

Digital Image Fundamentals

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Review

- DIP 之發展背景
- Applications
- What is DIP?
- DIP Methods
- DIP Components

DIP Methods

- Image acquisition
- Image enhancement
- Image restoration
- Color image processing
- Image compression
- Morphological image processing
- Segmentation
- Representation and description
- Recognition

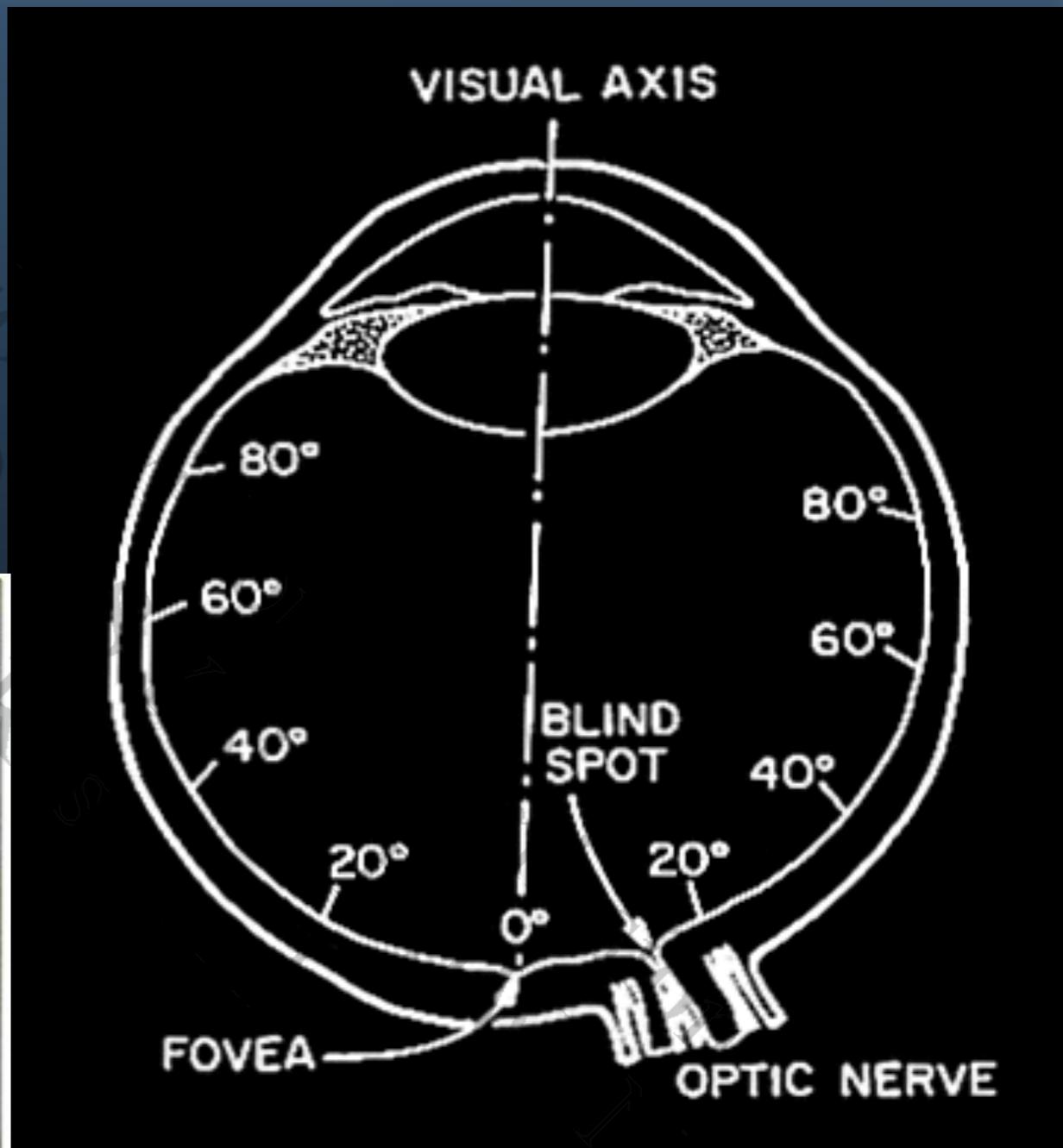
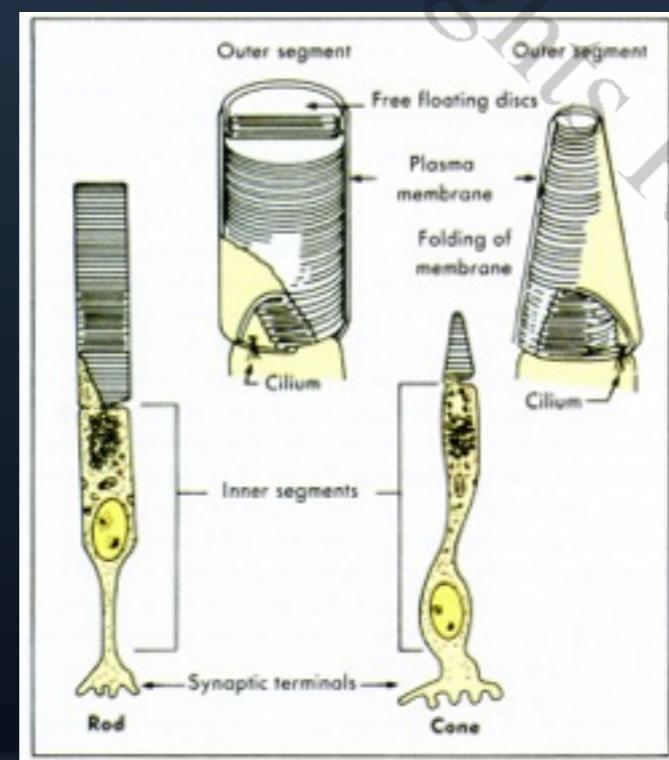
Visual Perception

