

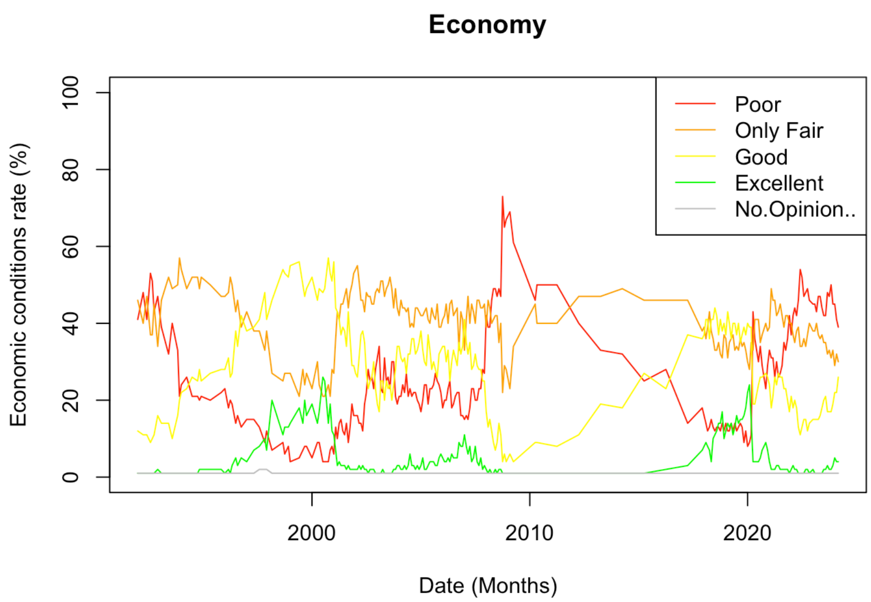
```
# Pre-processing
unique(df$No.Opinion..) # Look at unique values that might not be numbers
```

```
## [1] "1"  "*"  "--" "2"
```

```
unique(df$Excellent..) # Look at unique values that might not be numbers
```

```
##  [1] "4"  "5"  "3"  "2"  "1"  "*"  "7"  "9"  "15" "24" "22" "18" "16" "14" "13"
## [16] "11" "12" "10" "17" "8"  "6"  "19" "25" "26" "20"
```

```
df$No.Opinion..[df$No.Opinion.. %in% c("*", "--")] <- NA
df$Excellent..[df$Excellent.. %in% c("*", "--")] <- NA
df$No.Opinion.. <- na.locf(df$No.Opinion..) # Replace missing values by-last-observed-carried-forward
df$Excellent.. <- na.locf(df$Excellent..) # Replace missing values by-last-observed-carried-forward
df$No.Opinion.. <- as.numeric(df$No.Opinion..) # Format "No Opinion" field as numeric for TS analysis
df$Excellent.. <- as.numeric(df$Excellent..) # Format "Excellent" field as numeric for TS analysis
df$Date <- substring(df$Date, 1, 8) # Trim the date
df$Date <- ym(df$Date) # format the date as YYYYMM
df <- df[order(df$Date), ] # Sort data by date
df <- df[!duplicated(df$Date), ] # Remove duplicates
X <- df$Date

# TS Plots
plot(df$Poor..~X, type = "l", col = "red", ylim = c(0, 100), main = "Economy", xlab = "Date (Months)", ylab = "Ec
onomic conditions rate (%)")
lines(df$Only.fair..~X, type = "l", col = "orange")
lines(df$Good..~X, type = "l", col = "yellow")
lines(df$Excellent..~X, type = "l", col = "green")
lines(df$No.Opinion..~X, type = "l", col = "grey")
legend("topright", legend = c("Poor", "Only Fair", "Good", "Excellent", "No.Opinion.."), col = c("red", "orange",
"yellow", "green", "grey"), lty = 1)
```




```
# Irregular TS
data_xts <- xts(df$Good.., order.by = df$Date)
print(head(data_xts))
```

```
##            [,1]
## 1992-01-01   12
## 1992-04-01   11
## 1992-06-01   11
## 1992-08-01    9
## 1992-09-01   10
## 1992-10-01   11
```

```
# Resampling to annual data using interpolation
X <- seq(start(data_xts), end(data_xts), by = "month")
data_ts <- ts(na.approx(data_xts, xout = X), frequency = 12)

summary(data_ts)
```

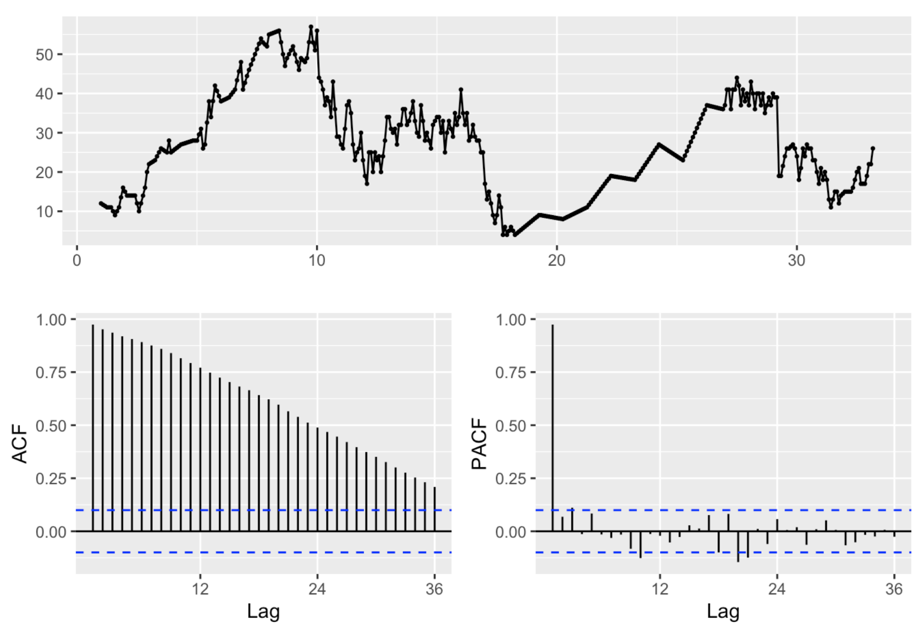
```
##        V1
##  Min.   : 4.00
##  1st Qu.:17.00
##  Median :26.00
##  Mean   :26.75
##  3rd Qu.:36.19
##  Max.   :57.00
```

```
n <- length(data_ts)
print(n)
```

```
## [1] 387
```

43% of monthly time instances were missing! Similarly, this adds limitations for our model reliability despite the size of the training data.

```
# Regular and partial autocorrelation function plots
data_ts %>%
  ggtsdisplay()
```



ACF's slow decay indicates non-stationarity of the monthly series.

```
# Augmented Dickey-Fuller Test for stationarity
print(adf.test(data_ts, alternative = "stationary"))
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data_ts
## Dickey-Fuller = -2.2027, Lag order = 7, p-value = 0.4916
## alternative hypothesis: stationary
```

Dickey-Fuller test's p-value greater than 0.01 fails to reject the null hypothesis of non-stationarity so we assume the monthly series is non-stationary. We difference the series to make it stationary.

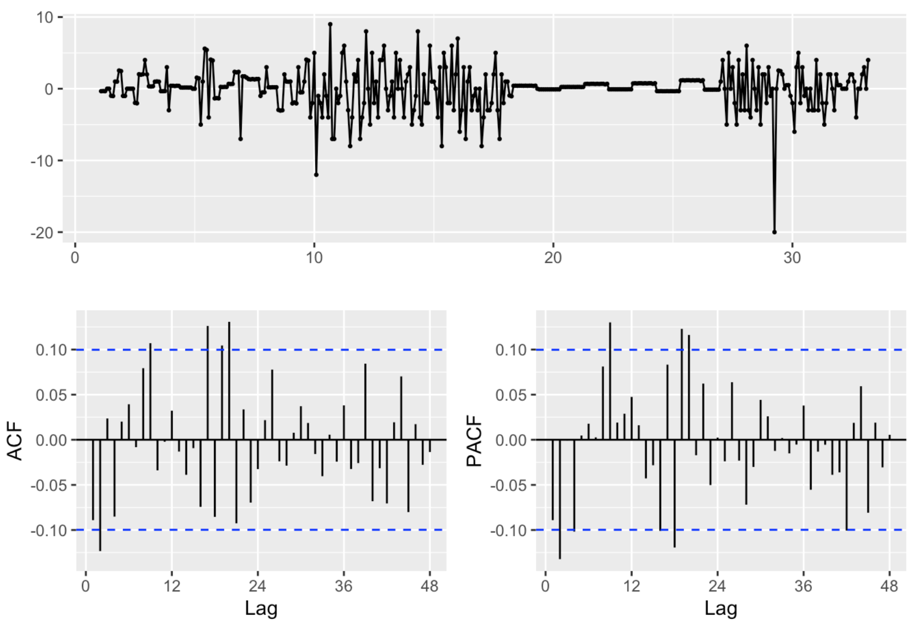
```
# First Differencing
data_ts_diff <- diff(data_ts)

# Augmented Dickey-Fuller test on the differenced series
print(adf.test(data_ts_diff, alternative = "stationary"))
```

```
## Warning in adf.test(data_ts_diff, alternative = "stationary"): p-value smaller
## than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data_ts_diff
## Dickey-Fuller = -6.6597, Lag order = 7, p-value = 0.01
## alternative hypothesis: stationary
```

Dickey-Fuller test's p-value is now much smaller than 0.01 rejecting the null hypothesis of non-stationarity. Therefore, we verify that the differenced series is stationary at 1% significance.

```
data_ts_diff %>%
  ggtsdisplay(lag.max = 48)
```



The ACF and PACF plots for the differenced series look complex. None exhibit patterns of exponential decay so they are not helpful in determining the orders of our model. Then, we rely on the extended autocorrelation function (EACF).

```
eacf(data_ts_diff)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 o x o o o o o o x o o  o  o  o
## 1 x x o o o o o o x o o  o  o  o
## 2 o x o x o o o o x o o  o  o  o
## 3 o x x x o o o o o o o  o  o  o
## 4 o x o x o o o o o o o  o  o  o
## 5 x x o x x o o o o o o  o  o  o
## 6 x x o x x o o o o o o  o  o  o
## 7 o x o o x o x o o o o  o  o  o
```

