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Contents

Recent Developments in Nearrings with Some Applications

Kuncham Syam Prasad[1], Kedukodi Babushri Srinivas[1], Panackal Harikrishnan[1] and Bhavanari Satyanarayana[2]

1) Department of Mathematics, Manipal Institute of Technology, Manipal University, Manipal – 576104, India

2) Department of Mathematics, Acharya Nagarjuna University, Nagarjuna Nagar – 522510, India

E-mail: syamprasad.k@manipal.edu, babushrisrinivas.k@manipal.edu, pk.harikrishnan@manipal.edu, bhavanari2002@yahoo.co.in

Abstract

Algebra is often described as the language of mathematics. Historically, the terms algebra and algorithm originate from the same source. The theory of nearrings is now a sophisticated theory in algebra. Nearrings are generalized rings: commutativity of addition is not assumed, and- more important- only one distributive law is required. Compared with a standard class of rings, endomorphism rings of abelian groups, one sees that rings describe "linear" maps on groups, while nearrings handle the general nonlinear case. Nearrings in which the nonzero elements form a multiplicative group are called nearfields. The connections between nearrings (especially nearfields) and geometry are well-known. In this review we present recent developments in nearrings and some applications of the notions involved.

Introduction

The algebraic systems like semigroups, groups and groupoids are equipped with one binary operation. For the last three decades, algebraic systems with two binary operations play a vital role in the various areas of research digital phenomena algebraic codes etc. One such important algebraic structure is 'nearring' which is equipped with two binary operations: addition and multiplication, satisfying all the ring axioms except possibly one of the distributive laws and commutativity of addition. The primary step towards nearrings was an axiomatic research done by Dickson [1, 2]. The theory of nearrings is now a sophisticated theory which has found numerous applications in various areas. Dickson showed that there do exist "fields with only one distributive law" (named nearfields). Nearfields showed up to be useful in coordinating certain important classes of geometric planes. Connections between other parts of nearrings (especially nearfields) and geometry come up at several places. Efficient block designs and codes can be constructed from finite near-rings. Many parts of the well-established theory of rings were transferred to nearrings and new nearring precise features were discovered, constructing up a theory of nearrings step by step (for example, Wedderburn – Artin Theorem for simple algebras). It is evident that every ring is a nearring. A natural example of a nearring (but not a ring) is given by the set M(G) of all mappings of an additively written group G (which is not necessarily Abelian) into G itself with usual addition of mappings: $(f + g)(x) = f(x) + g(x)$ and composition as multiplication operation: $(fg)(x) = f(g(x))$ for all $x \in G$, f, $g \in M(G)$. A *right nearring* N is an additive group (not necessarily abelian), a multiplicative semigroup and only distributive laws holds (we may consider multiplication distributes over addition from right side). In a similar way one can define a left nearring.

() A proper ideal P of N is a called *prime (*resp.*, semiprime)* if for all ideals I, J of N, $IJ \subseteq P \Rightarrow I \subseteq P$ or $J \subseteq P$ (resp., $I^2 \subseteq P$, implies $I \subseteq P$). An ideal P of N is said to be a *completely prime* (resp. *completely semiprime*) ideal if for a, $b \in N$, $ab \in P$, implies either $a \in P$ or $b \in P$ (respectively, $a^2 \in P$ implies $a \in P$). Ramakotaiah and Rao [5] studied the IFP (insertion of factors property) in nearrings [3,4].

Reddy and Satyanarayana introduced the concept of finite Goldie dimension in N-groups. The notions of f-prime ideals and related f-prime radical in nearrings, Goldie dimension in modules over nearrings and obtained various characterizations [6, 7].

Reddy and Satyanarayana studied the concepts of finite spanning dimension in modules over nearrings and various examples were presented [8, 9, 10, 11, and 12]. Some characterizations of the concepts of the f-prime radical of nearrings were presented by Satyanarayana and Weigandt [13]. (, [15], [16], [17]) The direct and inverse systems on modules over nearrings were introduced and characterizations for e-direct systems and inverse systems are obtained Bhavanari and Kuncham [14, 15, 16, and 17]. For more wide-ranging results on Goldie dimension of modules over nearrings, refer Kuncham and Bhavanari [18].

Matrix Nearrings

Matrix nearrings are introduced in Meldrum and Van der Walt [19]. The matrix nearring of a given nearring N is denoted by $M_n(N)$. The correspondence between the two-sided ideals in the given nearring N and the two sided ideals in the matrix nearring $M_n(N)$ are studied by Meldrum and Van der Walt [19]. For a nearring N with identity, we write N^n as the direct sum of n-copies of additive group N. Define $e_i = \left(0, \dots, \underset{i^{th}}{1}, \dots, 0\right)$. We define the i^{th} injective mapping and the j^{th} projection respectively are i_j: $N \to N^n$ as $i_j(a) = \left(0, \dots, \underset{i^{th}}{a}, \dots, 0\right)$ and π_j: $N^n \to N$ as $\pi_j(a_1, \dots, a_n) = a_j$. Define f^r: $N \to N$ as $f^r(s) = rs$ for all $s \in N$. The composition of these mappings denoted by $f_{ij}^r = i_i f^r \pi_j$ where $f_{ij}^r : N^n \to N^n$ for all $1 \le i, j \le n$, defined as $f_{ij}^r(a_1, \dots, a_n) = i_i f^r \pi_j(a_1, \dots, a_n) = i_i f^r(a_j) = i_i(ra_j) = \left(0, \dots, \underset{i^{th}}{ra_j}, \dots, 0\right)$.

