The Computational Nature of Phonological Generalizations

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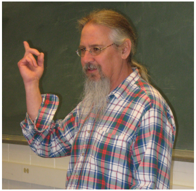
In this talk. . .

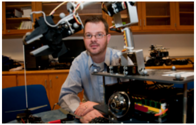
1. Explain why computational characterizations of language patterns matter.
2. Explain the subregular computational classes that phonological generalizations appear to belong to.
3. Provide some pyscholinguistic evidence that the boundaries of these computational classes are psychologically real.

Collaborators

- Prof. Jim Rogers (Earlham College)
- Prof. Herbert G. Tanner (UD)
- Prof. Bill Idsardi (UMCP)
- Dr. Regine YeeKing Lai, PhD 2012
- Cesar Koirala (PhD exp. 2013)
- Jane Chandlee (PhD exp. 2014)
- Adam Jardine (PhD exp. 2016)
- Amanda Payne (PhD exp. 2016)
- Huan Luo (PhD exp. 2017)
- Brian Gainor (LDC)



Jim



Bert

Regine

Cesar

Adam

Amanda

Unpictured
Bill, Jane, Huan, Brian

What are phonological generalizations?

1. Phonotactics
2. Phonological processes; i.e. *mappings*
3. Contrast (not part of this talk)

Phonotactics - Knowledge of word well-formedness

ptak thole hlad plast sram mgla vlas flitch dnom rtut

Halle, M. 1978. In *Linguistic Theory and Pyschological Reality*. MIT Press.

Phonotactics - Knowledge of word well-formedness

possible English words	impossible English words
thole	ptak
plast	hlad
flitch	sram
	mgla
	vlas
	dnom
	rtut

Question

How do English speakers know which of these words belong to different columns?

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This knowledge can be modeled as a stringset

Example

All possible English words are in the set; all impossible words are out of the set.

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$$mgl$$

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$$mgl \cdot \Sigma^*$$

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$$\overline{mgl \cdot \Sigma^*} \cap \overline{pt \cdot \Sigma^*} \cap \ldots$$

This knowledge can be modeled as a stringset

Example

A phonotactic constraint in Yawelmani Yokuts prohibits sequences of three consonants (*CCC).

All logically possible strings with no CCC sequence are in the set; all others with at least one CCC sequence are out of the set.

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Example

A phonotactic constraint in Yawelmani Yokuts prohibits sequences of three consonants (*CCC).

All logically possible strings with no CCC sequence are in the set; all others with at least one CCC sequence are out of the set.

$$\overline{\Sigma^* \cdot CCC \cdot \Sigma^*}$$

This knowledge can be modeled as a stringset

Example

Any markedness constraint in Optimality Theory.

All surface forms with zero violations are in the set; all surface forms with nonzero violations are out of the set.

Classifying Sets of Strings

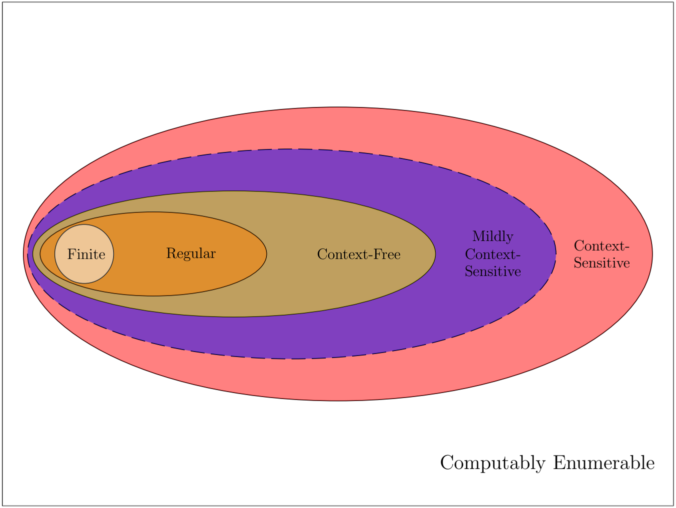
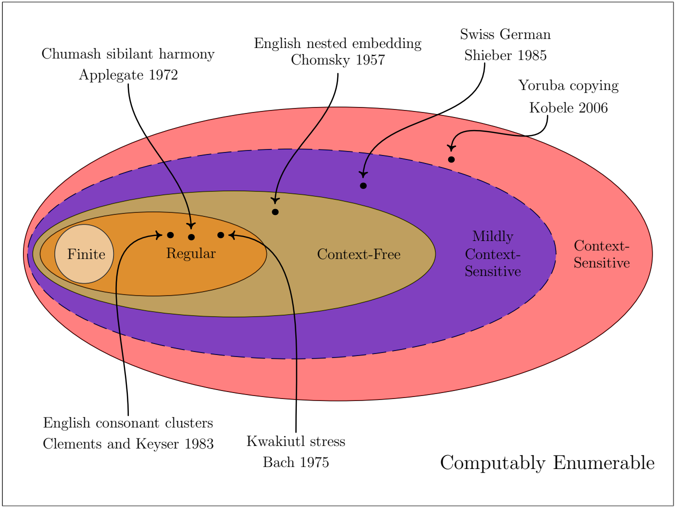