The EACF plot shows a triangle with vertex at (3, 4) and MA order no less than 2. Hence, we consider AR orders from 0 to 3 and MA orders from 2 to 4.

```r
for(p in 0: 3){
  for(q in 2: 4){
    model <- Arima(data_ts, order = c(p, 1, q), method = 'ML') # Maximum likelihood
    cat("\np =", p, "q =", q, ":\n")
    Y <- fitted(model)
    print(model)

    # Linear model
    se <- residuals(model, standardize = TRUE)
    data_lm <- data.frame(residuals = se, fitted = Y)
    model_lm <- lm(se ~ Y + I(Y^2), data = data_lm)

    # Breusch-Pagan Test
    print(bptest(model_lm))
  }
}
```

```
##
## p = 0 q = 2 :
## Series: data_ts
## ARIMA(0,1,2)
##
## Coefficients:
##           ma1      ma2
##       -0.1004  -0.1431
## s.e.   0.0511   0.0531
##
## sigma^2 = 7.878:  log likelihood = -945.11
## AIC=1896.22   AICc=1896.29   BIC=1908.09
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 11.01, df = 2, p-value = 0.004066
##
##
## p = 0 q = 3 :
## Series: data_ts
## ARIMA(0,1,3)
##
## Coefficients:
##           ma1      ma2     ma3
##       -0.1009  -0.1471  0.0169
## s.e.   0.0510   0.0541  0.0474
##
## sigma^2 = 7.896:  log likelihood = -945.05
## AIC=1898.1   AICc=1898.2   BIC=1913.92
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 10.869, df = 2, p-value = 0.004363
##
##
## p = 0 q = 4 :
## Series: data_ts
## ARIMA(0,1,4)
##
## Coefficients:
##           ma1      ma2     ma3      ma4
##       -0.0969  -0.1313  0.0267  -0.0578
## s.e.   0.0509   0.0518  0.0487   0.0478
##
## sigma^2 = 7.887:  log likelihood = -944.32
## AIC=1898.64   AICc=1898.8   BIC=1918.42
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 10.571, df = 2, p-value = 0.005065
##
##
```

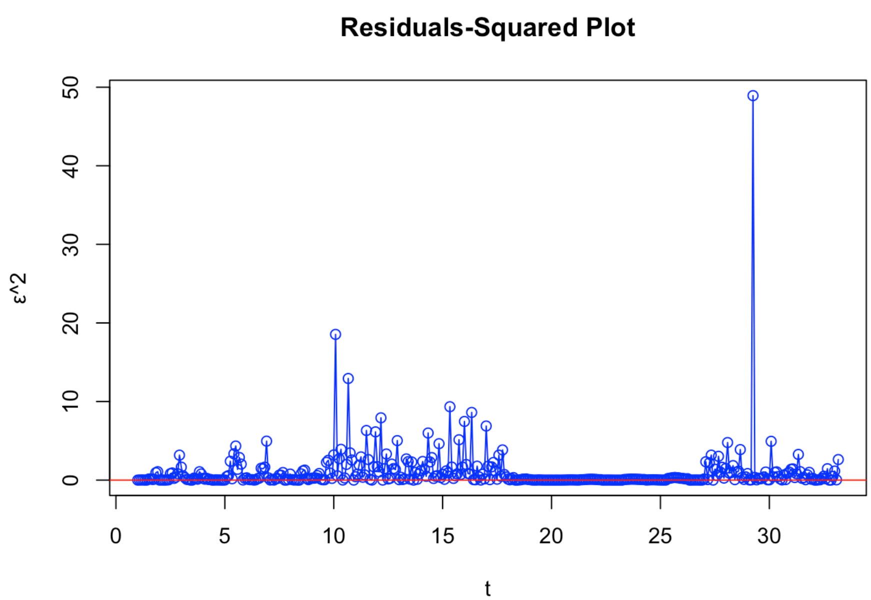
```
## p = 1 q = 2 :
## Series: data_ts
## ARIMA(1,1,2)
##
## Coefficients:
##           ar1     ma1      ma2
##       -0.3473  0.2446  -0.1781
## s.e.   0.4053  0.4034   0.0558
##
## sigma^2 = 7.889:  log likelihood = -944.87
## AIC=1897.74   AICc=1897.84   BIC=1913.56
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 10.533, df = 2, p-value = 0.005161
##
##
## p = 1 q = 3 :
## Series: data_ts
## ARIMA(1,1,3)
##
## Coefficients:
##           ar1     ma1      ma2      ma3
##       -0.8571  0.7563  -0.2313  -0.0930
## s.e.   0.1316  0.1425   0.0654   0.0644
##
## sigma^2 = 7.889:  log likelihood = -944.37
## AIC=1898.74   AICc=1898.9   BIC=1918.52
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 10.893, df = 2, p-value = 0.004311
##
##
## p = 1 q = 4 :
## Series: data_ts
## ARIMA(1,1,4)
##
## Coefficients:
##           ar1     ma1      ma2      ma3      ma4
##       -0.2816  0.1823  -0.1578  -0.0129  -0.0586
## s.e.   0.4811  0.4784   0.0718   0.0834   0.0509
##
## sigma^2 = 7.902:  log likelihood = -944.17
## AIC=1900.34   AICc=1900.56   BIC=1924.08
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 10.554, df = 2, p-value = 0.005108
##
##
## p = 2 q = 2 :
## Series: data_ts
## ARIMA(2,1,2)
##
## Coefficients:
##           ar1     ar2     ma1      ma2
##       -0.4293  0.2922  0.3260  -0.4665
## s.e.   0.2006  0.1858  0.1837   0.1693
##
## sigma^2 = 7.887:  log likelihood = -944.33
## AIC=1898.66   AICc=1898.82   BIC=1918.44
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 10.861, df = 2, p-value = 0.004382
##
##
## p = 2 q = 3 :
## Series: data_ts
```

```
## ARIMA(2,1,3)
##
## Coefficients:
##           ar1     ar2     ma1      ma2      ma3
##       -0.5812  0.2071  0.4812  -0.4034  -0.0407
## s.e.   0.5182  0.3558  0.5194   0.2908   0.1219
##
## sigma^2 = 7.905:  log likelihood = -944.27
## AIC=1900.55   AICc=1900.77   BIC=1924.28
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 10.962, df = 2, p-value = 0.004165
##
##
## p = 2 q = 4 :
## Series: data_ts
## ARIMA(2,1,4)
##
## Coefficients:
##           ar1      ar2     ma1     ma2      ma3      ma4
##       -0.5229  -0.8894  0.4301  0.7191  -0.1552  -0.1799
## s.e.   0.0771   0.0565  0.0883  0.0727   0.0545   0.0518
##
## sigma^2 = 7.8:  log likelihood = -941.33
## AIC=1896.66   AICc=1896.95   BIC=1924.35
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 10.557, df = 2, p-value = 0.005101
##
##
## p = 3 q = 2 :
## Series: data_ts
## ARIMA(3,1,2)
##
## Coefficients:
##           ar1     ar2      ar3     ma1      ma2
##       -0.4959  0.2430  -0.0267  0.3962  -0.4310
## s.e.   0.2777  0.2419   0.0744  0.2735   0.2071
##
## sigma^2 = 7.905:  log likelihood = -944.27
## AIC=1900.54   AICc=1900.76   BIC=1924.27
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 10.97, df = 2, p-value = 0.004148
##
##
## p = 3 q = 3 :
## Series: data_ts
## ARIMA(3,1,3)
##
## Coefficients:
##          ar1     ar2      ar3      ma1      ma2     ma3
##       0.6862  0.5147  -0.7068  -0.7859  -0.6047  0.8344
## s.e.  0.1080  0.1491   0.0956   0.0911   0.1327  0.0820
##
## sigma^2 = 7.667:  log likelihood = -938.15
## AIC=1890.31   AICc=1890.6   BIC=1918
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 10.858, df = 2, p-value = 0.004388
##
##
## p = 3 q = 4 :
## Series: data_ts
## ARIMA(3,1,4)
##
```

```
## Coefficients:
##           ar1      ar2     ar3     ma1     ma2      ma3      ma4
##       -0.6487  -0.9569  -0.128  0.5522  0.7702  -0.0365  -0.1940
## s.e.   0.2932   0.1645   0.287  0.2883  0.1467   0.2765   0.0567
##
## sigma^2 = 7.816:  log likelihood = -941.23
## AIC=1898.46   AICc=1898.84   BIC=1930.11
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 10.349, df = 2, p-value = 0.00566
```

White test's p-values less than 0.01 for all candidates reject the null hypothesis of constant variance of the residuals so the constancy of variance assumption has been violated at 1%-significance. The current best model is ARIMA(3, 1, 3) with significantly lower AIC (1890.31).

```
model <- Arima(data_ts, order = c(3, 1, 3), method = 'ML') # Current best mean model
ε <- rstandard(model) # Standardized residuals

plot(ε^2, type = 'o', col = 'blue', main = 'Residuals-Squared Plot', xlab = "t")
    abline(h = 0, col = "red")
```



The squared standardized residuals plot exhibits volatility clusters which calls for the need of a GARCH model. We consider the auto correlations of the squared residuals, absolute returns, and squared returns to help us determine GARCH orders.

```
# Autocorrelation of Squared Residuals and returns for GARCH parameters
Y <- fitted(model)
data_ts_return <- diff(log(data_ts)) # Returns

print("Residual-Squared:")
```

```
## [1] "Residual-Squared:"
```

```
eacf(ε^2)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 o o o o o o x o o o o  o  o  o
## 1 x o o o o o x o o o o  o  o  o
## 2 x x o o o o x o o o o  o  o  o
## 3 x o o o o o o o o o o  o  o  o
## 4 x o o x o o o o o o o  o  o  o
## 5 x o o x o o o o o o o  o  o  o
## 6 x x x x o x o o o o o  o  o  o
## 7 x o x o x x x o o o o  o  o  o
```