The nearring of $n \times n$ matrices over N is the subnearring of $M(N^n)$ generated by the set $\{f_{ij}^r : r \in N, 1 \le i, j \le n\}$, is called a *matrix nearring* (denoted by $M_n(N)$). Meldrum and Van der Walt [19], Booth and Groenewald [20], Groenewald [21, 22, 23], Bhavanari, Rao and Kuncham [24] studied prime ideals and related concepts in matrix nearrings. The matrix nearring $M_n(N)$ is a right nearring with identity. If N is a ring with identity, then $M_n(N)$ is the ring of $n \times n$ matrices over N. Bhavarani and Kuncham [25] investigated the relationship of two sided ideals of a nearring N to those of matrix nearring $M_n(N)$. If L is the subset of N, L^n denotes the Cartesian product of n copies of L (here, n is a positive integer). Meldrum and Van der Walt [19] obtained that if L is a left ideal of N then L^n is an ideal of the $M_n(N)$-group N^n. More significantly, if L is a left ideal of N, then $(L^n : N^n) = \{K \in M_n(N) \mid K\rho \in L^n \text{ for all } \rho \in N^n\}$ is a two sided ideal of $M_n(N)$. For an ideal I of $M_n(N)$, define $I_* = \{x \in N \mid x \in \text{im}(\pi_j K)$ for some ideal K of $M_n(N)$, $1 \le j \le n\}$ is an ideal of N, and if I is an ideal of N then the related ideal I^* in $M_n(N)$ is defined as $I^* = \{K \in M_n(N) \mid K\rho \in I^n \text{ for all } \rho \in N^n\}$. The correspondence theorem between the substructures of matrix nearring and its base nearring are established in Meldrum and Van der Walt [19]. Kuncham and Bhavanari [26] proved that if $M_n(N)$ is reduced (no non-zero nilpotent elements) then N has insertion of factors property. The concept of equiprime nearrings were another generalization of prime nearrings which was introduced in Veldsman [27]. A significant study of equiprime, 3-prime, strongly equiprime ideals of nearrings can be found in Booth and Groenewald [20, 28]. A nearring N is said to be *equiprime* if for all $0 \ne a \in N$ and $x, y \in N$, $arx = ary$ for all $r \in N$ implies $x = y$, and an ideal P of N is called an equiprime ideal of N if N/P is an equiprime nearring. A nearring N is called *strongly equiprime* if for all $0 \ne a \in N$, there exists a finite subset F of N such that $x, y \in N$, $afx = afy$ for all $y \in F$ implies $x = y$. (Bhavanari, Rao and Kuncham [24]) For any A, B $\in M_n(N)$, we say that A is j^{th} row equivalent to B if (f_{1j}^1 + ... + f_{nj}^1)A = (f_{1j}^1 + ... + f_{nj}^1)B. The concepts *strictly equiprime*, j^{th} row strictly equiprime in matrix nearrings was introduced, and obtained that if N is a strictly equiprime nearring with 1, then $M_n(N)$ is j^{th} row strictly equiprime for $1 \leq j \leq n$. The connections between prime ideal of N and the corresponding prime ideals of matrix nearrings are discussed. If I is prime left ideal of N then I^* is prime left ideal in $M_n(N)$ [25]. For any ideal I of $M_n(N)$-group N^n we denote $I_{**} = \{x \in N / x = \pi_j A$ for some $A \in I, 1 \leq j \leq n\}$, where π_j is the j^{th} projection map from N^n to N, is an ideal of N. Eventually, we attained Goldie dimension of N-group N is equal to that of $M_n(N)$-group N^n.

Gamma nearrings

The concept of gamma nearring is a generalization of both the concepts gamma ring and nearring, which was defined by Satyanarayana [29, 30, 31]. Later several authors like Booth [32, 33], Jun, Kim, Ozturk [34], Satyanarayana and Syam Prasad [4], and Selvaraj [35,36], studied different concepts like ideals, prime ideals, semiprime ideals, fuzzy ideals, fuzzy prime ideals etc. in gamma nearrings.

A natural example of a gamma nearring is given in Satyanarayana [31]. Let (M, +) be a group (not necessarily Abelian) and Γ be a non-empty set. Then M is said to be a Γ-nearring if there exists a mapping $M \times \Gamma \times M \rightarrow M$ (denote the image of (m_1, α_1, m_2) by $m_1\alpha_1 m_2$ for $m_1, m_2 \in M$ and $\alpha_1 \in \Gamma$) satisfying the following conditions: (i) $(m_1 + m_2)\alpha_1 m_3 = m_1\alpha_1 m_3 + m_2\alpha_1 m_3$ and (ii) $(m_1\alpha_1 m_2)\alpha_2 m_3 = m_1\alpha_1(m_2\alpha_2 m_3)$ for all $m_1, m_2, m_3 \in M$ and for all $\alpha_1, \alpha_2 \in \Gamma$. Furthermore, M is said to be a zero-symmetric Γ-nearring if $m\alpha 0 = 0$ for all $m \in M, \alpha \in \Gamma$ (where '0' is additive identity in M).