Figure: The Chomsky hierarchy

computably enumerable
|
context-sensitive
|
mildly context-sensitive
|
context-free
|
regular
|
finite

Classifying Sets of Strings



computably enumerable
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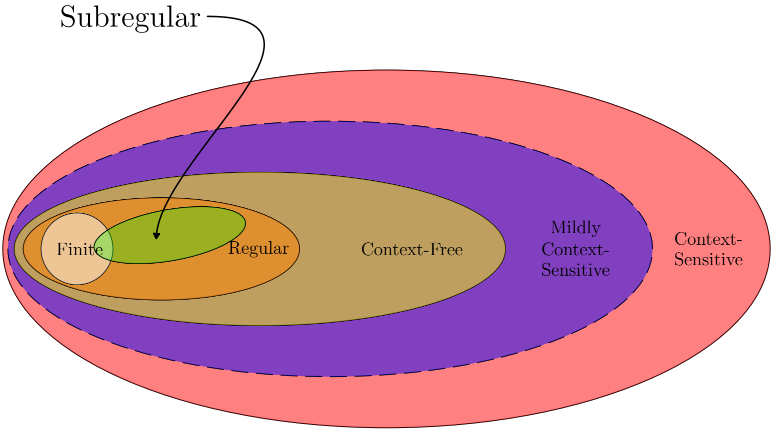
Figure: Natural language patterns in the hierarchy.

What is subregular?



There is room at the bottom

Better characterizations of phonological patterns

- Leads to stronger universals
- Leads to new hypotheses regarding what a *humanly* possible phonological pattern is

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Payoffs for better understanding *learning*

- Are the stronger universals useful for learning?

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Payoffs for better understanding *learning*

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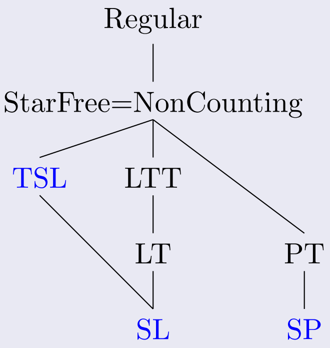
Payoffs for natural language processing

- Insights can be incorporated into NLP algorithms
- Factoring and composition may occur with lower complexity

Interesting *subregular* classes of stringsets



(McNaughton and Papert 1971, Rogers et al. 2010, 2012, Heinz et al. 2011)

LTT	Locally Threshold Testable	TSL	Tier-based Strictly Local
LT	Locally Testable	PT	Piecewise Testable
SL	Strictly Local	SP	Strictly Piecewise

Phonotactics - Knowledge of word well-formedness
Samala Version

ʃtojonowonowaʃ

stojonowonowaʃ

stojonowonowas

ʃtojonowonowas

pisotonosikiwat

pisotonoʃikiwat

sanisotonosikiwas

ʃanipisotonoʃikiwas

Phonotactics - Knowledge of word well-formedness
Samala Version

possible Samala words	impossible Samala words
ʃtojonowonowaʃ	stojonowonowaʃ
stojonowonowas	ʃtojonowonowas
pistonoskiwat	pisotonoʃikiwat
sanisotonoskiwas	ʃanipisotonoʃikiwas

1. Question: How do Samala speakers know which of these words belong to different columns?
2. By the way, *ʃtoyonowonowaʃ* means ‘it stood upright’

(Applegate 1972)

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Phonotactics - Knowledge of word well-formedness Language X

possible words of Language X	impossible words of Language X
ʃotkoʃ	sotkoʃ
ʃoʃkoʃ	ʃotkos
ʃosokoʃ	ʃoʃkos
soʃokos	soskoʃ
sokosos	
pitkol	
pisol	
piʃol	

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Sibilant sounds which begin and end words must agree (but not ones word medially).

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soʃkostoʃ	soskoʃ
	soksos
	piskol
	piʃkol

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Words must have an *even number* of sibilant sounds.

Typology

Attested Phonotactic Patterns

1. Words don't begin with mgl. (English)
2. Words don't contain both ʃ and s. (Samala)

Unattested Phonotactic Patterns

1. Words don't begin and end with disagreeing sibilants. (Language X = First/Last Harmony)
2. Words don't contain an even number of sibilants. (Language Y = *EVEN-Sibilants)

What's the explanation?