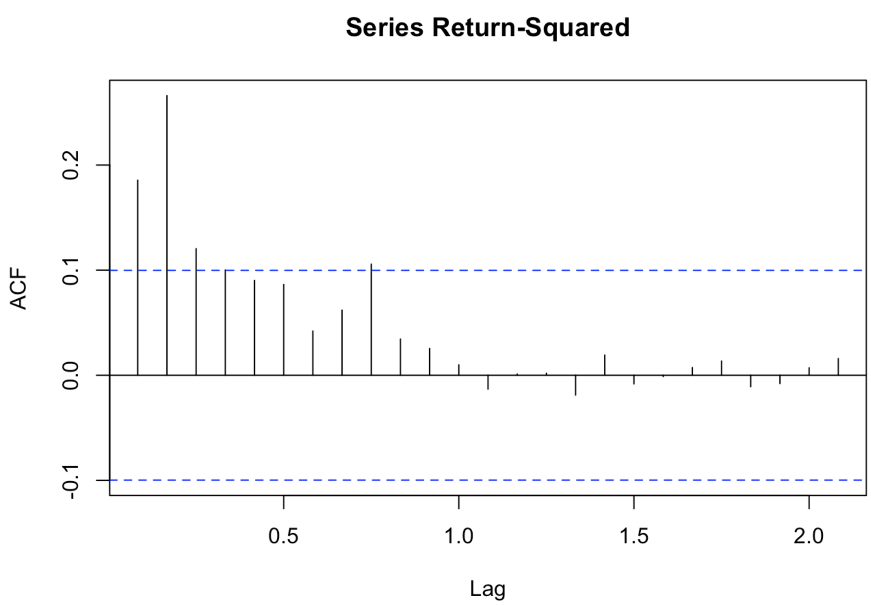
```
print("Absolute Returns:")
```

```
## [1] "Absolute Returns:"
```

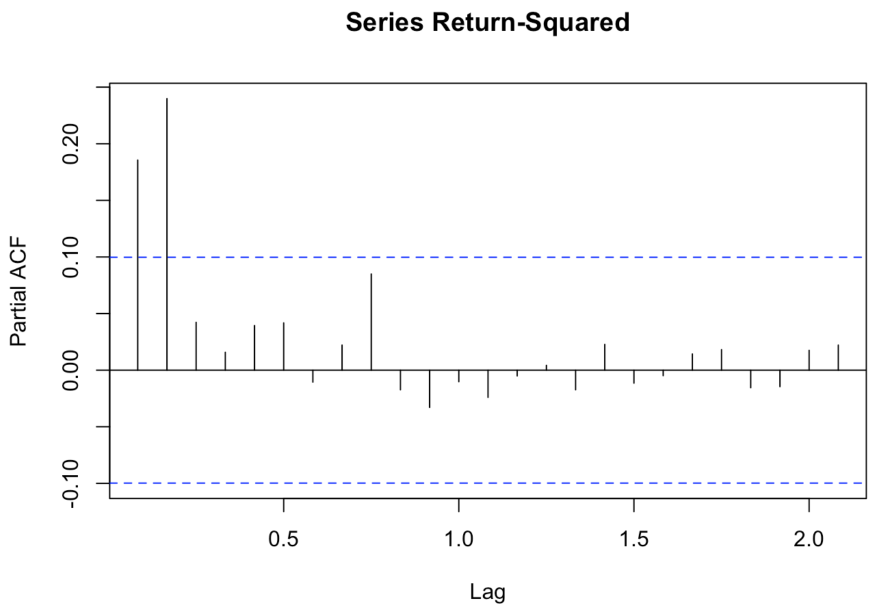
```
eacf(abs(data_ts_return))
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x x x x x x x  x  o  x
## 1 x x o o o o o o x o x  o  o  o
## 2 x o o o o o o o o o x  o  x  o
## 3 x x o o o o o o o o o  o  o  o
## 4 x o o x o o o o o o o  o  o  o
## 5 x x x x o o o o o o o  o  o  o
## 6 x x x x o o o o o o o  o  o  o
## 7 x o x x x o x o o o o  o  o  o
```

```
acf(data_ts_return^2, main = "Series Return-Squared")
```




```
pacf(data_ts_return^2, main = "Series Return-Squared")
```