If M is a Γ-nearring, then for $\alpha \in \Gamma$, define a binary operation '$*\alpha$' on M by $m_1 * \alpha m_2 = m_1\alpha m_2$ for all $m_1, m_2 \in M$. Then $(M, +, *\alpha)$ is a nearring. Conversely if (M, +) is a group and Γ is a set of binary operations on M such that

(i) (M, +, *) is a nearring for all $* \in \Gamma$, and
(ii) $(m_1 *_1 m_2) *_2 m_3 = m_1 *_1 (m_2 *_2 m_3)$ for all $*_1, *_2 \in \Gamma$, and for all $m_1, m_2, m_3 \in M$.

Fuzzy set theoretical aspects of nearrings:

Let μ be a fuzzy subset of a nearring N. Then μ is called a fuzzy ideal with thresholds of N, if for all $x, y, i \in N$, the conditions: $\alpha \vee \mu(x + y) \geq \beta \wedge \mu(x) \wedge \mu(y)$, $\alpha \vee \mu(-x) \geq \beta \wedge \mu(x)$, $\alpha \vee \mu(y + x - y) \geq \beta \wedge \mu(x)$, $\alpha \vee \mu(xy) \geq \beta \wedge \mu(x)$, $\alpha \vee \mu(x(y + i) - xy\} \geq \beta \wedge$

$\mu(i)$ (here, we call α as the lower threshold of N and β as the upper threshold of N (here, $\mu(0) \geq \beta$)) [37].

A fuzzy ideal μ of N is called (i) equiprime if $\alpha \vee \mu(a) \vee \mu(x\text{-}y) \geq \beta \wedge \ inf_{r \in N}\ \mu(arx - ary)$, for all x, y, a $\in$ N, (ii) 3-prime if for all a, b $\in$ N, $\alpha \vee \mu(a) \vee \mu(b) \geq \beta \wedge inf_{r \in N}\ \mu(arb)$, and (iii) c-prime $\alpha \vee \mu(a) \vee \mu(b) \geq \beta \wedge inf_{r \in N}\ \mu(ab)$. (One can observe that in case of commutative rings, all these concepts coincide) [38,39,40,41].

For a zero symmetric gamma nearring, the concepts fuzzy ideal, fuzzy prime ideals, and fuzzy cosets are defined. For a fuzzy ideal μ of a gamma nearring M, it is proved that there exist an order preserving bijective correspondence between the set $\boldsymbol{P}$ of all fuzzy ideal of σ of M such that $\sigma \supseteq \mu$ and $\sigma(0) = \mu(0)$ and the set Q of all fuzzy ideal θ of M/μ such that $\theta \supseteq \theta_\mu$ [42, 43].

Near algebras:

A near algebra is an algebraic system with two binary operations satisfying all of the axioms for a ring, except possibly one distributive law, and admitting a field as a left operator domain. This was introduced by Harold Brown [44]. A (left) near algebra over a field F is a linear space N over F on which N forms a semigroup under multiplication, multiplication is left distributive over addition, $a(kb) = k(ab)$ for $a, b, \in N$ and $k \in F$ (here 0 is the additive identity of N) and the vector space structure of N by (N, +), and, if N has a multiplicative identity (denoted by 1). A natural example of a near algebra over a field F which is not algebra is the system of all the transformations of a non-zero linear space over F into itself.

Nearrings of zero-preserving Lipschitz functions on metric spaces were introduced in Mark Farag and Brink van der Merwe [45], and identified a condition on the metric ensuring that the set of all such Lipschitz functions is a nearring. The explorations on the problems that arise from the lack of left distributivity in the resulting right nearring were established. The behaviour of the set of invertible Lipschitz functions, and investigations into the ideal structure of normed nearrings of Lipschitz functions were observed and related examples studied. Some results on primeness in the nearrings of Lipschitz functions can be found in Farag [46].

A normed nearring $(N, ||.||)$ is a nearring N with a function
$\|.\| : N \to \mathbb{R}^{+} \cup \{0\}$ such that

(i) $\|a\| = 0$ if and only if $a = 0$;

(ii) $\|a\| = \|-a\|$ for all $a \in N$;

(iii) $\|ab\| \leq \|a\|\|b\|$ for all $a, b \in N$;

(iv) $\|a+b\| \leq \|a\| + \|b\|$ for all $a, b \in N$.

Let (X_1, ρ_1) and (X_2, ρ_2) be metric spaces [47,48]. A function f from a metric space X_1 to a metric space X_2 is called *Lipschitz* if there exists a constant such that $K \geq 0$ for all $a, b \in X_1, \rho_2(f(a), f(b)) \leq K\rho_1(a, b)$. If $f : (X_1, \rho)_1 \to (X_2, \rho_2)$ is Lipschitz, then the *Lipschitz number* of f is defined as

$$\|f\|_L = \sup\left\{\frac{\rho_2(f(a), f(b))}{\rho_1(a,b)} \mid a, b \in X_1, a \neq b\right\}$$

For any metric space (X, ρ), denote L_X, the set of zero-preserving Lipschitz functions on X.

Mark Farag et al. [45] established that if ρ is a K-subadditive metric on X then $(L_X, +, \circ)$ where + is the point wise addition and $\circ$ is the composition of mappings is a zero-symmetric nearring with identity. $(L_X, ||.||_L)$ is called a normed nearring if $K = 1$.

