### 1 Variables

### 2 root

|    | var                 | symbol | documentation                      | type       | units | eqs |
|----|---------------------|--------|------------------------------------|------------|-------|-----|
| 2  | $F_{N,A}$           | F      | incidence matrix of directed graph | network    |       |     |
| 1  | $t_N$               | t      | time                               | frame      | s     |     |
| 88 | $to_N$              | to     | initial time                       | frame      | s     | 63  |
| 89 | $te_N$              | te     | end time                           | frame      | s     | 64  |
| 5  | #                   | value  | a value                            | constant   |       |     |
| 6  | $1\backslash 2$     | half   | one half                           | constant   |       | 1   |
| 7  | 0                   | zero   | zero                               | constant   |       | 2   |
| 8  | $\partial t_{N,dt}$ | dt     | differential time                  | *diffFrame | s     | 3   |

## 3 physical

|     | var            | symbol     | documentation                                                                             | type       | units | eqs |
|-----|----------------|------------|-------------------------------------------------------------------------------------------|------------|-------|-----|
| 30  | $F_{NS,AS}$    | F_NS_AS    | incidence matrix for species network                                                      | network    |       |     |
| 41  | $P_{N,A,dt}$   | P_N_A_dt   | projection of node to arc (used for mapping transport system material to application arc) | projection |       |     |
| 49  | $P_{NS,AS,dt}$ | P_NS_AS_dt | projection of node to arc (used for mapping transport system material to application arc) | projection |       |     |
| 9   | $r_{xN}$       | rx         | spatial coordinate x                                                                      | frame      | m     |     |
| 20  | $r_{y_{N}}$    | ry         | spatial coordinate y                                                                      | frame      | m     |     |
| 21  | $r_{zN}$       | rz         | spatial coordinate z                                                                      | frame      | m     |     |
| 104 | $r_N$          | r          | radius                                                                                    | frame      | m     |     |
| 10  | $n_{NS}$       | n          | species mass in moles                                                                     | state      | mol   | 65  |

|     | var        | symbol | documentation                             | type           | units                                                | eqs      |
|-----|------------|--------|-------------------------------------------|----------------|------------------------------------------------------|----------|
| 11  | $U_N$      | U      | internal energy                           | state          | $kg m^2 s^{-2}$                                      |          |
| 12  | $S_N$      | S      | entropy                                   | state          | $kg m^2 K^{-1} s^{-2}$                               |          |
| 13  | $V_N$      | V      | volume                                    | state          | $m^3$                                                |          |
| 92  | $m_N$      | m      | mass                                      | state          | kg                                                   |          |
| 66  | R          | R      | gas constant                              | constant       | $kg^{-1} m^{-2} mol K^{-1} s^2$                      | :        |
| 17  | $H_N$      | Н      | definition of enthalpy                    | secondaryState | $kg m^2 s^{-2}$                                      | 7        |
| 18  | $v_{xN}$   | vx     | definition of velocity in x direction     | secondaryState | $ms^{-1}$                                            | 8        |
| 22  | $A_{xN}$   | Ax     | area at location x                        | secondaryState | $m^2$                                                | 10       |
| 111 | $A_N$      | A      | definition of Helmholtz energy            | secondaryState | $kg m^2 s^{-2}$                                      | 90       |
| 112 | $G_N$      | G      | definition of Gibbs free energy           | secondaryState | $kg m^2 s^{-2}$                                      | 91       |
| 14  | $T_N$      | Т      | definition of temperature                 | effort         | K                                                    | 4        |
| 15  | $p_N$      | р      | definition of pressure                    | effort         | $ kg  m^{-1}  s^{-2} $ $ kg  m^2  mol^{-1}  s^{-2} $ | 5        |
| 31  | $\mu_{NS}$ | mu     | chemical potential from Gibbs free energy | effort         | $kg m^2 mol^{-1} s^{-2}$                             | 18<br>92 |