Both the squared residuals and absolute returns EACF plots show a triangle with vertex at (3, 3). For the the squared returns, the ACF plot tails off and the PACF plot shows 2 significant lags suggesting GARCH orders qg = 0 and pg = 2.

```
for(p in 0: 3){
  for(q in 0: 3){
    model_garch <- ugarchspec(variance.model = list(model = "sGARCH",
                                                    garchOrder = c(p, q), submodel = NULL,
                                                    external.regressors = NULL,
                                                    variance.targeting = FALSE),

                              mean.model = list(armaOrder = c(3, 3),
                                                external.regressors = NULL,
                                                include.mean = FALSE
                                                ),

                              distribution.model = "norm") # Stationary differenced series presumes uncorrelated
returns with zero mean/ no constant term

    GARCH_fit <- ugarchfit(spec = model_garch, data = data_ts_return)
    print(GARCH_fit)
  }
}
```

```
## Warning in .sgarchfit(spec = spec, data = data, out.sample = out.sample, :
## ugarchfit-->warning: solver failer to converge.
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(0,0)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Convergence Problem:
## Solver Message:
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
```

```
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(0,1)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##        Estimate  Std. Error    t value Pr(>|t|)
## ar1    1.362320    0.117790   11.56564 0.000000
## ar2   -0.545159    0.231205   -2.35791 0.018378
## ar3    0.017768    0.165094    0.10762 0.914294
## ma1   -1.454569    0.105941  -13.73002 0.000000
## ma2    0.508433    0.181759    2.79730 0.005153
## ma3    0.116011    0.155038    0.74828 0.454293
## omega  0.000054    0.000010    5.19843 0.000000
## beta1  0.996723    0.000707 1409.80794 0.000000
##
## Robust Standard Errors:
##        Estimate  Std. Error    t value Pr(>|t|)
## ar1    1.362320    0.300505    4.53343 0.000006
## ar2   -0.545159    0.638573   -0.85371 0.393264
## ar3    0.017768    0.394933    0.04499 0.964115
## ma1   -1.454569    0.294750   -4.93492 0.000001
## ma2    0.508433    0.505549    1.00571 0.314557
## ma3    0.116011    0.400413    0.28973 0.772023
## omega  0.000054    0.000016    3.34831 0.000813
## beta1  0.996723    0.000732 1361.44763 0.000000
##
## LogLikelihood : 265.8039
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.3358
## Bayes        -1.2538
## Shibata      -1.3366
## Hannan-Quinn -1.3033
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                    0.000641  0.9798
## Lag[2*(p+q)+(p+q)-1][17]  2.833496  1.0000
## Lag[4*(p+q)+(p+q)-1][29]  9.690853  0.9797
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic   p-value
## Lag[1]                      11.56 6.752e-04
## Lag[2*(p+q)+(p+q)-1][2]     33.69 1.150e-09
## Lag[4*(p+q)+(p+q)-1][5]     54.03 4.441e-15
## d.o.f=1
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale   P-Value
## ARCH Lag[2]     43.80 0.500 2.000 3.628e-11
## ARCH Lag[4]     51.73 1.397 1.611 5.118e-14
## ARCH Lag[6]     56.51 2.222 1.500 3.442e-15
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  2.2048
## Individual Statistics:
## ar1   0.03692
## ar2   0.12909
## ar3   0.17984
## ma1   0.05042
## ma2   0.26195
## ma3   0.32374
## omega 0.20825
```

```
## beta1 0.20628
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.89 2.11 2.59
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value      prob sig
## Sign Bias           0.2324 0.8163283
## Negative Sign Bias  3.4301 0.0006694 ***
## Positive Sign Bias  2.3807 0.0177703  **
## Joint Effect       17.6948 0.0005084 ***
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     117.1    3.841e-16
## 2    30     129.4    1.195e-14
## 3    40     131.9    5.084e-12
## 4    50     142.8    4.251e-11
##
##
## Elapsed time : 0.182843
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(0,2)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##        Estimate  Std. Error     t value Pr(>|t|)
## ar1    0.581312    0.241521     2.40688 0.016090
## ar2    0.623022    0.370625     1.68100 0.092762
## ar3   -0.690339    0.256497    -2.69142 0.007115
## ma1   -0.701361    0.215320    -3.25730 0.001125
## ma2   -0.680397    0.348295    -1.95351 0.050759
## ma3    0.814974    0.227126     3.58819 0.000333
## omega  0.000043    0.000007     6.27623 0.000000
## beta1  0.997402    0.000025 40526.46235 0.000000
## beta2  0.000000    0.000815     0.00001 0.999992
##
## Robust Standard Errors:
##        Estimate  Std. Error     t value Pr(>|t|)
## ar1    0.581312    0.571798  1.0166e+00 0.309325
## ar2    0.623022    0.843903  7.3826e-01 0.460355
## ar3   -0.690339    0.507585 -1.3600e+00 0.173815
## ma1   -0.701361    0.481336 -1.4571e+00 0.145085
## ma2   -0.680397    0.794374 -8.5652e-01 0.391710
## ma3    0.814974    0.479116  1.7010e+00 0.088944
## omega  0.000043    0.000009  4.6630e+00 0.000003
## beta1  0.997402    0.000063  1.5777e+04 0.000000
## beta2  0.000000    0.001075  8.0000e-06 0.999994
##
## LogLikelihood : 264.9492
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.3262
## Bayes        -1.2339
## Shibata      -1.3272
## Hannan-Quinn -1.2896
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
```

```
##                            statistic p-value
## Lag[1]                        0.3656  0.5454
## Lag[2*(p+q)+(p+q)-1][17]    4.7071  1.0000
## Lag[4*(p+q)+(p+q)-1][29]   11.0732  0.9243
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic   p-value
## Lag[1]                      8.096 4.437e-03
## Lag[2*(p+q)+(p+q)-1][5]    43.736 3.955e-12
## Lag[4*(p+q)+(p+q)-1][9]    54.899 4.008e-14
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale   P-Value
## ARCH Lag[3]      3.86 0.500 2.000 0.0494372
## ARCH Lag[5]     11.95 1.440 1.667 0.0021851
## ARCH Lag[7]     16.18 2.315 1.543 0.0005866
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  3.3995
## Individual Statistics:
## ar1   0.27263
## ar2   0.06659
## ar3   0.09008
## ma1   0.27222
## ma2   0.04643
## ma3   0.15106
## omega 0.20300
## beta1 0.20135
## beta2 0.20135
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.1 2.32 2.82
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                      t-value     prob sig
## Sign Bias          1.857e-04 0.999852
## Negative Sign Bias 3.476e+00 0.000568 ***
## Positive Sign Bias 1.803e+00 0.072211   *
## Joint Effect       1.562e+01 0.001355 ***
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     133.6    3.047e-19
## 2    30     158.0    1.042e-19
## 3    40     156.0    6.405e-16
## 4    50     165.3    1.598e-14
##
##
## Elapsed time : 0.1783969
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(0,3)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##        Estimate  Std. Error     t value Pr(>|t|)
```

```
## ar1    1.379082    0.143619  9.6024e+00 0.000000
## ar2   -0.571560    0.292709 -1.9527e+00 0.050860
## ar3    0.030104    0.206995  1.4544e-01 0.884368
## ma1   -1.471873    0.134972 -1.0905e+01 0.000000
## ma2    0.537799    0.232032  2.3178e+00 0.020461
## ma3    0.101485    0.199681  5.0823e-01 0.611289
## omega  0.000027    0.000037  7.1848e-01 0.472460
## beta1  0.000000    0.319672  0.0000e+00 1.000000
## beta2  0.016135    0.317676  5.0791e-02 0.959492
## beta3  0.982769    0.000085  1.1515e+04 0.000000
##
## Robust Standard Errors:
##        Estimate  Std. Error    t value Pr(>|t|)
## ar1    1.379082    0.511229    2.69758 0.006985
## ar2   -0.571560    1.107259   -0.51619 0.605719
## ar3    0.030104    0.717622    0.04195 0.966539
## ma1   -1.471873    0.515571   -2.85484 0.004306
## ma2    0.537799    0.886646    0.60655 0.544147
## ma3    0.101485    0.730541    0.13892 0.889516
## omega  0.000027    0.000019    1.36680 0.171689
## beta1  0.000000    0.098722    0.00000 1.000000
## beta2  0.016135    0.101652    0.15873 0.873882
## beta3  0.982769    0.000385 2554.77640 0.000000
##
## LogLikelihood : 265.7327
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.3250
## Bayes        -1.2226
## Shibata      -1.3263
## Hannan-Quinn -1.2844
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                           statistic p-value
## Lag[1]                    0.0006127  0.9803
## Lag[2*(p+q)+(p+q)-1][17] 2.8616947  1.0000
## Lag[4*(p+q)+(p+q)-1][29] 9.7275744  0.9789
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                           statistic   p-value
## Lag[1]                        11.68 0.0006308
## Lag[2*(p+q)+(p+q)-1][8]      63.75 0.0000000
## Lag[4*(p+q)+(p+q)-1][14]     74.39 0.0000000
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale  P-Value
## ARCH Lag[4]     7.557 0.500 2.000 0.005978
## ARCH Lag[6]    13.235 1.461 1.711 0.001339
## ARCH Lag[8]    15.753 2.368 1.583 0.001040
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  3.6352
## Individual Statistics:
## ar1   0.03659
## ar2   0.12930
## ar3   0.17828
## ma1   0.05118
## ma2   0.26202
## ma3   0.32195
## omega 0.19515
## beta1 0.19316
## beta2 0.19315
## beta3 0.19316
##
## Asymptotic Critical Values (10% 5% 1%)
```

```
## Joint Statistic:          2.29 2.54 3.05
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value      prob sig
## Sign Bias           0.2391 0.8111358
## Negative Sign Bias  3.4255 0.0006804 ***
## Positive Sign Bias  2.3940 0.0171492  **
## Joint Effect       17.7531 0.0004945 ***
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     115.0    9.330e-16
## 2    30     126.0    4.634e-14
## 3    40     130.1    9.980e-12
## 4    50     140.9    7.842e-11
##
##
## Elapsed time : 0.345068
```

```
## Warning in .sgarchfit(spec = spec, data = data, out.sample = out.sample, :
## ugarchfit-->warning: solver failer to converge.
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,0)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Convergence Problem:
## Solver Message:
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1     1.989314    0.008567  232.1967 0.000000
## ar2    -1.309804    0.003276 -399.7580 0.000000
## ar3     0.239411    0.037885    6.3194 0.000000
## ma1    -1.633985    0.024570  -66.5040 0.000000
## ma2     0.720537    0.029971   24.0414 0.000000
## ma3     0.096652    0.021949    4.4035 0.000011
## omega   0.000066    0.000023    2.8766 0.004019
## alpha1  0.237991    0.027012    8.8107 0.000000
## beta1   0.761009    0.022348   34.0533 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## ar1     1.989314    0.031956  62.2523 0.000000
## ar2    -1.309804    0.065539 -19.9852 0.000000
## ar3     0.239411    0.075239   3.1820 0.001463
## ma1    -1.633985    0.044928 -36.3689 0.000000
## ma2     0.720537    0.024217  29.7534 0.000000
## ma3     0.096652    0.024904   3.8809 0.000104
```

```
## omega   0.000066    0.000059   1.1219 0.261926
## alpha1  0.237991    0.052777   4.5094 0.000007
## beta1   0.761009    0.076639   9.9298 0.000000
##
## LogLikelihood : 379.8168
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.9213
## Bayes        -1.8291
## Shibata      -1.9224
## Hannan-Quinn -1.8848
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                       4.834 0.02791
## Lag[2*(p+q)+(p+q)-1][17]     8.112 0.93624
## Lag[4*(p+q)+(p+q)-1][29]    18.160 0.13817
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.02168  0.8829
## Lag[2*(p+q)+(p+q)-1][5]   0.25921  0.9877
## Lag[4*(p+q)+(p+q)-1][9]   0.60177  0.9972
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]   0.01782 0.500 2.000  0.8938
## ARCH Lag[5]   0.55974 1.440 1.667  0.8658
## ARCH Lag[7]   0.69745 2.315 1.543  0.9572
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  8.5606
## Individual Statistics:
## ar1    1.0244
## ar2    0.5865
## ar3    0.3547
## ma1    0.1842
## ma2    0.1412
## ma3    0.1299
## omega  0.1436
## alpha1 0.4141
## beta1  0.1320
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.1 2.32 2.82
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.8322 0.4058
## Negative Sign Bias  0.1714 0.8640
## Positive Sign Bias  0.6921 0.4893
## Joint Effect        0.8426 0.8393
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     41.25    2.232e-03
## 2    30     70.53    2.566e-05
## 3    40     71.62    1.118e-03
## 4    50     81.88    2.236e-03
##
##
```

```
## Elapsed time : 0.099931
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,2)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1     1.519239    0.150738 10.078644 0.000000
## ar2    -0.913228    0.224484 -4.068113 0.000047
## ar3     0.251395    0.120797  2.081144 0.037421
## ma1    -1.215522    0.151284 -8.034688 0.000000
## ma2     0.508429    0.226375  2.245957 0.024707
## ma3    -0.015809    0.082160 -0.192417 0.847416
## omega   0.000122    0.000024  5.150490 0.000000
## alpha1  0.231834    0.025285  9.168866 0.000000
## beta1   0.767165    0.369566  2.075854 0.037908
## beta2   0.000001    0.309167  0.000004 0.999997
##
## Robust Standard Errors:
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1     1.519239    0.245680  6.183811 0.000000
## ar2    -0.913228    0.303380 -3.010180 0.002611
## ar3     0.251395    0.141408  1.777803 0.075436
## ma1    -1.215522    0.149748 -8.117109 0.000000
## ma2     0.508429    0.139258  3.650978 0.000261
## ma3    -0.015809    0.110839 -0.142630 0.886582
## omega   0.000122    0.000179  0.680584 0.496135
## alpha1  0.231834    0.164296  1.411079 0.158221
## beta1   0.767165    1.237374  0.619994 0.535262
## beta2   0.000001    1.050178  0.000001 0.999999
##
## LogLikelihood : 372.613
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.8788
## Bayes        -1.7763
## Shibata      -1.8801
## Hannan-Quinn -1.8382
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                       5.881 0.01531
## Lag[2*(p+q)+(p+q)-1][17]     9.079 0.43852
## Lag[4*(p+q)+(p+q)-1][29]    19.901 0.05444
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                     0.03065  0.8610
## Lag[2*(p+q)+(p+q)-1][8]    0.41079  0.9979
## Lag[4*(p+q)+(p+q)-1][14]   0.65724  1.0000
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[4]    0.2090 0.500 2.000  0.6476
## ARCH Lag[6]    0.5969 1.461 1.711  0.8645
## ARCH Lag[8]    0.6551 2.368 1.583  0.9678
##
```

```
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  7.3459
## Individual Statistics:
## ar1    1.3265
## ar2    0.8031
## ar3    1.0001
## ma1    1.0871
## ma2    0.6388
## ma3    1.0515
## omega  0.3431
## alpha1 0.2827
## beta1  0.1044
## beta2  0.1025
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.29 2.54 3.05
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           1.2268 0.2206
## Negative Sign Bias  0.4202 0.6746
## Positive Sign Bias  0.8112 0.4177
## Joint Effect        1.5972 0.6600
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     40.74     0.002617
## 2    30     45.19     0.028186
## 3    40     49.85     0.114185
## 4    50     59.85     0.137664
##
##
## Elapsed time : 0.0921309
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,3)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1    -0.396941    0.149120 -2.661888 0.007770
## ar2     0.393871    0.119095  3.307200 0.000942
## ar3     0.459869    0.080504  5.712359 0.000000
## ma1     0.651261    0.144224  4.515610 0.000006
## ma2    -0.177719    0.161607 -1.099697 0.271464
## ma3    -0.517089    0.090790 -5.695449 0.000000
## omega   0.000152    0.000037  4.066016 0.000048
## alpha1  0.311728    0.039443  7.903360 0.000000
## beta1   0.687271    0.270962  2.536414 0.011199
## beta2   0.000000    0.373017  0.000001 1.000000
## beta3   0.000000    0.101972  0.000001 0.999999
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## ar1    -0.396941    0.185926  -2.1349 0.032766
## ar2     0.393871    0.114638   3.4358 0.000591
## ar3     0.459869    0.159105   2.8903 0.003848
## ma1     0.651261    0.154400   4.2180 0.000025
## ma2    -0.177719    0.158686  -1.1199 0.262738
## ma3    -0.517089    0.122138  -4.2336 0.000023
## omega   0.000152    0.000121   1.2553 0.209369
```

```
## alpha1  0.311728    0.063191   4.9331 0.000001
## beta1   0.687271    0.503721   1.3644 0.172445
## beta2   0.000000    0.634061   0.0000 1.000000
## beta3   0.000000    0.245135   0.0000 1.000000
##
## LogLikelihood : 386.8773
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.9476
## Bayes        -1.8348
## Shibata      -1.9491
## Hannan-Quinn -1.9028
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                          statistic   p-value
## Lag[1]                       3.647 5.617e-02
## Lag[2*(p+q)+(p+q)-1][17]    19.704 0.000e+00
## Lag[4*(p+q)+(p+q)-1][29]    31.413 4.599e-06
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                      0.1545  0.6942
## Lag[2*(p+q)+(p+q)-1][11]    1.3155  0.9904
## Lag[4*(p+q)+(p+q)-1][19]    2.0835  0.9994
## d.o.f=4
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[5]    0.7632 0.500 2.000  0.3823
## ARCH Lag[7]    1.1000 1.473 1.746  0.7295
## ARCH Lag[9]    1.1539 2.402 1.619  0.9092
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  6.8861
## Individual Statistics:
## ar1    0.41437
## ar2    0.29109
## ar3    0.09937
## ma1    0.08201
## ma2    0.28502
## ma3    0.13136
## omega  0.44024
## alpha1 0.55585
## beta1  0.17245
## beta2  0.14178
## beta3  0.14451
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.49 2.75 3.27
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias          1.19850 0.2315
## Negative Sign Bias 0.08795 0.9300
## Positive Sign Bias 0.96500 0.3352
## Joint Effect       1.76264 0.6231
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     56.49    1.372e-05
## 2    30     69.60    3.443e-05
## 3    40     80.32    1.095e-04
```

```
## 4    50      79.54    3.760e-03
##
##
## Elapsed time : 0.1612689
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(2,0)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## ar1      0.839608    0.000284   2954.1        0
## ar2      0.511160    0.000155   3301.9        0
## ar3     -0.587149    0.000179  -3289.3        0
## ma1     -0.898277    0.000414  -2169.7        0
## ma2     -0.585541    0.000177  -3309.4        0
## ma3      0.652448    0.000199   3273.8        0
## omega    0.000203    0.000000   2941.7        0
## alpha1   0.238301    0.000072   3309.1        0
## alpha2   0.329969    0.000112   2948.9        0
##
## Robust Standard Errors:
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1      0.839608   12.176459  0.068953  0.94503
