# Variables & Equations

### ProMo

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### 1 Variables

#### 2 root

|    | var   | symbol      | documentation                                | type    | units | eqs |
|----|---|-------------|--|---------|-------|-----|
| 11 | _   | I_tu        | identity mapping from $<$ t $>$ to $<$ u $>$ | network |       |     |
| 21 | $\begin{vmatrix} I_{t,u} \\ u_{N,t,u} \end{vmatrix}$      | u_Ntu       | input signal in control domain               | network |       |     |
| 9  | $oxed{S_{I,p}}$   | S_Ip        | selection matrix interface to control input  | network |       |     |
| 3  | $F^{source}_{N,I}$  | F_NI_source | incidence matrix NI source                   | network |       |     |
| 2  | $F_{N,A}$   | F           | incidence matrix                             | network |       |     |
| 19 | 4   | A_Npq       | mapping from inputs to outputs               | network |       |     |
| 7  | $egin{array}{c} A_{N,p,q} \ F^{source}_{N,A} \end{array}$ | F_NA_source | incidence matrix NA source                   | network |       |     |
| 5  | $F^{source}{}_{A,I}$                                      | F_AI_source | incidence matrix AI source                   | network |       |     |
| 17 | $cz_N$  | cz_N        | output from control                          | network |       |     |
| 13 | C   | S_Aq        | selection matrix arcs to outputs             | network |       |     |
| 14 | $egin{array}{c} S_{I,q} \ S_{N,p,q} \end{array}$          | S_Npu       | selection matrix for stacker                 | network |       |     |
| 16 | $mv_I$  | mv_I        | interface variable macro -> control          | network |       |     |

|     | var                | symbol     | documentation                                       | type    | units | eqs |
|-----|--------------------|------------|---|---------|-------|-----|
| 20  | $A_{N,t,u}$        | A_Ntu      | mapping from input elements to outputs              | network |       |     |
| 6   | $F^{sink}{}_{A,I}$ | F_AI_sink  | incidence matrix AI sink                            | network |       |     |
| 10  | $S_{I,q}$          | S_Iq       | selection matrix interface to control output        | network |       |     |
| 22  | $y_{N,t,u}$        | y_Ntu      | output signal in control domain                     | network |       |     |
| 8   | $F^{sink}_{N,A}$   | F_NA_sink  | incidence matrix NA sink                            | network |       |     |
| 4   | $F^{sink}_{N,I}$   | F_NI_sink  | incidence matrix NI sink                            | network |       |     |
| 15  | $S_{N,q,t}$        | S_Nqt      | selection matrix or splitter                        | network |       |     |
| 12  | $S_{A,p}$          | S_Ap       | selection matrix interface species-related measures | network |       |     |
| 18  | $cz_I$             | cz_I       | interface variable macro -> control                 | network |       |     |
| 27  | $I_{N,A}$          | I_NA       | identity mapping from $<$ N $>$ to $<$ A $>$        | network |       |     |
| 105 | $t^o$              | to         | starting time                                       | frame   | s     | 4   |
| 1   | t                  | t          | time  | frame   | s     |     |
| 107 | $\Delta t$         | t_interval | time interval                                       | frame   | s     | 6   |
| 106 | $t^e$              | te         | end time  | frame   | s     | 5   |

|     | var | symbol  | documentation            | type     | units | eqs |
|-----|-----|---------|--------------------------|----------|-------|-----|
| 104 |     | oneHalf | numerical value one half | constant |       | 3   |
|     | 0.5 |         |                          |          |       |     |
| 101 |     | value   | numerical value          | constant |       |     |
|     | #   |         |                          |          |       |     |
| 102 |     | zero    | numerical value zero     | constant |       | 1   |
|     | 0   |         |                          |          |       |     |
| 103 |     | one     | numerical value one      | constant |       | 2   |
|     | 1   |         |                          |          |       |     |

# 3 physical

|     | var   | symbol | documentation                        | type     | units                               | eqs |
|-----|---|--------|--------------------------------------|----------|-------------------------------------|-----|
| 23  |   | r_x    | x-coordinate                         | frame    | m                                   |     |
| 25  | $egin{array}{c} r_{xN} \ & & & \\ r_{zN} \ & & \end{array}$ | r_z    | z-coordinate                         | frame    | m                                   |     |
| 24  | $r_{yN}$  | r_y    | y-coordinate                         | frame    | m                                   |     |
| 110 | $V_N$   | v      | volume                               | state    | $m^3$                               | 7   |
| 111 | $n_{N,S}$   | n      | fundamental state – molar mass       | state    | mol                                 | 93  |
| 109 | $S_N$   | S      | fundamental state – internal entropy | state    | $kg  m^2  K^{-1}  s^{-2}$           |     |
| 144 | $C_N$   | С      | $fundamental\ state-charge$          | state    | A s                                 |     |
| 137 | $m_N$   | m      | mass                                 | state    | kg                                  | 30  |
| 108 | $U_N$   | U      | fundamental state – internal energy  | state    | $kgm^2s^{-2}$                       |     |
| 123 | R   | R      | gas constant                         | constant | $kg  m^2  mol^{-1}  K^{-1}  s^{-2}$ | 17  |
| 122 | $k^B$   | Boltz  | Boltzmann constant                   | constant | $kg  m^2  K^{-1}  s^{-2}$           |     |
| 121 | $N^A$   | Avo    | Avogadro constant                    | constant | $mol^{-1}$                          |     |
| 132 | $\lambda_S$   | Mm     | molecular masses                     | constant | $kgmol^{-1}$                        |     |