Continuous Image Characterization

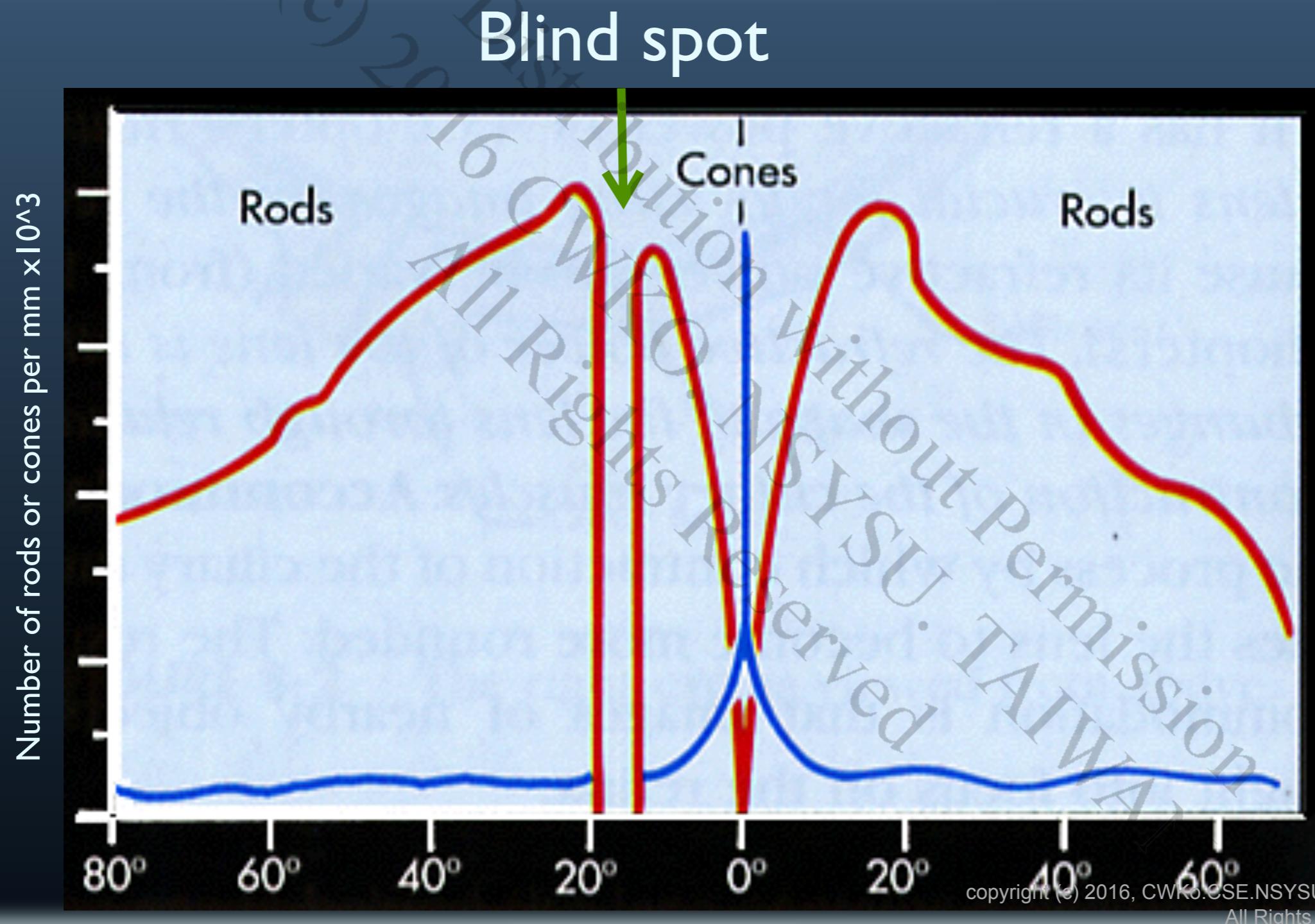
“Digital Image Processing”, William K. Pratt,
3rd ed., John Wiley & Sons. Inc., 2001

Structure of the Human Eye

- Cones
 - color : red, green, blue
- Rods
 - light



Cones & Rods



Blind Spot Test

<http://faculty.washington.edu/chudler/chvision.html>



To draw the blind spot tester on a piece of paper, make a small dot on the left side separated by about 6-8 inches from a small + on the right side. Close your right eye. Hold the image (or place your head from the computer monitor) about 20 inches away. With your left eye, look at the +. Slowly bring the image (or move your head) closer while looking at the +. At a certain distance, the dot will disappear from sight...this is when the dot falls on the blind spot of your retina. Reverse the process. Close your left eye and look at the dot with your right eye. Move the image slowly closer to you and the + should disappear.