Optimality Theory

1. Constraints like *#mgl and *[+strident,α anterior]…[+strident,$-\alpha$ anterior] are part of CON.
2. Constraints like *EVEN-Sibilants or *#[+strident,α anterior]…[+strident,$-\alpha$ anterior]# are not.

What's the explanation?

Phonetically-based Phonology

1. There are perceptual and/or articulatory reasons for constraints like *#mgl and *[+strident,α anterior]…[+strident,$-\alpha$ anterior].
2. There are no such reasons for constraints like *EVEN-Sibilants or *#[+strident,α anterior]…[+strident,$-\alpha$ anterior]# .

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What are those reasons?

First/Last Harmony

1. Long-distance assimilation is well-attested (Hansson, 2001; Rose & Walker, 2004)
2. Word edges in phonology are privileged positions (Beckman, 1997; Endress, Nespor & Mehler, 2009; Fougeron & Keating, 1997).

Question

What theory of perception or articulation prevents there from being harmony only in privileged positions?

First/Last Harmony

Are the memory requirements greater?

Given the pattern templates, the answer seems to be no.

	[s]	[ʃ]
[s]	✓	✗
[ʃ]	✗	✓

[. . . __. . . __. . .]

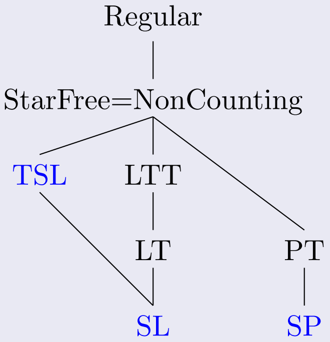
	[s]	[ʃ]
[s]	✓	✗
[ʃ]	✗	✓

[#__. . . __#]

*EVEN-Sibilants

- It's plausible to me at least that perception or articulation should be able to explain the absence of counting mod n patterns in phonology, but I haven't seen any explicit connection.
- Whatever it is, it *should* connect to the computational properties discussed here.

A computational explanation



1. Constraints like *#mgl are Strictly Local.
2. Constraints like *[+strident,α anterior]…[+strident,$-\alpha$ anterior] are Strictly Piecewise.
3. Constraints like First Last Harmony are Locally Testable.
4. Constraints like *EVEN-Sibilants are Counting (properly regular).

Strictly Local Stringsets

Substrings

String u is a substring of w iff $w \in \Sigma^* u \Sigma^*$. We write $u \preceq w$.

$$F_k(w) = \begin{cases} \{u \in \Sigma^* \mid u \preceq w \wedge |u| = k\} & |w| \geq k \\ \{w\} & \text{otherwise} \end{cases}$$

Example: $\alpha = abbcac$; $F_2(\alpha) = \{ab,\ bb,\ bc,\ ca,\ ac\}$.

Strictly Local (SL) Stringsets

$$L \in SL_k \iff \Big(\exists G \subseteq F_k(\rtimes \cdot \Sigma^* \cdot \ltimes)\Big)$$
$$\Big[\ L = L(G) = \big\{w \in \Sigma^* \mid (\forall u \in G)\ [u \not\preceq \rtimes \cdot w \cdot \ltimes]\big\}\Big]$$

Example: If $G = \{ac,\ ad\}$ then $\alpha \notin L(G)$.

(McNaughton and Papert 1971, Rogers and Pullum 2011)

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Strictly Piecewise Stringsets

Subsequences

String $u = \sigma_1 \cdots \sigma_n$ is a subsequence of w iff $w \in \Sigma^*\sigma_1\Sigma^* \cdots \Sigma^*\sigma_n\Sigma^*$. We write $u \sqsubseteq w$.

$$P_k(w) = \Big\{ u \in \Sigma^* \mid u \sqsubseteq w \wedge |u| \leq k \Big\}$$

Example: $\alpha = abcd$; $P_2(\alpha) = \{\lambda, a, b, c, d, ab, ac, ad, bc, bd, cd\}$.

Strictly Piecewise (SP) Stringsets

$$L \in SP_k \iff \Big(\exists G \subseteq P_k(\Sigma^*) \Big)$$
$$\Big[L = L(G) = \big\{ w \in \Sigma^* \mid (\forall u \in G)\ [u \not\sqsubseteq w] \big\} \Big]$$

Example: If $G = \{aa, ad\}$ then $\alpha \notin L(G)$.

Rogers et al. 2010, 2012)

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Example: If $G = \{aa, ad\}$ then $\alpha \notin L(G)$.

