Random Manhattan Indexing

Behrang Q. Zadeh and Siegfried Handschuh

National University of Ireland, Galway, Ireland
University of Passau, Lower Bavaria, Germany

Processing Natural Language Text

- For Computers, natural language text is simply a sequence of bits and bytes.

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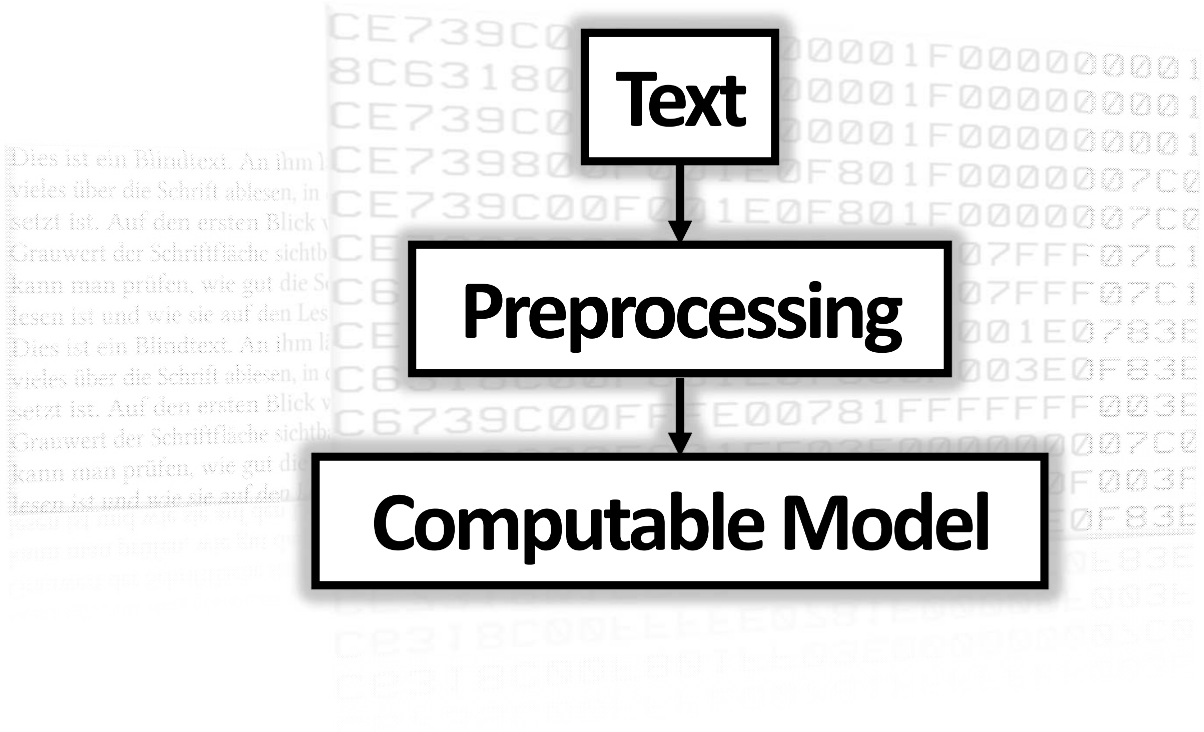


Vector Space Model (VSM)

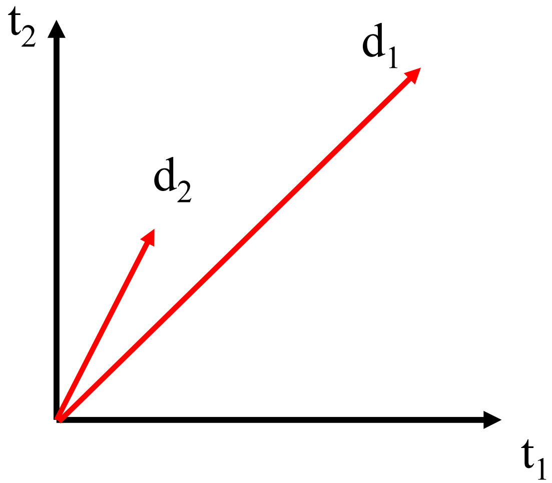
- Vector spaces are one of the models employed for text processing.
  - A Mathematical Structure $\langle V, \mathbb{R}, +, \cdot \rangle$ that satisfy certain axioms.
- Text elements are converted to real-valued vectors.
- Each dimension of the vector represents some information about text elements.

VSMs: an Example

- In a text-based information retrieval task:
  - Each dimension of the VSM represents an index term *t*.
  - Documents are represented by vectors.

Salton et. al. (1975)

VSMs: an Example



$$\longleftrightarrow \quad \begin{array}{c c} & \begin{array}{cc} t_1 & t_2 \end{array} \\ \begin{array}{c} d_1 \\ d_2 \end{array} & \begin{pmatrix} 4 & 4 \\ 1 & 2 \end{pmatrix} \end{array}$$

Salton et. al. (1975)

VSMs: an Example

- In a text-based information retrieval task:
    - Each dimension of the VSM represents an index term *t*.
    - Documents are represented by vectors.
    - Queries are treated as pseudo-documents.

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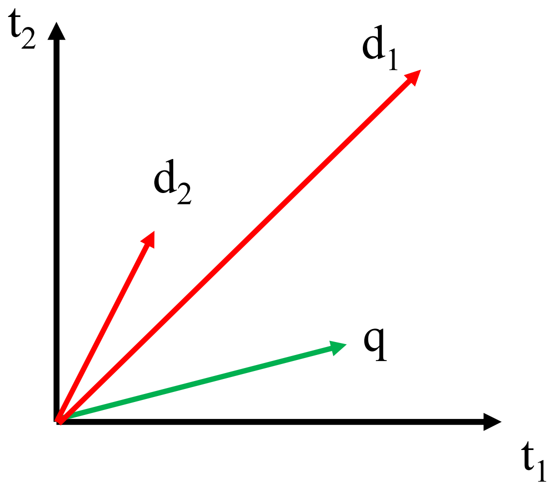
VSMs: an Example



$$
\longleftrightarrow
\quad
\begin{array}{c}
\\ d_1 \\ d_2 \\ q
\end{array}
\begin{array}{cc}
t_1 & t_2 \\
\end{array}
$$

$$
\begin{array}{c|cc}
 & t_1 & t_2 \\
\hline
d_1 & 4 & 4 \\
d_2 & 1 & 2 \\
q & 3 & 1
\end{array}
$$