For a (right) nearring (A, +, ×), for all $a, b \in A, k \in F$, the scalar multiplication $(ka).b = k(a.b)$, is called (right) *nearalgebra* over F [3]. As in nearrings, nearalgebras need not be zero-symmetric in general, as understood in the following example.

Let X be a vector space over field F. Then the set of all self-maps of X, is a near-algebra over F which is not zero-symmetric. Also, if R is any subalgebra of the F-algebra of all endomorphisms on X over a filed F, then the set of all affine transformations arising from elements of R and X, (denoted as $AF_R(X) := \{A_a \mid A \in R, a \in X\}$), where $A_a(x) := Ax + a$, is a sub-near -algebra of , which is also not zero-symmetric.

A *normed near-algebra* $(A, ||.||)$ over a field F with an absolute value $|.|$ is a near-algebra such that $(A, ||.||)$ is also a normed vector space over F with the norm $||.||$ satisfies the condition: $|| f.g || \leq || f || \, || g ||$ for all $f, g \in A$ [49].

Let **A** be a normed vector space over a normed field F. Then L_X is a normed near-algebra over F [45].

A random normed space ((or Sherstnev probabilistic normed spaces, briefly, a RN space) is a triple (X, μ, T) where X is a vector space, T is a continuous t-norm and μ is a mapping from X into D^+ (set of all distribution functions) with the following conditions holds [50]:

$(RN1)\, \mu_x(t) = \varepsilon_0(t)$ for all $t > 0$ if and only if $x = 0$ (0 is the null vector in X);

$(RN2)\, \mu_{\alpha x}(t) = \mu_x\left(\frac{t}{|\alpha|}\right)$ for all $x \in X$ and $\alpha \neq 0$;

$(RN3)\, \mu_{x+y}(t+s) \geq T\left(\mu_x(t), \mu_y(s)\right)$ for all $x, y \in X$ and $t, s \geq 0$

where μ_x denotes the value of μ at a point $x \in X$.

Let X be a normed linear space. Define a mapping $\mu_x(t) = \begin{cases} 0 & \text{if } t \leq 0 \\ \frac{t}{t+\|x\|} & \text{if } t > 0 \end{cases}$ and $T_p(x, y) = xy$ then (X, μ, T_p) is a random normed space.

In any random Normed space (X, μ, T) one can define the open and closed ball with centre $x \in X$ and radius $x < r < 1$ for all as $B_x(r,t) = \{y \in X; \mu_{x-y}(t) > 1-r\}$ and $B_x[r,t] = \{y \in X; \mu_{x-y}(t) \geq 1-r\}$, respectively.

It is obvious to note that every open ball in a random normed space is an open set and every RN-space is a Hausdorff space. Some of the classical theorems in Banach spaces like, Ascoli-Arzela theorem, Open mapping theorem, and Closed Graph theorem have been established in random Normed spaces.

A random normed algebra is a random Normed space with algebraic structure such that $\mu_{xy}(ts) \geq \mu_x(t)\mu_y(s)$ for all $x, y \in X$ and $t, s > 0$. For example, every normed algebra X defines a random normed algebra (X, μ, T_M), where $\mu_x(t) = \frac{t}{t+\|x\|}$ for all $t > 0$ and $T_M(x,y) = Min\{x,y\}$. Let (X, μ, T_M) and (Y, μ, T_M) be random normed algebras then

(ι) An linear mapping $f: A \to B$ is called a random homomorphism if $f(ab) = f(a)f(b)$ for all $a, b \in A$.

(ιι) An linear mapping $f: A \to A$ is called a random derivation if $f(ab) = f(a)b + af(b)$ for all $a, b \in A$.

Normed near-algebras defined and provides some characteristic examples on them, and proved the stability of random homomorphisms, Cauchy Jensen functional equations and random *-derivations in random Banach algebras [51].

Using the fixed point method Yeol Je Cho et al., [51] proved the Hyers – Ulam stability of random homomorphisms and random derivations the complete random Normed algebras associated with the Cauchy additive functional inequality and Cauchy-Jensen additive functional inequality respectively.

Let X be a *-algebra and (X, μ, T_M) be a random normed space then

(ι) (X, μ, T_M) is called a random normed *-algebra if $\mu_{xy}(st) \geq \mu_x(s).\mu_y(t)$, $\mu_{x^*}(t) = \mu_x(t)$ for all $x, y \in X$ and $s, t > 0$.

(ιι) A complete random normed *-algebra is called random Banach *-algebra.

An immediate example is, for every normed *-algebra $(X, ||.||)$ and define $\mu_x(t) = \begin{cases} \dfrac{t}{t+\|x\|} & \text{for all } t > 0 \\ 0 & \text{for all } t \leq 0 \end{cases}$ with $T_M(x, y) = Min\{x, y\}$ then (X, μ, T_M) is a random normed *-algebra.

Let $(X, ||.||)$ be a C*-algebra and μ be a random norm on X then the random Banach *-algebra is called an induced random C*- algebra.

Let (X, μ, T_M) and (X, μ', T_M) be random normed algebras then

(ι) an linear mapping $H : (X, \mu, T_M) \rightarrow (X, \mu', T_M)$ is called a random *-homomorphism if $H(x^*) = H(x^*)$ for all $x \in X$.