|      | var       | symbol | documentation           | type           | units        | eqs |
|------|-----------|--------|-------------------------|----------------|--------------|-----|
| 143  |           | rho    | density                 | secondaryState | $kg  m^{-3}$ | 36  |
| 1.10 | $ ho_N$   |        |                         |                | 9            |     |
| 149  | $A_{xzN}$ | Axz    | cross sectional are xz  | secondaryState | $m^2$        | 41  |
| 150  | 2214      | Ayz    | cross sectional area yz | secondaryState | $m^2$        | 42  |
|      | $A_{yzN}$ |        |                         |                |              |     |
| 148  |           | Axy    | cross sectional area xy | secondaryState | $m^2$        | 40  |
|      | $A_{xyN}$ |        |                         |                |              |     |

### 4 macroscopic

|     | var   | symbol | documentation  | type      | units             | eqs   |
|-----|---|--------|--|-----------|-------------------|-------|
| 210 |   | aq_z   | accumulation due to heat flow in z-direction                         | transport | $kgm^2s^{-3}$     | 105   |
| 152 | $\dot{q}_{zN}$  | fq_y   | heat flow in y-direction   | transport | $kg  m^2  s^{-3}$ | 44    |
| 208 | $\hat{q}_{yA}$  | aq_x   | accumulation due to heat flow in x-direction                         | transport | $kg  m^2  s^{-3}$ | 103   |
| 211 | $\dot{q}_{xN}$  | fw     | a fixed work flow to start with                                      | transport | $kg  m^2  s^{-3}$ | 106   |
| 151 | $\hat{w}_A$   | fq_x   | heat flow in x-direction   | transport | $kgm^2s^{-3}$     | 43    |
| 156 | $egin{aligned} \hat{q}_{xA} \ & \hat{n}^d_{zA,S} \end{aligned}$ | fnd_z  | diffusion flow in z-direction  | transport | $mol  s^{-1}$     | 48 91 |
| 158 |   | c_AS   | concentration in convective event-dynamic flow                       | transport | $m^{-3}  mol$     | 50    |
| 212 | $\dot{n}_{yN,S}^d$  | and_y  | accumulation due to diffusion in y-direction                         | transport | $mol  s^{-1}$     | 107   |
| 160 | $\hat{n}^c_{xA,S}$  | fnc_x  | molar convective flow in x-direction                                 | transport | $mol  s^{-1}$     | 52    |
| 206 | $\dot{H}_{yN}^d$  | aHnd_y | accumulation of enthalpy due to diffusional mass flow in y-direction | transport | $kgm^2s^{-3}$     | 101   |
| 214 | $\dot{w}_N$   | aw     | accumulation of enthalpy due to work flow                            | transport | $kg  m^2  s^{-3}$ | 109   |
| 155 | $\hat{n}_{yA,S}^d$  | fnd_y  | diffusion flow in y-direction  | transport | $mol  s^{-1}$     | 47 90 |
| 194 | $\dot{n}_{xN,S}^{c}$  | anc_x  | accumulation of molar mass due to convection                         | transport | $mol  s^{-1}$     | 87    |

|     | var                            | symbol    | documentation  | type       | units              | eqs   |
|-----|--------------------------------|-----------|--|------------|--------------------|-------|
| 159 | $\hat{V}_A$                    | fV        | volumetric flow in x-direction                                       | transport  | $m^3  s^{-1}$      | 51    |
| 154 | $\hat{n}^d_{xA,S}$             | fnd_x     | diffusion flow in x-direction  | transport  | $mol  s^{-1}$      | 46 89 |
| 207 | $\dot{H}^d_{zN}$               | aHnd_z    | accumulation of enthalpy due to diffusional mass flow in z-direction | transport  | $kg  m^2  s^{-3}$  | 102   |
| 195 | $\dot{n}^d_{xN,S}$             | and_x     | accumulation due to diffusion in x-direction                         | transport  | $mol  s^{-1}$      | 88    |
| 209 | $\dot{q}_{yN}$                 | aq_y      | accumulation due to heat flow in y-direction                         | transport  | $kgm^2s^{-3}$      | 104   |
| 213 | $\dot{n}_{zN,S}^d$             | and_z     | accumulation due to diffusion in z-direction                         | transport  | $mol  s^{-1}$      | 108   |
| 153 | $\hat{q}_{zA}$                 | fq_z      | heat flow in z-direction   | transport  | $kg  m^2  s^{-3}$  | 45    |
| 205 | $\dot{H}^d_{xN}$               | aHnd_x    | accumulation of enthalpy due to diffusional mass flow in x-direction | transport  | $kg  m^2  s^{-3}$  | 100   |
| 157 | $d_A$                          | d         | flow direction of convective flow                                    | transport  |                    | 49    |
| 204 | $\dot{H}^c_{xN}$               | aHnc_x    | accumulation of enthalpy due to convective mass flow in x-direction  | transport  | $kg  m^2  s^{-3}$  | 99    |
| 234 | $\hat{m}_{N,A}$                | fm        | convective mass flow   | transport  | $kg s^{-1}$        | 133   |
| 191 | $\hat{k}_{y}^{d,Fick}{}_{A,S}$ | kdAFick_y | Fick diffusivity in arc and y-direction                              | properties | $ms^{-1}$          | 84    |
| 186 | $k_{xA}^q$                     | kqA_x     | thermal conductivity in arc and x-direction                          | properties | $kg K^{-1} s^{-3}$ | 79    |
| 189 | $\rho_A$                       | rhoA      | density in arc   | properties | $kg m^{-3}$        | 82    |