Salton et. al. (1975)

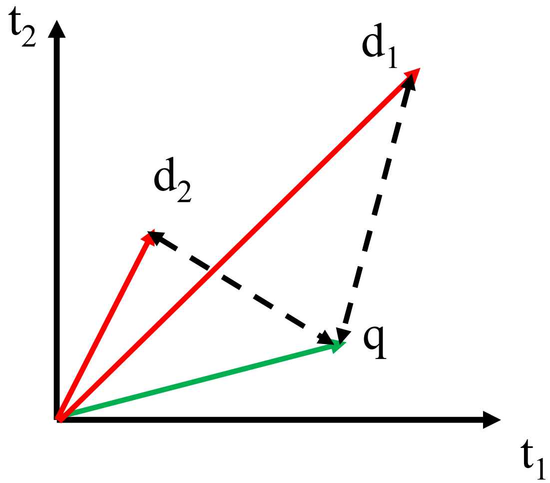
VSMs: an Example

- In a text-based information retrieval task:
  - Each dimension of the VSM represents an index term *t*.
  - Documents are represented by vectors.
  - Queries are treated as pseudo-documents.
  - Using a norm structure, a notion of distance is defined and used to assess the similarity between vectors, i.e. documents and queries.

Salton et. al. (1975)

VSMs: an Example

The L2 or Euclidean norm, $\|v\|_2 = \sqrt{\sum_i v_{\mathrm{i}}^{\,2}}$



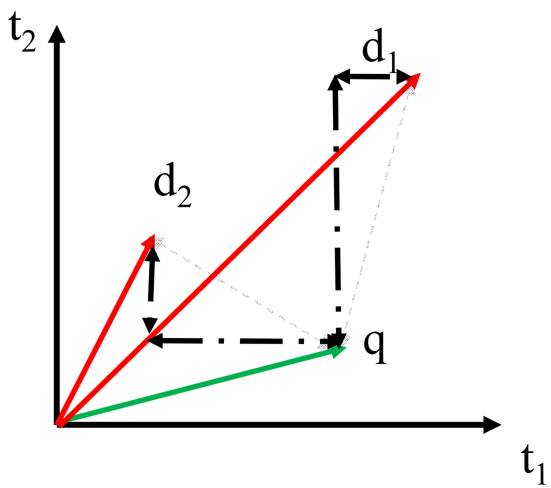
$$\begin{array}{cc} & \begin{array}{cc} t_1 & t_2 \end{array} \\ \begin{array}{c} d_1 \\ d_2 \\ q \end{array} & \begin{pmatrix} 4 & 4 \\ 1 & 2 \\ 3 & 1 \end{pmatrix} \end{array}$$

$$dist_2(d_1, q) = \|d_1 - q\|_2 = \sqrt{(4-3)^2 + (4-1)^2} = \sqrt{10}$$
$$dist_2(d_2, q) = \|d_2 - q\|_2 = \sqrt{(1-3)^2 + (2-1)^2} = \sqrt{5}$$

Salton et. al. (1975)

VSMs: an Example

The L1 norm, $\|v\|_1 = \sum_i |v_i|$



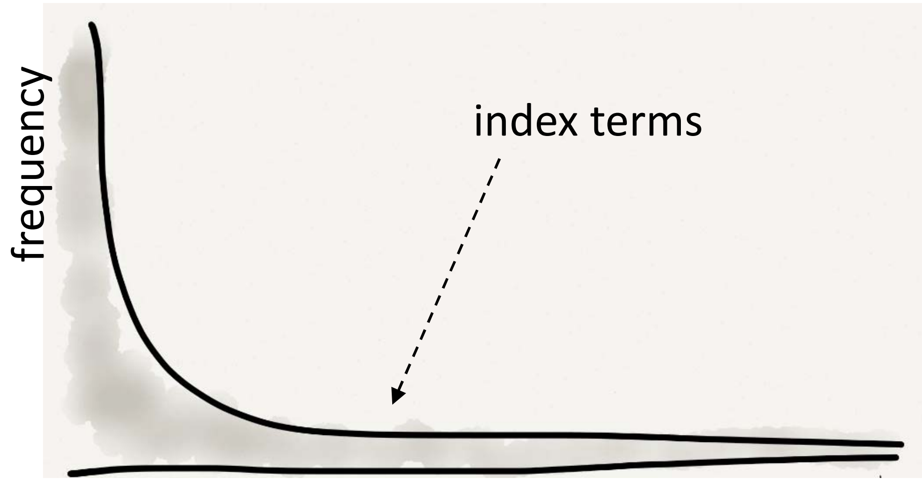
$$\begin{array}{c} \\ d_1 \\ d_2 \\ q \end{array} \begin{array}{c} \begin{array}{cc} t_1 & t_2 \end{array} \\ \begin{pmatrix} 4 & 4 \\ 1 & 2 \\ \boxed{3 \quad 1} \end{pmatrix} \end{array}$$

$$dist_1(d_1, q) = \|d_1 - q\|_1 = |(4-3)| + |(4-1)| = 4$$

$$dist_1(d_2, q) = \|d_2 - q\|_1 = |(1-3)| + |(2-1)| = 3$$

The dimensionality barrier

- Due to the Zipfian distribution of index terms, the number of index terms escalates when new documents are added.
- Adding new index terms requires adding additional dimensions to the vector space.