Rogers et al. 2010, 2012)

Locally Testable and Piecewise Testable Stringsets

Locally Testable (LT) Stringsets

These are finite boolean combinations of SL languages. It is also known:

$$L \in LT_k \iff \forall w, v \in \Sigma^* \left[F_k(w) = F_k(v) \Rightarrow \left[w \in L \Leftrightarrow v \in L \right] \right]$$

Piecewise Testable (PT) Stringsets

These are finite boolean combinations of SP languages. It is also known:

$$L \in PT_k \iff \forall w, v \in \Sigma^* \left[P_k(w) = P_k(v) \Rightarrow \left[w \in L \Leftrightarrow v \in L \right] \right]$$

Tiers: Ignoring inconsequential events

Tiers

A tier T is a subset of Σ.

Definition

The erasing (projection) function:

$$E_T(\sigma_1 \cdots \sigma_n) = \tau_1 \cdots \tau_n$$

where $\tau_i = \sigma_i$ iff $\sigma_i \in T$ and $\tau_i = \lambda$ otherwise

Example

If $\Sigma = \{a, b, c\}$ and $T = \{b, c\}$ then

$$E_T(aabaaacaaabaa) = bcb$$

(Heinz et al. 2011)

Typology of phonotactic patterns

Phonotactic Patterns derived from

- ✓ Adjacency constraints are SL
- ✓ Consonantal harmony are SP/TSL
- ✓ Consonantal disharmony are TSL
- ✓ Vowel harmony without neutral vowels are SP/TSL
- ✓ Vowel harmony with opaque vowels are TSL
- ✓ Vowel harmony with transparent vowels are SP/TSL
- ✓ Stress patterns are ... (attend Jim's talk next week!)

Heinz 2007, 2010, Rogers et al. 2010, Heinz et al. 2011

Learnability

1. SL_k, SP_k, and $TSL_{T,k}$ are provably identifiable in the limit from positive data by an incremental, set-driven, polytime learning algorithms.

   Garcia et al. 1991, Heinz 2007, 2010, Rogers et al. 2010

   Heinz et al. 2011, Heinz et al. 2012

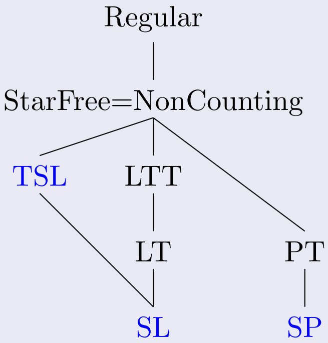
   - k (and T) must be known a priori.
   - k appears to be small for phonology (perhaps ≤ 5).

2. Provably-correct stochastic versions of these algorithms exist which learn probability distributions over stringsets.

   Jurafsky and Martin 2008, Heinz and Rogers 2010

3. Phonological features and syllables can be fully integrated into these algorithms without compromising their correctness. Heinz and Koirala 2010, Koirala et seq.

A computational explanation



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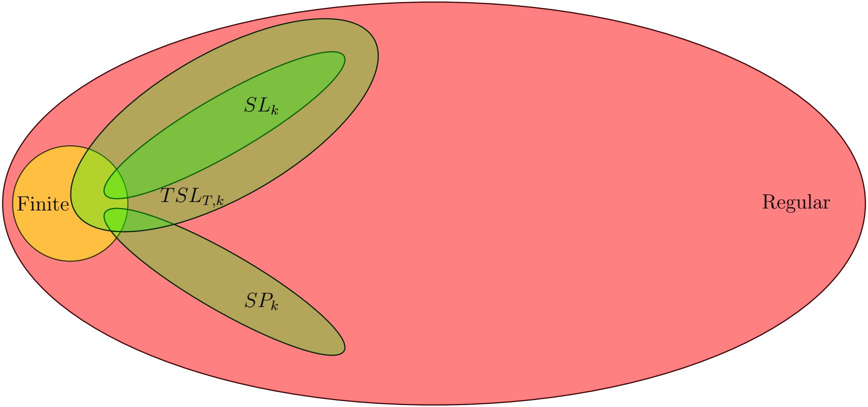


Can college students learn First Last Harmony?

Artificial language learning experiments

1. Subjects are exposed to training items (exemplars of a pattern).
2. Subjects are tested on novel items, some which exemplify the target pattern, and some which don't.
   - Which word do you think more likely belongs to the language you just heard?

Comparative artificial language learning experiments (Lai 2012, under review)

		Pattern Type	
		SL/SP/TSL (SH)	non-SL/SP/TSL (FL)
Outcomes	1	Learnable	Learnable
	2	Unlearnable	Unlearnable
	3	Learnable	Unlearnable
	4	Unlearnable	Learnable

- It is not possible to test for the *unlearnability* of some pattern.
- Instead, Lai (2012) tests the *comparative* learnability.

Methodology (Lai 2012, under review)

Subjects

66 adult native English speakers

All Stimuli

Training and test items were $C_1V.C_2V.C_3VC_4$ (tryisyllabic), containing 3 sibilants.