|     | var                                   | symbol      | documentation                                       | type       | units                       | eqs |
|-----|---------------------------------------|-------------|---|------------|-----------------------------|-----|
| 185 | $k_{zA}^c$                            | kcA_z       | convecive mass conductivity in arc and y-direction  | properties | $m^{-1} s$                  | 78  |
| 188 | $oxed{k_{zA}^q}$                      | kqA_z       | thermal conductivity in arc and z-direction         | properties | $kg K^{-1} s^{-3}$          | 81  |
| 181 | $k_{yA,S}^d$                          | kdA_y       | diffusivity in arc and y-direction                  | properties | $kg^{-1} m^{-4} mol^2 s$    | 74  |
| 184 | $egin{array}{c} k_{yA}^c \end{array}$ | kcA_y       | convective mass conductivity in arc and y-direction | properties | $m^{-1} s$                  | 77  |
| 193 | $h_{A,S}$                             | hA          | partial molar enthalpiies in arc                    | properties | $kg  m^2  mol^{-1}  s^{-2}$ | 86  |
| 187 | $k_{yA}^q$                            | kqA_y       | thermal conductivity in arc and y-direction         | properties | $kg K^{-1} s^{-3}$          | 80  |
| 182 | $k_{zA,S}^d$                          | kdA_z       | diffusivity in arc and z-direction                  | properties | $kg^{-1} m^{-4} mol^2 s$    | 75  |
| 183 | $k_{xA}^c$                            | kcA_x       | convective mass conductivity in arc and x diretion  | properties | $m^{-1} s$                  | 76  |
| 180 | $k^d_{xA,S}$                          | kdA_x       | diffusivity in arc and x-direction                  | properties | $kg^{-1} m^{-4} mol^2 s$    | 73  |
| 190 | $\hat{k}_{x}^{d,Fick}{}_{A,S}$        | kdAFick_x   | Fick's diffusivity in arc and x-direction           | properties | $ms^{-1}$                   | 83  |
| 192 | $\hat{k}_z^{d,Fick}{}_{A,S}$          | kdAFick_z   | Fick diffusivity in arc and z-direction             | properties | $ms^{-1}$                   | 85  |
| 219 | $R^e_{\ N}$                           | elResistant | electrical resistant                                | properties | $kg m^2 A^{-2} s^{-3}$      | 115 |
| 116 | $A_N$                                 | A           | Helmholtz energy                                    | state      | $kgm^2s^{-2}$               | 12  |
| 203 | $n^o{}_{N,S}$                         | no          | initial mass  | state      | mol                         | 98  |

|     | var   | symbol          | documentation                               | type           | units                       | eqs       |
|-----|---|-----------------|---|----------------|-----------------------------|-----------|
| 117 | $G_N$   | G               | Gibbs free energy                           | state          | $kgm^2s^{-2}$               | 13        |
| 216 | $H^o{}_N$   | Но              | initial enthalpy                            | state          | $kg  m^2  s^{-2}$           | 111       |
| 115 | $H_N$   | н               | Enthalpy                                    | state          | $kgm^2s^{-2}$               | 11<br>112 |
| 112 | $p_N$   | р               | thermodynamic pressure                      | effort         | $kgm^{-1}s^{-2}$            | 8         |
| 161 | $\mu^{o}{}_{N,S}$   | chemPotStandard | instantiating standard chemical potential   | effort         | $kg  m^2  mol^{-1}  s^{-2}$ | 53        |
| 113 | $T_N$   | Т               | temperature                                 | effort         | K                           | 9 121     |
| 114 |   | chemPot         | chemical potential                          | effort         | $kg m^2 mol^{-1} s^{-2}$    | 10 54     |
| 217 | $\mu_{N,S}$   | Ue              | electrical potential – voltage              | effort         | $kg  m^2  A^{-1} s^{-3}$    | 113       |
| 140 | $U^e{}_N$   | x               | mole fraction                               | secondaryState |                             | 33        |
| 119 | $x_{N,S}$   | v_y             | velocity in y-direction                     | secondaryState | $ms^{-1}$                   | 15        |
| 118 | $v_{yN}$  | v_x             | velocity in x-direction                     | secondaryState | $ms^{-1}$                   | 14        |
| 139 | $v_{xN}$  | nt              | total number of moles                       | secondaryState | mol                         | 32        |
| 124 | $n^t{}_N$   | Ср              | total heat capacity at constant pressure    | secondaryState | $kg  m^2  K^{-1}  s^{-2}$   | 18<br>117 |
| 141 | $egin{array}{ccc} C_{pN} & & & & \\ c_{pN} & & & & \end{array}$ | ср              | specific heat capacity at constant pressure | secondaryState | $m^2 K^{-1} s^{-2}$         | 34<br>120 |

