

HW 2: Factors and Constraints defined with Propositional Logic

1. Locally Testable Constraints (PROP(\triangleleft))

- (a) For each φ , write two words whose models satisfy it. (Assume $\Sigma = \{a, b, c\}$ and $\mathcal{M} = \langle D; <, a, b, c \rangle$.)
- $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa}$?
 - $\varphi \stackrel{\text{def}}{=} \neg \mathcal{M}_{aa}$?
 - $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \vee \mathcal{M}_{ab}$?
 - $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \Rightarrow \mathcal{M}_{ab}$?
 - $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \Leftrightarrow \mathcal{M}_{ab}$?
- (b) Write a constraint that says “All words with a NT sequence also have a CC sequence” (You may assume a model with features for nasals, consonants, etc.)
- (c) Prove that the constraint which says “Words have at least two *as*” is not LT.
- (d) You are born on planet Locally 2-Testable, where everyone’s DNA has a UG programmed for LT-2 constraints. You observe the word *aab*.
- Can you infer that *aaab* is a word in your language?
 - What about *ab*?

2. Piecewise Testable Constraints (PROP($<$))

- (a) For each φ , write two words whose models satisfy it. (Assume $\Sigma = \{a, b, c\}$ and $\mathcal{M} = \langle D; <, a, b, c \rangle$.)
- $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa}$?
 - $\varphi \stackrel{\text{def}}{=} \neg \mathcal{M}_{aa}$?
 - $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \vee \mathcal{M}_{ab}$?
 - $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \Rightarrow \mathcal{M}_{ab}$?
 - $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \Leftrightarrow \mathcal{M}_{ab}$?
- (b) You are born on planet Locally 2-Piecewise, where everyone’s DNA has a UG programmed for PT-2 constraints. You observe the word *aabcb*.
- Can you infer that *aaabcb* is a word in your language?
 - What about *ab*? *aabcb*? *ac*? *ca*?

3. Strictly Local Constraints (CNL(\triangleleft))

- (a) Consider a Strictly 2-Local stringset L which contains the words *aa* and *ab*. Using Suffix Substitution Closure, explain what other words must be in L .
- (b) Consider the constraint $*s \dots f$. Show this is not SL_k for any k .
- (c) You are born on planet Strictly 2-Local, where everyone’s DNA has a UG programmed for SL-2 constraints. You observe the word *aab*.
- Can you infer that *aaab* is a word in your language?
 - What about *ab*?
 - Do inhabitants of planet Locally 2-Testable generalize in the same way (see above)?

4. Strictly Piecewise Constraints ($\text{CNL}(<)$)

- (a) Consider a Strictly 2-Piecewise stringset L which contains the words aa and ab . Using subsequence closure, explain what other words must be in L .
- (b) Consider the constraint $*nt$. Show this is not SP_k for any k .
- (c) You are born on planet Strictly 2-Local, where everyone's DNA has a UG programmed for SL-2 constraints. You observe the word $aabcb$.
 - i. Can you infer that $aaabcb$ is a word in your language?
 - ii. What about ab ? $aabbc$? ac ? ca ?
 - iii. Do inhabitants of planet Piecewise 2-Testable generalize in the same way (see above)?