Vowel Harmony and Subsequentiality

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MOL @ Sofia, Bulgaria
August 9, 2013

This talk

- In this talk we propose the tightest computational characterization currently known for vowel harmony patterns, and by extension, for phonological patterns more generally.
- Specifically, we show how 'pathological' phonological patterns can be distinguished from attested ones with *sub*regular computational boundaries.

Outline

Characterizing Phonology

Subsequentiality

Harmony

Results

Discussion

The computational nature of phonological generalizations

Phonological processes can be modeled with mappings from underlying lexical representations to surface representations.

Question

- What kind of maps are these?

First answer: They are regular (Johnson 1972, Koskiennimi 1983, Kaplan and Kay 1994)

Important!

While this result was shown with SPE-style and two-level grammars, the fact remains:

The mappings *themselves* are regular regardless of the grammatical formalism used (SPE, 2-level, OT, GP)

(at least until a bonafide phonological pattern is found that is not describable with SPE or 2-level grammars)

Classifying Sets of Strings

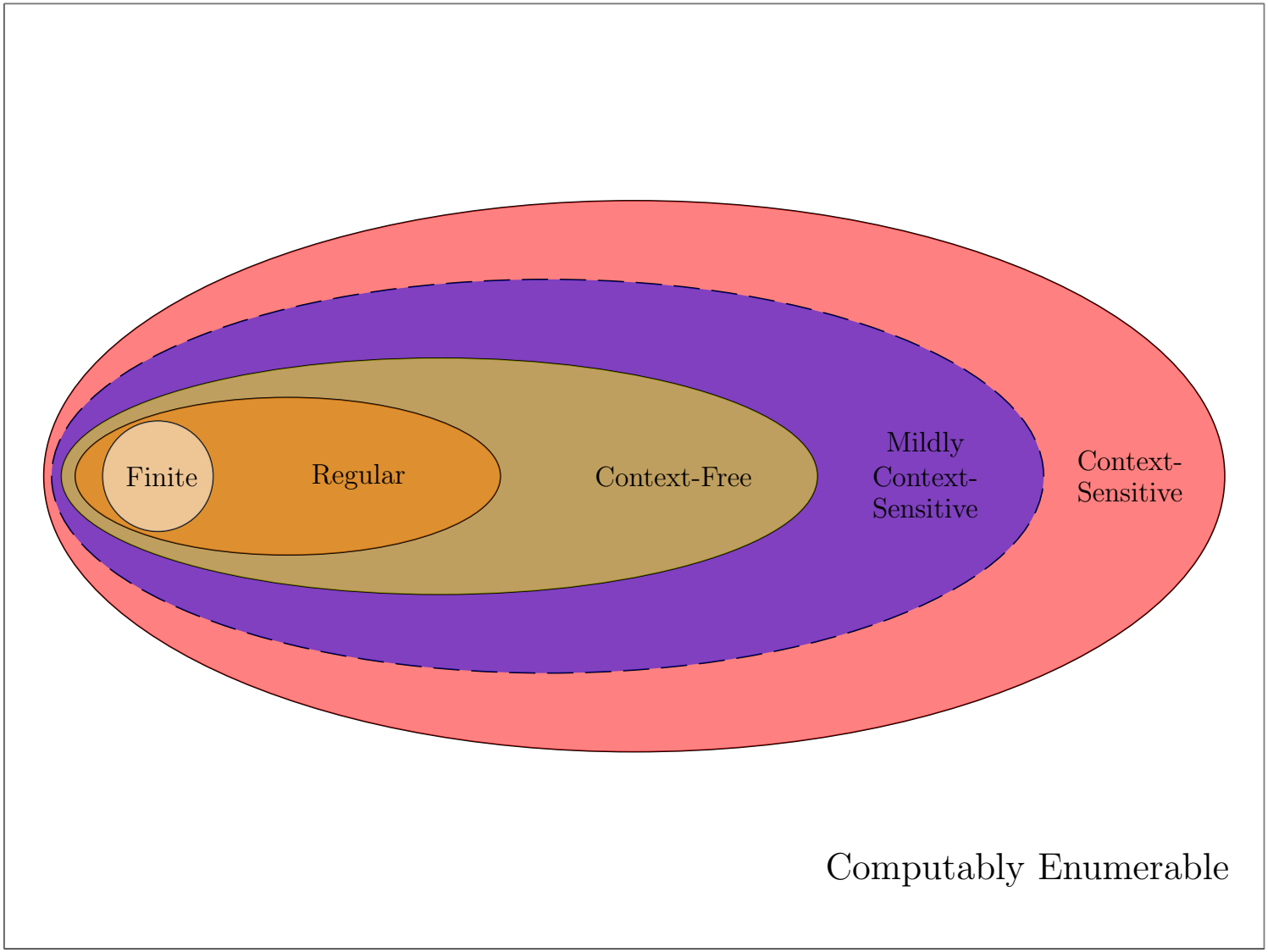


Figure: The Chomsky hierarchy

Classifying Sets of Strings

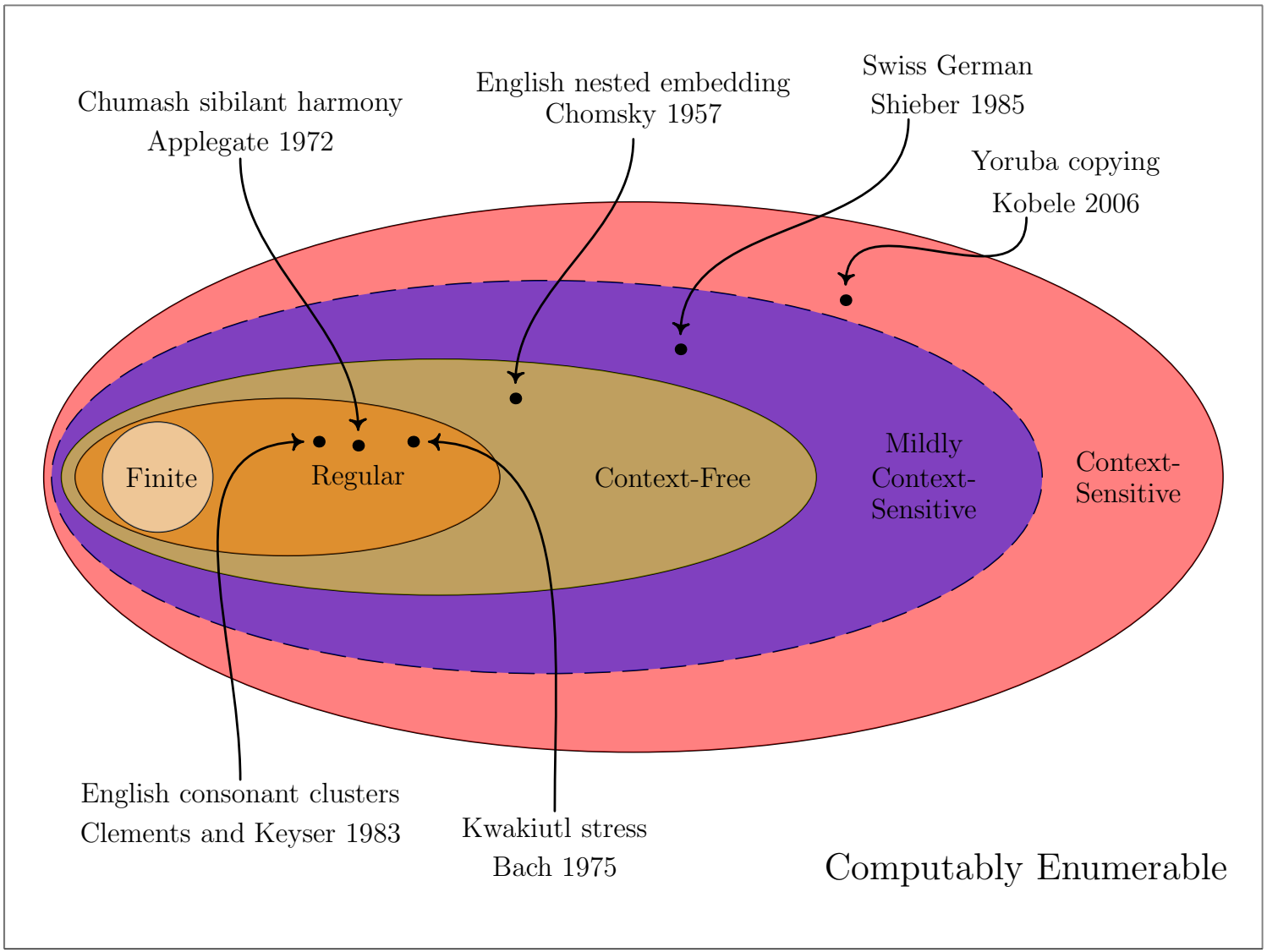
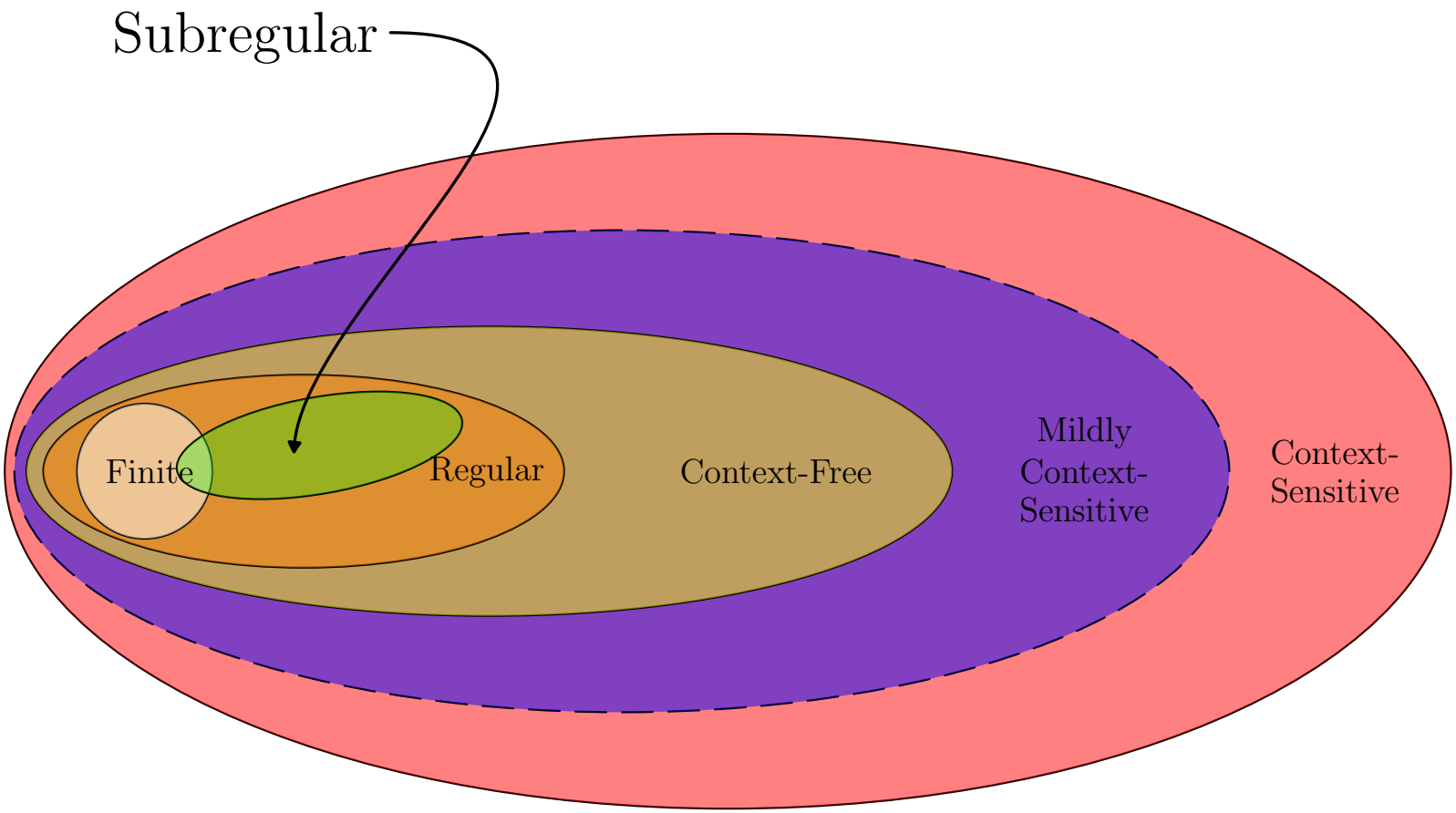