|     | var  | symbol         | documentation                              | type              | units                     | eqs |
|-----|--|----------------|--|-------------------|---------------------------|-----|
| 120 |  | v_z            | velocity in z-direction                    | secondaryState    | $ms^{-1}$                 | 16  |
| 222 | $egin{array}{c} v_{zN} \\ T^{ref}{}_{N} \end{array}$ | T_ref          | reference temperature                      | secondaryState    | K                         | 119 |
| 125 | $C_{VN}$   | CV             | total heat capacity at constant volume     | secondaryState    | $kg  m^2  K^{-1}  s^{-2}$ | 19  |
| 142 | $c_{VN}$   | cV             | specific heat capacity at constant volume  | secondaryState    | $m^2 K^{-1} s^{-2}$       | 35  |
| 138 | $c_{N,S}$  | С              | molar concentration                        | secondaryState    | $m^{-3}  mol$             | 31  |
| 136 | $h_{N,S}$  | h              | partial molar enthalpies                   | secondaryState    | $kg m^2 mol^{-1} s^{-2}$  | 29  |
| 202 | $	ilde{n}_{N,S}$                                     | np             | link variable np to interface macroscopic  | conversion        | $m^{-3}  mol  s^{-1}$     | 97  |
| 220 | $\dot{U}^e{}_A$                                      | dUe            | Kirkhoffs first law                        | diffState         | $kg m^2 A^{-1} s^{-3}$    | 116 |
| 215 | $\dot{H}_N$  | dH             | accumulation of enthalpy                   | diffState         | $kg  m^2  s^{-3}$         | 110 |
| 196 | $\dot{n}_{N,S}$                                      | an             | differential mass balance without reaction | diffState         | $mol  s^{-1}$             | 92  |
| 218 | $I^e{}_N$  | current        | current definition                         | internalTransport | A                         | 114 |
| 223 | $T^n{}_N$  | T_meas_norming | value to norm measurement of temperature   | observation       | K                         | 122 |
| 224 | $ar{T}_N$  | T_meas         | temperature measurement                    | observation       |                           | 123 |

# 5 reactions

|     | var                 | symbol      | documentation                                 | type           | units                       | eqs |
|-----|---------------------|-------------|---|----------------|-----------------------------|-----|
| 26  |                     | N           | stoichiometric matrix                         | constant       |                             |     |
| 197 | $N_{S,K}$ $E^a{}_K$ | Ea          | Arrhenius activation energy                   | constant       | $kg  m^2  mol^{-1}  s^{-2}$ |     |
| 198 | $K^o{}_K$           | Ко          | Arrhenius frequency factor                    | constant       | $m^{-3}  mol  s^{-1}$       |     |
| 167 | $T_{N,p}$           | Т           | link variable T to interface reactions        | effort         | K                           | 60  |
| 163 | $c_{N,S,p}$         | С           | link variable c to interface reactions        | secondaryState | $m^{-3}  mol$               | 56  |
| 165 | $x_{N,S,p}$         | x           | link variable <b>x</b> to interface reactions | secondaryState |                             | 58  |
| 168 | $f_{N,S,K,p}$       | factor      | factor for probability computation            | conversion     |                             | 61  |
| 169 | $\xi_{N,K,p}$       | probability | probability of reaction to take place         | conversion     |                             | 62  |
| 200 | $	ilde{n}_{N,S,q}$  | np          | production from reaction set                  | conversion     | $m^{-3}  mol  s^{-1}$       | 95  |
| 199 | $K_{N,K,p}$         | K           | Arrhenius reaction "constant"                 | conversion     | $m^{-3}  mol  s^{-1}$       | 94  |

# 6 macroscopic-reactions

|     | var         | symbol | documentation   | type | units         | eqs |
|-----|-------------|--------|---|------|---------------|-----|
| 164 | $\_x_{I,S}$ | _x     | $\begin{array}{c} \mbox{link variable x to interface macroscopic } > \mbox{reactions} \\ \mbox{with source:} \mbox{node} \end{array}$ | get  |               | 57  |
| 166 | $\_T_I$     | _Т     | link variable T to interface macroscopic »> reactions with source:node  | get  | K             | 59  |
| 162 | $\_c_{I,S}$ | _c     | link variable c to interface macroscopic »> reactions with source:node  | get  | $m^{-3}  mol$ | 55  |