Blind Spot Test

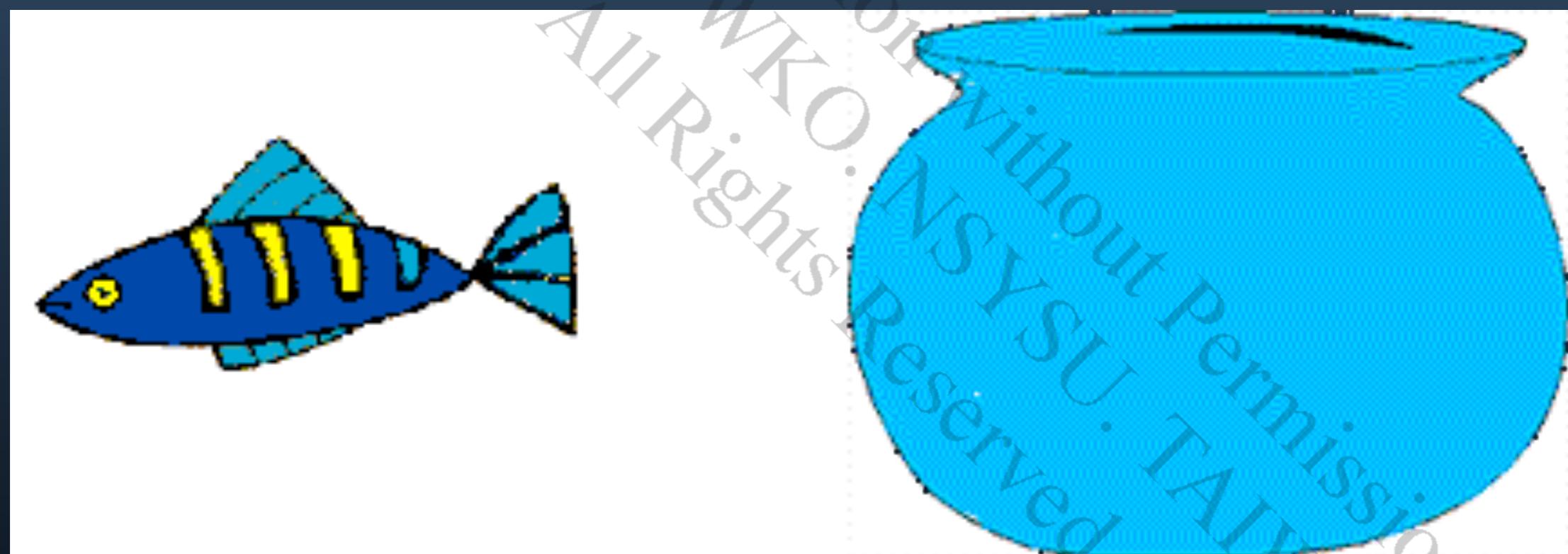
<http://faculty.washington.edu/chudler/chvision.html>



9 8 7 6 5 4 3 2 1

Close your right eye. With your left eye, look at the numbers on the right side, starting with the number "1." You should be able to see the "sad face" in your peripheral vision. Keep your head still, and with your left eye, look at the other numbers.

Creating an afterimage

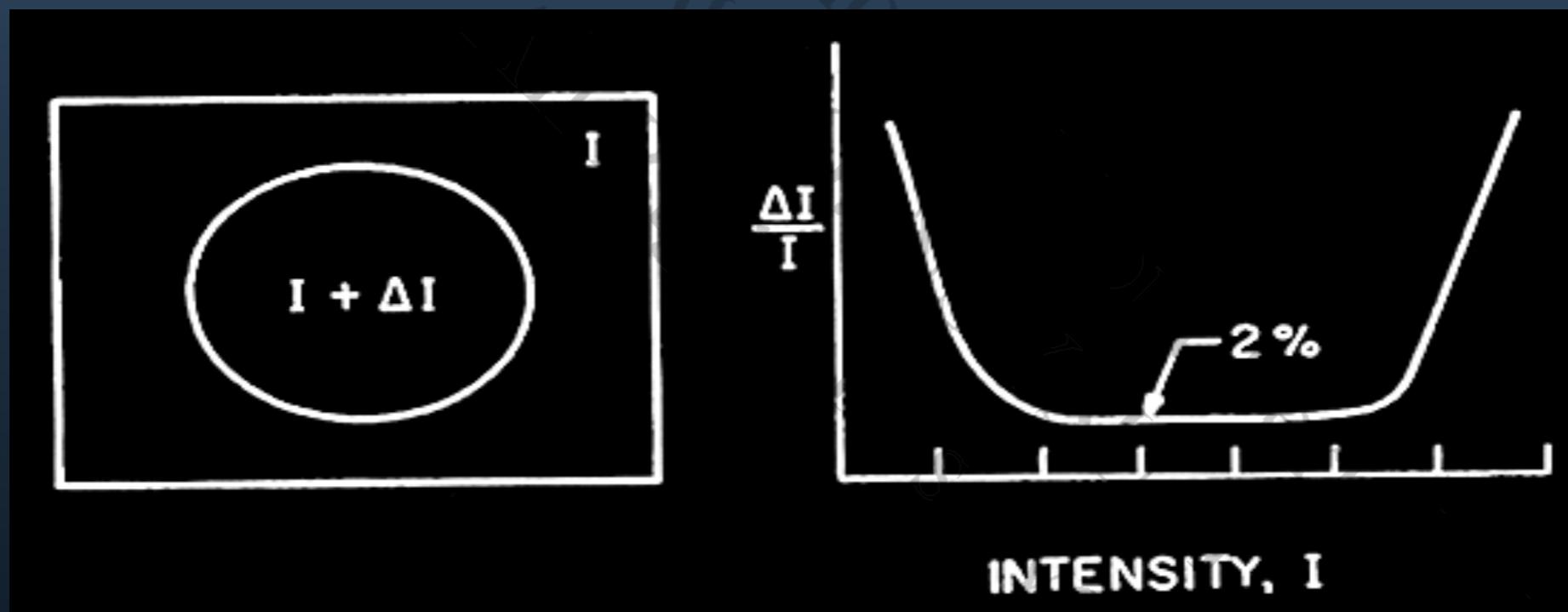


Visual Phenomena

- Contrast Sensitivity
- The response of the eye to changes in the intensity of illumination is known to be nonlinear.

