A Concatenation Operation to Derive Autosegmental Graphs

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July 26 2015

Introduction

- What kind of structures are autosegmental phonological representations (APRs; Goldsmith, 1976)?
- Current conception: top-down constraints on graph structures
- Our result: these fundamental properties emerge from concatenation of an alphabet of primitives (Engelfriet and Vereijken, 1997; Courcelle et al., 2012)
- This makes them directly comparable to strings

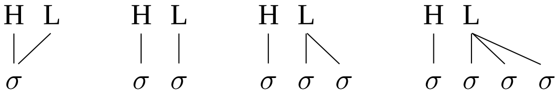
Introduction

Overview

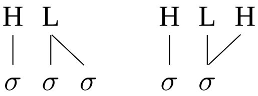
- Why APRs & what are APRs
- Representation of APRs as labeled mixed graphs
- Top-down, axiomatic definition of APRs
- Constructive derivation of axioms from concatenation & primitives
- Empirical consequences
- Discussion

APRs

- What are APRs?
- Mende (Leben, 1973; Goldsmith, 1976): fewer attested tone combinations than possible
  F, HL, HLL, HLLL, ...
  *HF, *HLHL, *HHLL, *HLLH, ...
- Answer: tone melodies independent from syllables

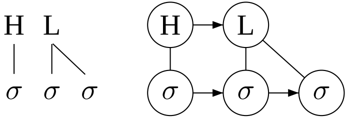


APRs



- Fundamentally, APRs are:
  - Ordered tiers of like autosegments
  - Association lines connecting items on distinct tiers
- Set of valid APRs usu. defined *axiomatically* (Goldsmith, 1976; Sagey, 1986; Bird and Klein, 1990; Kornai, 1995)

APGs

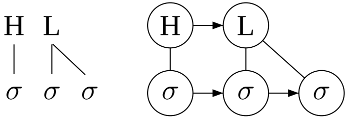


- To represent APRs, we can use (simple) *labeled mixed graphs*

$$G = \langle V, E, A, \ell \rangle$$

- $GR(\Sigma)$
  - $V = \{0, 1, ..., n\}$
  - $E \subseteq \mathcal{P}_{=2}(V)$
  - $A \subseteq V \times V$
  - $\ell : V \to \Sigma$

APGs



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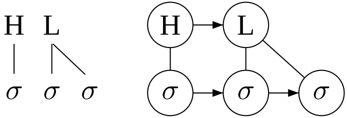
$$G = \langle V, E, A, \ell \rangle$$

- Why graphs?
  - Graphs explicitly encode all information in APRs
  - Deep literature on pattern matching, characterizing & learning graph sets, graph transductions, etc. (Courcelle et al., 2012; Engelfriet and Hoogeboom, 2001; López et al., 2012; Ferrera, 2013, inter alia)

APGs

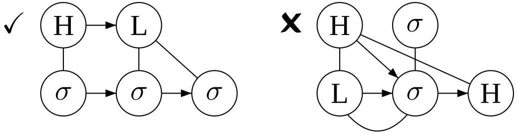


- To represent APRs, we can use (simple) *labeled mixed graphs*

$$G = \langle V, E, A, \ell \rangle$$

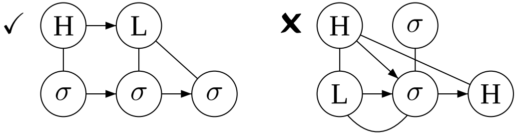
- To select *well-formed* autosegmental phonological graphs (APGs), there are two options
  - Axiomatically (top-down approach)
  - Constructively (bottom-up approach)

Axiomatic approach: $APG(\Sigma, T) \subset GR(\Sigma)$



- We first define the basic autosegmental structure in $APG(\Sigma, T)$
- We consider two *tiers*

Axiomatic approach: $APG(\Sigma, T) \subset GR(\Sigma)$

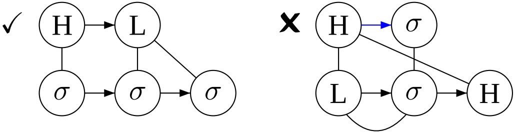


- Let $\preccurlyeq$ be the reflexive, transitive closure of A

Axiom (Tier structure)

*V is partitioned into two sets V_0, V_1 which are each totally ordered by $\preccurlyeq$. V_0 and V_1 are the **tiers** of G.*