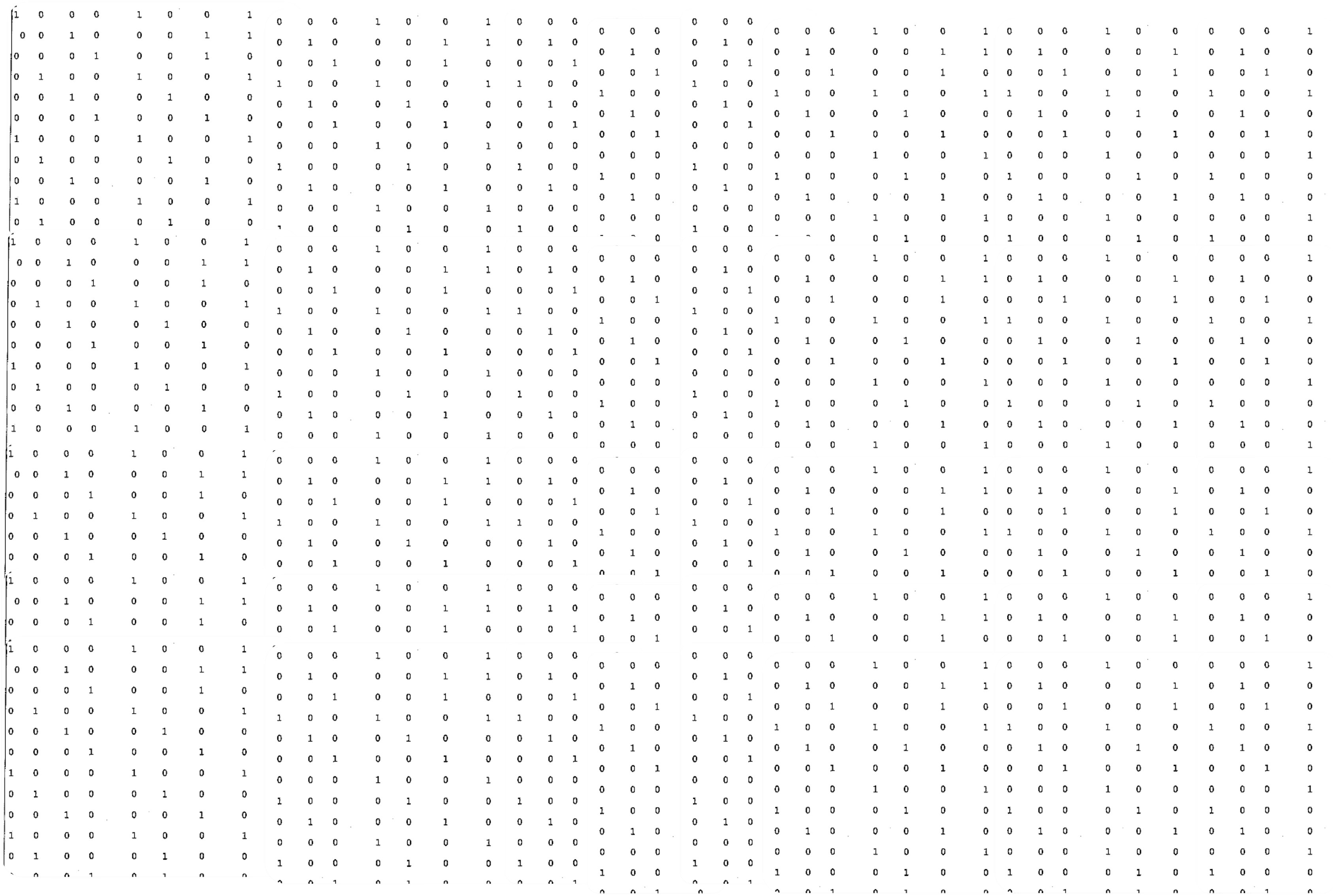


The dimensionality barrier

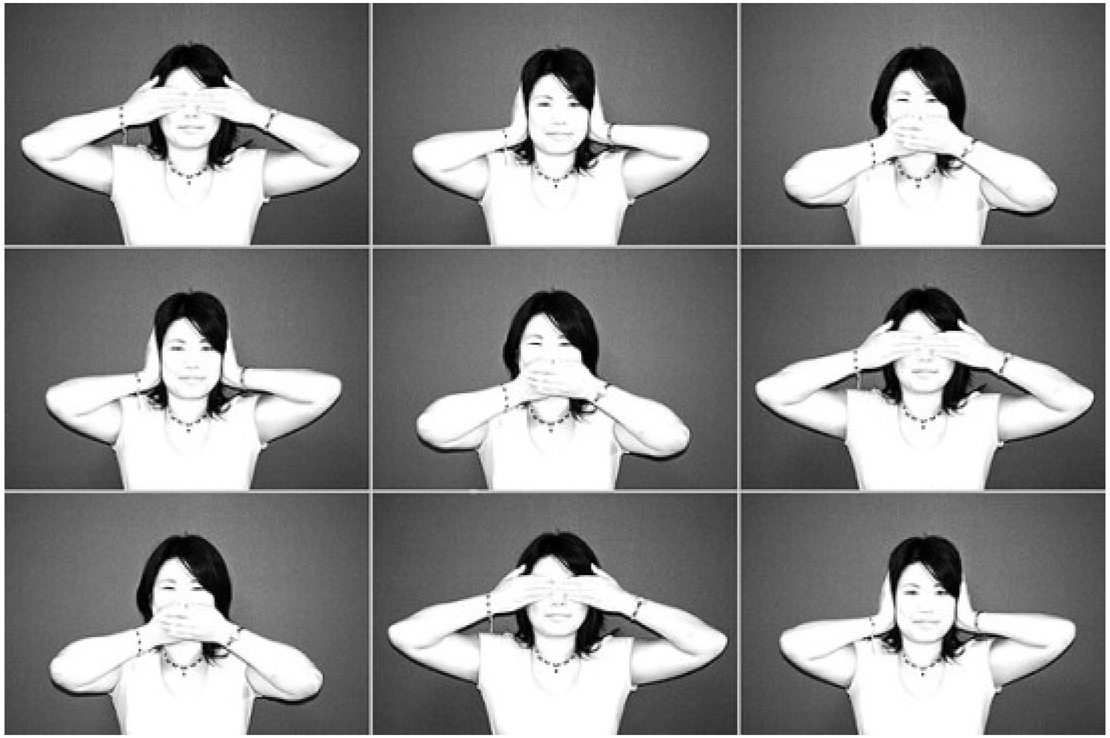
- Due to the Zipfian distribution of index terms, the number of index terms escalates when new documents are added.
- Adding new index terms requires adding additional dimensions to the vector space.
- Therefore, VSMs are extremely high-dimensional and sparse:

“THE CURSE OF DIMENSIONALITY”

The dimensionality barrier



Overcoming the dimensionality barrier



Overcoming the dimensionality barrier

- Dimensionality Reduction Techniques are employed to resolve the problem.