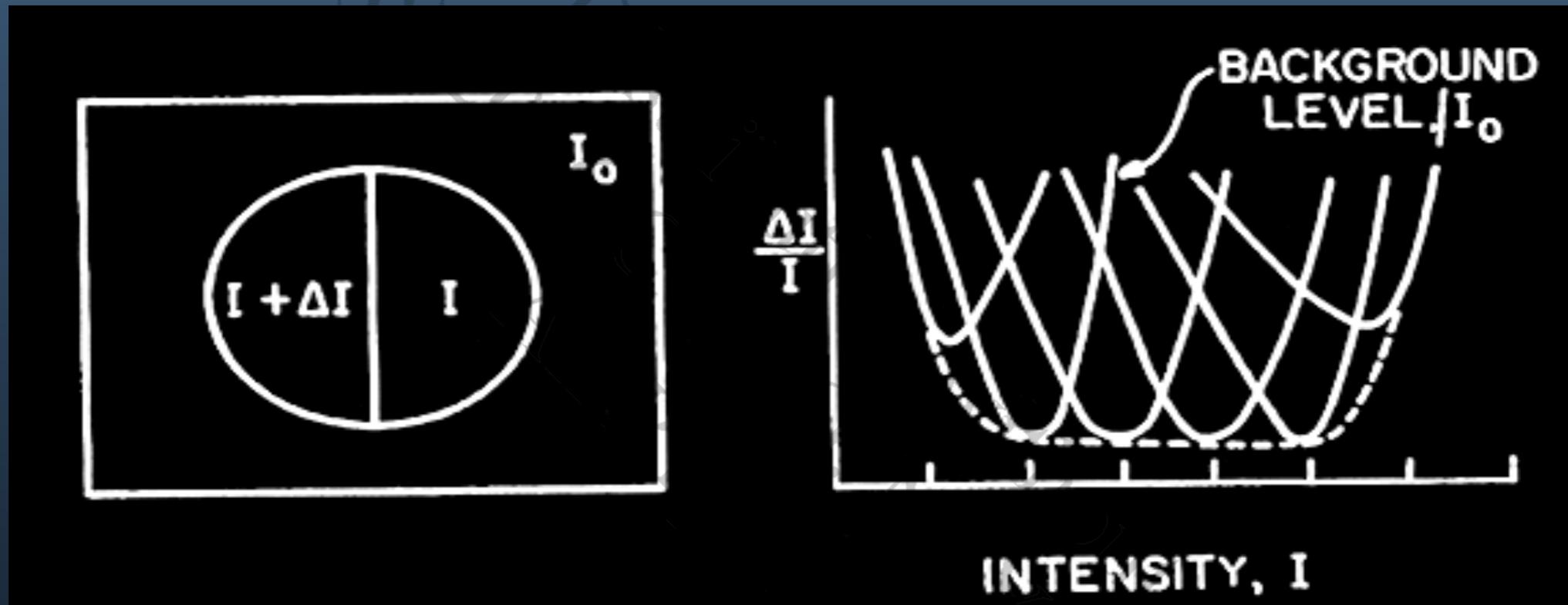
Visual Phenomena (1)

- Contrast Sensitivity
 - ΔI : 剛好使人能區別



$\frac{\Delta I}{I}$ = Weber fraction (ratio)
= 0.02 for wide ranges of values of I

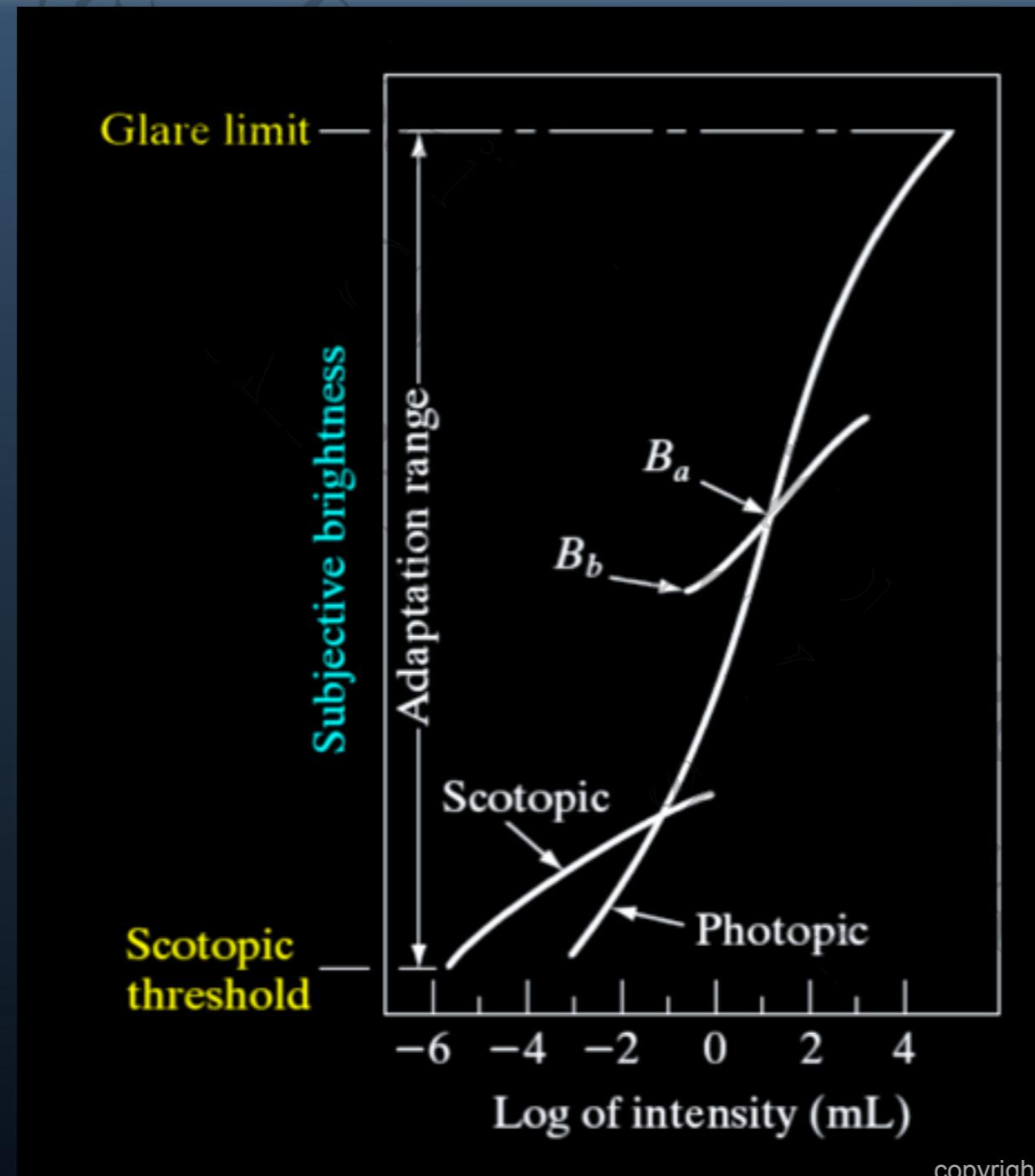
Visual Phenomena (1)