Figure: Natural language patterns in the hierarchy.

Second Answer: They are *sub*regular.



Why do we want stronger characterizations?

Better characterizations of phonological patterns

- Leads to stronger universals
- Leads to new hypotheses regarding what a *humanly* possible phonological pattern is, which is in principle testable with artificial language learning experiments (Lai 2012, Jäger and Rogers 2012)

Why do we want stronger characterizations?

Payoffs for better understanding *learning*

- These computational properties can help solve the learning problem (Heinz 2009, 2010).

Why do we want stronger characterizations?

Payoffs for natural language processing

- Insights can be incorporated into NLP algorithms
- Factoring and composition may occur with lower complexity

Overview of Results

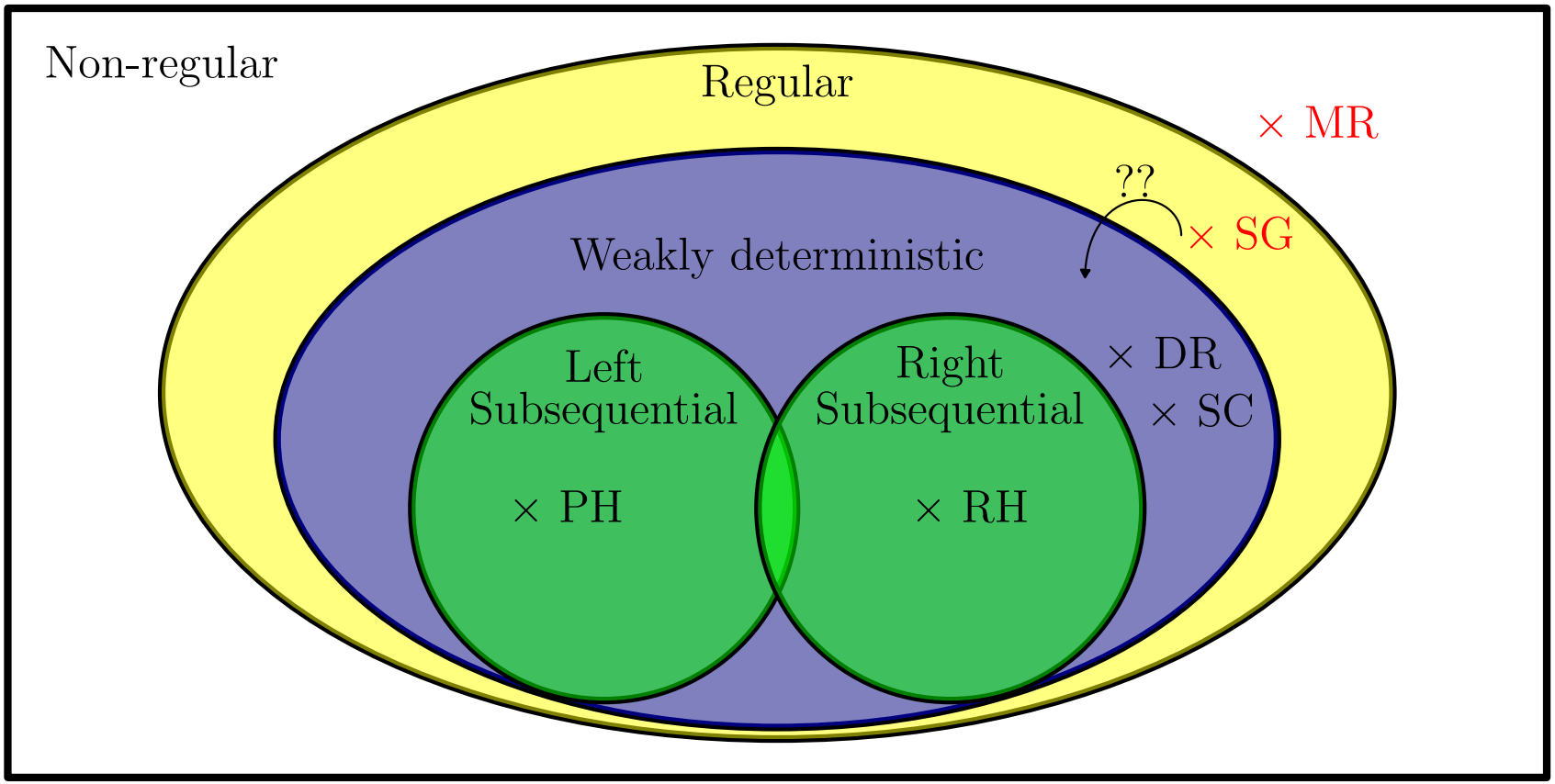


Figure: Hierarchies of transductions with the results of this paper shown. PH=progressive harmony, RH=regressive harmony, DR=dominant/recessive harmony, SC=stem control harmony, SG=sour grapes harmony, and MR=majority rules harmony.

Related Work

It has been shown that the following are left or right subsequential.

- Nevins' 2010 actual vowel harmony analyses in his VH typology (Gainor et al. 2012)
- synchronically attested metathesis patterns in Beth Hume's database, including long-distance ones, (Chandlee et al. 2012, Chandlee and Heinz 2012)
- the typology of partial reduplication in Riggle (2006) (Chandlee and Heinz 2012)
- All local phonological patterns whose trigger and target fall within a span of length k (Chandlee, in progress)
- long distance consonantal harmony and disharmony (Luo 2013 MS, Payne 2013 MS)

The only robust exception seems to be unbounded tone plateauing (Jardine 2013, MS)—but this only establishes Yip's (2002) and Hyman's (2011) point that tone is different from segmental phonology.

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Ostensive definition of 'subsequential'

Informally, subsequential transducers are *weighted* acceptors that are deterministic on the input, and where the weights are strings and multiplication is concatenation.