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Dimensionality Reduction → Selection Process (based on a heuristic)

Dimensionality Reduction → Transformation-based methods

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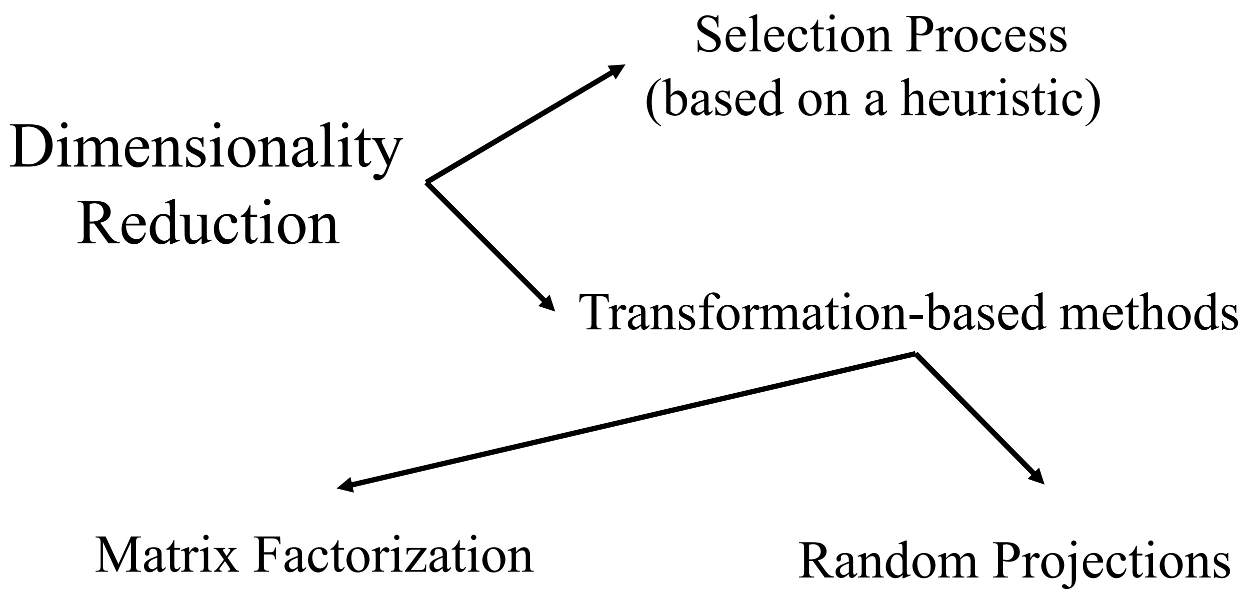

Overcoming the dimensionality barrier

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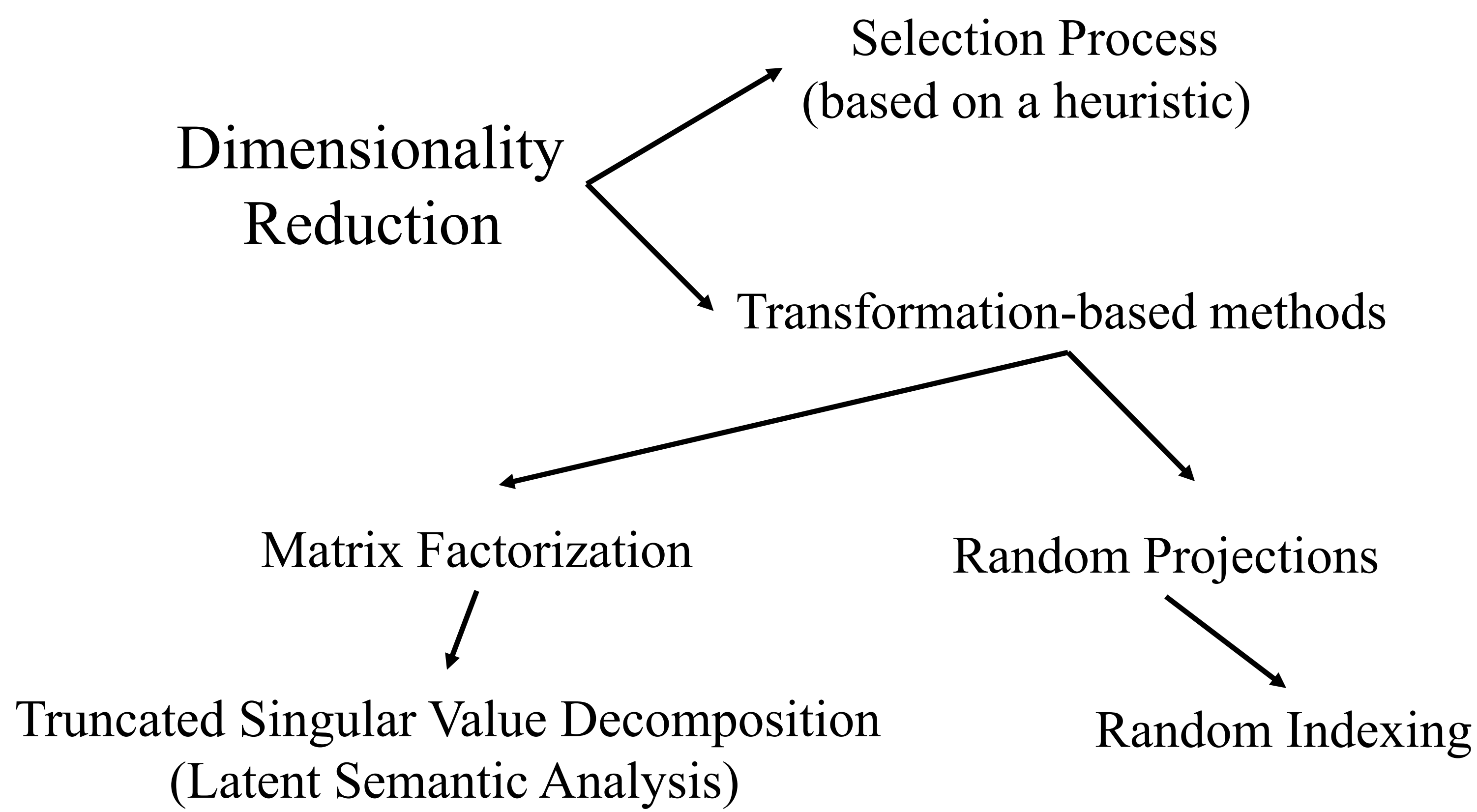

Limitations of Matrix Factorization

- Computation of Singular Value Decomposition (SVD) is process-intensive, i.e. $O(mn^2)$.
- Every time a VSM is updated, SVD must be recomputed:
  - Not suitable for big-text data analytics.
  - Not suitable for frequently updated text-data, e.g. blogs, tweeter analysis, etc.