(ιι) an linear mapping $D : (X, \mu, T_M) \rightarrow (X, \mu', T_M)$ is called a random *-derivation if $D(xy) = D(x)y + xD(y)$, $D(x^*) = D(x)^*$ for all $x, y \in X$.

Using the fixed point method, Yeol Je Cho et al. [51] proved the Hyers – Ulam stability of the Cauchy- Jensen functional equation in random Banach *-algebras, random *-derivations in random Banach *-algebras. Bernardo and Harikrishnan have provided comprehensive overview of probabilistic normed spaces [50].

Network applications of nearrings:
A graph G is said to be a *prime graph* of R (denoted by PG(R)) if the elements of ring are the vertices and the edge set defined as $E = \{\overline{xy} \mid xRy = 0 \text{ or } yRx = 0, \text{ and } x \neq y\}$. Various prime graphs on $\mathbb{Z}_n$, (where n is a positive integer) the ring of integers modulo n are provided with characterizations [52,53].

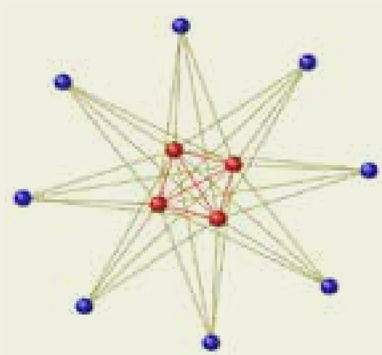
The prime graph of a nearring N defined as a graph with the elements of N as the vertices and any two $x, y \in N$ are adjacent if and only if $xny = 0$ or $ynx = 0$ for all $n \in N$. If N is a commutative ring then the zero-divisor graph of N is a subgraph of the prime graph of N [54]. For a fixed ideal I of N, the 'graph of a nearring N with respect to I' (denoted as $G_1(N)$). The graph $G_1(N)$ with each element of N as a vertex, and two distinct vertices x and y are connected by an edge if and only if $xNy \subseteq$ or $yNx \subseteq I$. The ideal symmetry of $G_1(N)$ implies the symmetry obtained by the automorphism group of $G_1(N)$. Beck [55] linked a commutative ring R to a graph by using the elements of R as vertices and any two vertices x, y are adjacent if and only if $xy = 0$. Anderson and Livingston [56] proposed an improved method of combining a commutative ring to a graph by presenting the concept of a zero-divisor graph of a commutative ring. In a zero-divisor graph of a commutative ring R we consider the vertices as the set of non-zero divisors of R, and there is an edge between the vertices x, y if and only if $x \neq y$ and $xy = 0$ (where xy is the product in the ring R). The zero-divisor graphs are highly symmetric. The associated notions are studied by Godsil and Royle [57], Bhavanari, Kuncham and Kedukodi [54]. For example, a human being is symmetric but only with respect to a specific vertical plane. We obtained that if I is a 3-prime ideal of a zero-symmetric nearring N, then I plays a role in $G_1(N)$ similar to the role of specific vertical plane in case of a human being. It is evident that the notions are coincides in case of rings. Further, it is obtained that if the nearring N has n elements and I be an ideal of N then the inequality holds: (i) $I \leq k(G_I(N)) \leq \lambda(G_I(N)) \leq \delta(G_I(N)) \leq n\text{-}1$; (ii) if N is zero-symmetric and I is a minimal 3-prime ideal of N, then $|I| \leq k(G_I(N)) \leq \delta(G_I(N))$.

The notion of fuzzy graph of a nearring N with respect to a level ideal depicts graphically the fuzzy character which is concealed algebraically in the examples of 3-prime fuzzy ideals of a nearring N. The idea comes from facts that: to depict fuzziness graphically, a natural mathematical tool is the fuzzy graph, and 3-prime fuzzy ideal of a nearring is characterized by its level ideals in the interval $(\alpha,\beta]$. The notion of fuzzy graph of a nearring N with respect to a level ideal μ_t denoted by (N, μ, σ, t).

If N is a zero-symmetric nearring and μ is 3-prime fuzzy ideal of N, then (N, μ, σ, t) has a special type of symmetry, called as the ideal symmetry of (N, μ, σ, t) [58]. Let $\mu: N \to (0; 1]$ be a fuzzy ideal of N with thresholds α and β. Define a mappingσ: $N \times N \to [0, 1]$ as follows: $\sigma(x, y) = \mu(x) \wedge \mu(y)$, if $x \neq y$ and ($xNy \subseteq \mu_t$ or $yNx \subseteq \mu_t$),

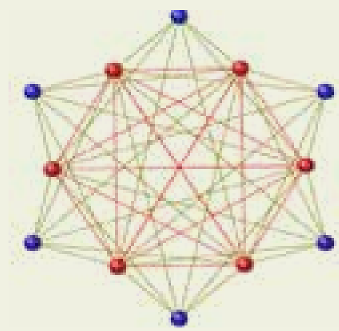
$\sigma(x, y) = 0$, otherwise. Then the fuzzy graph (N, μ, σ, t) is called fuzzy graph of N with respect to the level ideal μ_t. We provide some networking applications.