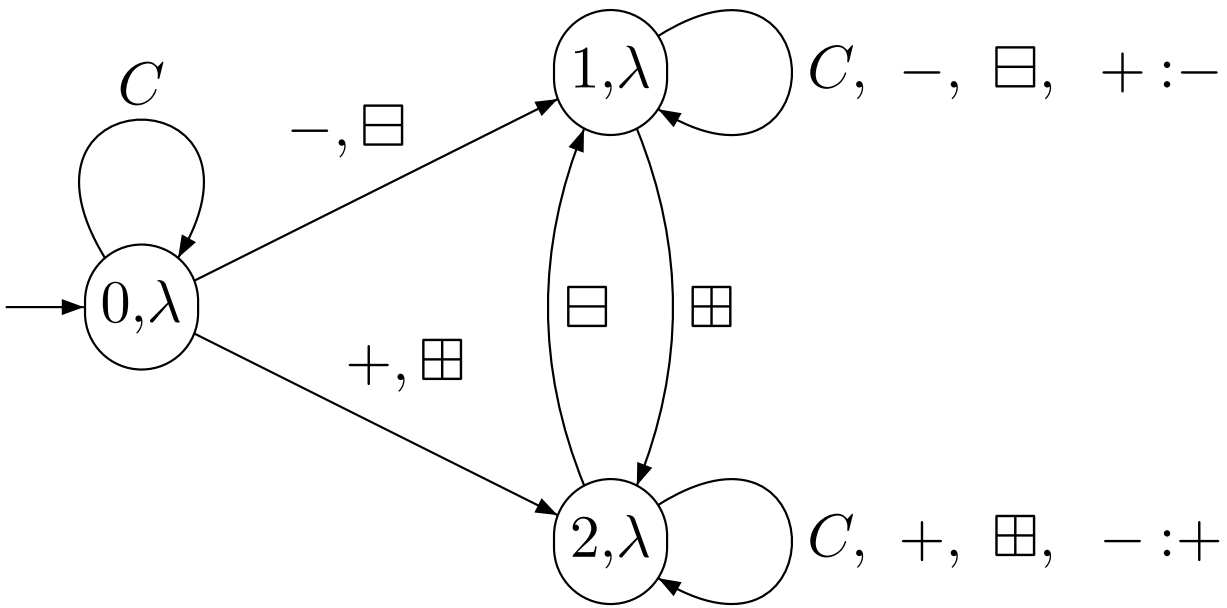
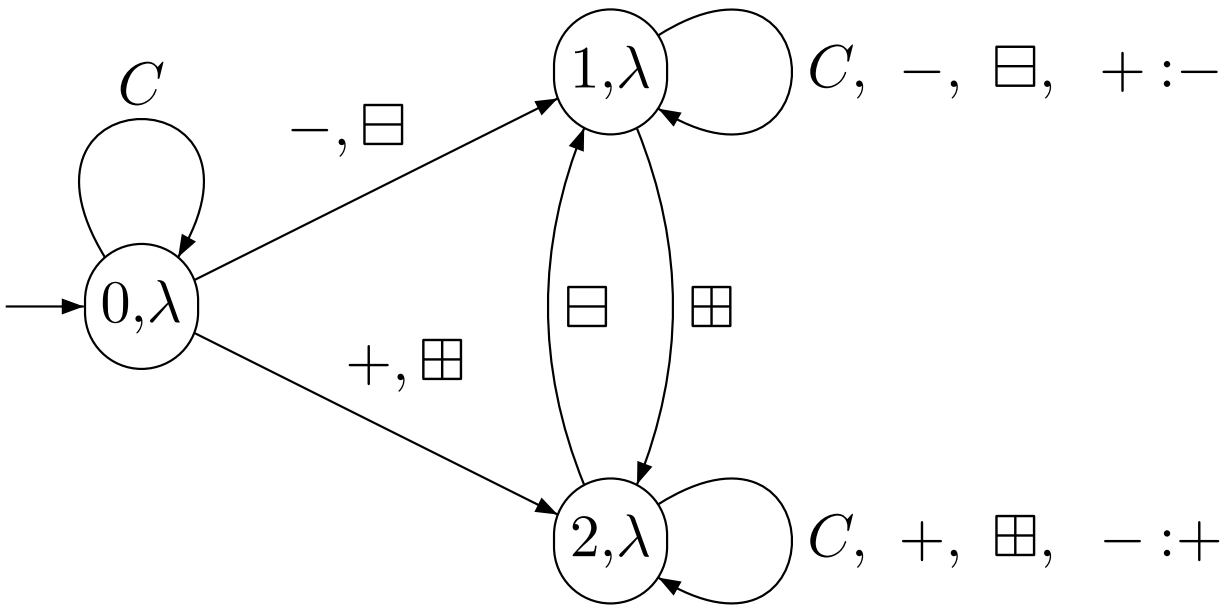


Figure: A subsequential transducer which recognizes iterative, progressive harmony.

(Schützenberger 1977, Mohri 1997, Roche and Schabes 1999)

Ostensive definition of 'subsequential'

input		+		-		-	-
state	0	→	2	→	2	→	2
output							



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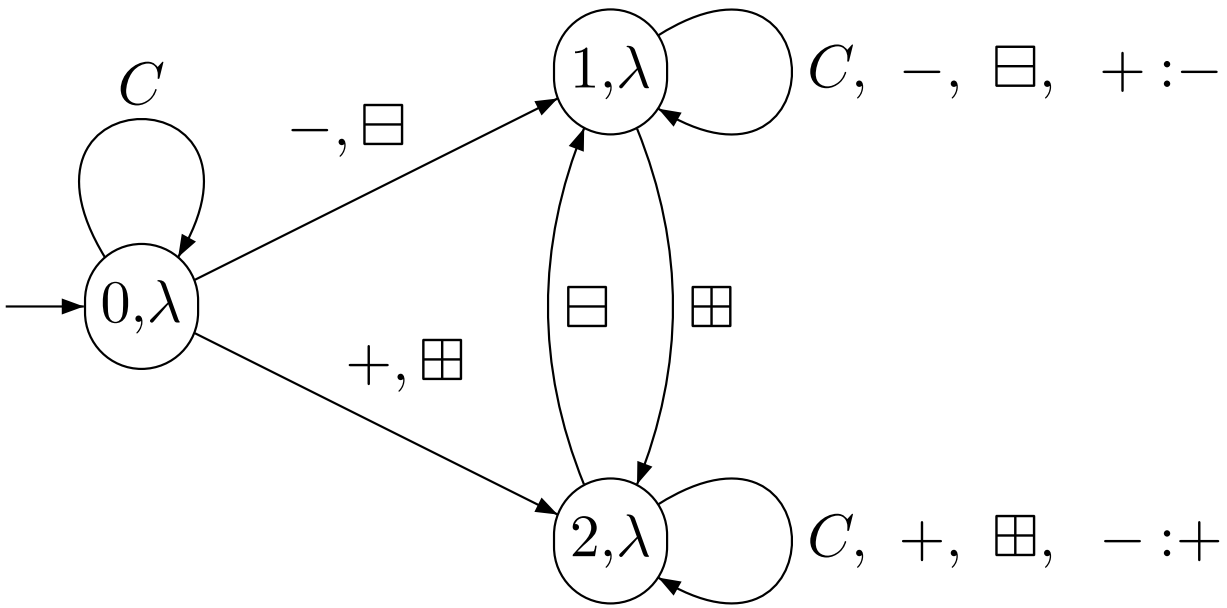
output		+						



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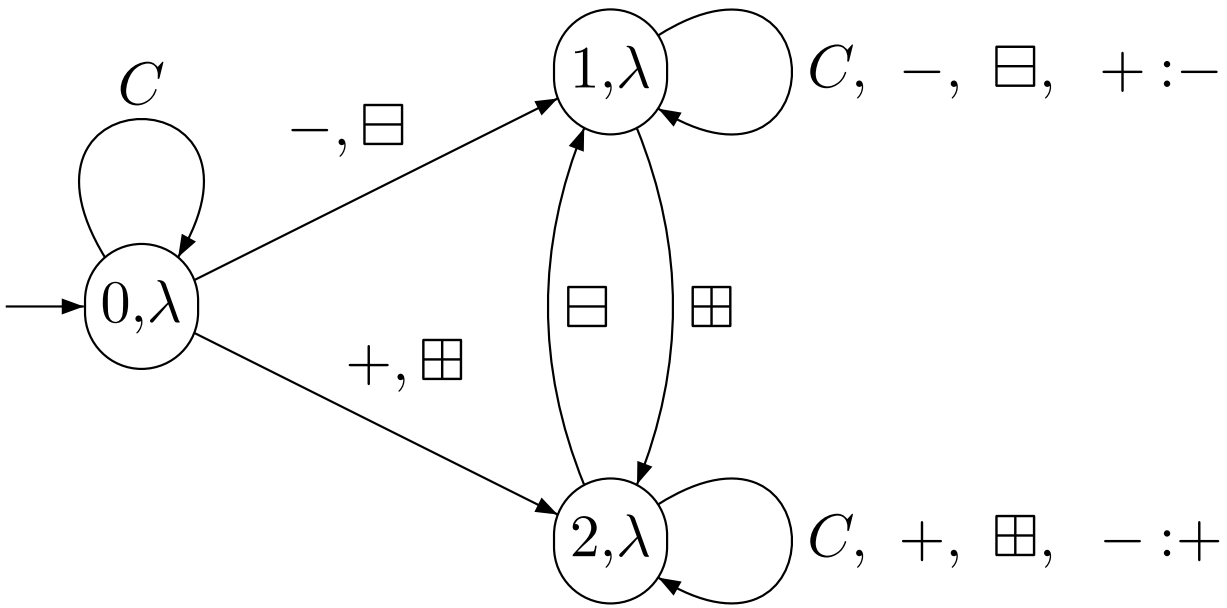
output		+		+				