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Solution: Random projection methods skip the computation of transformations (eigenvectors).

Random Projections: Random Indexing

- VSM Dimensionality is decided independent of the number of index terms, i.e. dimensionality of a VSM is fixed.

- Algorithm:
  - Assign each index term to a randomly generated "index vector".
    - Most elements of index vectors are 0 and only a few +1 and -1, e.g. $r^{t1} = (0, 0, 1, 0, -1)$, $r^{t2} = (1, 0, -1, 0, 0)$, etc.
  - Assign each document to an empty "context vector".
  - Construct the VSM incrementally by the accumulation of index vectors to context vectors.

Random Indexing (Example)

Munich is the capital and largest city of the German state of Bavaria.

Document 1

Random Indexing (Example)

Munich is the capital and largest city of the German state of Bavaria.

Document 1

Munich
Capital
German
Bavaria
Largest
City
State

Index Terms

Random Indexing (Example)

Munich is the capital and largest city of the German state of Bavaria.

Document 1

D1 = (0, 0, 0, 0, 0)

Munich: (1,-1,0,0,0)
Capital: (0,0,1,0,-1)
Germany: (1,0,0,-1,0)
Bavaria: (0,0,0,1,-1)
Largest: (0,1,0,-1,0)
City: (1,0,-1,0,0)
State: (1,0,0,0,-1)

Index Terms

Random Indexing (Example)

Munich is the capital and largest city of the German state of Bavaria.

Document 1

D1 = (4, 0, 0, -1, -3)

Munich: (1,-1,0,0,0)
Capital: (0,0,1,0,-1)
Germany: (1,0,0,-1,0)
Bavaria: (0,0,0,1,-1)
Largest: (0,1,0,-1,0)
City: (1,0,-1,0,0)
State: (1,0,0,0,-1)

Index Terms

Munich	(	1	,-1	,0	,0	,0	)	+
Capital	(	0	,0	,1	,0	,-1	)	+
Germany	(	1	,0	,0	,-1	,0	)	+
Bavaria	(	0	,0	,0	,1	,-1	)	+
Largest	(	0	,1	,0	,-1	,0	)	+
City	(	1	,0	,-1	,0	,0	)	+
State	(	1	,0	,0	,0	,-1	)	
D1 =	**(**	**4**	**,0**	**, 0**	**, -1**	**, -3**	**)**	

Random Indexing (Example)

Munich is the capital and largest city of the German state of Bavaria.

Document 1

D1 = (4, 0, 0, -1, -3)

Munich is a beautiful, historical city.

Document 2

Munich: (1,-1,0,0,0)
Capital: (0,0,1,0,-1)
Germany: (1,0,0,-1,0)
Bavaria: (0,0,0,1,-1)
Largest: (0,1,0,-1,0)
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Index Terms

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Document 2

D2 = (0, 0, 0, 0, 0)

Munich: (1,-1,0,0,0)
Capital: (0,0,1,0,-1)
Germany: (1,0,0,-1,0)
Bavaria: (0,0,0,1,-1)
Largest: (0,1,0,-1,0)
City: (1,0,-1,0,0)
State: (1,0,0,0,-1)
Beautiful: (0,0,1,-1,0)
Historical: (0,1,0,0,-1)

Index Terms

Random Indexing (Example)

Munich is the capital and largest city of the German state of Bavaria.

Document 1

D1 = (4, 0, 0, -1, -3)

Munich is a beautiful, historical city.

Document 2

D2 = (2,0,0, -1,-1)

Munich: (1,-1,0,0,0)
Capital: (0,0,1,0,-1)
Germany: (1,0,0,-1,0)
Bavaria: (0,0,0,1,-1)
Largest: (0,1,0,-1,0)
City: (1,0,-1,0,0)
State: (1,0,0,0,-1)
Beautiful: (0,0,1,-1,0)
Historical: (0,1,0,0,-1)

Index Terms

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D1 = (4, 0, 0, -1, -3)

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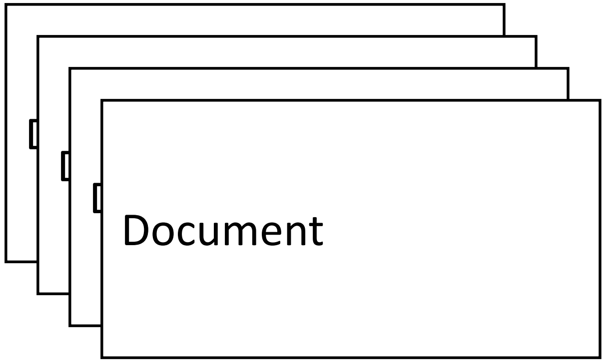