Consider $N = Z_{12}$, the ring of integers modulo 12. Then N is a commutative ring with unit element 1. Let $x, y \in (0.2, 0.8)$ with $x \neq y$. Define a fuzzy subset $\mu: Z_{12} \to [0, 1]$ by $\mu(a) = \begin{cases} 0.9 \; if \; a = 6 \\ 0.8 \; if \; a = 0 \\ x \; if \; a \in \{3, 9\} \\ y \; if \; a \in \{2, 4, 8, 10\} \\ 0.2 \; elsewhere \end{cases}$. Take thresholds $\alpha = a \wedge b$ and $\beta = a \vee b$. Then μ is a fuzzy ideal of Z_{12}. Let $t = \beta$. To get a sketch of (N, μ, σ, β), choose some values of x and y. Choose $x = 0.6$ and $y = 0.4$. Then $\alpha = 0.4$ and $\beta = 0.6$. We have $t = \beta = 0.6$ and $\mu_t = \{0, 3, 6, 9\}$. We form a table for σ as follows. Refer the network for $x > y$.



σ(x,y)	y = 0	y = 1	y = 2	y = 3	y = 4	y = 5	y = 6	y = 7	y = 8	y = 9	y = 10	y = 11
x = 0	0	0.2	0.4	0.6	0.4	0.2	0.8	0.2	0.4	0.6	0.4	0.2
x = 1	0.2	0	0	0.2	0	0	0.2	0	0	0.2	0	0
x = 2	0.4	0	0	0.4	0	0	0.4	0	0	0.4	0	0
x = 3	0.6	0.2	0.4	0	0.4	0.2	0.6	0.2	0.4	0.6	0.4	0.2
x = 4	0.4	0	0	0.4	0	0	0.4	0	0	0.4	0	0
x = 5	0.2	0	0	0.2	0	0	0.2	0	0	0.2	0	0
x = 6	0.8	0.2	0.4	0.6	0.4	0.2	0	0.2	0.4	0.6	0.4	0.2
x = 7	0.2	0	0	0.2	0	0	0.2	0	0	0.2	0	0
x = 8	0.4	0	0	0.4	0	0	0.4	0	0	0.4	0	0
x = 9	0.6	0.2	0.4	0.6	0.4	0.2	0.6	0.2	0.4	0	0.4	0.2
x = 10	0.4	0	0	0.4	0	0	0.4	0	0	0.4	0	0
x = 11	0.2	0	0	0.2	0	0	0.2	0	0	0.2	0	0
deg(y)	4.4	0.8	1.6	4.2	1.6	0.8	4.4	0.8	1.6	4.2	1.6	0.8

Now we choose x = 0.4 and y = 0.6. Then α = 0.4 and β = 0.6. We have t = β = 0.6 and μ_t = {0, 2, 6, 8, 10}, the network diagram and the table for x < y is given below.



σ(x,y)	y = 0	y = 1	y = 2	y = 3	y = 4	y = 5	y = 6	y = 7	y = 8	y = 9	y = 10	y = 11
x = 0	0	0.2	0.6	0.4	0.6	0.2	0.8	0.2	0.6	0.4	0.6	0.2
x = 1	0.2	0	0.2	0	0.2	0	0.2	0	0.2	0	0.2	0
x = 2	0.6	0.2	0	0.4	0.6	0.2	0.6	0.2	0.6	0.4	0.6	0.2
x = 3	0.4	0	0.4	0	0.4	0	0.4	0	0.4	0	0.4	0
x = 4	0.6	0.2	0.6	0.4	0	0.2	0.6	0.2	0.6	0.4	0.6	0.2
x = 5	0.2	0	0.2	0	0.2	0	0.2	0	0.2	0	0.2	0
x = 6	0.8	0.2	0.6	0.4	0.6	0.2	0	0.2	0.6	0.4	0.6	0.2
x = 7	0.2	0	0.2	0	0.2	0	0.2	0	0.2	0	0.2	0
x = 8	0.6	0.2	0.6	0.4	0.6	0.2	0.6	0.2	0	0.4	0.6	0.2
x = 9	0.4	0	0.4	0	0.4	0	0.4	0	0.4	0	0.4	0
x = 10	0.6	0.2	0.6	0.4	0.6	0.2	0.6	0.2	0.6	0.4	0	0.2
x = 11	0.2	0	0.2	0	0.2	0	0.2	0	0.2	0	0.2	0
deg(y)	4.8	1.2	4.6	2.4	4.6	1.2	4.8	1.2	4.6	2.4	4.6	1.2

For a zero symmetric nearring N, if μ is 3-prime fuzzy ideal of N then (μ_t, μ, σ, t) is complete subgraph of (N, μ, σ, t) [38,58].

Applications to rough sets

Pawlak [59] introduced the concept of a rough set. The theory of rough sets is an extension of the notion set theory, in which a subset of a universe is designated by a pair of crisp sets called the lower approximation and the upper approximation. The concept of a rough set has numerous applications in data analysis. The notion of reference point was introduced and studied in Kedukodi [38], Kedukodi, Kuncham and Bhavanari [40]. The notion of a reference point provides local approximations for a subset of the universe. The reference point naturally provides a rough approximations framework, wherein some approximations are possible on the same set. An example of rough approximations framework arises with the help of an equiprime ideal of a nearring.