- C_1 & C_4: sibilants
- C_2 & C_3: either sibilant or [k]

Training

40 words × 5 repetitions = 200 words. Subjects listened and repeated each word. 3 Training Conditions:

SH: [s. . . s. . . s], [ʃ. . . ʃ. . . ʃ]

FL: [s. . . s. . . s], [ʃ. . . ʃ. . . ʃ], [s. . . ʃ. . . s], [ʃ. . . s. . . ʃ]

Control: No training

Testing (Lai 2012, under review)

Two alternative forced choice

Words were presented in pairs (minimally different)

E.g. [sakisis] vs. [ʃakisis]

- In the FL and SH conditions, subjects had to answer "Which word do you think belongs to the language you just heard?"
- In the control condition, they were asked "Which word do you prefer?"
- 48 pairs in total

Stimuli (Lai 2012, under review)

Three Stimuli Types

FL/SH	[sokosos]
*FL/*SH	[sokosoʃ]
FL/*SH	[sokoʃos, ʃokosoʃ]

- These 3 types of stimuli were pitted against each other and generated 3 types of pairings.
  (a) FL/*SH vs. *FL/*SH (also includes *FL/*SH vs. FL/*SH)
  (b) FL/SH vs. *FL/*SH (also includes *FL/*SH vs. FL/SH)
  (c) FL/*SH vs. FL/SH (also includes FL/SH vs. FL/*SH)
- The order of presentation was counter-balanced across types

Data Analysis (Lai 2012, under review)

The dependent variable for each pairing is different, so they were analyzed separately

(a) FL/*SH vs. *FL/*SH
Rate of choosing FL/*SH

(b) FL/SH vs. *FL/*SH
Rate of choosing FL/SH

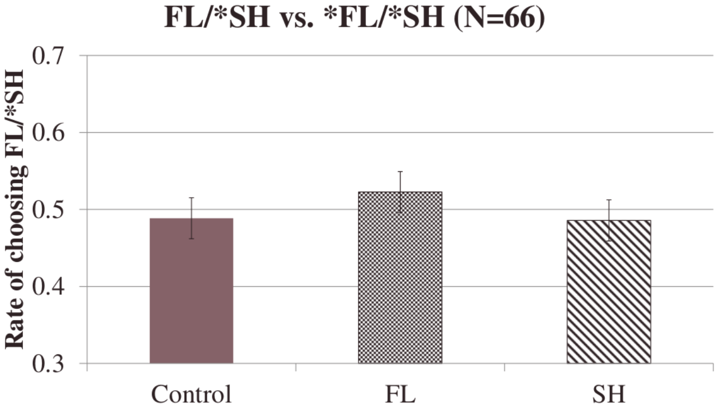
(c) FL/*SH vs. FL/SH
Rate of choosing FL/SH

Predictions (Lai 2012, under review)

If subjects internalized the pattern they were exposed to during training, they should perform as follows.

	Pairs		
Conditions	FL/*SH vs. *FL/*SH	FL/SH vs. *FL/*SH	FL/SH vs. FL/*SH
SH	No preference	FL/SH	FL/SH
FL	FL/*SH	FL/SH	No preference
Control	No preference	No preference	No preference

Results (Lai 2012, under review)



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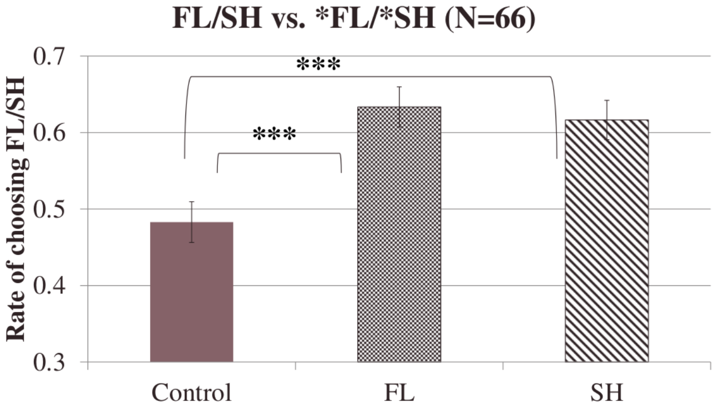