Axiomatic approach: $APG(\Sigma, T) \subset GR(\Sigma)$



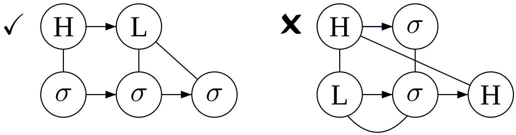
- Let T be a partition $\{T_t, T_m\}$ on Σ

Axiom (Like tier labels)

Partition of V into tiers respects partition T on Σ

- Let V_m correspond to T_m, likewise V_t, T_t
- Let's set $T_m = \{\text{H}, \text{L}\}$, $T_t = \{\sigma\}$

Axiomatic approach: $APG(\Sigma, T) \subset GR(\Sigma)$



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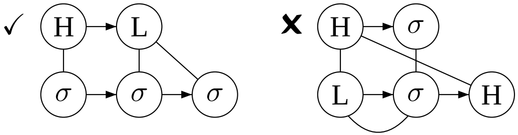

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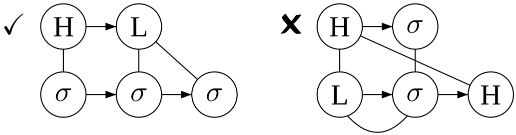

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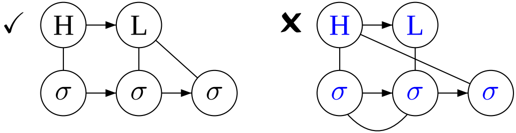
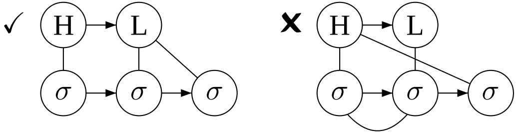


- Associations can only be between tiers

Axiom (Associations between tiers)

For all $\{x, y\} \in E$, $x \not\lessdot y$ *and* $y \not\lessdot y$.

Axiomatic approach: $APG(\Sigma, T) \subset GR(\Sigma)$



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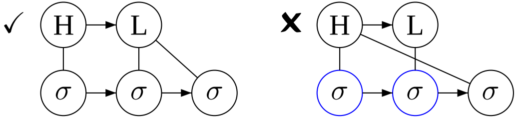
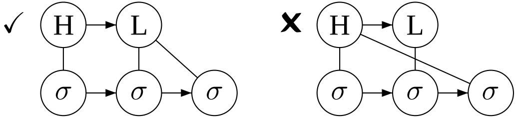


- No line crossing (Goldsmith, 1976; Hammond, 1988; Coleman and Local, 1991)

Axiom (NCC)

For all $u, v, x, y \in V$, if $\{u, x\}, \{v, y\} \in E$ and $u \preccurlyeq v$, then $x \preccurlyeq y$.

Axiomatic approach: $APG(\Sigma, T) \subset GR(\Sigma)$



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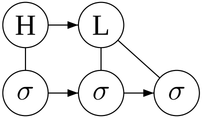
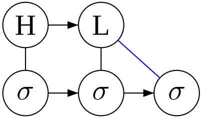
✓ 

✓ 

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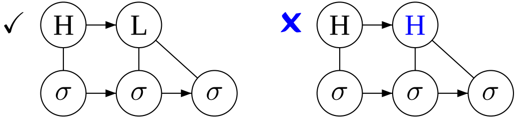
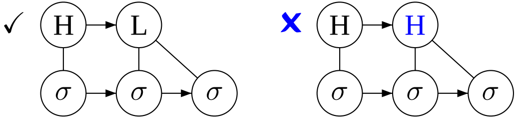


- The Obligatory Contour Principle Leben (1973); Goldsmith (1976); McCarthy (1986); Odden (1986)

Axiom (OCP)

For tier V_m, for all $x, y \in V_m$, $(x, y) \in A$ implies $\ell(x) \neq \ell(y)$.