$$\frac{d}{dI} \log I = \frac{1}{I} \rightarrow d[\log I] = \frac{dI}{I} \rightarrow \Delta[\log I] = \frac{\Delta I}{I}$$

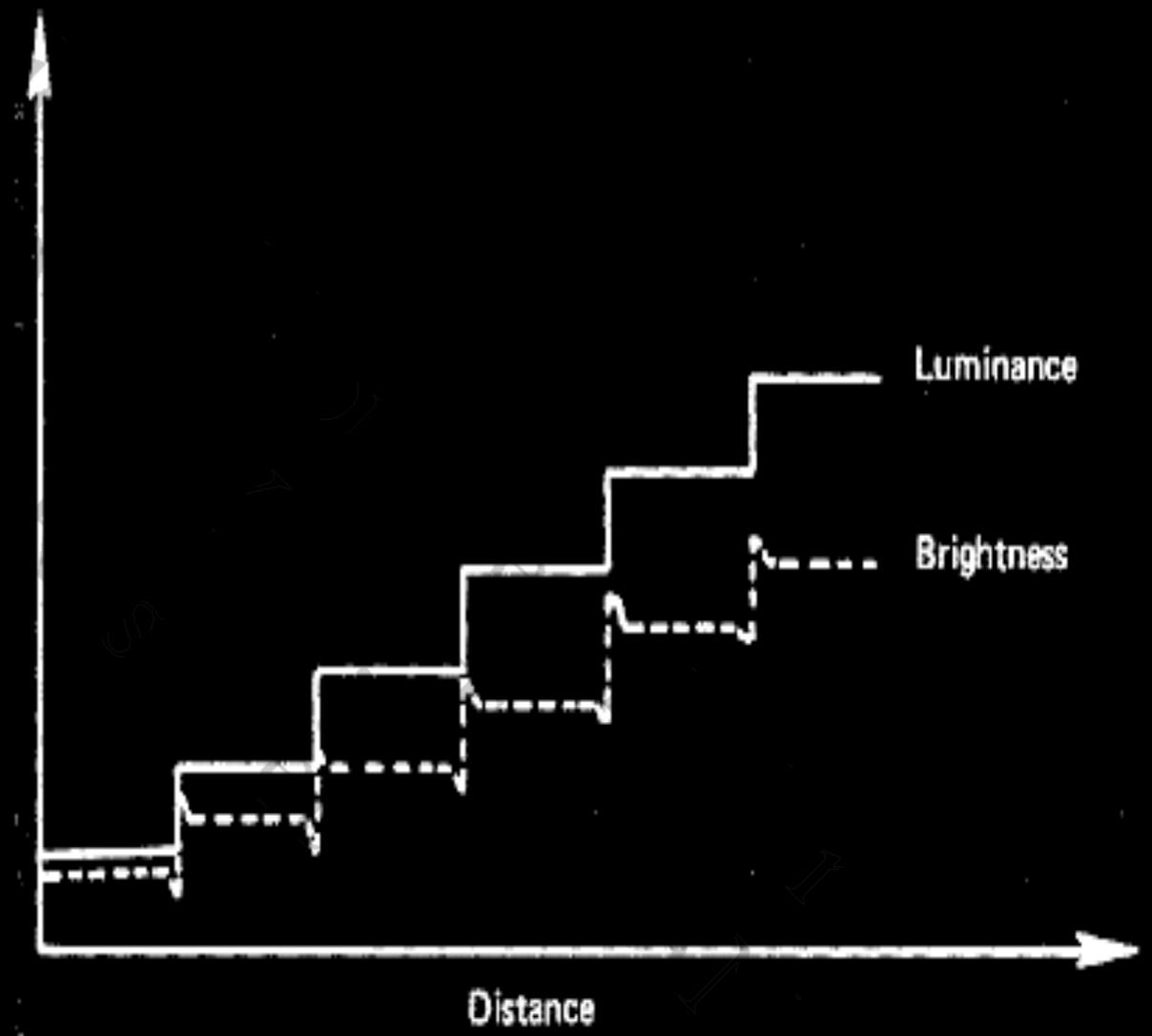
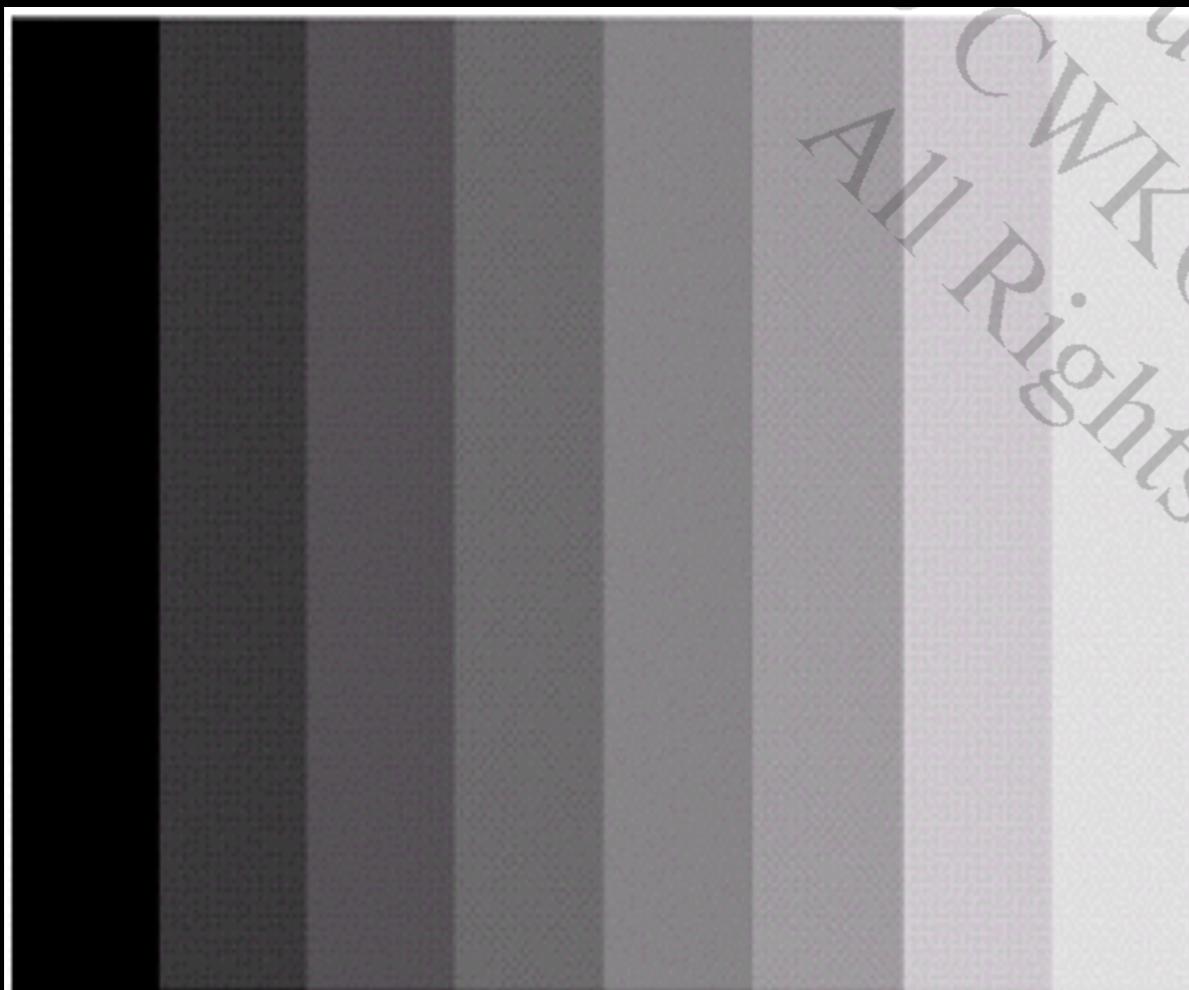
人的視覺用 log. model 較準確