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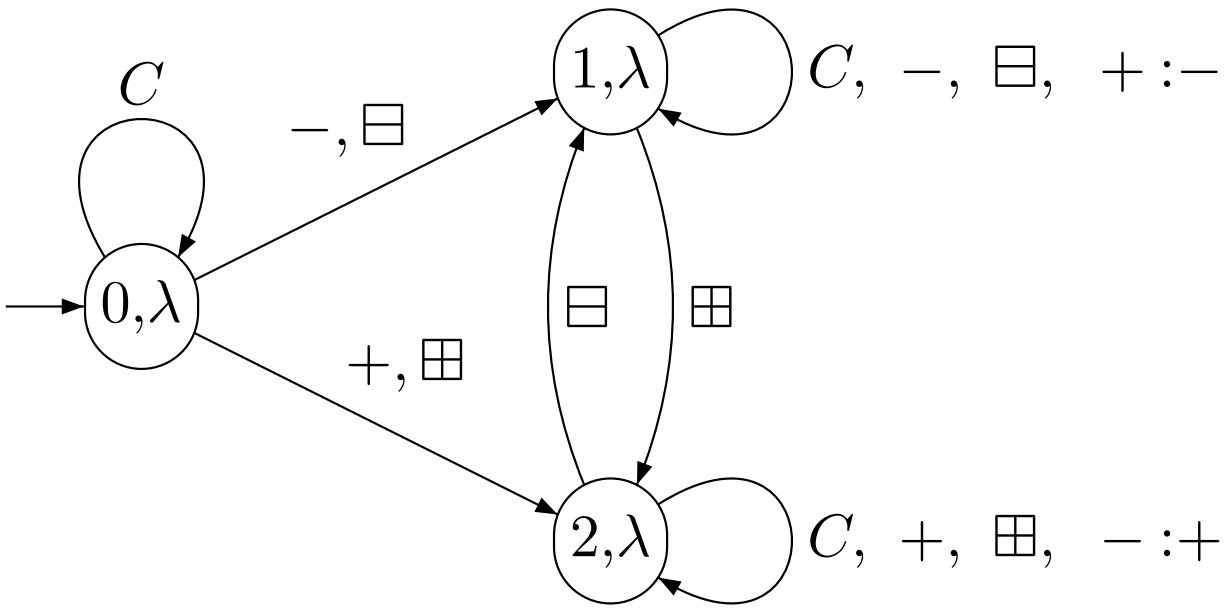
state	0	→	2	→	2	→	2
output		+		+		+	



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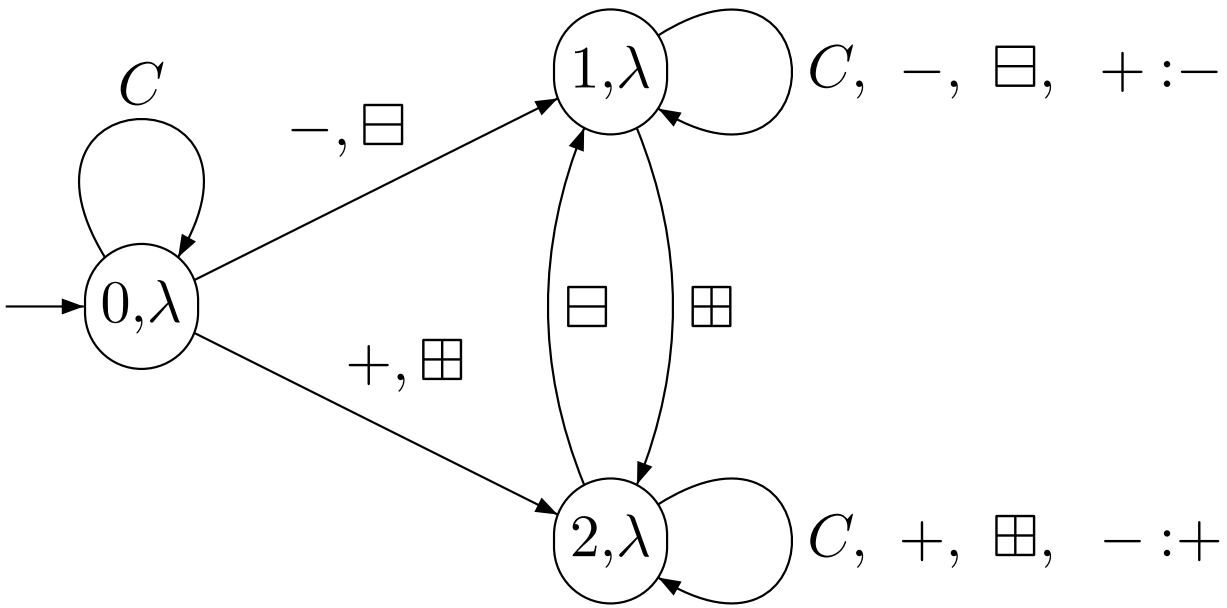

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Ostensive definition of 'subsequential'

input		+		-		-		-	
state	0	→	2	→	2	→	2		
output		+		+		+		+	λ

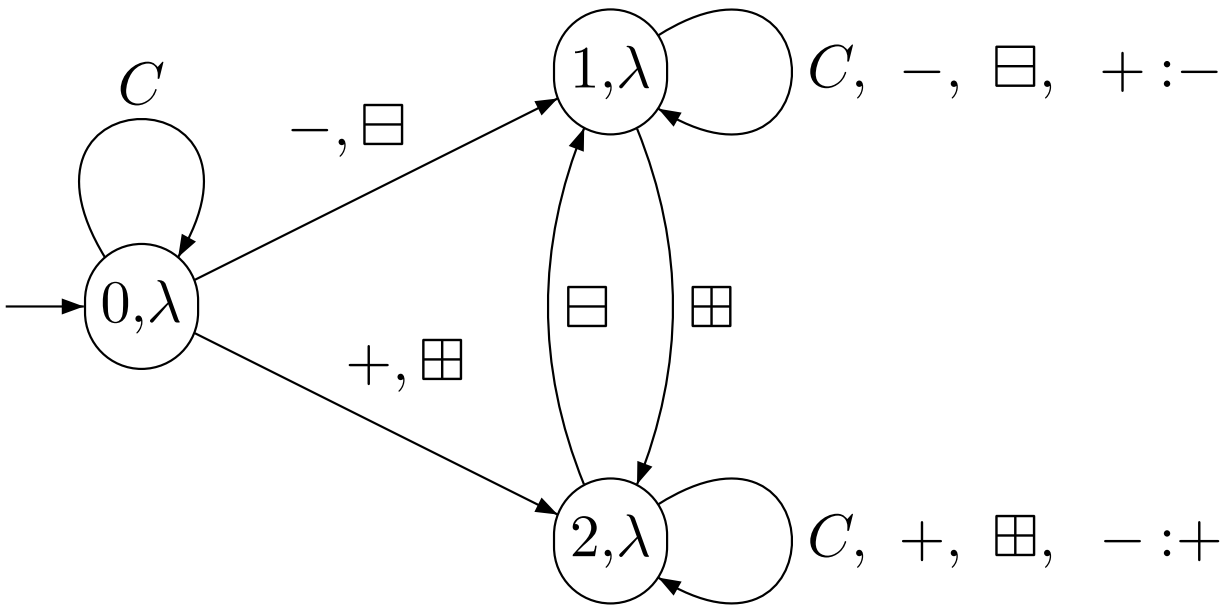

Figure: A subsequential transducer which recognizes iterative, progressive harmony.

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Left and Right subsequential

Definition (Left subsequential)

The class of functions recognized by subsequential transducers are called *left subsequential*. Denote this class LSF.

Definition (Right subsequential)

The reverse of f is $f^r = \{x^r, y^r) \mid (x, y) \in f\}$. A function f is *right subsequential* iff f^r is left subsequential. Denote this class RSF.

Notation

For any right subsequential function f, there exists a subsequential transducer T which recognizes f reading and writing the input and output string *from right to left*.

Lemma

Let f^r be right subsequential. Then there exists T recognizing f such that

$$(\forall x \in X^*)[f^r(x) = T(x^r)^r]. \tag{1}$$

- If T reads and writes left-to-right then we write $\overrightarrow{T}$.
- If T reads and writes right-to-left then we write $\overleftarrow{T}$.

Some facts

Theorem (Mohri 1997)

The following hold:

1. $LSF, RSF \subsetneq RR$
   (RR denotes the class of regular relations).
2. $RSF^r = LSF$.
3. *LSF and RSF are incomparable.*

Canonical forms and learnability

Left subsequential functions have canonical transducers determined by sets of "good tails."

$$TL_f(x) = \{(y, v) \mid f(xy) = uv \wedge u = lcp(f(xX^*))\} . \tag{2}$$

Theorem (Oncina et al. 1993)

$f \in LSF \Leftrightarrow \{TL_f(x) \mid x \in X^*\}$ has finite cardinality.

Theorem (Oncina et al. 1993)

Left subsequential functions are identifiable in the limit from positive data.

Weak Determinism

Theorem (Elgot and Mezei 1965)

Let $T : X^* \to Y^*$ be a function. Then $T \in RR$ iff there exists $L : X^* \to Z^* \in LSF$, and $R : Z^* \to Y^* \in RSF$ with $X \subseteq Z$ such that $T = R \circ L$.

Definition

A regular function T is *weakly deterministic* iff there exists $L : X^* \to X^* \in LSF$, and $R : X^* \to X^* \in RSF$ such that L is not length-increasing and $T = R \circ L$. The class of weakly deterministic functions is denoted WD.