## 4 control

## 5 macroscopic

|    | var              | symbol | documentation                                      | type      | units           | eqs |
|----|------------------|--------|----------------------------------------------------|-----------|-----------------|-----|
| 54 | $\hat{q}_{N,dt}$ | fq     | heat flow                                          | transport | $kg m^2 s^{-3}$ | 39  |
| 81 | $\hat{V}_{A,dt}$ | fV     | volumetric flow                                    | transport | $m^3 s^{-1}$    | 83  |
| 82 | $d_A$            | dir    | direction of flow relative to reference coordinate | transport |                 | 57  |

|    | var                 | symbol  | documentation                                           | type           | units                    | eqs |
|----|---------------------|---------|---------------------------------------------------------|----------------|--------------------------|-----|
| 83 | $S_{NS,AS}$         | S_NS_AS | selection of the flow sources                           | transport      |                          | 58  |
| 84 | $c_{AS}$            | c_AS    | concentration in the convective flows                   | transport      | $m^{-3}  mol$            | 59  |
| 85 | $\hat{n}^v_{NS,dt}$ | fnv     | convective species flow                                 | transport      | $mol  s^{-1}$            | 60  |
| 86 | $\hat{n}^d_{NS,dt}$ | fnd     | diffusional species flows                               | transport      | $mol  s^{-1}$            | 82  |
| 57 | $P_{NS,KS}$         | P_NS_KS | projection of node species onto reaction species        | projection     |                          |     |
| 58 | $P_{K,NK}$          | P_K_NK  | projection reaction on node x reactions                 | projection     |                          |     |
| 59 | $P_{N,NK}$          | P_N_NK  | projection node to node x reactions                     | projection     |                          |     |
| 74 | $P_{NK,KS}$         | P_NK_KS | projection node x conversion to conversion x species    | projection     |                          |     |
| 56 | $n^0{}_{NS}$        | n0      | initial condition                                       | state          | mol                      | 41  |
| 87 | $\dot{n}_{NS,dt}$   | dn      |                                                         | state          | $mol  s^{-1}$            | 62  |
| 62 | $N_{K,KS}$          | N_KS_K  | stoichiometry for reaction k                            | constant       |                          |     |
| 64 | $K^{o}{}_{K,dt}$    | Ко      | Arrhenius prexponential factor (matrix)                 | constant       | $m^{-3}  mol  s^{-1}$    |     |
| 65 | $E^A{}_K$           | ka      | Arrhenius activation energy                             | constant       | $kg^{-1} m^{-2} mol s^2$ |     |
| 61 | $c_{KS}$            | c_KS    | molar concentrations assigned to reaction K             | conversion     | $m^{-3}  mol$            | 43  |
| 63 | $N_{NS,NK}$         | N_NS_NK | global stoichiomety block matrix                        | conversion     |                          | 44  |
| 67 | $T_{NK}$            | T_NK    | temperature for reactions                               | conversion     | K                        | 45  |
| 68 | $K_{NK,dt}$         | K       | reaction "constants"                                    | conversion     | $m^{-3}  mol  s^{-1}$    | 46  |
| 69 | $c^o{}_{KS}$        | c_KS_o  | norming concentration – probability must have no units! | conversion     | $m^{-3}  mol$            | 47  |
| 71 | $x_{KS}$            | x_KS    | normed concentration little like more fractions         | conversion     |                          | 49  |
| 75 | $\phi_{NK}$         | phi     | probability for the reactions to occur                  | conversion     |                          | 52  |
| 76 | $\xi_{NK,dt}$       | xi      | dyamic extend of reaction                               | conversion     | $m^{-3}  mol  s^{-1}$    | 53  |
| 77 | $	ilde{n}_{NS,dt}$  | pn      | production term                                         | conversion     | $mol  s^{-1}$            | 54  |
| 60 | $c_{NS}$            | С       | molar concentration                                     | secondaryState | $m^{-3}  mol$            | 42  |