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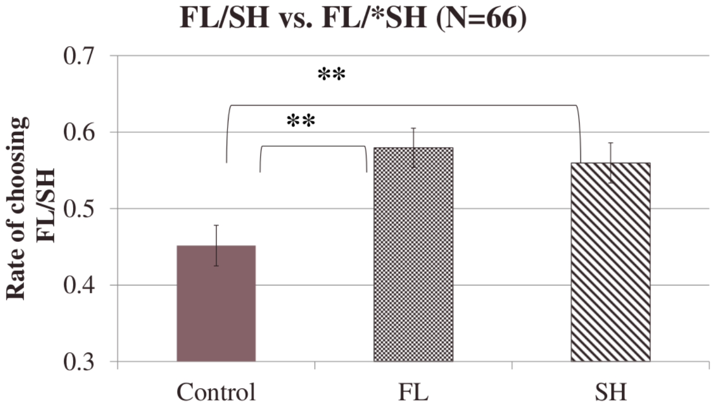

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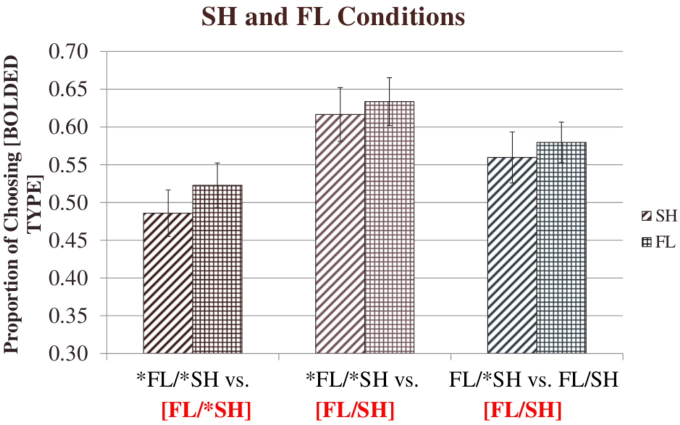

Discussion (Lai 2012, under review)

1. Subjects in the SH condition behaved as if they had internalized the SH pattern.
2. Subjects in the FL condition behaved as if they had internalized the SH pattern, *not* the FL pattern!

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Conclusion

The heavy bias for SH can be understood if only phonotactic patterns which can be modeled as SL, SP, or TSL stringsets are the humanly learnable ones.

Explaining Allomorphy

Lithuanian Verbal Prefixes exhibit allomorphy.

at-eiti	‘to arrive’
at-imti	‘to take away’
at-neʃti	‘to bring’
at-leisti	‘to forgive’
at-likti	‘to complete’
at-koːpti	‘to rise’
at-praʃiːti	‘to ask’
at-kurti	‘to reestablish’
ad-bekti	‘to run up’
ad-gauti	‘to get back’
ad-bukti	‘to become blunt’
ad-gimti	‘to be born again’

Explaining Allomorphy

Lithuanian Verbal Prefixes exhibit allomorphy.

ap-eiti	to circumvent
ap-ieʃkoːti	to search everywhere
ap-akti	to become blind
ap-moːkiːti	to train
ap-temdiːti	to obscure
ap-ʃaukti	to proclaim
ab-gauti	to deceive
ab-ʒʲureti	to have a look at
ab-ʒelti	to become overgrown
ab-dauʒiːti	to damage
ab-draskiːti	to tear

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- Phonology is the theory that posits a single lexical representation /ap/ and a mapping which maps a /p/ which is immediately followed by a voiced obstruent to [b].

This knowledge can be represented with string relations

Example

In Lithuanian, voiceless stops become voiced when immediately followed by voiced obstruents. Every pair (u, w) such that u maps to w is in the set and every other pair is out.

$$
\begin{array}{lcl}
\text{ateiti} & \mapsto & \text{ateiti} \\
\text{atbekti} & \mapsto & \text{adbekti} \\
\text{apmoːkiːti} & \mapsto & \text{apmoːkiːti} \\
\text{apgauti} & \mapsto & \text{abgauti} \\
 & \vdots &
\end{array}
$$

Phonology is regular (Kaplan and Kay 1994)

1. Optional, left-to-right, right-to-left, and simultaneous application of SPE-style rules $A \longrightarrow B \;/\; C \_\_D$ (where A,B,C,D are regular expressions) *describe regular relations*, provided the rule cannot reapply to the locus of its structural change.
2. Rule ordering is functional composition (finite-state transducer composition).
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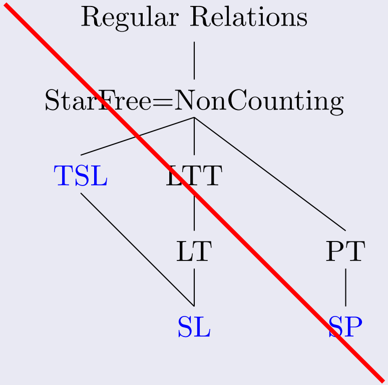
Therefore, phonological mappings are regular relations.

Regardless of whether they are described with SPE or OT grammars.