Why alphabet-preservation alone isn't sufficient

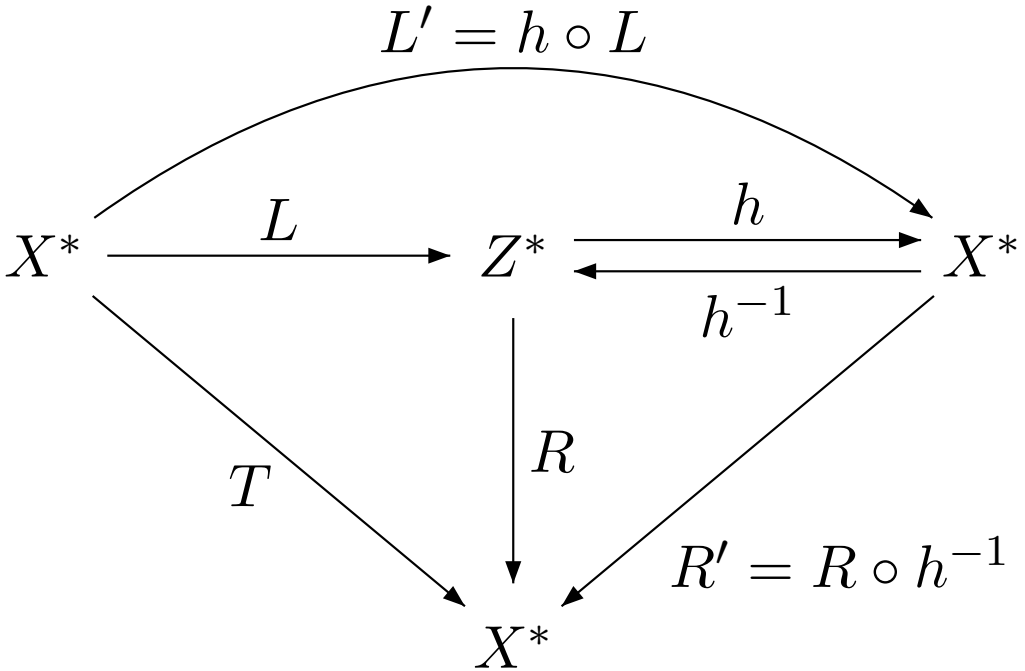

Figure: Decompositions of regular function T with $X \subseteq Z$.

Corollary to Elgot and Mezei 1965

Corollary

$LSF, RSF \subseteq WD \subseteq RR.$

Vowel harmony patterns analyzed will be witnesses separating LSF, RSF from WD and will suggest a separation for WD and RR.

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Ostensive example of Vowel Harmony

	noun	genitive	gloss
a.	ip	ip-in	*rope*
b.	el	el-in	*and*
c.	son	son-un	*end*
d.	pul	pul-un	*stamp*

Table: Examples illustrating a fragment of the Vowel harmony from Turkish (Nevins 2010:32).

Example phonological analysis

(a)

w	$f(w)$
/ip-un/	[ip-in]
/el-un/	[el-in]
/son-un/	[son-un]
/pul-un/	[pul-un]
...	

(b)

w	$f(w)$
/−C+C/	[−C−C]
/C+C+C/	[C+C+C]
...	

Table: Examples showing fragments of the phonological function describing Turkish back harmony assuming the underlying genitive morpheme is /-un/.

Illustrating the mappings examined here

	w	PH(w)	RH(w)	DR(w)	SG(w)	MR(w)
a.	/+ − −/	[+ + +]	[− − −]	[+ + +]	[+ + +]	[− − −]
b.	/− + +/	[− − −]	[+ + +]	[+ + +]	[− − −]	[+ + +]
c.	/− − −/	[− − −]	[− − −]	[− − −]	[− − −]	[− − −]
d.	/− + −/	[− − −]	[− − −]	[+ + +]	[− − −]	[− − −]
e.	/+ − ⊟/	[+ + ⊟]	[− − ⊟]	[+ + ⊟]	[+ − ⊟]	[− − ⊟]
f.	/+ ⊖ −/	[+ ⊖ +]	[− ⊖ −]	[+ ⊖ +]	[+ ⊖ +]	[− ⊖ −]

Table: Example mappings of underlying forms (w) given by progressive harmony (PH), regressive harmony (RH), dominant/recessive harmony (DR), and 'pathological' sour grapes harmony (SG), and majority rules harmony (MR). Symbols [+] indicates a [+F] vowel and [−] indicates a [−F] vowel where "F" is the feature harmonizing. Symbols [⊟] and [⊖] are [−F] vowels that are opaque and transparent, respectively.

Coverage of these examples.

- Nevins (2010) provides a typological survey of dozens of VH patterns and concludes they all can be analyzed as progressive or regressive harmony with underspecification. Gainor et al. (2012) confirm his analyses are left or right subsequential mappings.
- Other linguists prefer analyzing VH patterns as progressive or regressive harmony *without* underspecification.
- Others still prefer to analyze VH in terms of dominant/recessive or stem control.
- Only one non-subsequential VH example has come to our attention (which is NOT included in Nevins 2010.) This is Yaka (Hyman 1998).

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Note

Two additional properties of these harmony patterns:

1. They are same-length
2. They are *sequential*: They can be described by a subsequential transducer whose output function for every state is λ

While both of these (independent) properties are strong; they are not true of other phonological processes such as epenthesis, and deletion (and even substitution) and therefore we make no use of these properties here.

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Progressive and Regressive Harmony

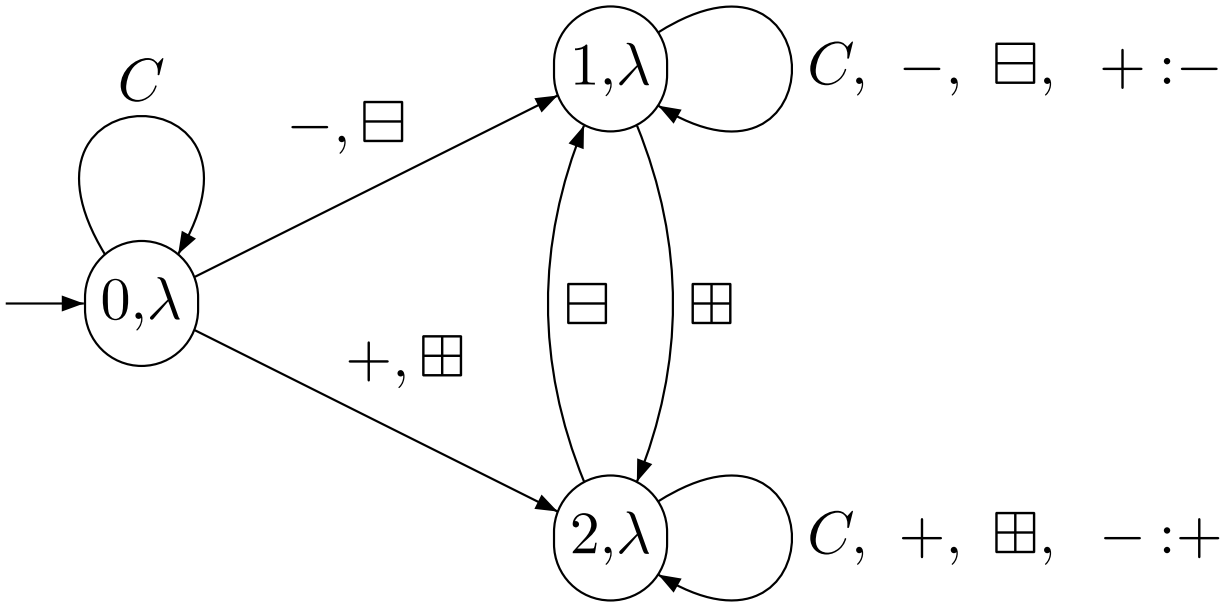


Figure: A subsequential transducer T. $\overrightarrow{T}$ recognizes iterative, progressive harmony and $\overleftarrow{T}$ recognizes iterative, regressive harmony.

Theorem

PH is left subsequential and RH is right subsequential.