## 6 materialDB

|     | var                    | symbol      | documentation                                                  | type     | units                     | eqs |
|-----|------------------------|-------------|----------------------------------------------------------------|----------|---------------------------|-----|
| 94  | $\lambda_S$            | mm          | molecular masses                                               | constant | $kg  mol^{-1}$            |     |
| 106 | $k_{granulate_{dt}}$   | k_granulate | kinetic constant for granulation dynamics                      | constant | $s^{-1}$                  |     |
| 26  | $c_{p_N}$              | ср          | heat capacity at constant pressure                             | property | $kg  m^2  K^{-1}  s^{-2}$ | 14  |
| 45  | $k^{q_x}{}_{A,dt}$     | kq_x        | heat transfer coefficient for an event-dynamic transfer system | property | $kg K^{-1} s^{-3}$        | 31  |
| 93  | $k^{d_x}{}_{NS,AS,dt}$ | kd_x        | diffusional mass diffusivity per area.                         | property | $kg^{-1} m^{-4} mol^2 s$  | 81  |
| 95  | $k^{c_x}{}_{N,A,dt}$   | kc_x        | convective mass diffusivity per area                           | property | $m^{-1} s$                | 76  |
| 96  | $ ho_N$                | rho         | mass density                                                   | property | $kg m^{-3}$               | 71  |
| 98  | $k^{v_x}_{N,A,dt}$     | kv_x        | convective volume flow per area                                | property | $kg^{-1} m^2 s$           | 78  |

# 7 mixing

|     | var             | symbol | documentation          | type      | units    | eqs |
|-----|-----------------|--------|------------------------|-----------|----------|-----|
| 113 | $y_N$           | у      | mixing index           | state     |          |     |
| 114 | β               | beta   | mixing beta parameter  | constant  |          |     |
| 115 | $\gamma$        | gamma  | mixing gamma parameter | constant  |          |     |
| 116 | f               | f      | stirrer frequency      | constant  | $s^{-1}$ |     |
| 117 | $a_{N,dt}$      | a      | stirrer amplitude      | constant  |          |     |
| 118 | $\alpha_{N,dt}$ | alpha  | stirrer function       | parameter |          | 93  |

## 8 fluid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|

## 9 solid

|     | var               | symbol | documentation          | type       | units     | eqs |
|-----|-------------------|--------|------------------------|------------|-----------|-----|
| 101 | $N_N$             | N      | number of particles    | state      |           |     |
| 110 | $N_{tN,dt}$       | dN     | population balance PDE | state      | $s^{-1}$  | 89  |
| 108 | $	ilde{N}_{N,dt}$ | pN     | granulation dynamics   | conversion | $ms^{-1}$ | 87  |

# 10 liquid

| var symbol documentation type units e |
|---------------------------------------|
|---------------------------------------|

## 11 gas

| V | var e | symbol | documentation | type | units | eqs |
|---|-------|--------|---------------|------|-------|-----|
|---|-------|--------|---------------|------|-------|-----|

# 12 control-gas

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

# 13 gas-control

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

### 14 control-liquid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

### 15 liquid-control

| var | symbol  | documentation | type  | units  | eas |
|-----|---------|---------------|-------|--------|-----|
| Vai | Бу шьог | documentation | oy pe | diffes | eqs |

#### 16 control-materialDB

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

#### 17 materialDB-control

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

### 18 control-mixing

|  |       |           |               | I    |        |      |
|--|-------|-----------|---------------|------|--------|------|
|  | var   | symbol    | documentation | type | units  | eas  |
|  | 7 002 | 5,1115-01 |               | JPC  | GIIIOS | o qo |

### 19 mixing-control

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

#### 20 control-solid

|    |                   | T      |               |      |       | 1   |  |  |
|----|-------------------|--------|---------------|------|-------|-----|--|--|
|    | var               | symbol | documentation | type | units | eqs |  |  |
| 21 | solid-control     |        |               |      |       |     |  |  |
|    | var               | symbol | documentation | type | units | eqs |  |  |
| 22 | gas-materialD     | В      |               |      |       |     |  |  |
|    | var               | symbol | documentation | type | units | eqs |  |  |
| 23 | 23 materialDB-gas |        |               |      |       |     |  |  |
|    | var               | symbol | documentation | type | units | eqs |  |  |
| 24 | gas-mixing        |        |               |      |       |     |  |  |
|    | var               | symbol | documentation | type | units | eqs |  |  |
| 25 | mixing-gas        |        |               |      |       |     |  |  |
|    | var               | symbol | documentation | type | units | eqs |  |  |
|    | -                 |        |               | ı    | 1     |     |  |  |