Munich: (1,-1,0,0,0)
Capital: (0,0,1,0,-1)
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Largest: (0,1,0,-1,0)
City: (1,0,-1,0,0)
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Beautiful: (0,0,1,-1,0)
Historical: (0,1,0,0,-1)
.
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.
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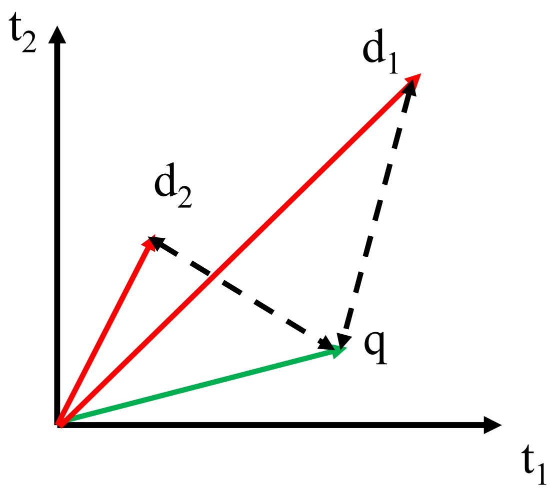
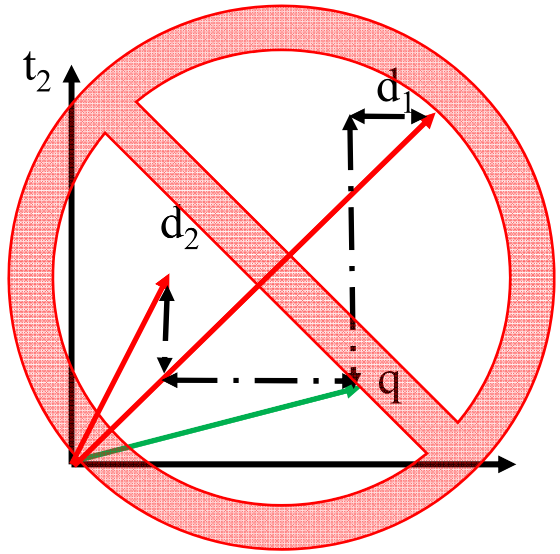
Index Terms

FIXED DIMENSION

Limitation of Random Indexing

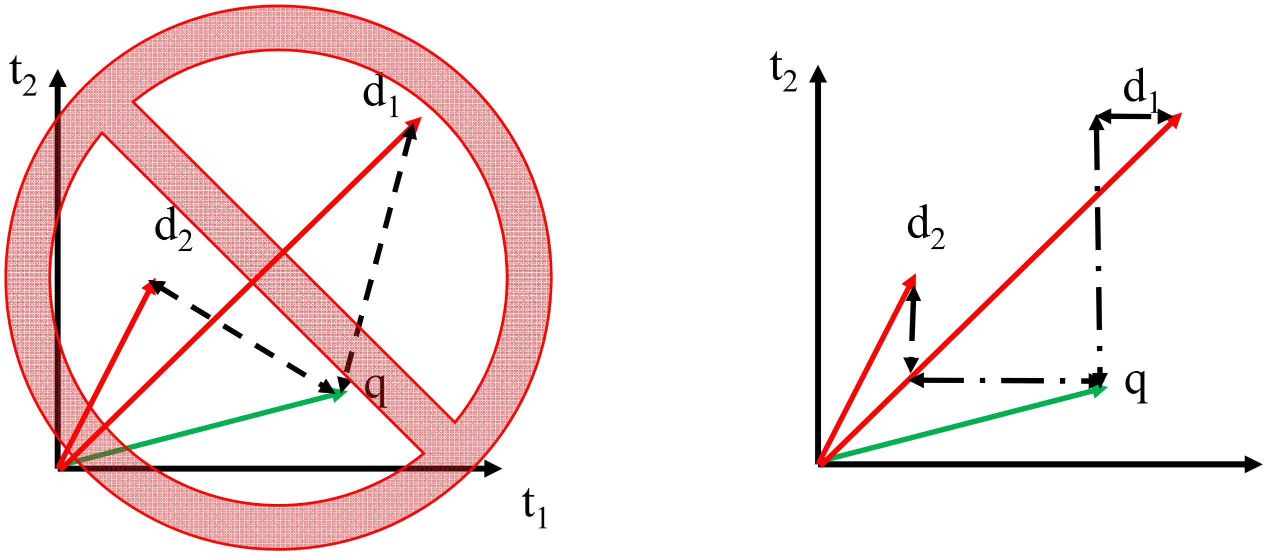
- Random Indexing employs a Gaussian Random Projection.
- A Gaussian Random Projection can only be used for the estimation of Euclidean distances.





Random Manhattan Indexing

- In an applications we may want to estimate the L1 (Manhattan) distance between vectors.
- Random Manhattan Indexing (RMI) can be used to estimate the Manhattan distances.



The RMI method

- RMI employs Cauchy Random Projections
- RMI is an incremental technique (similar to RI):
  - Each index term is assigned to an index vectors.
  - Index vectors are high-dimensional and randomly generated with the following distribution:

$$r_{ij} = \begin{cases} \frac{1}{U_1} & \text{With probability } \frac{s}{2} \\ 0 & \text{With probability } 1 - s \\ -\frac{1}{U_2} & \text{With probability } \frac{s}{2} \end{cases}$$

U_1 and U_2 are independent uniform random variables in (0, 1).

Indyk (2000), Li (2008)

The RMI method

- Assign documents to context vectors.
- Create context vectors incrementally.
- Estimate the Manhattan distance between vectors using the following (non-linear) equation:

$$\widehat{dist}_1(u,v) = \exp\left(\frac{1}{m}\sum_{i=1}^{m} \ln(|u_i - v_i|)\right)$$

m is the dimension of the RMI-constructed VSM

RMI (Example)

Munich is the capital and largest city of the German state of Bavaria.

Document 1

D1 = (1.3,5.89,-6.7,-3.5,-8)

Munich: (0,0,1.0,-2.1,0)
Capital: (0,1.49,0,-1.2,0)
German: (0,0,0,1.9,-1.8)
Bavaria: (1.3,0,0,0,-3.9)
Largest: (0,0,1.6,-2.1,0)
City: (0,2.5,0,0,-2.3)
State: (0,1.9,-9.3,0,0

Index Terms

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State: (0,1.9,-9.3,0,0

Index Terms

Use the non-linear estimator to assess similarities:

$$\widehat{dist}_1(u,v) = \exp\left(\frac{1}{m}\sum_{i=1}^{m} \ln(|u_i - v_i|)\right)$$

RMI's parameters

- The dimension of the RMI-constructed VSM.
- Number of non-zero elements in index vectors.

RMI's parameters

- The dimension of the RMI-constructed VSM:
  - It is independent of the number of index terms.
  - It is decided by the probability and the maximum expected distortion in distances.
  - For fixed set of documents, larger dimension results in less distortion.
  - According to our experiment, m=400 is suitable for most applications.