Illustrative Example

$$\text{PH} = \overrightarrow{T}$$

input		+		-		-		-	
state	0	→	2	→	2	→	2		
output		+		+		+		+	λ

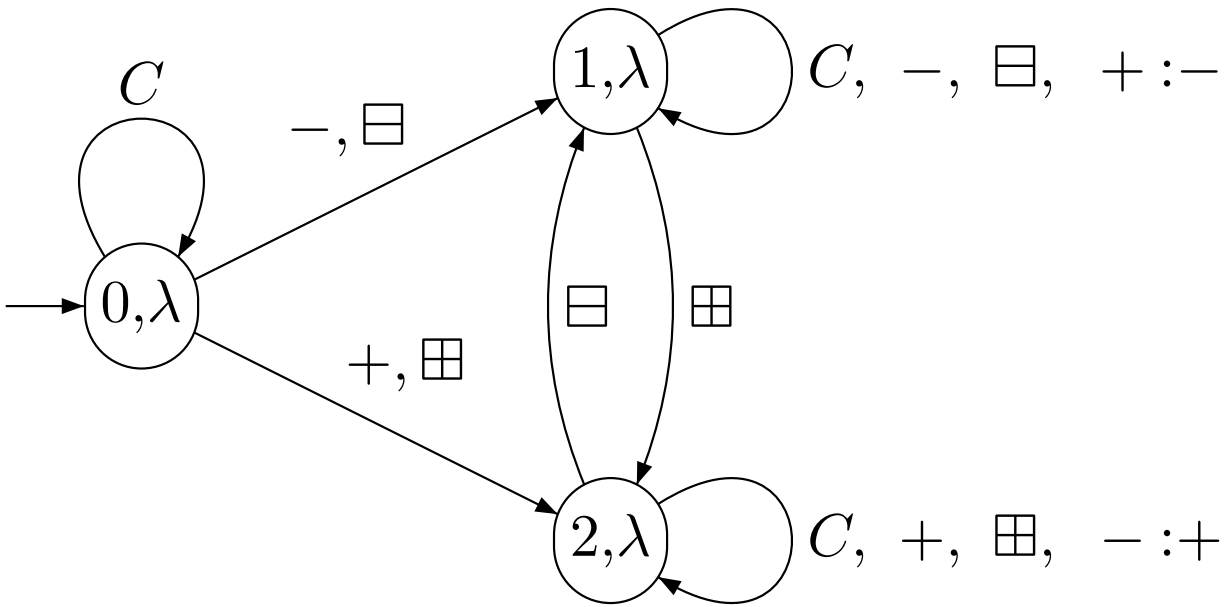
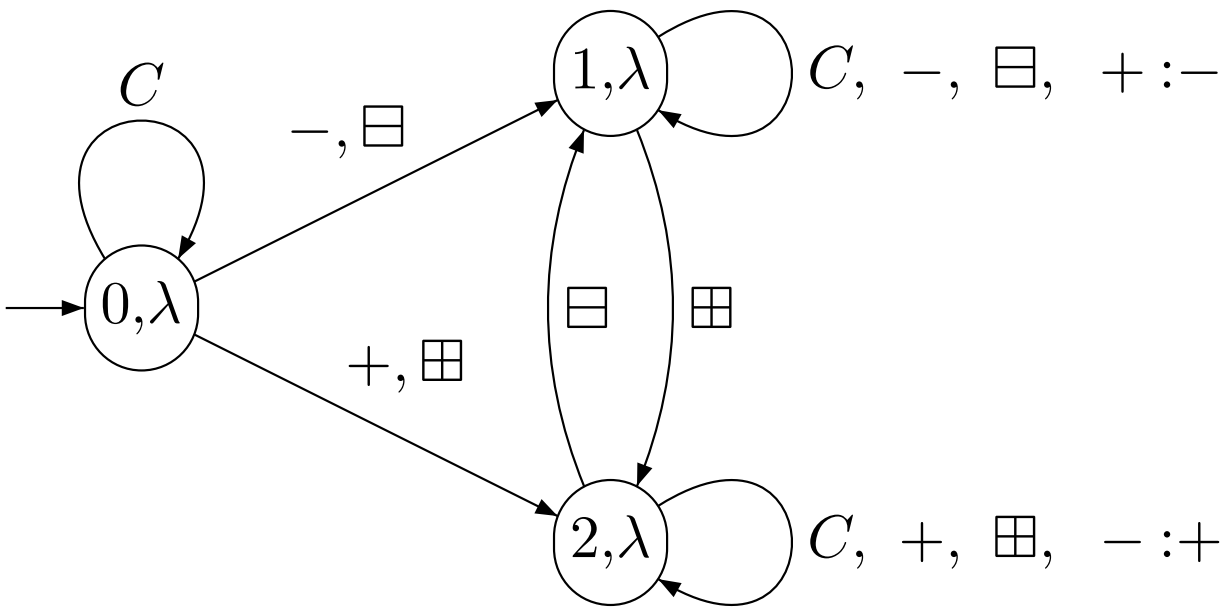


Figure: A subsequential transducer T.

Illustrative Example

$$\text{RH} = \overleftarrow{T} \Leftrightarrow \forall x \in X^*[\overleftarrow{T} = (\overrightarrow{T}(x^r))^r]$$

$$\text{input } x \quad -\boxminus--+$$



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Illustrative Example

$$\text{RH} = \overleftarrow{T} \Leftrightarrow \forall x \in X^*[\overleftarrow{T} = (\overrightarrow{T}(x^r))^r]$$

$$\begin{array}{ll} \text{input } x & - \boxminus - - + \\ x^r & + - - \boxminus - \end{array}$$

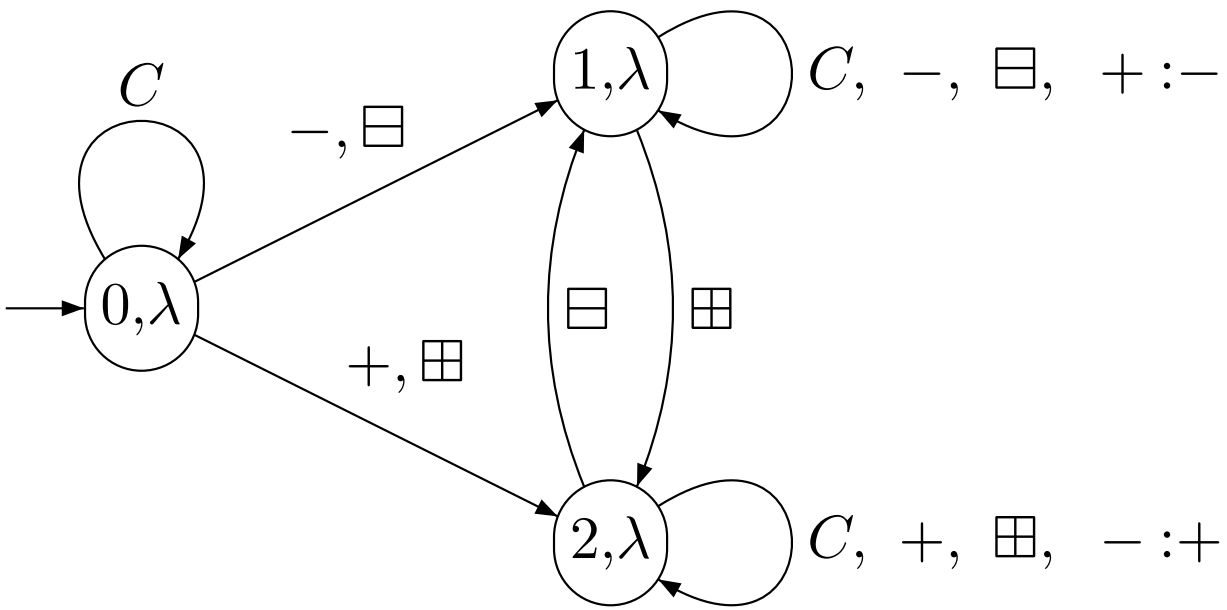

Figure: A subsequential transducer T.

Illustrative Example

$$\text{RH} = \overleftarrow{T} \Leftrightarrow \forall x \in X^*[\overleftarrow{T} = (\overrightarrow{T}(x^r))^r]$$

input x	$-\boxminus--+$
x^r	$+--\boxminus-$
$\overrightarrow{T}(x^r)$	$+++\boxminus-$

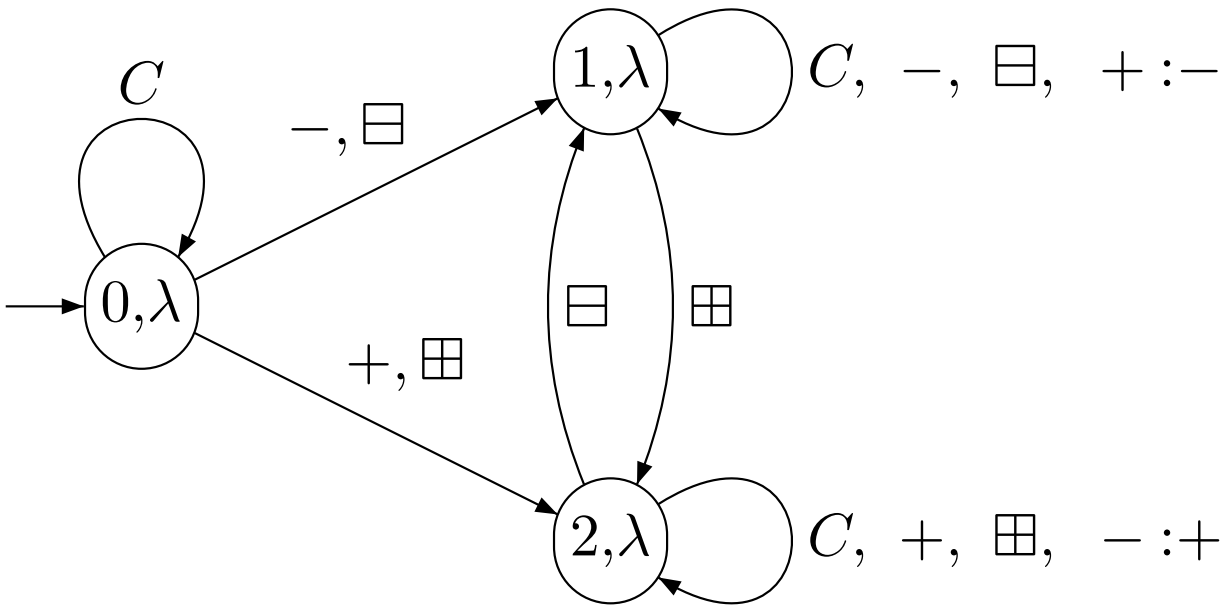

Figure: A subsequential transducer T.