Axiomatic approach: $APG(\Sigma, T) \subset GR(\Sigma)$



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Axiomatic approach: $APG(\Sigma, T) \subset GR(\Sigma)$

- $APG(\Sigma, T)$: graphs in $GR(\Sigma)$ obeying these axioms

Axioms	1) Tier structure	4) NCC
	2) Like tier labels	5) OCP
	3) Associations between tiers	

- These properties can all be derived from concatenation

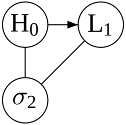
Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

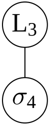
- Engelfriet and Vereijken (1997); Courcelle et al. (2012): two methods of graph concatenation:

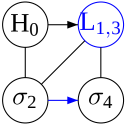
  > *One way is to "glue" them together, by identifying some of their vertices. The other way is to "bridge" them (or rather, "bridge the gap between them"), by adding edges between their vertices. (Courcelle et al., 2012, p. 6)*

- We define a specialized version of this machinery (using both "gluing" and "bridging") to generate the set of APGs

Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

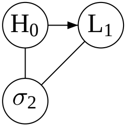
$G_1 =$ 

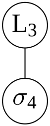
$G_2 =$ 

$G_1 \circ G_2 =$ 

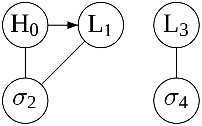
Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

“Gluing” and **“bridging”**

$G_1 =$ 

$G_2 =$ 

- Identify the ‘ends’ of the tiers
- Merge identical end nodes on V_m
- Draw arcs between any other pair of end nodes

$G_{1,2} =$ 

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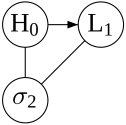
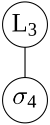
“Gluing” and **“bridging”**

$G_1 =$ 

$G_2 =$ 

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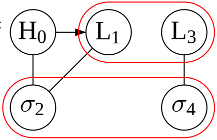
$G_{1,2} =$ 

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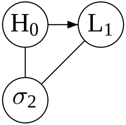
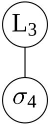
$G_1 =$ 

$G_2 =$ 

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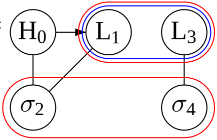
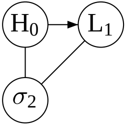
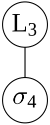
$G_{1,2} =$ 

Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

“Gluing” and **“bridging”**

$G_1 =$ 

$G_2 =$ 

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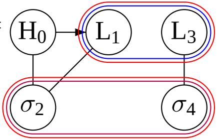
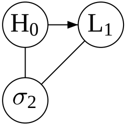
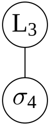
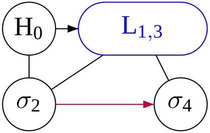
$G_{1,2} =$ 

Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

"Gluing" and **"bridging"**

$G_1 =$ 

$G_2 =$ 

$G_1 \circ G_2 =$ 

Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

The identity graph

- Let $G_\lambda = \langle \emptyset, \emptyset, \emptyset, \emptyset \rangle$

Theorem (G_λ is identity)

For any $G \in GR(\Sigma)$, $G \circ G_\lambda = G_\lambda \circ G = G$.

Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

Graph primitives

- Engelfriet and Vereijken (1997): graph sets can interpret string sets
- Concatenation of symbols = concatenation of graph primitives

Definition

An *alphabet of graph primitives* over $GR(\Sigma)$ is a finite set Γ of symbols and a naming function $g : \Gamma \to GR(\Sigma)$.

Definition

For $w \in \Gamma^*$, $g(w) \stackrel{\text{def}}{=}$

- G_λ if $w = \lambda$
- $g(u) \circ g(\gamma)$ if $w = u\gamma, u \in \Gamma^*, \gamma \in \Gamma$

Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

APG graph primitives

Definition (APG graph primitive)

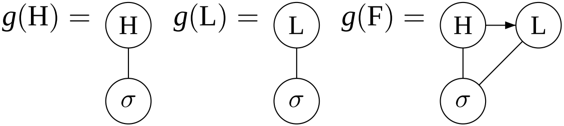
For Σ and $T = \{T_t, T_m\}$, an *APG graph primitive* is a graph $G \in GR(\Sigma)$ for which

a. V_t is a singleton set $\{v_t\}$

b. V_m is totally ordered by $\preccurlyeq$

c. All $e \in E$ are of the form $\{v_m, v_t\}$, $v_m \in V_m$

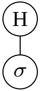
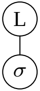
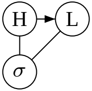
$$\Gamma = \{\mathrm{H}, \mathrm{L}, \mathrm{F}\}$$



Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

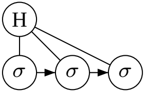
Graph primitives

Def. of APG prim.
a. $V_t = \{v_t\}$
b. $\preccurlyeq$ totally orders V_m
c. All $e \in E = \{v_m, v_t\}$ for $v_m \in V_m$