Subjective brightness vs. Intensity

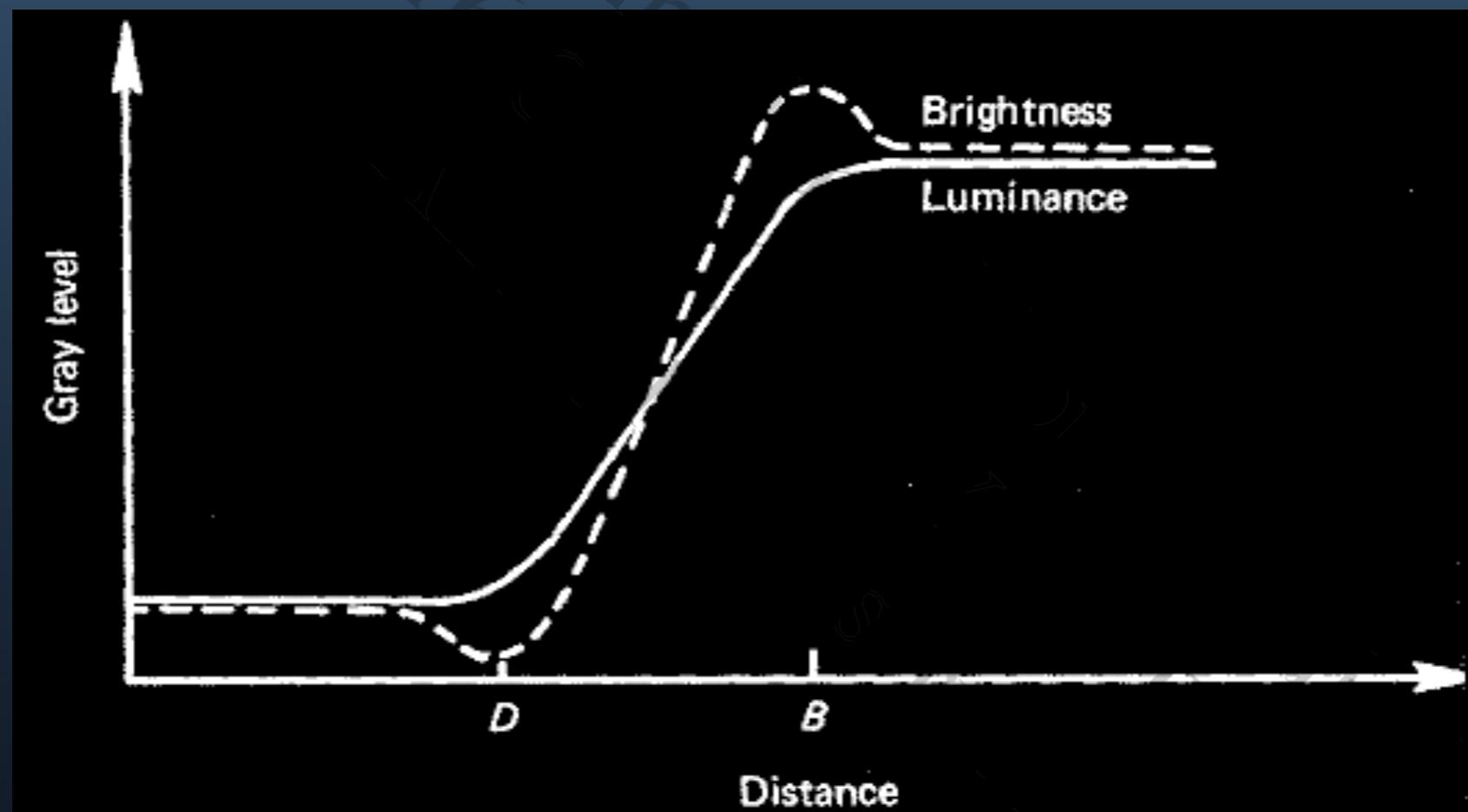
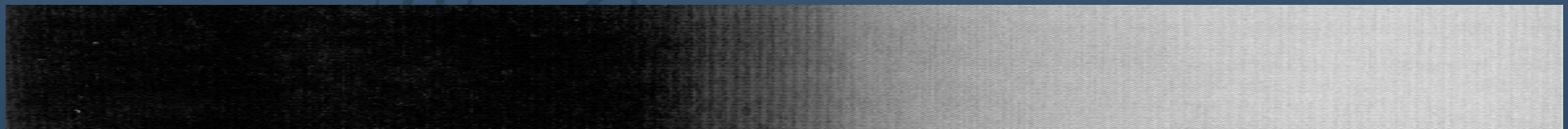