Regular sets $\neq$ Regular relations



There are no similar subregular hierarchies for relations

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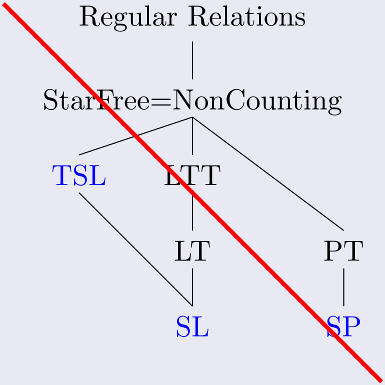

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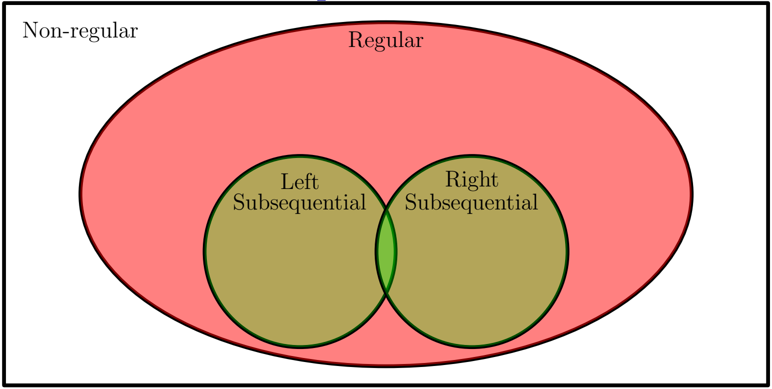
Subsequential Functions



- Subsequential finite-state transducers T are deterministic on the input.
  - Left subsequential functions are mappings obtained by applying T left-to-right to a word.
  - Right subsequential functions are mappings obtained by applying T right-to-left to a word.

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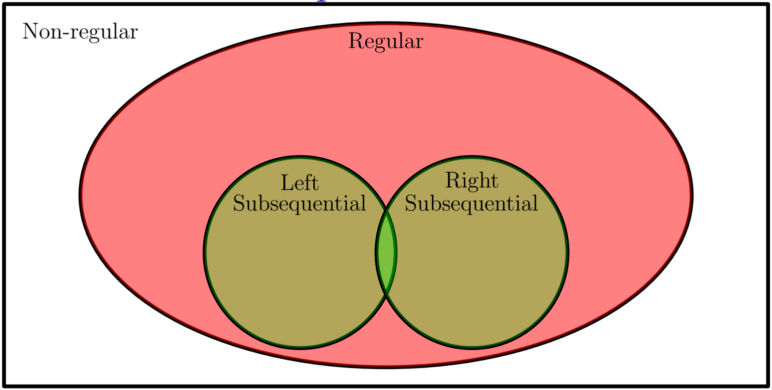

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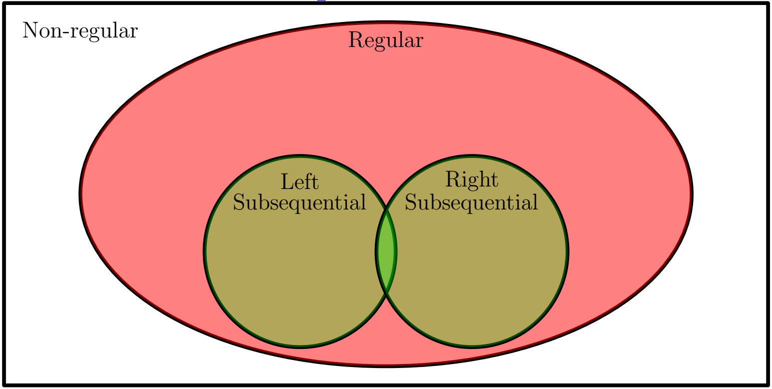

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Survey of results for segmental phonology