RMI's parameters

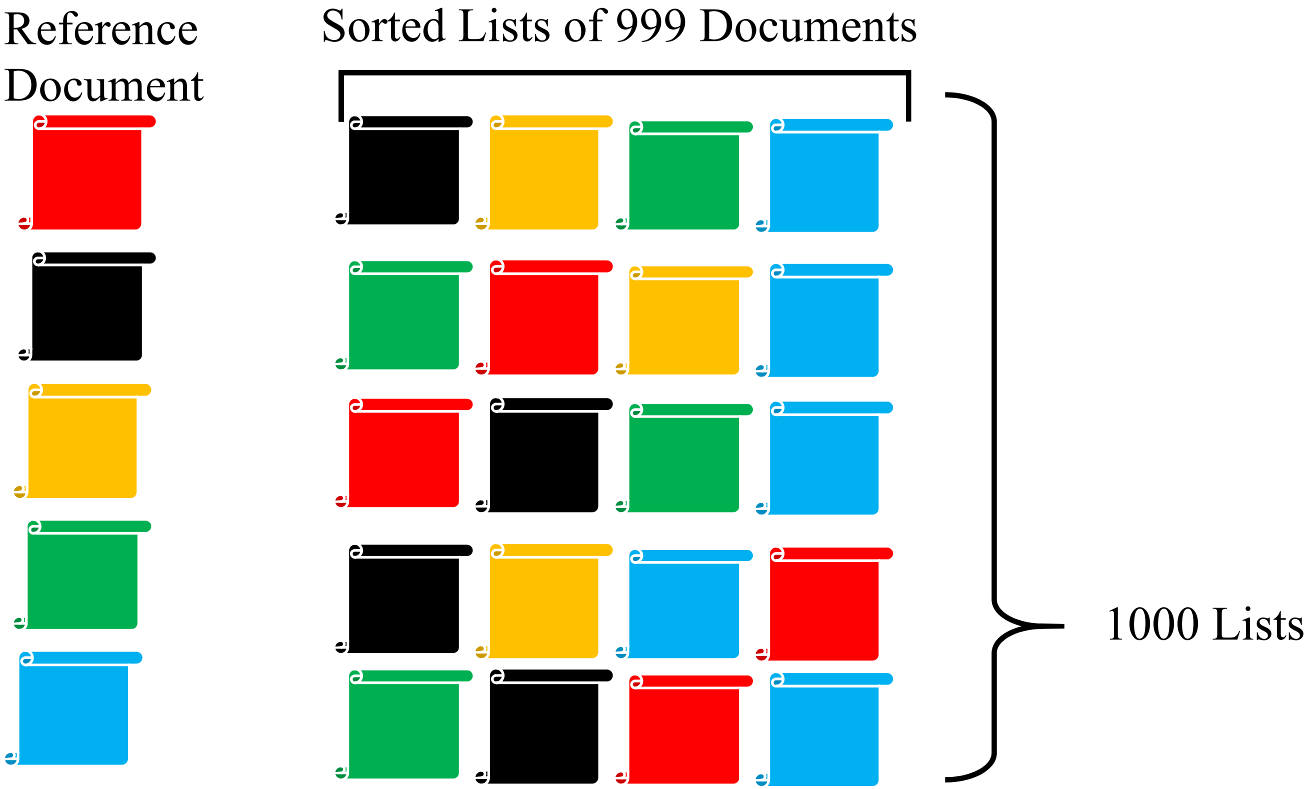
- The dimension of the RMI-constructed VSM.
- Number of non-zero elements in index vectors:
  - It is decided by the number of index terms and the sparsity of VSM at its original high-dimension.
  - We suggest $s = \frac{1}{O(\sqrt{\beta n})}$, where β is the sparseness of original high-dimensional VSM and n is the number index terms.
  - β is often considered to be around 0.0001 to 0.01.

Experimental Results

- We designed an experiment that shows the ability of RMI in preserving L1 distances:
  - A VSM is first constructed from the INEX-Wikipedia 2009 collection at its original high dimension (dimensionality of 2.53 million).
  - We choose a random list of 1000 random articles from the corpus.
  - The L1 distance of each document to the rest of documents in the list are calculated.
  - Documents are sorted by the calculated L1 distances.
  - The result is 1000 lists of 999 sorted documents.