Visual Phenomena (2)

- Mach Band Effect



Visual Phenomena (2)



For the designer, perfect fidelity of edge contours
can be sacrificed !

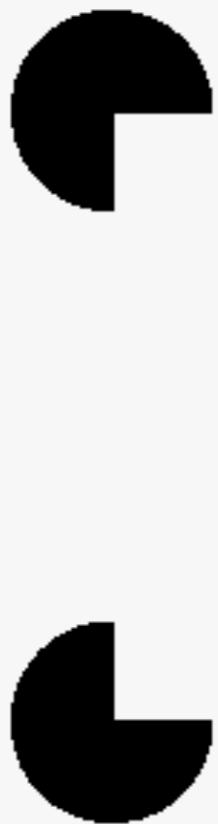
Visual Phenomena (3)

- Simultaneous Contrast

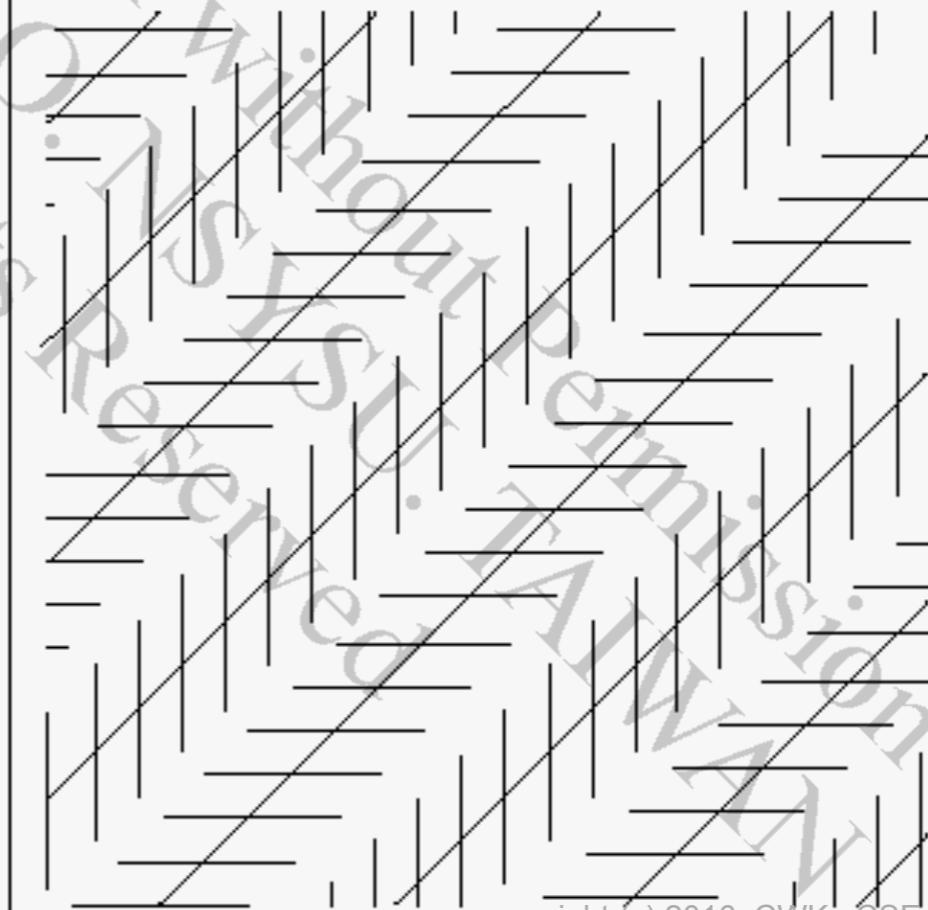
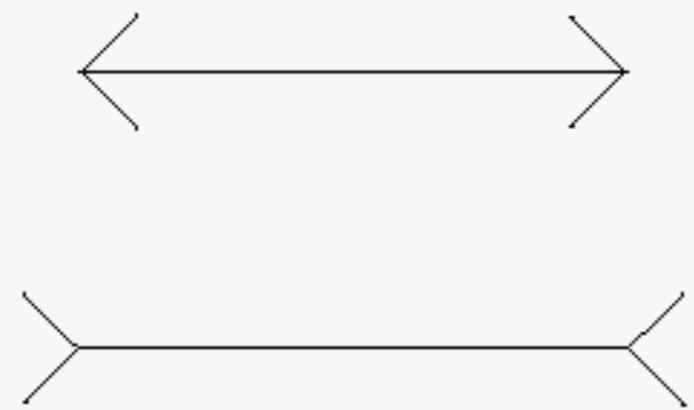
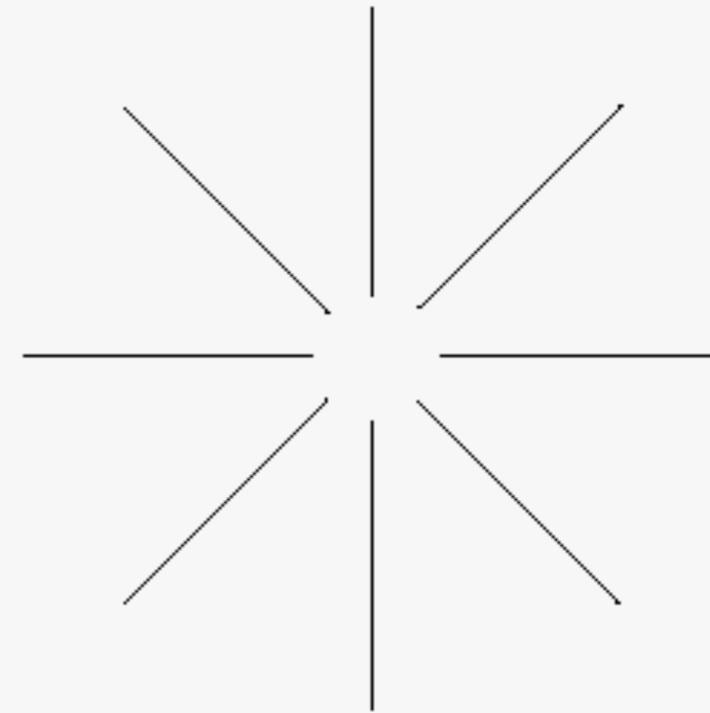