## ar2      0.511160    3.414726  0.149693  0.88101
## ar3     -0.587149    3.991862 -0.147087  0.88306
## ma1     -0.898277   21.769945 -0.041262  0.96709
## ma2     -0.585541    4.172327 -0.140339  0.88839
## ma3      0.652448    4.504463  0.144845  0.88483
## omega    0.000203    0.002973  0.068273  0.94557
## alpha1   0.238301    1.334610  0.178555  0.85829
## alpha2   0.329969    4.806763  0.068647  0.94527
##
## LogLikelihood : 120.5063
##
## Information Criteria
## ------------------------------------
##
## Akaike       -0.57775
## Bayes        -0.48552
## Shibata      -0.57881
## Hannan-Quinn -0.54118
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                       1.287  0.2566
## Lag[2*(p+q)+(p+q)-1][17]     6.865  1.0000
## Lag[4*(p+q)+(p+q)-1][29]    15.368  0.4245
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.01855  0.8917
## Lag[2*(p+q)+(p+q)-1][5]   0.04578  0.9996
## Lag[4*(p+q)+(p+q)-1][9]   0.06682  1.0000
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]  0.001436 0.500 2.000  0.9698
## ARCH Lag[5]  0.022147 1.440 1.667  0.9985
## ARCH Lag[7]  0.035011 2.315 1.543  0.9999
```

```
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  251055771
## Individual Statistics:
## ar1    0.1244
## ar2    0.1244
## ar3    0.1244
## ma1    0.1244
## ma2    0.1244
## ma3    0.1243
## omega  0.1244
## alpha1 0.1244
## alpha2 0.1244
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.1 2.32 2.82
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           1.1450 0.2529
## Negative Sign Bias  0.1687 0.8661
## Positive Sign Bias  1.4179 0.1570
## Joint Effect        2.4636 0.4819
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     171.0    1.815e-26
## 2    30     228.3    7.794e-33
## 3    40     277.4    1.016e-37
## 4    50     349.5    5.851e-47
##
##
## Elapsed time : 0.3076329
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(2,1)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## ar1      1.981532    0.017222 115.0560 0.000000
## ar2     -1.288396    0.015508 -83.0788 0.000000
## ar3      0.227209    0.038467   5.9065 0.000000
## ma1     -1.642300    0.028367 -57.8954 0.000000
## ma2      0.734357    0.034215  21.4632 0.000000
## ma3      0.087446    0.022131   3.9514 0.000078
## omega    0.000081    0.000031   2.5890 0.009627
## alpha1   0.178354    0.056275   3.1693 0.001528
## alpha2   0.084875    0.069423   1.2226 0.221487
## beta1    0.735771    0.034138  21.5530 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1      1.981532    0.029368  67.47146 0.000000
## ar2     -1.288396    0.077860 -16.54766 0.000000
## ar3      0.227209    0.081740   2.77965 0.005442
## ma1     -1.642300    0.058117 -28.25868 0.000000
## ma2      0.734357    0.038068  19.29044 0.000000
## ma3      0.087446    0.021199   4.12501 0.000037
## omega    0.000081    0.000084   0.97239 0.330857
## alpha1   0.178354    0.074404   2.39711 0.016525
```

```
## alpha2  0.084875    0.121599   0.69799 0.485184
## beta1   0.735771    0.111692   6.58751 0.000000
##
## LogLikelihood : 380.5113
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.9197
## Bayes        -1.8173
## Shibata      -1.9210
## Hannan-Quinn -1.8791
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                       4.550 0.03292
## Lag[2*(p+q)+(p+q)-1][17]     7.897 0.97237
## Lag[4*(p+q)+(p+q)-1][29]    17.776 0.16584
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                    0.002885  0.9572
## Lag[2*(p+q)+(p+q)-1][8]   0.569300  0.9946
## Lag[4*(p+q)+(p+q)-1][14]  0.970657  0.9997
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[4]    0.2847 0.500 2.000  0.5936
## ARCH Lag[6]    0.8611 1.461 1.711  0.7879
## ARCH Lag[8]    0.9524 2.368 1.583  0.9312
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  8.671
## Individual Statistics:
## ar1    0.9335
## ar2    0.4935
## ar3    0.2687
## ma1    0.1817
## ma2    0.1333
## ma3    0.1161
## omega  0.1679
## alpha1 0.5752
## alpha2 0.2800
## beta1  0.1422
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.29 2.54 3.05
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.9007 0.3683
## Negative Sign Bias  0.2743 0.7840
## Positive Sign Bias  0.6567 0.5117
## Joint Effect        0.9005 0.8253
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     37.01    7.923e-03
## 2    30     67.26    7.111e-05
## 3    40     79.70    1.302e-04
## 4    50     98.97    3.125e-05
##
##
```

```
## Elapsed time : 0.120501
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(2,2)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1     0.163481    0.112774  1.449633 0.147161
## ar2    -0.167063    0.133037 -1.255762 0.209202
## ar3     0.611241    0.089881  6.800579 0.000000
## ma1     0.102993    0.119490  0.861941 0.388720
## ma2     0.145776    0.129461  1.126029 0.260153
## ma3    -0.520012    0.093709 -5.549208 0.000000
## omega   0.000194    0.000090  2.147469 0.031756
## alpha1  0.185248    0.064585  2.868284 0.004127
## alpha2  0.157655    0.103035  1.530110 0.125990
## beta1   0.656095    0.307817  2.131444 0.033053
## beta2   0.000001    0.220805  0.000004 0.999997
##
## Robust Standard Errors:
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1     0.163481    0.104407  1.565797 0.117396
## ar2    -0.167063    0.142677 -1.170920 0.241631
## ar3     0.611241    0.138059  4.427386 0.000010
## ma1     0.102993    0.148276  0.694602 0.487305
## ma2     0.145776    0.139550  1.044617 0.296200
## ma3    -0.520012    0.171225 -3.037016 0.002389
## omega   0.000194    0.000257  0.754605 0.450486
## alpha1  0.185248    0.069382  2.669959 0.007586
## alpha2  0.157655    0.183261  0.860279 0.389635
## beta1   0.656095    0.769948  0.852130 0.394142
## beta2   0.000001    0.530686  0.000002 0.999999
##
## LogLikelihood : 383.2707
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.9289
## Bayes        -1.8161
## Shibata      -1.9304
## Hannan-Quinn -1.8842
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic   p-value
## Lag[1]                      5.157 2.316e-02
## Lag[2*(p+q)+(p+q)-1][17]    14.759 1.110e-16
## Lag[4*(p+q)+(p+q)-1][29]    26.709 3.693e-04
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.01675  0.8970
## Lag[2*(p+q)+(p+q)-1][11]   1.15269  0.9941
## Lag[4*(p+q)+(p+q)-1][19]   1.90241  0.9996
## d.o.f=4
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[5]    0.8119 0.500 2.000  0.3676
## ARCH Lag[7]    1.2891 1.473 1.746  0.6779
```

```
## ARCH Lag[9]    1.3870 2.402 1.619  0.8716
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  7.3601
## Individual Statistics:
## ar1    0.3530
## ar2    0.1755
## ar3    0.1950
## ma1    0.3093
## ma2    0.1588
## ma3    0.1686
## omega  0.4619
## alpha1 0.7888
## alpha2 0.2832
## beta1  0.1799
## beta2  0.1633
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.49 2.75 3.27
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias          0.57536 0.5654
## Negative Sign Bias 0.35971 0.7193
## Positive Sign Bias 0.05821 0.9536
## Joint Effect       0.47679 0.9240
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     54.93    2.380e-05
## 2    30     65.09    1.381e-04
## 3    40     84.47    3.373e-05
## 4    50     94.05    1.152e-04
##
##
## Elapsed time : 0.1949601
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(2,3)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error    t value Pr(>|t|)
## ar1     1.981533    0.017489 113.299081 0.000000
## ar2    -1.288397    0.017251 -74.683559 0.000000
## ar3     0.227210    0.037816   6.008318 0.000000
## ma1    -1.642300    0.027838 -58.994113 0.000000
## ma2     0.734358    0.033854  21.691643 0.000000
## ma3     0.087446    0.022095   3.957696 0.000076
## omega   0.000081    0.000044   1.868452 0.061699
## alpha1  0.178354    0.038899   4.585075 0.000005
## alpha2  0.084876    0.092353   0.919040 0.358075
## beta1   0.735768    0.455790   1.614267 0.106469
## beta2   0.000002    0.352451   0.000005 0.999996
## beta3   0.000001    0.097558   0.000009 0.999993
##
## Robust Standard Errors:
##         Estimate  Std. Error    t value Pr(>|t|)
## ar1     1.981533    0.032224  61.492212 0.000000
## ar2    -1.288397    0.076374 -16.869550 0.000000
## ar3     0.227210    0.085306   2.663451 0.007734
```

```
## ma1    -1.642300    0.071410 -22.998053 0.000000
## ma2     0.734358    0.056434  13.012731 0.000000
## ma3     0.087446    0.022372   3.908727 0.000093
## omega   0.000081    0.000261   0.312071 0.754987
## alpha1  0.178354    0.160175   1.113496 0.265496
## alpha2  0.084876    0.597420   0.142070 0.887024
## beta1   0.735768    2.326647   0.316235 0.751824
## beta2   0.000002    1.529887   0.000001 0.999999
## beta3   0.000001    0.235065   0.000004 0.999997
##
## LogLikelihood : 380.5113
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.9094
## Bayes        -1.7864
## Shibata      -1.9112
## Hannan-Quinn -1.8606
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      4.550 0.03292
## Lag[2*(p+q)+(p+q)-1][17]    7.897 0.97237
## Lag[4*(p+q)+(p+q)-1][29]   17.776 0.16583
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                   0.002885  0.9572
## Lag[2*(p+q)+(p+q)-1][14]  0.970656  0.9997
## Lag[4*(p+q)+(p+q)-1][24]  1.599479  1.0000
## d.o.f=5
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[6]     0.1207 0.500 2.000  0.7282
## ARCH Lag[8]     0.2038 1.480 1.774  0.9710
## ARCH Lag[10]    0.3968 2.424 1.650  0.9913
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  9.434
## Individual Statistics:
## ar1    0.9335
## ar2    0.4935
## ar3    0.2687
## ma1    0.1817
## ma2    0.1333
## ma3    0.1161
## omega  0.1679
## alpha1 0.5752
## alpha2 0.2800
## beta1  0.1422
## beta2  0.1322
## beta3  0.1353
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.69 2.96 3.51
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.9007 0.3683
## Negative Sign Bias  0.2743 0.7840
## Positive Sign Bias  0.6567 0.5117
## Joint Effect        0.9005 0.8253
##
##
```

```
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     37.01    7.923e-03
## 2    30     67.26    7.111e-05
## 3    40     79.70    1.302e-04
## 4    50     98.97    3.125e-05
##
##
## Elapsed time : 0.138525
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(3,0)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1     0.397119    0.079573    4.9906 0.000001
## ar2     0.489936    0.051416    9.5289 0.000000
## ar3     0.099708    0.025964    3.8403 0.000123
## ma1    -0.023635    0.005593   -4.2257 0.000024
## ma2    -0.550190    0.000992 -554.5574 0.000000
## ma3    -0.435415    0.003080 -141.3715 0.000000
## omega   0.000809    0.000166    4.8573 0.000001
## alpha1  0.263798    0.044331    5.9507 0.000000
## alpha2  0.288739    0.055820    5.1727 0.000000
## alpha3  0.446462    0.062122    7.1869 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1     0.397119    0.303528    1.3083 0.190756
## ar2     0.489936    0.209476    2.3389 0.019342
## ar3     0.099708    0.111493    0.8943 0.371159
## ma1    -0.023635    0.016516   -1.4311 0.152405
## ma2    -0.550190    0.005062 -108.6896 0.000000
## ma3    -0.435415    0.008566  -50.8302 0.000000
## omega   0.000809    0.000476    1.6998 0.089169
## alpha1  0.263798    0.110311    2.3914 0.016784
## alpha2  0.288739    0.109709    2.6319 0.008492
## alpha3  0.446462    0.168991    2.6419 0.008243
##
## LogLikelihood : 383.5002
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.9352
## Bayes        -1.8328
## Shibata      -1.9365
## Hannan-Quinn -1.8946
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                          statistic   p-value
## Lag[1]                       9.504 2.051e-03
## Lag[2*(p+q)+(p+q)-1][17]    25.225 0.000e+00
## Lag[4*(p+q)+(p+q)-1][29]    42.362 2.552e-11
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                      0.0761  0.7827
## Lag[2*(p+q)+(p+q)-1][8]     0.8942  0.9808
## Lag[4*(p+q)+(p+q)-1][14]    6.6497  0.5517
```

```
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[4]    0.1353 0.500 2.000  0.7130
## ARCH Lag[6]    0.7498 1.461 1.711  0.8203
## ARCH Lag[8]    0.8096 2.368 1.583  0.9502
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  6.4793
## Individual Statistics:
## ar1    0.06084
## ar2    0.06252
## ar3    0.06446
## ma1    0.03885
## ma2    0.04684
## ma3    0.04393
## omega  0.82305
## alpha1 2.61118
## alpha2 0.61714
## alpha3 0.47393
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.29 2.54 3.05
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias          0.25945 0.7954
## Negative Sign Bias 0.06586 0.9475
## Positive Sign Bias 0.40972 0.6822
## Joint Effect       0.19210 0.9789
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     73.48    2.403e-08
## 2    30     76.75    3.432e-06
## 3    40    101.88    1.601e-07
## 4    50    120.48    5.934e-08
##
##
## Elapsed time : 0.6151769
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(3,1)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## ar1     0.186096    0.111199  1.67353 0.094223
## ar2    -0.134569    0.134947 -0.99720 0.318668
## ar3     0.600180    0.092000  6.52367 0.000000
## ma1     0.087685    0.111881  0.78373 0.433199
## ma2     0.124951    0.126983  0.98400 0.325116
## ma3    -0.543047    0.093182 -5.82781 0.000000
## omega   0.000262    0.000138  1.89405 0.058219
## alpha1  0.194312    0.063026  3.08307 0.002049
## alpha2  0.108569    0.082288  1.31938 0.187041
## alpha3  0.125871    0.119683  1.05170 0.292937
## beta1   0.570248    0.126197  4.51872 0.000006
##
```

```
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## ar1     0.186096    0.159740  1.16499 0.244022
## ar2    -0.134569    0.202928 -0.66314 0.507244
## ar3     0.600180    0.152181  3.94385 0.000080
## ma1     0.087685    0.144301  0.60765 0.543419
## ma2     0.124951    0.204161  0.61202 0.540523
## ma3    -0.543047    0.209274 -2.59490 0.009462
## omega   0.000262    0.000454  0.57755 0.563565
## alpha1  0.194312    0.076318  2.54610 0.010894
## alpha2  0.108569    0.124977  0.86872 0.385003
## alpha3  0.125871    0.402856  0.31245 0.754701
## beta1   0.570248    0.427873  1.33275 0.182614
##
## LogLikelihood : 384.0633
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.9330
## Bayes        -1.8202
## Shibata      -1.9345
## Hannan-Quinn -1.8883
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                          statistic  p-value
## Lag[1]                       5.286 0.021494
## Lag[2*(p+q)+(p+q)-1][17]    15.547 0.000000
## Lag[4*(p+q)+(p+q)-1][29]    27.149 0.000252
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                    0.004977  0.9438
## Lag[2*(p+q)+(p+q)-1][11]  1.467462  0.9860
## Lag[4*(p+q)+(p+q)-1][19]  2.502037  0.9982
## d.o.f=4
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[5]     1.050 0.500 2.000  0.3054
## ARCH Lag[7]     1.486 1.473 1.746  0.6264
## ARCH Lag[9]     1.580 2.402 1.619  0.8377
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  7.2113
## Individual Statistics:
## ar1    0.3481
## ar2    0.2064
## ar3    0.1278
## ma1    0.3278
## ma2    0.1818
## ma3    0.2026
## omega  0.5454
## alpha1 1.3658
## alpha2 0.4762
## alpha3 0.1745
## beta1  0.2177
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.49 2.75 3.27
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.5481 0.5840
## Negative Sign Bias  0.2704 0.7870
## Positive Sign Bias  0.1278 0.8983
```

```
## Joint Effect        0.4977 0.9194
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     60.32    3.442e-06
## 2    30     77.21    2.941e-06
## 3    40     88.82    9.379e-06
## 4    50     93.79    1.232e-04
##
##
## Elapsed time : 0.3306892
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(3,2)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## ar1     1.941981    0.043572   44.5694 0.000000
## ar2    -1.297932    0.075010  -17.3034 0.000000
## ar3     0.265101    0.051399    5.1577 0.000000
## ma1    -1.614467    0.008948 -180.4276 0.000000
## ma2     0.733231    0.008787   83.4451 0.000000
## ma3     0.084656    0.004325   19.5734 0.000000
## omega   0.000167    0.000053    3.1574 0.001592
## alpha1  0.200249    0.060981    3.2838 0.001024
## alpha2  0.081112    0.061342    1.3223 0.186073
## alpha3  0.203974    0.062044    3.2876 0.001011
## beta1   0.152130    0.131993    1.1526 0.249091
## beta2   0.361535    0.111615    3.2391 0.001199
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## ar1     1.941981    0.080868  24.0142 0.000000
## ar2    -1.297932    0.148436  -8.7441 0.000000
## ar3     0.265101    0.134455   1.9717 0.048648
## ma1    -1.614467    0.035135 -45.9509 0.000000
## ma2     0.733231    0.054882  13.3600 0.000000
## ma3     0.084656    0.048324   1.7518 0.079800
## omega   0.000167    0.000121   1.3818 0.167033
## alpha1  0.200249    0.092199   2.1719 0.029862
## alpha2  0.081112    0.077248   1.0500 0.293711
## alpha3  0.203974    0.140503   1.4517 0.146572
## beta1   0.152130    0.120635   1.2611 0.207280
## beta2   0.361535    0.173007   2.0897 0.036644
##
## LogLikelihood : 384.5243
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.9302
## Bayes        -1.8072
## Shibata      -1.9320
## Hannan-Quinn -1.8814
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                       4.007 0.04531
## Lag[2*(p+q)+(p+q)-1][17]     7.804 0.98158
## Lag[4*(p+q)+(p+q)-1][29]    18.099 0.14234
## d.o.f=6
## H0 : No serial correlation
```

```
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                             statistic p-value
## Lag[1]                       0.007275  0.9320
## Lag[2*(p+q)+(p+q)-1][14]  1.514794  0.9974
## Lag[4*(p+q)+(p+q)-1][24]  2.627277  0.9999
## d.o.f=5
##
## Weighted ARCH LM Tests
## ------------------------------------
##              Statistic Shape Scale P-Value
## ARCH Lag[6]     0.1766 0.500 2.000  0.6743
## ARCH Lag[8]     0.2691 1.480 1.774  0.9572
## ARCH Lag[10]    0.9042 2.424 1.650  0.9480
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  9.1267
## Individual Statistics:
## ar1    1.1176
## ar2    0.6054
## ar3    0.2697
## ma1    0.3333
## ma2    0.2295
## ma3    0.1356
## omega  0.2145
## alpha1 1.2339
## alpha2 0.7812
## alpha3 0.2219
## beta1  0.2234
## beta2  0.1593
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.69 2.96 3.51
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           1.0821 0.2799
## Negative Sign Bias  0.3086 0.7578
## Positive Sign Bias  0.6702 0.5031
## Joint Effect        1.2292 0.7460
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     35.97     0.010649
## 2    30     51.41     0.006339
## 3    40     56.49     0.034646
## 4    50     78.25     0.004981
##
##
## Elapsed time : 0.1532431
##
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(3,3)
## Mean Model   : ARFIMA(3,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## ar1     0.219077    0.106646  2.05425 0.039951
## ar2    -0.126551    0.132868 -0.95246 0.340864
## ar3     0.570465    0.102563  5.56210 0.000000
```