1. Mappings described by simultaneous application of SPE-style rules $A \longrightarrow B \ / \ C \_\_ D$, where A, B, C, and D are feature bundles (or $\varnothing$) are left subsequential (Koirala 2010, MS).
2. All the iterative vowel harmony patterns described by Nevins (2010) are left or right subsequential (Gainor et al. 2012).
3. All the synchronically attested metathesis patterns, including long-distance ones, in Beth Hume's NSF-funded metathesis database are left or right subsequential (Chandlee et al. 2012).
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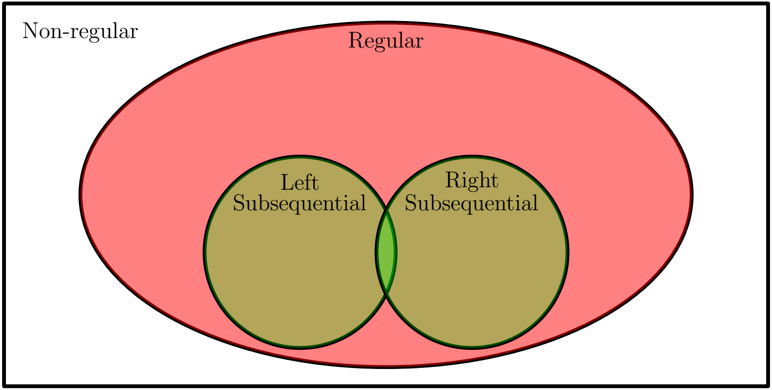
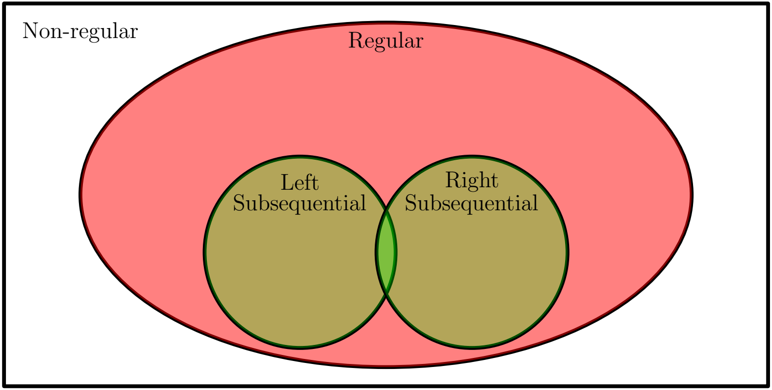
What is not subsequential?



7. The “Majority Rules” vowel harmony pattern is not regular and the “Sour Grapes” vowel harmony pattern is neither left nor right subsequential (Heinz and Lai, 2012 MS).

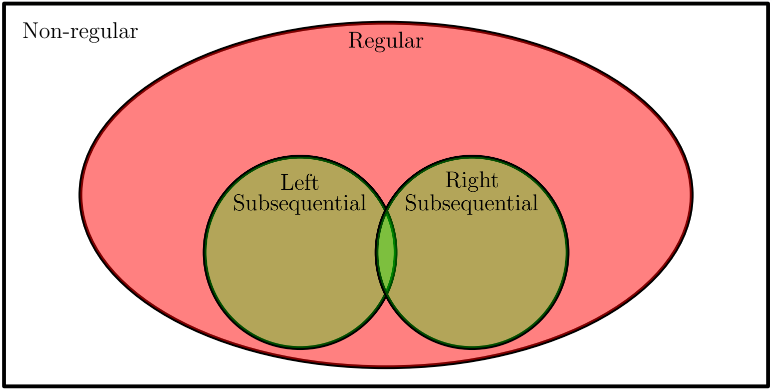
Suprasegmental phonology



8. Unbounded Tone Plateauing is neither left nor right subsequential (Jardine, 2012 MS).

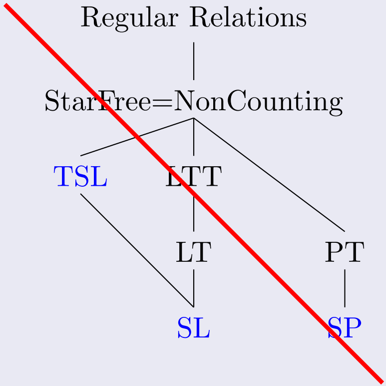
- Paraphasing Yip (2001) and Hyman (2011): “Tone can do everything segmental phonology can do and more!”

The future...



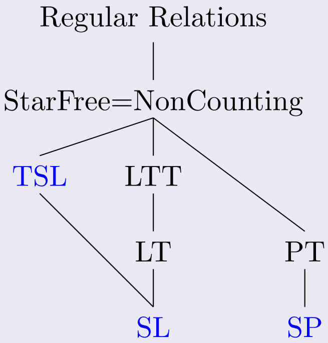
- There are stronger properties than "being left or right subsequential".
- Chandlee's in progress thesis (exp. 2014) aims to define Strictly Local *mappings* to capture local phonological processes, including many of the ones mentioned above.
- Chandlee and Koirala (2013, PLC) present the first *learning* results following this line of research.

Regular sets $\neq$ Regular relations



There are no similar subregular hierarchies for relations

Regular sets may inform Regular relations



There are no similar subregular hierarchies for relations (yet)

Conclusions

1. Computational analysis of stringsets and string mappings (ongoing) is yielding natural classes of pattern complexity.
2. When phonological patterns are studied through this lens, strong computational properties are revealed, which:
   2.1 appear to make the right kind of cuts between attested and unattested patterns.
   2.2 appear to draw the right distinctions between segmental and suprasegmental phenomenon.
   2.3 are strong enough to make learning possible from positive evidence.
   2.4 make experimentally testable predictions.

THANK YOU