

## 1 Variables

### 2 root

|   | var       | symbol       | documentation                   | type     | units | tokens | eqs      |
|---|-----------|--------------|---------------------------------|----------|-------|--------|----------|
| 8 | $F_{N,A}$ | <b>F</b>     | directed graph incidence matrix | network  |       | []     |          |
| 1 | $t$       | <b>t</b>     | time                            | frame    | $s$   | []     |          |
| 6 | $t_o$     | <b>to</b>    | starting time                   | frame    | $s$   | []     | <b>3</b> |
| 7 | $t_e$     | <b>te</b>    | end time                        | frame    | $s$   | []     | <b>4</b> |
| 3 | $\#$      | <b>value</b> | numerical value                 | constant |       | []     |          |
| 4 | 1         | <b>one</b>   | numerical value 1               | constant |       | []     | <b>1</b> |
| 5 | 0         | <b>null</b>  | numerical value 0               | constant |       | []     | <b>2</b> |

### 3 Properties

|    | var              | symbol    | documentation                          | type        | units | tokens | eqs                |
|----|------------------|-----------|--|-------------|-------|--------|--------------------|
| 57 | $M^{A,\alpha}_N$ | M_A_alpha | norming factor token A mechanism alpha | constant    |       | []     | <a href="#">51</a> |
| 58 | $M^{A,\beta}_N$  | M_A_beta  | norming factor token A mechanism beta  | constant    |       | []     | <a href="#">52</a> |
| 59 | $M^{B,\gamma}_N$ | M_B_gamma | norming factor token B mechanism gamma | constant    |       | []     | <a href="#">53</a> |
| 60 | $M^{B,\delta}_N$ | M_B_delta | norming factor token B mechanism delta | constant    |       | []     | <a href="#">54</a> |
| 69 | $M^A_N$          | M_A       | stack of M matrices token A            | propertyOut |       | []     | <a href="#">55</a> |
| 70 | $M^B_N$          | M_B       | stack of M matrices token B            | propertyOut |       | []     | <a href="#">56</a> |
| 71 | $M_N$            | M         | stack of M matrices token A and B      | propertyOut |       | []     | <a href="#">57</a> |

## 4 Control

|    | var         | symbol   | documentation   | type              | units    | tokens | eqs   |
|----|-------------|----------|---|-------------------|----------|--------|-------|
| 67 | $y_A$       | y        | controller output   | controlOut        |          | []     | 49 50 |
| 61 | $m_A$       | m        | measurement   | measureIn         |          | []     | 61 62 |
| 65 | $I_{N,D}$   | I_N_D    | Identity to shift from differential space to integral space | network           |          | []     |       |
| 66 | $I_{A,D}$   | I_A_D    | identity to shift from differential space to arc            | network           |          | []     |       |
| 55 | $x_N$       | x        | controller state  | state             |          | []     | 48    |
| 56 | $x_N^o$     | xo       | controller state initial condition                          | state             |          | []     | 44    |
| 50 | $A_{N,D}$   | A        | dynamic matrix A  | constant          | $s^{-1}$ | []     |       |
| 51 | $B_{A,D}$   | B        | input matrix C  | constant          | $s^{-1}$ | []     |       |
| 52 | $C_{N,A}$   | C        | output matrix C   | constant          |          | []     |       |
| 53 | $D_{N,A}$   | D        | event matrix D  | constant          |          | []     |       |
| 62 | $y_{sA}$    | setpoint | set point   | constant          |          | []     | 45    |
| 63 | $e_A$       | e        | control error   | constant          |          | []     | 46    |
| 68 | $D_A$       | D_A      | event diagonal matrix D                                     | constant          |          | []     |       |
| 64 | $\dot{x}_D$ | dx       | differential state  | differentialState | $s^{-1}$ | []     | 47    |