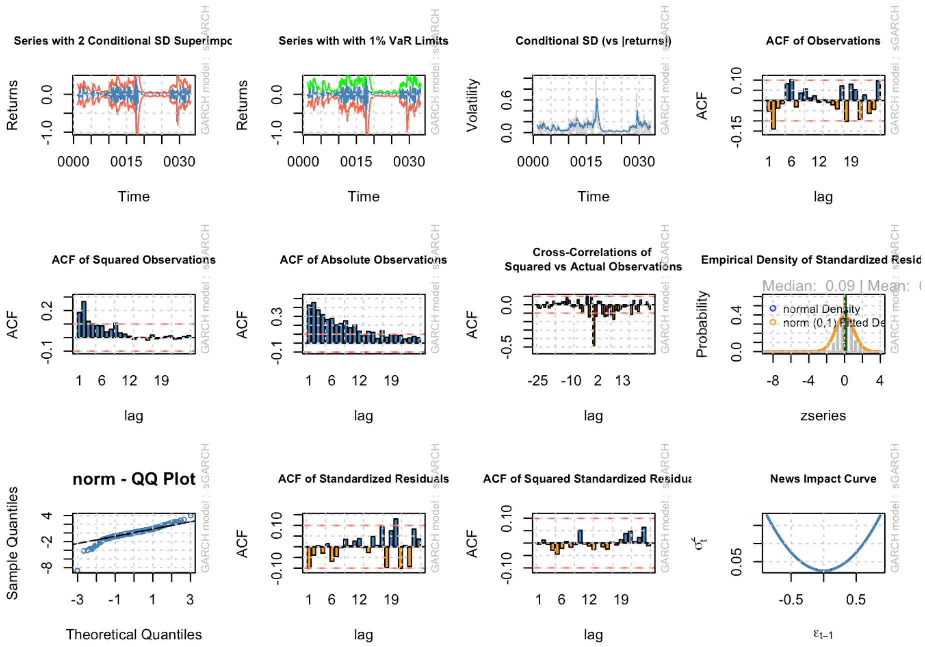
```
## ma1     0.082701    0.104505  0.79136 0.428732
## ma2     0.119736    0.129668  0.92341 0.355796
## ma3    -0.503019    0.083923 -5.99380 0.000000
## omega   0.000406    0.000121  3.35465 0.000795
## alpha1  0.203113    0.059396  3.41964 0.000627
## alpha2  0.170215    0.060893  2.79531 0.005185
## alpha3  0.343285    0.055463  6.18947 0.000000
## beta1   0.141578    0.075774  1.86843 0.061702
## beta2   0.000000    0.135848  0.00000 1.000000
## beta3   0.140809    0.051077  2.75682 0.005837
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## ar1     0.219077    0.105674  2.07314 0.038159
## ar2    -0.126551    0.146455 -0.86409 0.387536
## ar3     0.570465    0.193530  2.94768 0.003202
## ma1     0.082701    0.109468  0.75548 0.449961
## ma2     0.119736    0.160995  0.74372 0.457045
## ma3    -0.503019    0.184097 -2.73236 0.006288
## omega   0.000406    0.000279  1.45143 0.146661
## alpha1  0.203113    0.062580  3.24565 0.001172
## alpha2  0.170215    0.105534  1.61289 0.106768
## alpha3  0.343285    0.073893  4.64572 0.000003
## beta1   0.141578    0.269499  0.52534 0.599347
## beta2   0.000000    0.339705  0.00000 1.000000
## beta3   0.140809    0.124636  1.12976 0.258578
##
## LogLikelihood : 388.7143
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.9467
## Bayes        -1.8135
## Shibata      -1.9489
## Hannan-Quinn -1.8939
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic   p-value
## Lag[1]                       5.95 1.472e-02
## Lag[2*(p+q)+(p+q)-1][17]    17.06 0.000e+00
## Lag[4*(p+q)+(p+q)-1][29]    29.55 2.814e-05
## d.o.f=6
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.00131  0.9711
## Lag[2*(p+q)+(p+q)-1][17]  2.75472  0.9917
## Lag[4*(p+q)+(p+q)-1][29]  6.07793  0.9939
## d.o.f=6
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[7]     0.1580 0.500 2.000  0.6910
## ARCH Lag[9]     0.2466 1.485 1.796  0.9632
## ARCH Lag[11]    0.9629 2.440 1.677  0.9443
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  8.5939
## Individual Statistics:
## ar1    0.3368
## ar2    0.2132
## ar3    0.1316
## ma1    0.3315
## ma2    0.1981
## ma3    0.1221
## omega  0.5399
## alpha1 2.6878
## alpha2 1.0483
```

```
## alpha3 0.2449
## beta1  0.3784
## beta2  0.2365
## beta3  0.2508
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.89 3.15 3.69
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.4317 0.6662
## Negative Sign Bias  0.2896 0.7723
## Positive Sign Bias  0.1578 0.8747
## Joint Effect        0.3515 0.9501
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     57.21    1.059e-05
## 2    30     68.97    4.184e-05
## 3    40     85.30    2.652e-05
## 4    50     89.91    3.306e-04
##
##
## Elapsed time : 0.2106459
```