# 26 liquid-materialDB

| var | symbol | documentation | type | units | eqs |
|-----|--------|---------------|------|-------|-----|
|     |        |               |      |       |     |

### 27 materialDB-liquid

|     | var                  | symbol | documentation                                 | type      | units                       | eqs |
|-----|----------------------|--------|-----------------------------------------------|-----------|-----------------------------|-----|
| 46  | $k^{q_x}{}_{A,dt}$   | kq_x   | link to get heat transfer coefficient         | transform | $kg K^{-1} s^{-3}$          | 32  |
| 99  | $k^{v_x}_{N,A,dt}$   | kv_x   | link to get volumetric flow coefficient       | transform | $kg^{-1}m^2s$               | 79  |
| 100 | $k^{d_x}_{NS,AS,dt}$ | kd_x   | link to get diffusional mass flow coefficient | transform | $kg^{-1}  m^{-4}  mol^2  s$ | 80  |

## 28 liquid-mixing

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

## 29 mixing-liquid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

## 30 material DB-mixing

| var symbol documentation type diffus |  |  | var | symbol |  | type | units | eqs |
|--------------------------------------|--|--|-----|--------|--|------|-------|-----|
|--------------------------------------|--|--|-----|--------|--|------|-------|-----|

## $31 \quad \text{mixing-materialDB}$

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

#### 32 materialDB-solid

|     | var                  | symbol      | documentation                        | type      | units    | eqs |
|-----|----------------------|-------------|--------------------------------------|-----------|----------|-----|
| 107 | $k_{granulate_{dt}}$ | k_granulate | link to kinetic granulation constant | transform | $s^{-1}$ | 86  |

### 33 solid-materialDB

|  |  |  | var | symbol | documentation | type | units | eqs |
|--|--|--|-----|--------|---------------|------|-------|-----|
|--|--|--|-----|--------|---------------|------|-------|-----|

### 34 mixing-solid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

### 35 solid-mixing

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|  |     |        |               | J I  |       | 1   |

## 36 gas-liquid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

### 37 gas-solid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

# 38 liquid-solid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

## 39 Equations

### 39.1 Model equations

| no | equation                                                                                                                                                   | documentation                                                      | layer                |
|----|------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|----------------------|
| 1  | $1\backslash 2:=Set(\#,-)$                                                                                                                                 | one half                                                           | root                 |
| 2  | 0 := Set(#, -)                                                                                                                                             | zero                                                               | root                 |
| 3  | $\partial t_{N,dt} := diffSpace(t_N)$                                                                                                                      | differential time                                                  | root                 |
| 4  | $T_N := \frac{\partial U_N}{\partial S_N}$                                                                                                                 | definition of temperature                                          | physical             |
| 5  | $p_N := \left( -\frac{\partial U_N}{\partial V_N} \right)$                                                                                                 | definition of pressure                                             | physical             |
| 7  | $H_N := U_N + p_N \cdot V_N$                                                                                                                               | definition of enthalpy                                             | physical             |
| 8  | $v_{xN} := \frac{\partial r_{xN}}{\partial t_N}$                                                                                                           | definition of velocity in x direction                              | physical             |
| 10 | $A_{xN} := r_{y_N} \cdot r_{z_N}$                                                                                                                          | area at location x                                                 | physical             |
| 14 | $c_{p_N} := \frac{\partial H_N}{\partial T_N}$                                                                                                             | heat capacity at constant pressure                                 | materialDB           |
| 18 | $\mu_{NS} := \frac{\partial U_N}{\partial n_{NS}}$                                                                                                         | definition of chemical potential                                   | physical             |
| 31 | $k^{q_x}{}_{A,dt} := \left(-P_{N,A,dt}\right) \stackrel{N}{\star} \left(\left(V_N\right)^{-1} \cdot \frac{\partial U_N}{\partial T_N} \cdot v_{xN}\right)$ | heat transfer coefficient for an event-<br>dynamic transfer system | materialDB           |
| 32 | $k^{q_x}{}_{A,dt} := k^{q_x}{}_{A,dt}$                                                                                                                     | link to get heat transfer coefficient                              | materialDB »> liquid |
| 39 | $\hat{q}_{N,dt} := F_{N,A} \stackrel{A}{\star} \left( (-k^{q_x}{}_{A,dt}) \cdot A_{xN} \cdot F_{N,A} \stackrel{N}{\star} T_N \right)$                      | heat flow                                                          | macroscopic          |
| 41 | $n^0{}_{NS} := Set(n_{NS}, -)$                                                                                                                             | initial condition                                                  | macroscopic          |
| 42 | $c_{NS} := (V_N)^{-1} \odot n_{NS}$                                                                                                                        | molar concentration                                                | macroscopic          |