## 5 System

|    | var                    | symbol          | documentation                                 | type               | units     | tokens | eqs   |
|----|------------------------|-----------------|---|--------------------|-----------|--------|-------|
| 76 | $u_A$                  | u               | [0-1]-normed control signal                   | controlIn          |           | []     | 63 70 |
| 74 | $z_N$                  | measure         | normed measurement of pi A alpha              | measureOut         |           | []     | 59    |
| 75 | $\underline{z}_A$      | measure_set_A   | measurement vector for A                      | measureOut         |           | []     | 60    |
| 25 | $\hat{x}^{A,\alpha}_N$ | fx_A_alpha      | netflow of token A due to mechanism alpha     | transport          | $ms^{-1}$ | []     | 11 71 |
| 26 | $\hat{x}^{A,\beta}_N$  | fx_A_beta       | net flow of token A due to mechanism beta     | transport          | $ms^{-1}$ | []     | 12 72 |
| 27 | $\hat{y}^{B,\gamma}_N$ | fy_B_gamma      | netflow of token B due to mechanism gamma     | transport          | $s^{-1}$  | []     | 14 73 |
| 28 | $\hat{y}^{B,\delta}_N$ | fy_B_delta      | netflow of token B due to mechansim beta      | transport          | $s^{-1}$  | []     | 15 74 |
| 73 | $I_{zN,A}$             | I_measure       | unidirectional graph for interface connection | network            |           | []     |       |
| 36 | $D_{N,A}$              | D               | difference operator                           | differenceOperator |           | []     |       |
| 9  | $x_N$                  | x               | state token A                                 | state              | $m$       | []     | 20    |
| 10 | $y_N$                  | y               | state token B                                 | state              |           | []     | 21    |
| 11 | $x^o_N$                | xo              | initial condition for state x                 | state              | $m$       | []     | 5     |
| 12 | $y^o_N$                | yo              | initial condition for state y                 | state              |           | []     | 6     |
| 34 | $s$                    | s               | mixed state                                   | state              |           | []     | 31    |
| 13 | $K^{A,\alpha}_A$       | K_A_alpha       | conductivity token A mechanism alpha          | constant           | $s^{-1}$  | []     |       |
| 14 | $K^{A,\beta}_A$        | K_A_beta        | conductivity token A mechanism beta           | constant           | $s^{-1}$  | []     |       |
| 15 | $K^{B,\gamma}_A$       | K_B_gamma       | conductivity token B mechanism gamma          | constant           | $s^{-1}$  | []     |       |
| 16 | $K^{B,\delta}_A$       | K_B_delta       | conductivity token B mechanism delta          | constant           | $s^{-1}$  | []     |       |
| 17 | $M^{A,\alpha}_N$       | M_A_alpha       | norming factor token A mechanism alpha        | constant           |           | []     |       |
| 18 | $M^{A,\beta}_N$        | M_A_beta        | norming factor token A mechanism beta         | constant           |           | []     |       |
| 19 | $M^{B,\gamma}_N$       | M_B_gamma       | norming factor token B mechanism gamma        | constant           |           | []     |       |
| 20 | $M^{B,\delta}_N$       | M_B_delta       | norming factor token B mechanism delta        | constant           |           | []     |       |
| 72 | $\pi^{A,\alpha,o}_N$   | pi_A_alpha_norm | norming factor for pi A alpha                 | constant           | $m$       | []     | 58    |

*Continued on next page*

|    | var                     | symbol     | documentation                             | type              | units     | tokens | eqs   |
|----|-------------------------|------------|---|-------------------|-----------|--------|-------|
| 29 | $\dot{x}_N$             | dx         | diferential balance for token A           | differentialState | $ms^{-1}$ | []     | 16 32 |
| 30 | $\dot{y}_N$             | dy         | differential balance for token B          | differentialState | $s^{-1}$  | []     | 17 33 |
| 35 | $\dot{x}y$              | dxy        | mixed stack of the two accumulation terms | differentialState |           | []     | 34    |
| 21 | $\pi^{A,\alpha}_N$      | pi_A_alpha | effort for A mechanism alpha              | secondaryState    | $m$       | []     | 7 27  |
| 22 | $\pi^{A,\beta}_N$       | pi_A_beta  | effort for A mechanism beta               | secondaryState    | $m$       | []     | 8 28  |
| 23 | $\pi^{B,\gamma}_N$      | pi_B_gamma | effort for B mechanism gamma              | secondaryState    |           | []     | 9 29  |
| 24 | $\pi^{B,\delta}_N$      | pi_B_delta | effort for B mechanism delta              | secondaryState    |           | []     | 10 30 |
| 31 | $\underline{\pi}^A_N$   | pi_A_stack | effort for token A stack                  | secondaryState    | $m$       | []     | 24    |
| 32 | $\underline{\pi}^B_N$   | pi_B_stack | effort for token B stack                  | secondaryState    |           | []     | 25    |
| 33 | $\underline{\pi}^{A,B}$ | pi_stack   | effort for token A, B stack               | secondaryState    |           | []     | 26    |
| 77 | $M^A_N$                 | M_A        | stack of M matrices for token A           | propertyIn        |           | []     | 64 67 |
| 78 | $M^B_N$                 | M_B        | stack of M matrices for token B           | propertyIn        |           | []     | 65 68 |
| 79 | $M_N$                   | M          | stack of all M matrices                   | propertyIn        |           | []     | 66 69 |