We compare the models with standard parameters in terms of AIC, Weighted Ljung-Box test results which indicate uncorrelated residuals (aim of GARCH modeling), and Adjusted Pearson Goodness-of-Fit test results. Some 0-order models did not even converge. The model that best weights these three criteria is ARIMA(3, 1, 3)-GARCH(3, 2) with AIC = -1.9302, Weighted Ljung-Box Test on Standardized Residuals p-value > 0.01 and Adjusted Pearson Goodness-of-Fit Test passing for 2/ 4 groups. We further run model diagnostics.

```
# Model Diagnosis
model_garch <- ugarchspec(variance.model = list(model = "sGARCH",
                                                    garchOrder = c(3, 2), submodel = NULL,
                                                    external.regressors = NULL,
                                                    variance.targeting = FALSE),

                              mean.model = list(armaOrder = c(3, 3),
                                                external.regressors = NULL,
                                                include.mean = FALSE
                                                ),
                          distribution.model = "norm"
                          )

GARCH_fit <- ugarchfit(spec = model_garch, data = data_ts_return)

plot(GARCH_fit, which = 'all')
```

```
##
## please wait...calculating quantiles...
```



The most relevant outputs include the ACF of observations which shows that most patterns in the returns of the monthly series were captured by the GARCH model; the ACF of squared observations which indicates that the model has captured most of the heteroscedasticity; and the normal QQ plot which shows slight deviations from the line indicating potential departures from normality.

```
# Linear model on Squared Std. GARCH Residuals
e <- GARCH_fit@fit$residuals # ARIMA-GARCH residuals
X_lm <- 1: length(e)
data_lm <- data.frame(squared_residuals = e^2, time_index = X_lm)
model_lm <- lm(e^2 ~ X_lm, data = data_lm)

# Breusch-Pagan Test
print(bptest(model_lm))
```

```
##
##  studentized Breusch-Pagan test
##
## data:  model_lm
## BP = 0.34397, df = 1, p-value = 0.5575
```

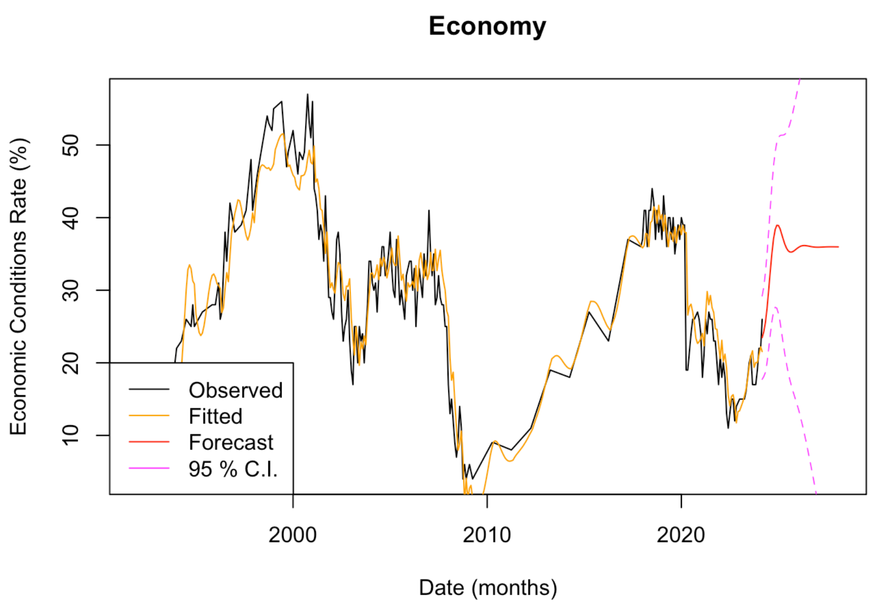
White test's p-value > 0.01 now fails to reject the null hypothesis of constant variance of the residuals so the constancy of variance assumption has been satisfied by incorporating GARCH.

```
# Normality Test
e_ts <- ts(e, frequency = 12) # Series Residual
shapiro.test(e_ts)
```