Illustrative Example

$$\text{RH} = \overleftarrow{T} \Leftrightarrow \forall x \in X^*[\overleftarrow{T} = (\overrightarrow{T}(x^r))^r]$$

input x	$-\boxminus--+$
x^r	$+--\boxminus-$
$\overrightarrow{T}(x^r)$	$+++\boxminus-$
$(\overrightarrow{T}(x^r))^r$	$-\boxminus+++$

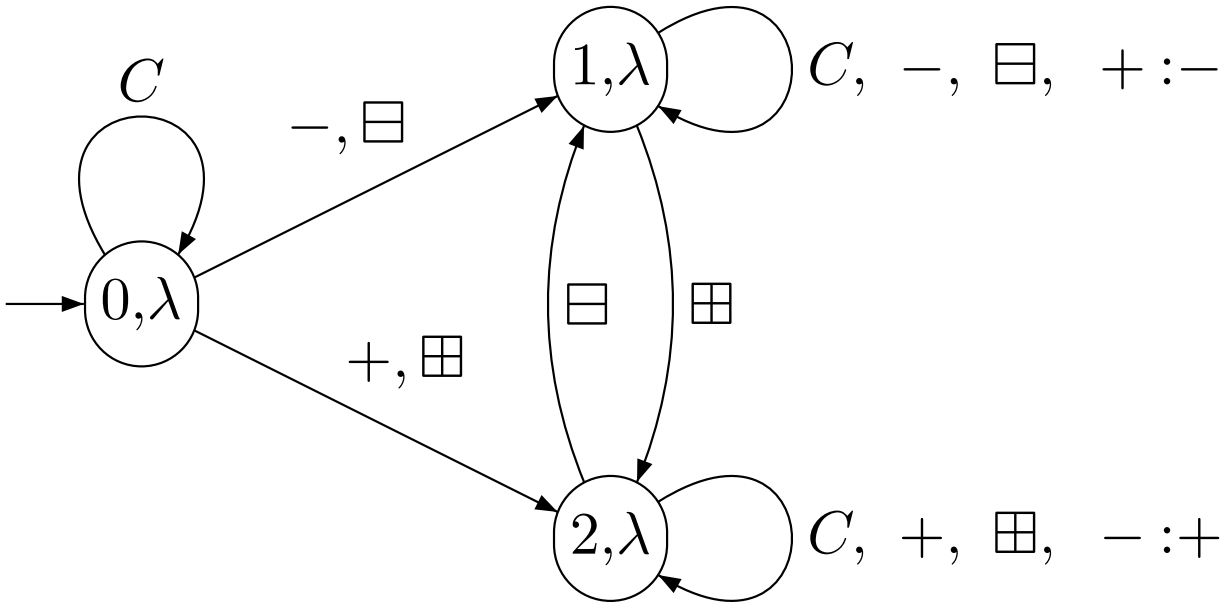

Figure: A subsequential transducer T.

The regular boundary

$$MR(w) = \begin{cases} +^{|w|} \text{ if } |w|_{+F} > |w|_{-F} \\ -^{|w|} \text{ if } |w|_{-F} > |w|_{+F} \end{cases} \tag{3}$$

Theorem

Majority Rules is not regular.

Proof.

We show that the intersection of a regular set with the image of the inverse of MR is not regular. □

This seems to be widely known. For example, see Riggle (2004).

The subsequential boundary

$$SG(+-^n) = + +^n \wedge SG(+ -^n \boxminus) = + -^n \boxminus \tag{4}$$

Theorem

Sour Grapes is regular but neither left nor right subsequential.

Proof (sketch).

The proof shows that for all distinct $n, m \in \mathbb{N}$ the tails of $+^n$ is not the same as the tails of $+-^m$, which implies that the canonical left subsequential transducer would have infinitely many states. □

A nondeterministic transducer for SG

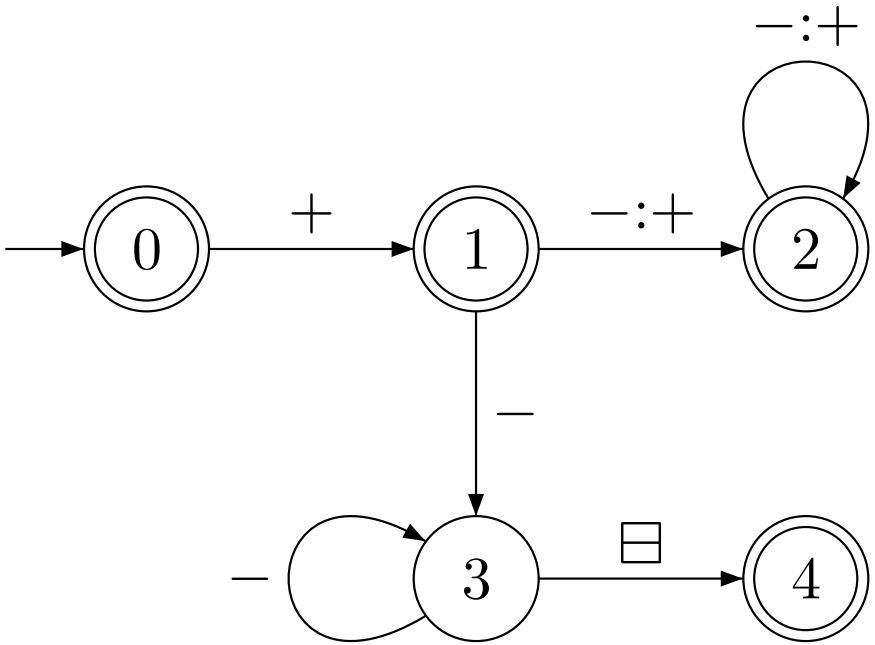

Figure: A non-deterministic transducer which recognizes this important fragment of SG harmony.

The weakly deterministic boundary

$$\forall w \in \{+,-\}^*,$$
$$DR(w) = \begin{cases} +^{|w|} & \text{if } (\exists\, 0 \leq i \leq |w|)[w_i = +] \\ -^{|w|} & \text{otherwise} \end{cases} \tag{5}$$

Theorem

DR harmony is neither left nor right subsequential.

Proof (sketch).

As with SG, one can show for all distinct $n, m \in \mathbb{N}$ the tails of ^{n} is not the same as the tails of $-^m$, which implies infinitely many states. □

DR is weakly deterministic

Theorem

DR harmony is weakly deterministic.

Proof.

We show that for all $w \in \{+, -\}^*$ it is the case that $DR(w) = \overleftarrow{T_{PHP}} \circ \overrightarrow{T_{PHP}}(w)$. □

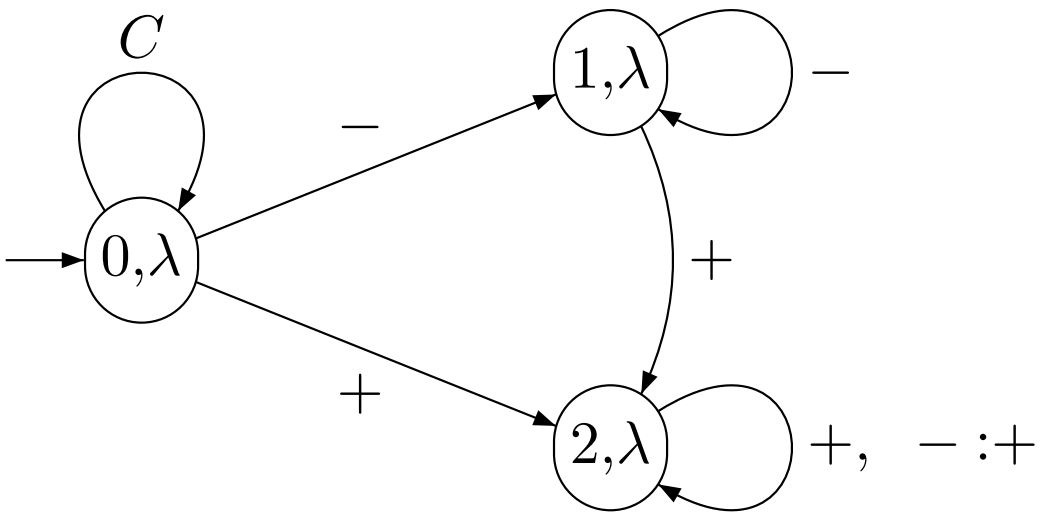

Figure: The subsequential transducer T_{PHP} which recognizes iterative, progressive harmony where only the + value spreads.

Contrasting DR with SG

Derivation of $DR(- - - + - - -)$

input $- - - + - - -$

Derivations of $SG(+ - - - -\boxminus)$ and $SG(+ - - - -)$

input $+ - - - -\boxminus$ $+ - - - -$

Contrasting DR with SG

Derivation of $DR(- - - + - - -)$

input $- - - + - - -$

$\xrightarrow[T_{PHP}]{}$ $- - - + + + +$

Derivations of $SG(+ - - - - \boxminus)$ and $SG(+ - - - -)$

input $+ - - - - \boxminus$ $+ - - - -$

Contrasting DR with SG

Derivation of $DR(- - - + - - -)$

input	$- - - + - - -$
$\xrightarrow{T_{PHP}}$	$- - - + + + +$
$\xleftarrow{T_{PHP}}$	$+ + + + + + +$

Derivations of $SG(+ - - - -\boxminus)$ and $SG(+ - - - -)$

input	$+ - - - -\boxminus$	$+ - - - -$

Contrasting DR with SG

Derivation of $DR(- - - + - - -)$

input	$- - - + - - -$
$\overrightarrow{T_{PHP}}$	$- - - + + + +$
$\overleftarrow{T_{PHP}}$	$+ + + + + + +$

Derivations of $SG(+ - - - -\boxminus)$ and $SG(+ - - - -)$

input	$+ - - - -\boxminus$	$+ - - - -$
$\overrightarrow{T}$	$+ \underline{?}\ \underline{?}\ \underline{?}\ \underline{?} \boxminus$	$+ \underline{?}\ \underline{?}\ \underline{?}\ \underline{?}$