| no | equation                                                                                           | documentation                                           | layer       |
|----|----------------------------------------------------------------------------------------------------|---------------------------------------------------------|-------------|
| 43 | $c_{KS} := P_{NS,KS} \stackrel{NS}{\star} c_{NS}$                                                  | molar concentrations assigned to reaction K             | macroscopic |
| 44 | $N_{NS,NK} := P_{NS,KS} \overset{KS}{\star} N_{K,KS} \overset{K}{\star} P_{K,NK}$                  | global stoichiomety block matrix                        | macroscopic |
| 45 | $T_{NK} := P_{N,NK} \stackrel{N}{\star} T_N$                                                       | temperature for reactions                               | macroscopic |
| 46 | $K_{NK,dt} := K^{o}_{K,dt} \odot exp(E^{A}_{K} \odot (R \cdot T_{NK})^{-1})$                       | reaction "constants"                                    | macroscopic |
| 47 | $c^o{}_{KS} := Set(c_{KS}, -)$                                                                     | norming concentration – probability must have no units! | macroscopic |
| 49 | $x_{KS} := \left(c^o{}_{KS}\right)^{-1} \cdot c_{KS}$                                              | normed concentration little like more fractions         | macroscopic |
| 52 | $\phi_{NK} := P_{NK,KS} \overset{KS}{\star} \left( \prod \left( x_{KS}^{N_{K,KS}} \right) \right)$ | probability for the reactions to occur                  | macroscopic |
| 53 | $\xi_{NK,dt} := K_{NK,dt} \cdot \phi_{NK}$                                                         | dyamic extend of reaction                               | macroscopic |
| 54 | $	ilde{n}_{NS,dt} := V_N \odot \left( N_{NS,NK} \stackrel{NK}{\star} \xi_{NK,dt} \right)$          | production term                                         | macroscopic |
| 57 | $d_A := \operatorname{sign}\left(F_{N,A} \stackrel{N}{\star} p_N\right)$                           | direction of flow relative to reference co-<br>ordinate | macroscopic |
| 58 | $S_{NS,AS} := 1 \backslash 2 \cdot (F_{NS,AS} - d_A \odot  F_{NS,AS} )$                            | selection of the flow sources                           | macroscopic |
| 59 | $c_{AS} := S_{NS,AS} \stackrel{NS}{\star} c_{NS}$                                                  | concentration in the convective flows                   | macroscopic |
| 60 | $\hat{n}_{NS,dt}^{v} := F_{NS,AS} \overset{AS}{\star} \left( \hat{V}_{A,dt} \odot c_{AS} \right)$  | convective species flow                                 | macroscopic |
| 62 | $\dot{n}_{NS,dt} := \hat{n}_{NS,dt}^v + \hat{n}_{NS,dt}^d + \tilde{n}_{NS,dt}$                     | differential species balances                           | macroscopic |
| 63 | $to_N := Set(t_N, -)$                                                                              | initial time                                            | root        |
| 64 | $te_N := Set(t_N, -)$                                                                              | end time                                                | root        |