## 6 Equations

## 7 Generic

| no | equation   | documentation                             | layer  |
|----|--|---|--------|
| 1  | $1 := \text{Instantiate}(\#, \#)$  | numerical value 1                         | root   |
| 2  | $0 := \text{Instantiate}(\#, \#)$  | numerical value 0                         | root   |
| 3  | $t_o := \text{Instantiate}(t, \#)$   | starting time                             | root   |
| 4  | $t_e := \text{Instantiate}(t, \#)$   | end time                                  | root   |
| 5  | $x_N^o := \text{Instantiate}(x_N, \#)$   | initial condition for state x             | System |
| 6  | $y_N^o := \text{Instantiate}(y_N, \#)$   | initial condition for state y             | System |
| 7  | $\pi^{A,\alpha}_N := M^{A,\alpha}_N \cdot x_N$   | effort for B mechanism alpha              | System |
| 8  | $\pi^{A,\beta}_N := M^{A,\beta}_N \cdot x_N$   | effort for A mechanism beta               | System |
| 9  | $\pi^{B,\gamma}_N := M^{B,\gamma}_N \cdot y_N$   | effort for B mechanism gamma              | System |
| 10 | $\pi^{B,\delta}_N := M^{B,\delta}_N \cdot y_N$   | effort for B mechanism delta              | System |
| 11 | $\hat{x}^{A,\alpha}_N := F_{N,A} \overset{A}{\star} \left( u_A \cdot K^{A,\alpha}_A \cdot D_{N,A} \overset{N}{\star} \pi^{A,\alpha}_N \right)$ | netflow of token A due to mechanism alpha | System |
| 12 | $\hat{x}^{A,\beta}_N := F_{N,A} \overset{A}{\star} \left( K^{A,\beta}_A \cdot D_{N,A} \overset{N}{\star} \pi^{A,\beta}_N \right)$              | net flow of token A due to mechanism beta | System |
| 14 | $\hat{y}^{B,\gamma}_N := F_{N,A} \overset{A}{\star} \left( K^{B,\gamma}_A \cdot D_{N,A} \overset{N}{\star} \pi^{B,\gamma}_N \right)$           | netflow of token B due to mechanism gamma | System |
| 15 | $\hat{y}^{B,\delta}_N := F_{N,A} \overset{A}{\star} \left( K^{B,\delta}_A \cdot D_{N,A} \overset{N}{\star} \pi^{B,\delta}_N \right)$           | netflow of token B due to mechansim beta  | System |

*Continued on next page*

| no | equation   | documentation                             | layer   |
|----|--|---|---------|
| 16 | $\dot{x}_N := \hat{x}^{A,\alpha}_N + \hat{x}^{A,\beta}_N$                              | diferential balance for token A           | System  |
| 17 | $\dot{y}_N := \hat{y}^{B,\gamma}_N + \hat{y}^{B,\delta}_N$                             | differential balance for token B          | System  |
| 20 | $x_N := \int_{t_o}^{t_e} \dot{x}_N dt + x_o_N$   | state token A                             | System  |
| 21 | $y_N := \int_{t_o}^{t_e} \dot{y}_N dt + y_o_N$   | state token B                             | System  |
| 24 | $\underline{\pi}^A_N := \text{Stack}(\pi^{A,\alpha}_N, \pi^{A,\beta}_N)$               | effort for token A stack                  | System  |
| 25 | $\underline{\pi}^B_N := \text{Stack}(\pi^{B,\gamma}_N, \pi^{B,\delta}_N)$              | effort for token B stack                  | System  |
| 26 | $\underline{\pi}^{A,B} := \text{MixedStack}(\underline{\pi}^A_N, \underline{\pi}^B_N)$ | effort for token A, B stack               | System  |
| 27 | $\pi^{A,\alpha}_N := \text{Instantiate}(\pi^{A,\alpha}_N, \#)$                         | effort for B mechanism alpha              | System  |
| 28 | $\pi^{A,\beta}_N := \text{Instantiate}(\pi^{A,\beta}_N, \#)$                           | effort for A mechanism beta               | System  |
| 29 | $\pi^{B,\gamma}_N := \text{Instantiate}(\pi^{B,\gamma}_N, \#)$                         | effort for B mechanism gamma              | System  |
| 30 | $\pi^{B,\delta}_N := \text{Instantiate}(\pi^{B,\delta}_N, \#)$                         | effort for B mechanism delta              | System  |
| 31 | $s := \text{MixedStack}(x_N, y_N)$   | mixed state                               | System  |
| 32 | $\dot{x}_N := \text{Instantiate}(\dot{x}_N, 0)$  | diferential balance for token A           | System  |
| 33 | $\dot{y}_N := \text{Instantiate}(\dot{y}_N, 0)$  | differential balance for token B          | System  |
| 34 | $\dot{xy} := \text{MixedStack}(\dot{x}_N, \dot{y}_N)$                                  | mixed stack of the two accumulation terms | System  |
| 44 | $x_o_N := \text{Instantiate}(x_N, \#)$   | controller state initial condition        | Control |
| 45 | $y_{sA} := \text{Instantiate}(m_A, \#)$  | set point                                 | Control |
| 46 | $e_A := m_A - y_{sA}$  | control error                             | Control |