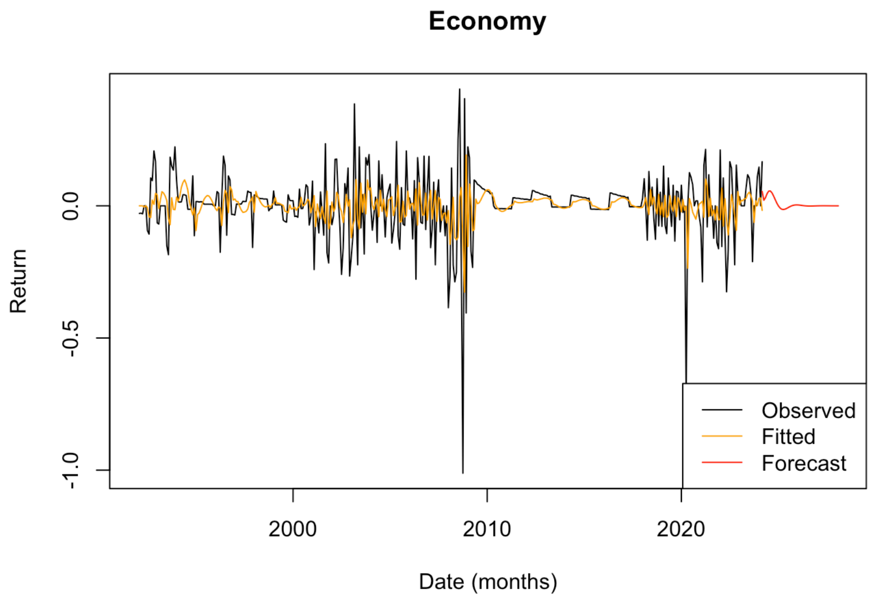
```
##
##  Shapiro-Wilk normality test
##
## data:  e_ts
## W = 0.86233, p-value < 2.2e-16
```

Shapiro Wilk test's p-value much smaller than 0.01 rejects the null hypothesis of normally distributed residuals so the normality assumption is violated (limitation).

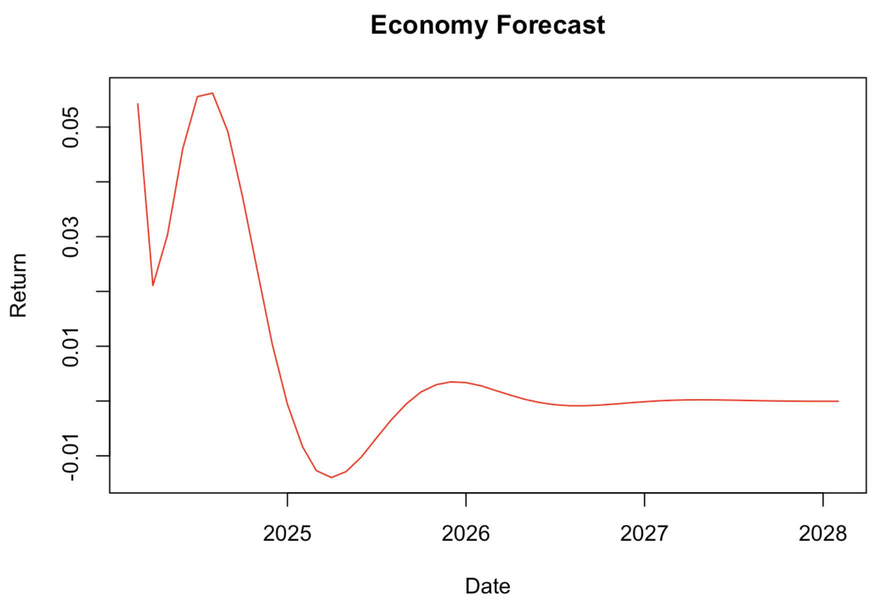
```
# Forecasts

R <- fitted(GARCH_fit) # Fitted returns
model_garch_h = ugarchforecast(fitORspec = GARCH_fit, n.ahead = 48) # Four years into the future
R_h <- fitted(model_garch_h) # Return forecast
H <- seq(from = end(data_xts), length.out = 48, by = "month")
V <- GARCH_fit@fit$sigma^2 # Fitted volatility
V_h <- sigma(model_garch_h)^2 # Volatility forecast

Y_h_first = 16 # Initial estimate
C <- data_ts[1] * exp(cumsum(R)) # Fitted cumulative levels
C_h <- Y_h_first * exp(cumsum(R_h)) # Cumulative level forecasts
VC_h <- Y_h_first / 2 * exp(cumsum(V_h)) # Cumulative Volatility forecast

# Scaling factor
Y_lm <- lm(data_ts[-1] ~ C)
Y <- coef(Y_lm)[2] * C + coef(Y_lm)[1]
Y_h <- coef(Y_lm)[2] * C_h + coef(Y_lm)[1]
W_h <- coef(Y_lm)[2] * VC_h + coef(Y_lm)[1]

# 85% CI
u <- Y_h + 1.96 * W_h
l <- Y_h - 1.96 * W_h

plot(X, data_ts, type = 'l', main = "Economy", xlab = "Date (months)", ylab = "Economic Conditions Rate (%)", xli
m = range(c(X, H)))
lines(X[-1], Y, col = "orange")
lines(H, Y_h, col = "red")
matlines(H, cbind(l, u), col = "magenta", lty = 2, lwd = 0.8)
legend("bottomleft", legend = c("Observed", "Fitted", "Forecast", "95 % C.I."), col = c("black", "orange", "red",
"magenta"), lty = 1)
```



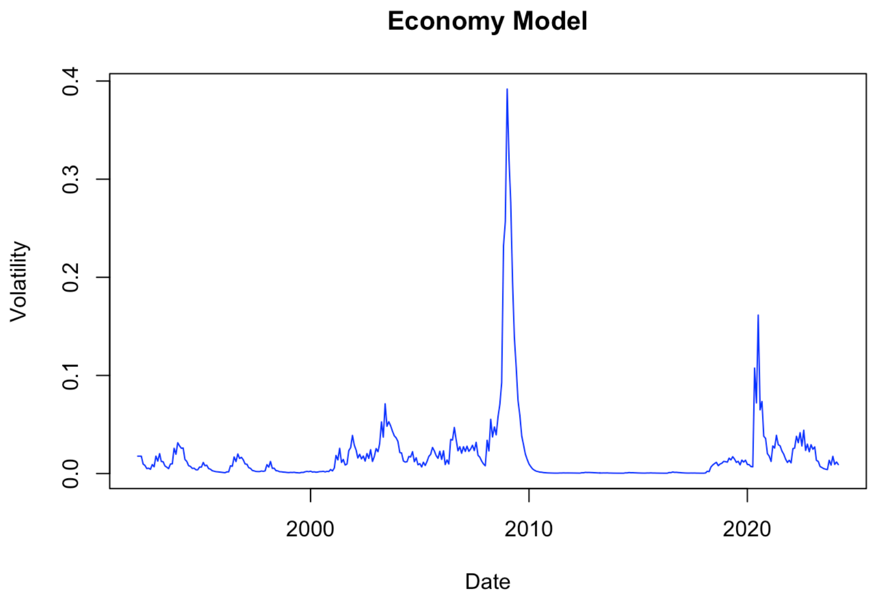

```
plot(X[-1], data_ts_return, type = 'l', main = "Economy", xlab = "Date (months)", ylab = "Return", xlim = range(
c(X, H)))
lines(H, R_h, col = "red")
lines(X[-1], R, col = "orange")
legend("bottomright", legend = c("Observed", "Fitted", "Forecast"), col = c("black", "orange", "red"), lty = 1)
```




```
plot(H, R_h, type = 'l', main = "Economy Forecast", xlab = "Date", ylab = "Return", col = "red")
```




```
plot(X[-1], V, type = 'l', main = "Economy Model", xlab = "Date", ylab = "Volatility", col = "blue")
```



The original series has been reconstructed by fitting the cumulative levels of the ARIMA-GARCH returns. The return forecast shows an overall decrease, that is, the “Good” rating % for the economic conditions of the United States are projected to increase less and less until 2028. The good economic ratings are expected to peak at 37 % of the population next year with 95 % confidence. The model exhibits volatility highs proper of financial data. They correspond to the years 2008 and 2020, coinciding with the observed drops in economic ratings at the beginning of Barak Obama’s term and the pandemic.

Conclusions

We were able to successfully simulate the satisfaction of Americans with the U.S. using a yearly seasonal model as well as make confident predictions on future ratings for the U.S. economy. The economy model was able to capture the patterns in the returns of the economic ratings successfully. The variance for the model residuals was found constant which increases our confidence in the predictions. Furthermore, the residuals were found random which indicates that our models successfully captured the underlying auto-correlations and dynamics in the data.

The missing months still coaxed us into underestimating the variability of our data which might have misled our conclusions. More robust methods (e.g. K-nearest neighbors (KNN) or Expectation Maximization (EM)) are part of our future studies to handle the missing values if collecting more data becomes impossible. On the other hand, the residuals were found to be not normally distributed which could potentially lead to incorrect inferences about our model parameters and specification. Future directions include employing different model distributions such as t-distribution especially to improve the goodness of fit for the GARCH model, removing potential outliers, and applying transformations such as Box-Cox to try to make the distribution more normal. We hope that our study provides insight on factors that may improve the satisfaction with the U.S.

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