| no | equation                                                                                                                                                                                                                           | documentation                                 | layer                                  |
|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------|----------------------------------------|
| 65 | $n_{NS} := \int_{to_N}^{te_N} \dot{n}_{NS,dt} \ dt_N + n^0_{NS}$                                                                                                                                                                   | integration of differential species balances  | macroscopic                            |
| 71 | $\rho_N := m_N \cdot (V_N)^{-1}$                                                                                                                                                                                                   | mass density                                  | materialDB                             |
| 76 | $k^{c_x}{}_{N,A,dt} := \left(-P_{N,A,dt}\right) \stackrel{N}{\star} \left(\lambda_S \stackrel{S \in NS}{\star} \left(\mu_{NS}\right)^{-1}\right) \cdot \left(V_N\right)^{-1} \cdot \frac{\partial U_N}{\partial p_N} \cdot v_{xN}$ | convective mass diffusivity per area          | materialDB                             |
| 78 | $k^{v_x}_{N,A,dt} := \left(\rho_N\right)^{-1} \cdot k^{c_x}_{N,A,dt}$                                                                                                                                                              | convective volume flow per area               | materialDB                             |
| 79 | $k^{v_x}{}_{N,A,dt} := k^{v_x}{}_{N,A,dt}$                                                                                                                                                                                         | link to get volumetric flow coefficient       | ${\rm material DB} \gg > {\rm liquid}$ |
| 80 | $k^{d_x}{}_{NS,AS,dt} := k^{d_x}{}_{NS,AS,dt}$                                                                                                                                                                                     | link to get diffusional mass flow coefficient | materialDB »> liquid                   |
| 81 | $k^{d_x}{}_{NS,AS,dt} := (-P_{NS,AS,dt}) $ $\left(v_{xN} \cdot (V_N)^{-1} \odot (\mu_{NS})^{-1}\right) \cdot \frac{\partial U_N}{\partial \mu_{NS}}$                                                                               | diffusional mass diffusivity per area.        | materialDB                             |
| 82 | $\hat{n}_{NS,dt}^{d} := A_{xN} \odot F_{NS,AS} \overset{AS}{\star} \left( \left( -k^{d_x}{}_{NS,AS,dt} \right) \cdot F_{NS,AS} \overset{NS}{\star} \mu_{NS} \right)$                                                               | diffusional species flows                     | macroscopic                            |
| 83 | $\hat{V}_{A,dt} := F_{N,A} \stackrel{A}{\star} (-k^{v_x}_{N,A,dt}) \cdot A_{xN} \cdot F_{N,A} \stackrel{N}{\star} p_N$                                                                                                             | volumetric flow                               | macroscopic                            |
| 86 | $k_{granulate_{dt}} := k_{granulate_{dt}}$                                                                                                                                                                                         | link to kinetic granulation constant          | ${\rm materialDB} \gg > {\rm solid}$   |
| 87 | $\tilde{N}_{N,dt} := k_{granulate_{dt}} \cdot r_N$                                                                                                                                                                                 | granulation dynamics                          | solid                                  |
| 89 | $N_{tN,dt} := \frac{\partial  \tilde{N}_{N,dt}}{\partial  r_N}$                                                                                                                                                                    | population balance PDE                        | solid                                  |
| 90 | $A_N := U_N - T_N \cdot S_N$                                                                                                                                                                                                       | definition of Helmholtz energy                | physical                               |
| 91 | $G_N := U_N + p_N \cdot V_N - T_N \cdot S_N$                                                                                                                                                                                       | definition of Gibbs free energy               | physical                               |
| 92 | $\mu_{NS} := \frac{\partial G_N}{\partial n_{NS}}$                                                                                                                                                                                 | chemical potential from Gibbs free energy     | physical                               |
| 93 | $\alpha_{N,dt} := a_{N,dt} \cdot sin(t_N \cdot f)$                                                                                                                                                                                 | stirrer function                              | mixing                                 |

#### 39.2 Instantiations