Experimental Results

The set of 1000 random documents

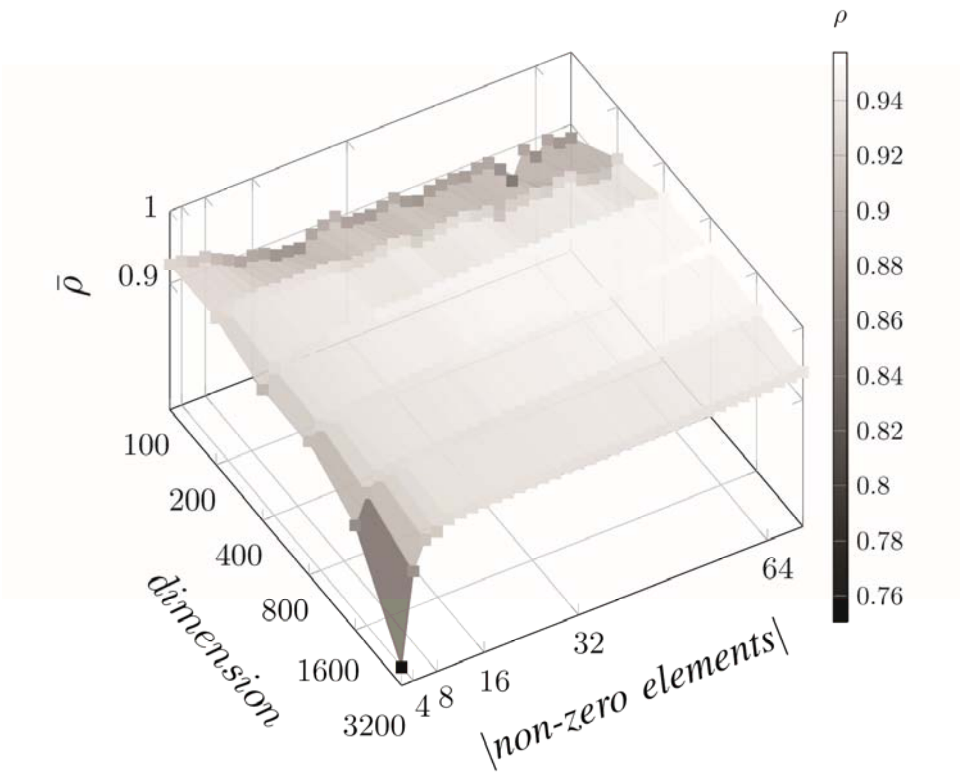


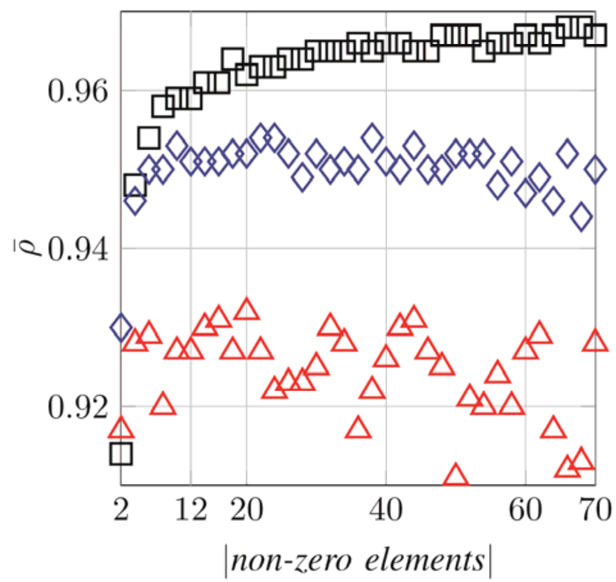
Experimental Results

- Construct the VSM using RMI method and repeat the sorting process.
  - Use different dimensionalities.
  - User different number of non-zero elements.
- Compare the sorted lists obtained from the VSM constructed at the original high dimension and the RMI-constructed VSM at reduced dimensionality.
  - Spearman's rank correlation (ρ) for comparison
- EXPECTATION: similar sorted lists from both VSM, i.e. $\rho = 1$.

Experimental Results

- Observed correlation:



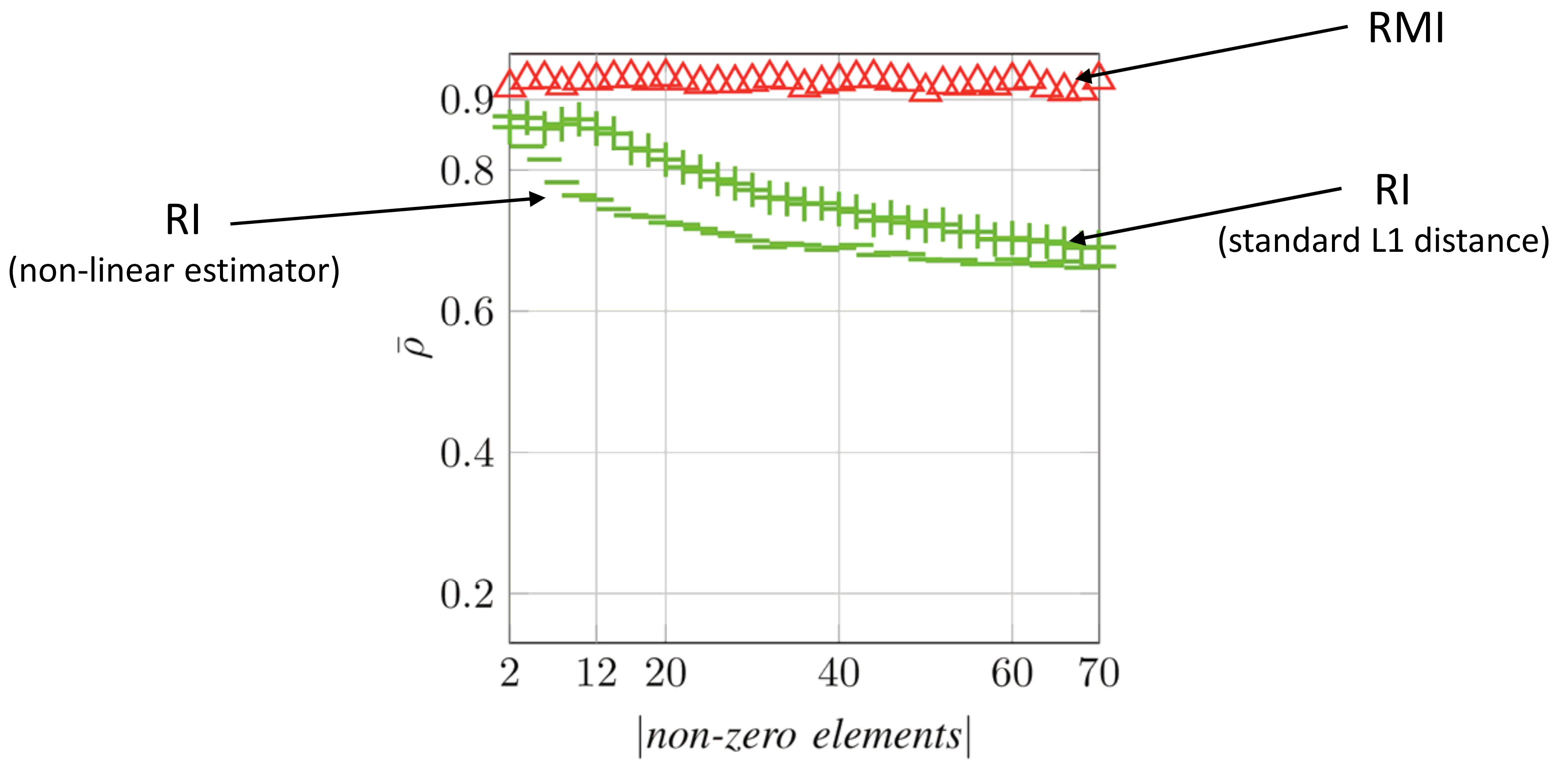


red triangles: dimension = 200
blue diamonds: dimension = 400
black squares: dimension = 800

For the baseline, we report ρ = 0.1375 for lists of documents sorted randomly.

Experimental Results

- Experiments support the earlier claim that RI do not preserve the L1 distance:



Discussion

- We proposed an incremental, scalable method for the construction of the L1 normed VSMs.
- We show the ability of the method in preserving the distances between documents.

- Is this possible to avoid floating-point calculation?
- What is the performance of RMI within the context of Information Retrieval benchmarks?

THANKS FOR YOUR ATTENTION!

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