# 7 reactions-macroscopic

|     | var          | symbol | documentation   | type | units                 | eqs |
|-----|--------------|--------|---|------|-----------------------|-----|
| 201 | $\_np_{I,S}$ | _np    | link variable np to interface reactions »> macroscopic with source:node | get  | $m^{-3}  mol  s^{-1}$ | 96  |

# 8 Equations

### 9 Generic

| no | equation   | documentation            | layer       |
|----|--|--------------------------|-------------|
| 1  | $0 := \mathbf{Instantiate}(\#, \#)$                  | numerical value zero     | root        |
| 2  | $1 := \mathbf{Instantiate}(\#, \#)$                  | numerical value one      | root        |
| 3  | $0.5 := \mathbf{Instantiate}(\#, \#)$                | numerical value one half | root        |
| 4  | $t^o := \mathbf{Instantiate}(t, \#)$                 | starting time            | root        |
| 5  | $t^e := \mathbf{Instantiate}(t, \#)$                 | end time                 | root        |
| 6  | $\Delta t := \mathbf{Instantiate}(t,\#)$             | time interval            | root        |
| 7  | $V_N := r_{xN} \cdot r_{yN} \cdot r_{zN}$            | volume                   | physical    |
| 8  | $p_N := \frac{\partial U_N}{\partial V_N}$           | thermodynamic pressure   | physical    |
| 9  | $T_N := \frac{\partial U_N}{\partial S_N}$           | temperature              | macroscopic |
| 10 | $\mu_{N,S} := \frac{\partial U_N}{\partial n_{N,S}}$ | chemical potential       | macroscopic |
| 11 | $H_N := U_N - p_N \cdot V_N$                         | Enthalpy                 | macroscopic |
| 12 | $A_N := U_N - T_N \cdot S_N$                         | Helmholtz energy         | macroscopic |
| 13 | $G_N := U_N + p_N \cdot V_N - T_N \cdot S_N$         | Gibbs free energy        | macroscopic |
| 14 | $v_{xN} := \frac{\partial r_{xN}}{\partial t}$       | velocity in x-direction  | macroscopic |
| 15 | $v_{yN} := \frac{\partial r_{yN}}{\partial t}$       | velocity in y-direction  | macroscopic |

| no | equation   | documentation                               | layer       |
|----|--|---|-------------|
| 16 | $v_{zN} := \frac{\partial r_{zN}}{\partial t}$                                 | velocity in z-direction                     | macroscopic |
| 17 | $R := N^A \cdot k^B$   | gas constant                                | physical    |
| 18 | $C_{pN} := \frac{\partial H_N}{\partial T_N}$                                  | total heat capacity at constant pressure    | macroscopic |
| 19 | $C_{VN} := \frac{\partial U_N}{\partial T_N}$                                  | total heat capacity at constant volume      | macroscopic |
| 29 | $h_{N,S} := H_N \cdot \left( n_{N,S} \right)^{-1}$                             | partial molar enthalpies                    | macroscopic |
| 30 | $m_N := \lambda_S \stackrel{S}{\star} n_{N,S}$                                 | mass  | macroscopic |
| 31 | $c_{N,S} := (V_N)^{-1} \cdot n_{N,S}$  | molar concentration                         | macroscopic |
| 32 | $n^{t}{}_{N} := \mathbf{reduceSum}\left(n_{N,S}, S\right)$                     | total number of moles                       | macroscopic |
| 33 | $x_{N,S} := \left(n^t{}_N\right)^{-1} \cdot n_{N,S}$                           | mole fraction                               | macroscopic |
| 34 | $c_{pN} := C_{pN} \cdot (m_N)^{-1}$  | specific heat capacity at constant pressure | physical    |
| 35 | $c_{VN} := C_{VN} \cdot \left( m_N \right)^{-1}$                               | specific heat capacity at constant volume   | macroscopic |
| 36 | $\rho_N := (V_N)^{-1} \cdot m_N$   | density                                     | physical    |
| 40 | $A_{xyN} := r_{xN} . r_{yN}$   | cross sectional area xy                     | physical    |
| 41 | $A_{xzN} := r_{xN} \cdot r_{zN}$   | cross sectional are xz                      | physical    |
| 42 | $A_{yzN} := r_{yN} \cdot r_{zN}$   | cross sectional area yz                     | physical    |
| 43 | $\hat{q}_{xA} := k_{xA}^q \cdot A_{yzN} \cdot F_{N,A} \stackrel{N}{\star} T_N$ | heat flow in x-direction                    | macroscopic |
| 44 | $\hat{q}_{yA} := k_{yA}^q \cdot A_{xzN} \cdot F_{N,A} \stackrel{N}{\star} T_N$ | heat flow in y-direction                    | macroscopic |