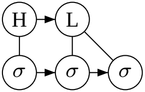
$\Gamma = \{\mathrm{H}, \mathrm{L}, \mathrm{F}\}$ $g(\mathrm{H}) =$  $g(\mathrm{L}) =$  $g(\mathrm{F}) =$ 

- $APG(\Gamma) = \{g(w) | w \in \Gamma^*\}$

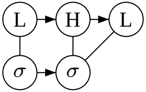
$g(\mathrm{HHH})$



$g(\mathrm{HLL})$



$g(\mathrm{LF})$



$\ldots$

Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

APG prim.	a. $V_t = \{v_t\}$	Ax.	1) Tier struct.	4) NCC
	b. $\preccurlyeq$ totally orders V_m		2) Tier labels	5) OCP
	c. All $e \in E = \{v_m, v_t\}$ for $v_m \in V_m$		3) Assoc.	

Theorem

Every $G \in APG(\Gamma)$ follows Ax. 1 – 5

- *APG graph primitives have two tiers V_t and V_m and concatenation preserves this*
- *NCC from singleton V_t*
- *OCP from merging of melody nodes*

Theorem

$\circ$ is associative over $APG(\Gamma)$

- *Concatenation preserves tier structure; proof similar of associativity over Γ^**

Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

APG prim.
a. $V_t = \{v_t\}$
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Ax.
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- *APG graph primitives have two tiers V_t and V_m and concatenation preserves this*
- *NCC from singleton V_t*
- *OCP from merging of melody nodes*

Theorem

$\circ$ is associative over $APG(\Gamma)$

- *Concatenation preserves tier structure; proof similar of associativity over Γ^**

Constructive approach: $APG(\Gamma) \subset GR(\Sigma)$

APG prim.	a. $V_t = \{v_t\}$	Ax.	1) Tier struct.	4) NCC
	b. $\preccurlyeq$ totally orders V_m		2) Tier labels	5) OCP
	c. All $e \in E = \{v_m, v_t\}$ for $v_m \in V_m$		3) Assoc.	

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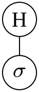
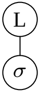
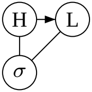
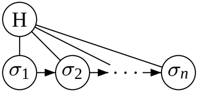
Concatenation and empirical phenomena

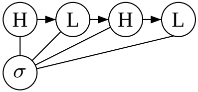
- $APG(\Gamma)$ also naturally captures facts of phonology
- Unbounded spreading, but no ‘unbounded contouring’

$g(\mathrm{H}) =$ 

$g(\mathrm{L}) =$ 

$g(\mathrm{F}) =$ 

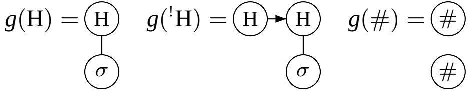
- $g(\mathrm{H}^n) =$ 

- No $w \in \Gamma^*$ s.t. $g(w) =$ 

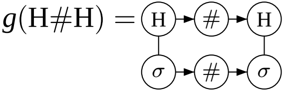
- (unless added to Γ)

Concatenation and empirical phenomena

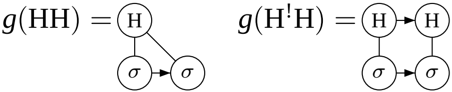
- Most OCP violations at morpheme boundaries or signaled by downstep (Hyman, 2014)



- Adjacent Hs across boundaries (Aghem; Hyman, 2014)



- Adjacent Hs tautomorphemically (Kishambaa; Odden, 1986)



Discussion

- g is direct way of relating strings to APRs
- Cognitive interpretation: how humans relate linear speech stream to APRs
- $(APG(\Gamma), \circ)$ as free monoid
- Multi-tier representations: n-ary partitions on Σ
- Full feature geometry may require more structure on partition
- Less restrictive version w.r.t. OCP could include second concatenation operation without 'gluing'

Conclusion

- We defined APRs as $APG(\Gamma)$ parallel to strings
- Maintains key properties of APRs
- Positive empirical implications (e.g. no unbounded contouring)
- *Restrictive* theory of APRs (w.r.t OCP)

Acknowledgements

We thank three reviewers for their insightful comments and suggestions. Adam Jardine acknowledges support from a University of Delaware Graduate Research Fellowship.