Let a $\in$ U and E_a be an equivalence relation associated with a [58]. We call a as the reference point. Suppose E_a partitioned U into equivalence classes $[x]_a = \{y \in U: xE_a y\}$. For $A \subseteq U$, the conditional probability that an element y is in A due to the fact that $y \in [x]_a$ is denoted by $P(A \mid [x]_a)$ and defined as follows:

$$P(A|[x]_a) = \frac{|\ A \cap [x]_a\ |}{|\ [x]_a\ |}.$$

Let v, w $\in$ [0, 1] are two parameters such that v > w. Let $A \subseteq U$ and a $\in$ U. The lower approximation of A with respect to a is defined as

$$l_{\{\nu,\, a\}}(A) = \{x \in U \mid P(A|[x]_a) \geq \nu\}$$

and the upper approximation of A with respect to a is defined as

$$u_{\{\omega,\, a\}}(A) = \{x \in U \mid P(A|[x]_a) > \omega\}$$

Consider U = Z_{12} (the ring of integer modulo 12).

Define $\mu: Z_{12} \to [0, 1]$ by

$$\mu(x) = \begin{cases} 0.8 \text{ } if \text{ } x \in \{\bar{0}, \bar{6}\} \\ 0.6 \text{ } if \text{ } x \in \{\bar{2}, \bar{4}, \bar{8}, \overline{10}\} \\ 0.2 \text{ } elsewhere \end{cases}$$

Let a $\in$ U and t $\in$ [0, 0.8]. Define $U_e(\mu, t, a) = \{(x, y) \in R \times R \mid \mu(arx - ary) \geq t$ for all $r \in R\}$. Now $E_a = U_e(\mu, t, a)$ is an equivalence relation on U. Denote the equivalences classes by $[x]_a = \{y \in U \mid xE_a y\}$. Let v = 0.75 and w = 0.25. Take a = 2 and t = 0.8. Then $E_2 = U_e(\mu, 0.8, 2) = \{ (\bar{0},\bar{0}), (\bar{1},\bar{1}), (\bar{2},\bar{2}), (\bar{3},\bar{3}), (\bar{4},\bar{4}), (\bar{5},\bar{5}), (\bar{6},\bar{6}), (\bar{7},\bar{7}), (\bar{8},\bar{8}), (\bar{9},\bar{9}), (\overline{10},\overline{10}), (\overline{11},\overline{11}), (\bar{0},\bar{3}), (\bar{3},\bar{0}), (\bar{0},\bar{6}), (\bar{6},\bar{0}), (\bar{0},\bar{9}), (\bar{9},\bar{0}), (\bar{3},\bar{6}), (\bar{6},\bar{3}), (\bar{3},\bar{9}), (\bar{9},\bar{3}), (\bar{1},\bar{4}), (\bar{4},\bar{1}), (\bar{1},\bar{7}), (\bar{7},\bar{1}), (\bar{1},\overline{10}), (\overline{10},\bar{1}), (\bar{4},\bar{7}), (\bar{7},\bar{4}), (\bar{4},\overline{10}), (\overline{10},\bar{4}), (\bar{2},\bar{5}), (\bar{5},\bar{2}), (\bar{2},\bar{8}), (\bar{8},\bar{2}), (\bar{2},\overline{11}), (\overline{11},\bar{2}), (\bar{5},\bar{8}), (\bar{8},\bar{5}), (\bar{5},\overline{11}), (\overline{11},\bar{5})\}$.

Now $E_2 = U_e(\mu, 0.8, 2)$ partitions $U = Z_{12}$ into the following three equivalence classes:

$\bar{0}$		$\bar{1}$		$\bar{2}$
$\bar{3}$		$\bar{4}$		$\bar{5}$
$\bar{6}$		$\bar{7}$		$\bar{8}$
$\bar{9}$		$\overline{10}$		$\overline{11}$

$u_{\{0.25,\ 2\}}(A) = \{x \in Z_{12} : \dfrac{|A \cap [x]_2|}{|[x]_2|} > 0.25\} = \{\bar{0},\bar{3},\bar{6},\bar{9}\}$.

$u_{\{0.25,\ 2\}}(B) = \{x \in Z_{12} : \dfrac{|B \cap [x]_2|}{|[x]_2|} > 0.25\} = \{\bar{0},\bar{3},\bar{6},\bar{9}\}$.

Note that $A \nsubseteq u_{\{0.25,\ 2\}}(A)$ and $B \nsubseteq u_{\{0.25,\ 2\}}(B)$. Now $A \cup B = \{\bar{0},\bar{1},\bar{3},\bar{4},\bar{6}\}$and $u_{\{0.25,\ 2\}}(A \cup B) = \{x \in Z_{12} : \dfrac{|(A \cup B) \cap [x]_2|}{|[x]_2|} > 0.25\} = \{\bar{0},\bar{1},\bar{3},\bar{4},\bar{6},\bar{7},\bar{9},\overline{10}\}$. Hence $u_{\{0.25,\ 2\}}(A) \cup u_{\{0.25,\ 2\}}(B) = \{\bar{0},\bar{3},\bar{6},\bar{9}\} \subsetneq \{\bar{0},\bar{1},\bar{3},\bar{4},\bar{6},\bar{7},\bar{9},\ \overline{10}\} = u_{\{0.25,\ 2\}}(A \cup B)$.
Take a = 1 and t = 0.8. Then

$E_1 = U_e(\mu, 0.8, 1)$
$= \{(\bar{0},\bar{0}),(\bar{1},\bar{1}),(\bar{2},\bar{2}),(\bar{3},\bar{3}),(\bar{4},\bar{4}),(\bar{5},\bar{5}),(\bar{6},\bar{6}),(\bar{7},\bar{7}),(\bar{8},\bar{8}),(\bar{9},\bar{9}),(\overline{10},\overline{10}),(\overline{11},\overline{11}),$
$(\bar{0},\bar{6}),(\bar{6},\bar{0}),(\bar{1},\bar{7}),(\bar{7},\bar{1}),(\bar{2},\bar{8}),(\bar{8},\bar{2}),(\bar{3},\bar{9}),(\bar{9},\bar{3}),(\bar{4},\overline{10}),(\overline{10},\bar{4}),(\bar{5},\overline{11}),(\overline{11},\bar{5})\}$.