| no | equation  | documentation  | layer       |
|----|---|--|-------------|
| 45 | $\hat{q}_{zA} := k_{zA}^q \cdot A_{xyN} \cdot F_{N,A} \stackrel{N}{\star} T_N$  | heat flow in z-direction                             | macroscopic |
| 46 | $\hat{n}^d_{xA,S} := \hat{k}^{d,Fick}_{x}{}_{A,S} \cdot A_{yzN} \cdot F_{N,A} \stackrel{N}{\star} c_{N,S}$  | Fick diffusion flow in x-direction                   | macroscopic |
| 47 | $\hat{n}_{yA,S}^d := \hat{k}_y^{d,Fick}{}_{A,S} \cdot A_{xzN} \cdot F_{N,A} \stackrel{N}{\star} c_{N,S}$  | Fick diffusion flow in y-direction                   | macroscopic |
| 48 | $\hat{n}_{zA,S}^d := \hat{k}_z^{d,Fick}{}_{A,S} \cdot (A_{xyN} \cdot F_{N,A}) \stackrel{N}{\star} c_{N,S}$  | Fick diffusion flow in z-direction                   | macroscopic |
| 49 | $d_A := \mathbf{sign}\left(F_{N,A} \overset{N}{\star} p_N\right)$   | flow direction of convective flow                    | macroscopic |
| 50 | $c_{A,S} := (0.5 \cdot (F_{N,A} - d_A \cdot  F_{N,A} )) \stackrel{N}{\star} c_{N,S}$  | concentration in convective event-<br>dynamic flow   | macroscopic |
| 51 | $\hat{V}_A := (\rho_A)^{-1} \cdot k_{xA}^c \cdot A_{yzN} \cdot F_{N,A} \stackrel{N}{\star} p_N$   | volumetric flow in x-direction                       | macroscopic |
| 52 | $\hat{n}^c_{xA,S} := \hat{V}_A \cdot c_{A,S}$   | molar convective flow in x-direction                 | macroscopic |
| 53 | $\mu^o{}_{N,S} := \mathbf{Instantiate}(\mu_{N,S}, \#)$  | instantiating standard chemical potential            | macroscopic |
| 54 | $\mu_{N,S} := \mu^o{}_{N,S} + R \cdot T_N \cdot \ln\left(x_{N,S}\right)$  | chemical potential standard model with mole fraction | macroscopic |
| 61 | $f_{N,S,K,p} := x_{N,S,p}(( N_{S,K} ))$   | factor for probability computation                   | reactions   |
| 62 | $\xi_{N,K,p} := \prod_S f_{N,S,K,p}$  | probability of reaction to take place                | reactions   |
| 73 | $k_{xA,S}^d := I_{N,A} * \left( \left( \mu_{N,S} \right)^{-1} \cdot \left( v_{xN} \cdot \left( \left( V_N \right)^{-1} \cdot \frac{\partial U_N}{\partial \mu_{N,S}} \right) \right) \right)$ | diffusivity in arc and x-direction                   | macroscopic |
| 74 | $k_{yA,S}^d := I_{N,A} * \left( (\mu_{N,S})^{-1} \cdot \left( v_{yN} \cdot \left( (V_N)^{-1} \cdot \frac{\partial U_N}{\partial \mu_{N,S}} \right) \right) \right)$                           | diffusivity in arc and y-direction                   | macroscopic |
| 75 | $k_{zA,S}^d := I_{N,A} * \left( (\mu_{N,S})^{-1} \cdot \left( v_{zN} \cdot \left( (V_N)^{-1} \cdot \frac{\partial U_N}{\partial \mu_{N,S}} \right) \right) \right)$                           | diffusivity in arc and z-direction                   | macroscopic |
| 76 | $k_{xA}^c := I_{N,A} \stackrel{N}{\star} \left( \left( \lambda_S \stackrel{S}{\star} (\mu_{N,S})^{-1} \right) \cdot (V_N)^{-1} \cdot \frac{\partial U_N}{\partial p_N} \cdot v_{xN} \right)$  | convective mass conductivity in arc and x diretion   | macroscopic |