*Continued on next page*

| no | equation  | documentation                          | layer      |
|----|---|--|------------|
| 47 | $\dot{x}_D := A_{N,D} \overset{N}{\star} x_N + B_{A,D} \overset{A}{\star} e_A$  | differential state                     | Control    |
| 48 | $x_N := \int_{t_o}^{t_e} I_{N,D} \overset{D}{\star} \dot{x}_D dt$   | controller state                       | Control    |
| 49 | $y_A := C_{N,A} \overset{N}{\star} x_N + I_{A,D} \overset{D}{\star} \left( I_{N,D} \overset{N}{\star} D_{N,A} \overset{A}{\star} e_A \right)$ | controller out put                     | Control    |
| 50 | $y_A := C_{N,A} \overset{N}{\star} x_N + D_A \cdot e_A$   | controller out put                     | Control    |
| 51 | $M^{A,\alpha}_N := \text{Instantiate}(M^{A,\alpha}_N, \#)$  | norming factor token A mechanism alpha | Properties |
| 52 | $M^{A,\beta}_N := \text{Instantiate}(M^{A,\beta}_N, \#)$  | norming factor token A mechanism beta  | Properties |
| 53 | $M^{B,\gamma}_N := \text{Instantiate}(M^{B,\gamma}_N, \#)$  | norming factor token B mechanism gamma | Properties |
| 54 | $M^{B,\delta}_N := \text{Instantiate}(M^{B,\delta}_N, \#)$  | norming factor token B mechanism delta | Properties |
| 55 | $M^A_N := \text{Stack}(M^{A,\alpha}_N, M^{A,\beta}_N)$  | stack of M matrices token A            | Properties |
| 56 | $M^B_N := \text{Stack}(M^{B,\gamma}_N, M^{B,\delta}_N)$   | stack of M matrices token B            | Properties |
| 57 | $M_N := \text{Stack}(M^A_N, M^B_N)$   | stack of M matrices token A and B      | Properties |
| 58 | $\pi^{A,\alpha,o}_N := \text{Instantiate}(\pi^{A,\alpha}_N, \#)$  | norming factor for pi A alpha          | System     |
| 59 | $z_N := (\pi^{A,\alpha,o}_N)^{-1} \cdot \pi^{A,\alpha}_N$   | normed measurement of pi A alpha       | System     |
| 60 | $\underline{z}_A := I_{zN,A} \overset{N}{\star} z_N$  | measurement vector for A               | System     |
| 61 | $m_A := \text{Instantiate}(m_A, \#)$  | measurement                            | Control    |
| 63 | $u_A := \text{Instantiate}(u_A, \#)$  | [0-1]-normed control signal            | System     |

*Continued on next page*



| no | equation  | documentation                             | layer  |
|----|---|---|--------|
| 64 | $M^A_N := \text{Stack}(M^{A,\alpha}_N, M^{A,\beta}_N)$                | stack of M matrices for token A           | System |
| 65 | $M^B_N := \text{Stack}(M^{B,\gamma}_N, M^{B,\delta}_N)$               | stack of M matrices for token B           | System |
| 66 | $M_N := \text{Stack}(M^A_N, M^B_N)$                                   | stack of all M matrices                   | System |
| 71 | $\hat{x}^{A,\alpha}_N := \text{Instantiate}(\hat{x}^{A,\alpha}_N, -)$ | netflow of token A due to mechanism alpha | System |
| 72 | $\hat{x}^{A,\beta}_N := \text{Instantiate}(\hat{x}^{A,\beta}_N, -)$   | net flow of token A due to mechanism beta | System |
| 73 | $\hat{y}^{B,\gamma}_N := \text{Instantiate}(\hat{y}^{B,\gamma}_N, -)$ | netflow of token B due to mechanism gamma | System |
| 74 | $\hat{y}^{B,\delta}_N := \text{Instantiate}(\hat{y}^{B,\delta}_N, -)$ | netflow of token B due to mechansim beta  | System |

## 8 Interface Link Equation

| no | equation              | documentation      | layer                           |
|----|-----------------------|--------------------|---------------------------------|
| 62 | $m_A := \mathbf{z}_A$ | interface equation | System $\rightarrow$ Control    |
| 67 | $M^A_N := M^A_N$      | interface equation | Properties $\rightarrow$ System |
| 68 | $M^B_N := M^B_N$      | interface equation | Properties $\rightarrow$ System |
| 69 | $M_N := M_N$          | interface equation | Properties $\rightarrow$ System |
| 70 | $u_A := y_N$          | interface equation | Control $\rightarrow$ System    |