Now $E_1 = U_e(\mu, 0.8, 1)$ partitions $U = Z_{12}$ into the following six equivalence classes:

$\bar{0}$		$\bar{1}$		$\bar{2}$		$\bar{3}$		$\bar{4}$		$\bar{5}$
$\bar{6}$		$\bar{7}$		$\bar{8}$		$\bar{9}$		$\overline{10}$		$\overline{11}$

Let $A = \{\bar{0},\bar{1},\bar{2},\bar{3},\bar{4},\bar{6}\}$. Suppose v = 0.75 and w = 0.25. Then $l_{\{0.75,\ 1\}}(A) = \{x \in Z_{12} : \dfrac{|A \cap [x]_1|}{|[x]_1|} \geq 0.75\} = \{\bar{0},\bar{6}\}$.

$u_{\{0.25,\ 1\}}(A) = \{x \in Z_{12} : \dfrac{|A \cap [x]_1|}{|[x]_1|} > 0.25\} = \{\bar{0},\bar{1},\bar{2}\ \bar{3},\bar{4},\bar{6},\bar{7},\bar{8},\bar{9},\ \overline{10}\}$. Hence $(u_{\{0.25,\ 1\}}(A))^c = \{\bar{5},\overline{11}\} \subsetneq \{\bar{0},\bar{1},\bar{2}\ \bar{3},\bar{4},\bar{6},\bar{7},\bar{8},\bar{9},\ \overline{10},\ \overline{11}\} = (l_{\{0.75,\ 1\}}(A))^c$. Now let us compute the lower and upper approximations of the set $A = \{\bar{0},\bar{1},\bar{2}\ \bar{3},\bar{4},\bar{6}\}$ with respect to the reference point a = 2.

$$l_{\{0.75,\,2\}}(A) = \{x \in Z_{12} : \frac{|A \cap [x]_2|}{|[x]_2|} \geq 0.75\} = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\}.$$

$$u_{\{0.25,\,1\}}(A) = \{x \in Z_{12} : \frac{|A \cap [x]_2|}{|[x]_2|} > 0.25\} = \{\bar{0}, \bar{1}, \bar{3}, \bar{4}, \bar{6}, \bar{7}, \bar{9}, \overline{10}\}.$$ Note that distinct reference points, namely a = 1 and a = 2, yield distinct lower and upper approximations for the set $A = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}\ \bar{4}, \bar{6}\}$ in the universe Z_{12}.

Consider $U = Z_{12}$ (the ring of integers modulo 12).

Further reading and interesting applications

One element that has appeared recently as an application of nearrings is the use of planar and other nearrings to develop designs and codes. Authors like Roland Eggestberger, Gerhard Wagner, Peter Fuchs, Gunter Pilz, Jim Clay and Wen F Ke have contributed a quantum research of planar nearrings and related applications. Planar nearrings are useful to develop several balanced incomplete block designs (BIB designs). Efficient error-correcting codes (linear as well as non-linear) are obtained from nearrings. Well-known concepts of geometry like collineation, dilatation, translation etc. are handled with the help of nearrings giving rise to a "dictionary" between geometry and group theory. One of the applications of group theory is in the study of symmetry. There are several results which explain how conservation laws of physical systems arise from their symmetries under various transformations. Non-linear transformations were handled using algorithms involving nearring generators wherein group theoretical algorithms could not be applied. The GAP package SONATA (abbreviated as: Systems Of Nearrings And Their Applications). The SONATA package provides (i) methods for the construction and analysis of finite nearrings, (ii) approaches for constructing all endomorphisms and all fixed-point-free automorphisms of a given group, (iii) constructing the nearring of polynomial functions of the group, (iv) obtaining the functions to get solvable fixed-point-free automorphism groups on abelian groups, nearfields, planar nearrings, in addition block designs from those, etc. The link between nearrings and automata, nearrings and dynamical systems can be found in Pilz [3]. For more details about nearrings and applications, one may visit nearrings webpage http://www.algebra.uni-linz.ac.at/Nearrings/. For more details on orders in primitive rings, refer Anh and Marki [60]; about the concepts on primeness, refer [61], [62], [63], [64], [65], and [66]. For Related fuzzy concepts, refer [67], [71], [74] [82], [83], [94], [96], [97], and [100]. The concepts on commutativity results, dualizing Goldie dimension in modules and N-groups were studied in [69], [70], [72], [73], [75], [76], [84], [85], [86], [87], [88], [89], [92] and [95]. Some connections to normed operators were found in [81]. The concepts on ideals of matrix nearings, intermediate ideals in matrix nearrings, and related radicals were found in [77], [78], [79], [80], and [98]. Some important graph theoretical aspects related to ideal symmetry, prime ideals were found in [68], [91], and [93].

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