| no | equation   | documentation                                       | layer       |
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| 77 | $k_{yA}^c := I_{N,A} \stackrel{N}{\star} \left( \left( \lambda_S \stackrel{S}{\star} (\mu_{N,S})^{-1} \right) \cdot (V_N)^{-1} \cdot \frac{\partial U_N}{\partial p_N} \cdot v_{yN} \right)$ | convective mass conductivity in arc and y-direction | macroscopic |
| 78 | $k_{zA}^c := I_{N,A} * \left( \left( \lambda_S * (\mu_{N,S})^{-1} \right) \cdot (V_N)^{-1} \cdot \frac{\partial U_N}{\partial p_N} \cdot v_{zN} \right)$                                     | convecive mass conductivity in arc and y-direction  | macroscopic |
| 79 | $k_{xA}^{q} := I_{N,A} * \left( \left( V_{N} \right)^{-1} . C_{pN} . v_{xN} \right)$   | thermal conductivity in arc and x-direction         | macroscopic |
| 80 | $k_{yA}^q := I_{N,A} \stackrel{N}{\star} \left( (V_N)^{-1} \cdot C_{pN} \cdot v_{yN} \right)$  | thermal conductivity in arc and y-direction         | macroscopic |
| 81 | $k_{zA}^q := I_{N,A} \stackrel{N}{\star} \left( (V_N)^{-1} \cdot C_{pN} \cdot v_{zN} \right)$  | thermal conductivity in arc and z-direction         | macroscopic |
| 82 | $ \rho_A := I_{N,A} \overset{N}{\star} \rho_N $  | density in arc                                      | macroscopic |
| 83 | $\hat{k}_x^{d,Fick}{}_{A,S} := I_{N,A} \stackrel{N}{\star} \left( v_{xN} \cdot \frac{\partial U_N}{\partial \mu_{N,S}} \cdot (n_{N,S})^{-1} \right)$   | Fick's diffusivity in arc and x-direction           | macroscopic |
| 84 | $\hat{k}_y^{d,Fick}{}_{A,S} := I_{N,A} \stackrel{N}{\star} \left( v_{yN} \cdot \frac{\partial U_N}{\partial \mu_{N,S}} \cdot (n_{N,S})^{-1} \right)$   | Fick diffusivity in arc and y-direction             | macroscopic |
| 85 | $\hat{k}_z^{d,Fick}{}_{A,S} := I_{N,A} \stackrel{N}{\star} \left( v_{zN} \cdot \frac{\partial U_N}{\partial \mu_{N,S}} \cdot (n_{N,S})^{-1} \right)$   | Fick diffusivity in arc and z-direction             | macroscopic |
| 86 | $h_{A,S} := I_{N,A} \stackrel{N}{\star} h_{N,S}$   | partial molar enthalpiies in arc                    | macroscopic |
| 87 | $\dot{n}_{xN,S}^c := F_{N,A} \stackrel{A}{\star} \hat{n}_{xA,S}^c$   | accumulation of molar mass due to convection        | macroscopic |
| 88 | $\dot{n}^d_{xN,S} := F_{N,A} \stackrel{A}{\star} \hat{n}^d_{xA,S}$   | accumulation due to diffusion in x-direction        | macroscopic |
| 89 | $\hat{n}_{xA,S}^d := k_{xA,S}^d \cdot (A_{yzN} \cdot F_{N,A}) \overset{N}{\star} \mu_{N,S}$  | Fick diffusion flow in x-direction                  | macroscopic |
| 90 | $\hat{n}_{yA,S}^d := k_{yA,S}^d \cdot (A_{yzN} \cdot F_{N,A}) \stackrel{N}{\star} \mu_{N,S}$   | Fick diffusion flow in y-direction                  | macroscopic |
| 91 | $\hat{n}_{zA,S}^d := k_{zA,S}^d \cdot (A_{xyN} \cdot F_{N,A}) \stackrel{N}{\star} \mu_{N,S}$   | mass diffusion flow in z-direction                  | macroscopic |

| no  | equation   | documentation  | layer       |
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| 92  | $\dot{n}_{N,S} := \dot{n}_{xN,S}^c + \dot{n}_{xN,S}^d + V_N \cdot \tilde{n}_{N,S}$                             | differential mass balance without reaction                           | macroscopic |
| 93  | $n_{N,S} := \int_{t^o}^{t^e} \dot{n}_{N,S} \ dt + n^o{}_{N,S}$   | fundamental state – molar mass                                       | macroscopic |
| 94  | $K_{N,K,p} := K^{o}_{K} \cdot \exp\left((-E^{a}_{K}) \cdot (R \cdot T_{N,p})^{-1}\right)$                      | Arrhenius reaction "constant"  | reactions   |
| 95  | $\tilde{n}_{N,S,q} := A_{N,p,q} \star \left( N_{S,K} \star \left( K_{N,K,p} \cdot \xi_{N,K,p} \right) \right)$ | production from reaction set   | reactions   |
| 98  | $n^o{}_{N,S} := \mathbf{Instantiate}(n_{N,S}, \#)$   | initial mass   | macroscopic |
| 99  | $\dot{H}_{xN}^c := F_{N,A} \stackrel{A}{\star} \left( \hat{n}_{xA,S}^c \stackrel{S}{\star} h_{N,S} \right)$    | enthalpy accumulation due to convective flow in x-direction          | macroscopic |
| 100 | $\dot{H}_{xN}^d := F_{N,A} \stackrel{A}{\star} \left( \hat{n}_{xA,S}^d \stackrel{S}{\star} h_{N,S} \right)$    | accumulation of enthalpy due to diffusional mass flow in x-direction | macroscopic |
| 101 | $\dot{H}_{yN}^d := F_{N,A} \stackrel{A}{\star} \left( \hat{n}_{yA,S}^d \stackrel{S}{\star} h_{N,S} \right)$    | accumulation of enthalpy due to diffusional mass flow in y-direction | macroscopic |
| 102 | $\dot{H}_{zN}^d := F_{N,A} \stackrel{A}{\star} \left( \hat{n}_{zA,S}^d \stackrel{S}{\star} h_{N,S} \right)$    | accumulation of enthalpy due to diffusional mass flow in z-direction | macroscopic |
| 103 | $\dot{q}_{xN} := F_{N,A} \stackrel{A}{\star} \hat{q}_{xA}$   | accumulation due to heat flow in x-direction                         | macroscopic |
| 104 | $\dot{q}_{yN} := F_{N,A} \stackrel{A}{\star} \hat{q}_{yA}$   | accumulation due to heat flow in y-direction                         | macroscopic |
| 105 | $\dot{q}_{zN} := F_{N,A} \stackrel{A}{\star} \hat{q}_{zA}$   | accumulation due to heat flow in z-direction                         | macroscopic |
| 106 | $\hat{w}_A := \mathbf{Instantiate}(\hat{q}_{xA}, \#)$  | a fixed work flow to start with                                      | macroscopic |
| 107 | $\dot{n}_{yN,S}^d := F_{N,A} \stackrel{A}{\star} \hat{n}_{yA,S}^d$   | accumulation due to diffusion in y-direction                         | macroscopic |