Contrasting DR with SG

Derivation of $DR(- - - + - - -)$

input	$- - - + - - -$
$\overrightarrow{T_{PHP}}$	$- - - + + + +$
$\overleftarrow{T_{PHP}}$	$+ + + + + + +$

Derivations of $SG(+ - - - -\boxminus)$ and $SG(+ - - - -)$

input	$+ - - - -\boxminus$	$+ - - - -$
$\overrightarrow{T}$	$+ \underline{?}\ \underline{?}\ \underline{?}\ \underline{?} \boxminus$	$+ \underline{?}\ \underline{?}\ \underline{?}\ \underline{?}$
$\overleftarrow{T'}$	$+ - - - -\boxminus$	$+ + + + +$

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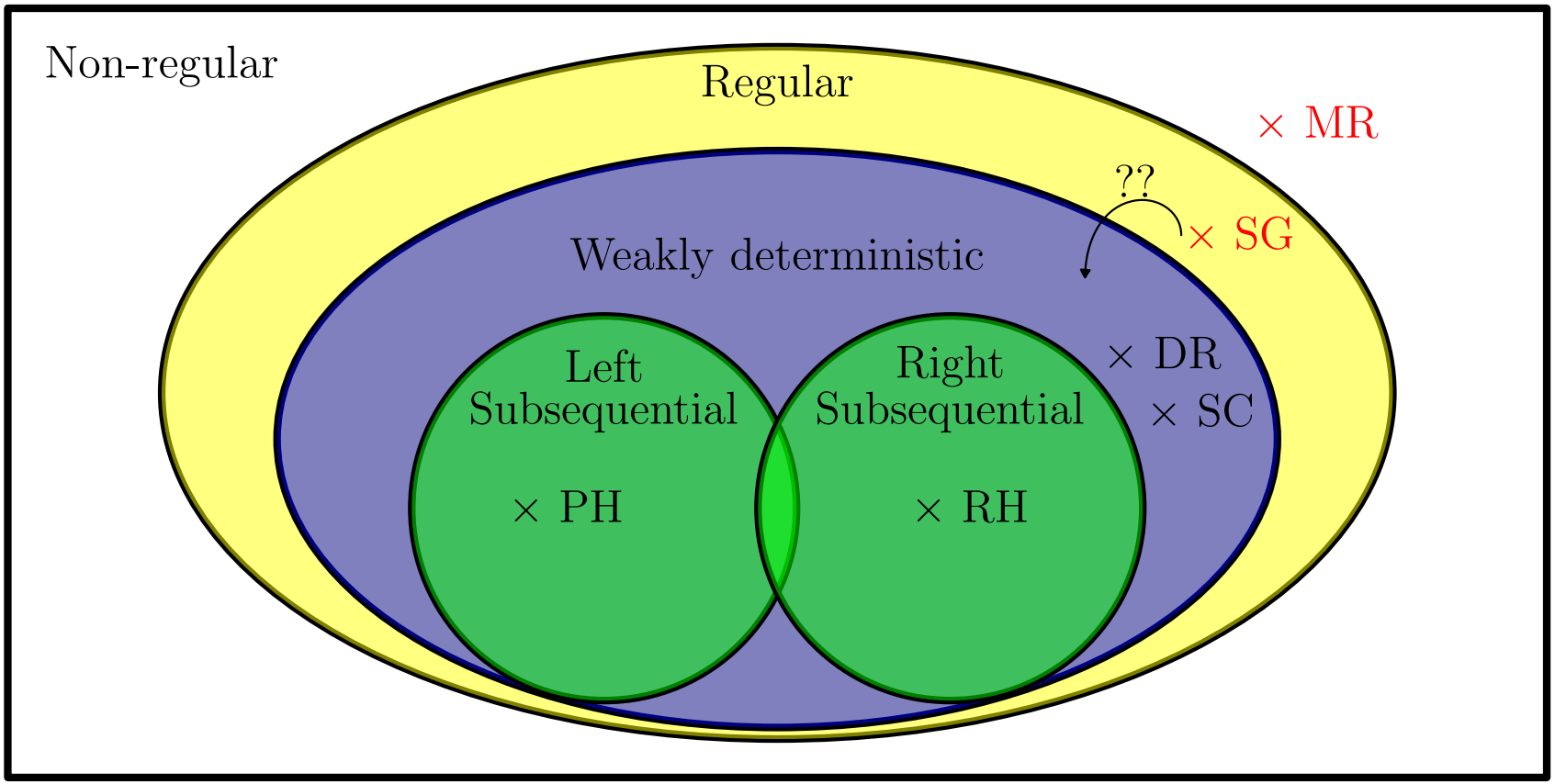
Review of Results



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Open Questions

- Is Sour Grapes weakly determinstic or not?
- Is there a hierarchy of 'weak determinism' depending on how much mark up is allowed?
- Is Yaka truly non-subsequential? Hyman's analysis of Yaka is to to somewhat controversial and appears to be unique (Hyman, p.c.). It merits further study.

Conclusion

1. It is possible to characterize nearly all known analyses of vowel harmony as weakly deterministic; the analyses of some linguists would characterize them as either left or right subsequential.
2. If correct, this is significant computational constraint on what a possible vowel harmony pattern is, and we would expect payoffs in many related areas including
   - learning
   - NLP
   - psycholinguistic research

Almost time for lunch - Thank you!

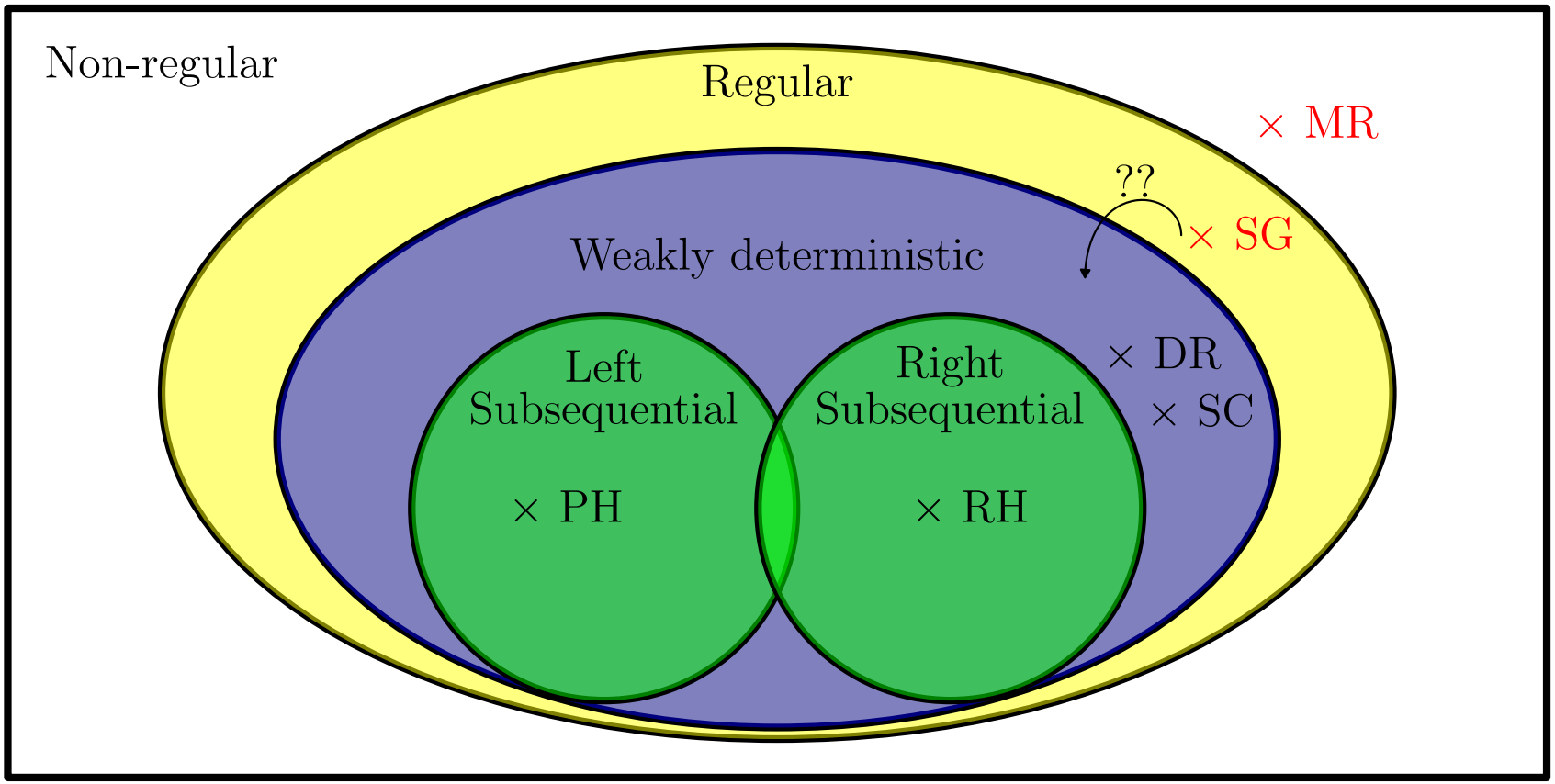

Figure: Hierarchies of transductions with the results of this paper shown. PH=progressive harmony, RH=regressive harmony, DR=dominant/recessive harmony, SC=stem control harmony, SG=sour grapes harmony, and MR=majority rules harmony.