| no  | equation  | documentation  | layer       |
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| 108 | $\dot{n}^d_{zN,S} := F_{N,A} \overset{A}{\star} \hat{n}^d_{zA,S}$   | accumulation due to diffusion in z-direction         | macroscopic |
| 109 | $\dot{w}_N := F_{N,A} \stackrel{A}{\star} \hat{w}_A$  | accumulation of enthalpy due to work flow            | macroscopic |
| 110 | $\dot{H}_N := \dot{H}_{xN}^c + \dot{H}_{xN}^d + \dot{H}_{yN}^d + \dot{H}_{zN}^d + \dot{q}_{xN} + \dot{q}_{yN} + \dot{q}_{zN} + \dot{w}_N$ | accumulation of enthalpy                             | macroscopic |
| 111 | $H^o{}_N := \mathbf{Instantiate}(H_N, \#)$  | initial enthalpy                                     | macroscopic |
| 112 | $H_N := \int_{t^o}^{t^e} \dot{H}_N \ dt + H^o{}_N$  | Enthalpy   | macroscopic |
| 113 | $U^e{}_N := \left(C_N\right)^{-1} . U_N$  | electrical potential – voltage                       | macroscopic |
| 114 | $I^e{}_N := \frac{dC_N}{dt}$  | current definition                                   | macroscopic |
| 115 | $R^{e}{}_{N} := (I^{e}{}_{N})^{-1} . U^{e}{}_{N}$   | electrical resistant                                 | macroscopic |
| 116 | $\dot{U}^e{}_A := F_{N,A} \stackrel{A}{\star} (R^e{}_N . I^e{}_N)$  | Kirkhoffs first law                                  | macroscopic |
| 117 | $C_{pN} := m_N \cdot c_{pN}$  | total heat capacity at constant pressure             | macroscopic |
| 119 | $T^{ref}{}_N := \mathbf{Instantiate}(T_N, \#)$  | reference temperature                                | macroscopic |
| 120 | $c_{pN} := \mathbf{Instantiate}(c_{pN}, \#)$  | constant specific heat capacity at constant pressure | macroscopic |
| 121 | $T_N := H_N \cdot (C_{pN})^{-1} + T^{ref}{}_N$  | temperature from constant heat capacity              | macroscopic |
| 122 | $T^n{}_N := \mathbf{Instantiate}(T_N, \#)$  | value to norm measurement of temperature             | macroscopic |
| 123 | $\bar{T}_N := T_N \cdot \left(T^n{}_N\right)^{-1}$  | temperature measurement                              | macroscopic |

| no  | equation                                 | documentation        | layer       |
|-----|--|----------------------|-------------|
| 133 | $\hat{m}_{N,A} := \hat{V}_A \cdot  ho_N$ | convective mass flow | macroscopic |

# 10 Interface Link Equation

| no | equation   | documentation      | layer                         |
|----|--|--------------------|-------------------------------|
| 55 | $\_c_{I,S} := F^{source}{}_{N,I} \overset{N}{\star} c_{N,S}$   | interface equation | macroscopic -> reactions      |
| 56 | $c_{N,S,p} := \left(F^{sink}_{N,I} \cdot \_c_{I,S}\right) \stackrel{I}{\star} S_{I,p}$   | interface equation | reactions                     |
| 57 | $x_{I,S} := F^{source}_{N,I} \overset{N}{\star} x_{N,S}$   | interface equation | macroscopic -> re-<br>actions |
| 58 | $x_{N,S,p} := (F^{sink}_{N,I} \cdot \_x_{I,S}) \overset{I}{\star} S_{I,p}$   | interface equation | reactions                     |
| 59 |  | interface equation | macroscopic -> reactions      |
| 60 | $T_{N,p} := (F^{sink}_{N,I} \cdot \_T_I) \overset{I}{\star} S_{I,p}$   | interface equation | reactions                     |
| 96 | $\_np_{I,S} := \mathbf{reduceSum}\left(\left(\left(F^{source}_{N,I} \stackrel{N}{\star} \tilde{n}_{N,S,q}\right).S_{I,q}\right), q\right)$ | interface equation | reactions -> macroscopic      |
| 97 | $\tilde{n}_{N,S} := F^{source}{}_{N,I} \stackrel{I}{\star} \_np_{I,S}$   | interface equation | macroscopic                   |