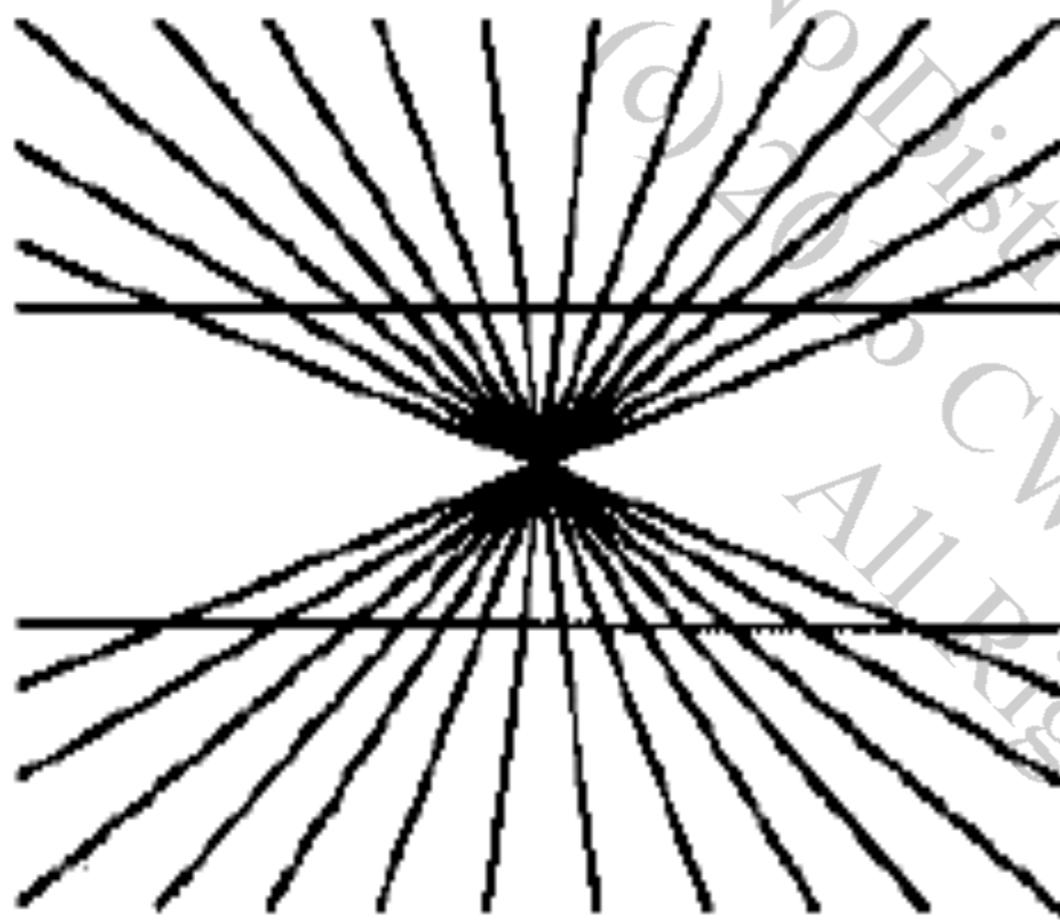
Visual Phenomena (3)

Illusion 錯覺！

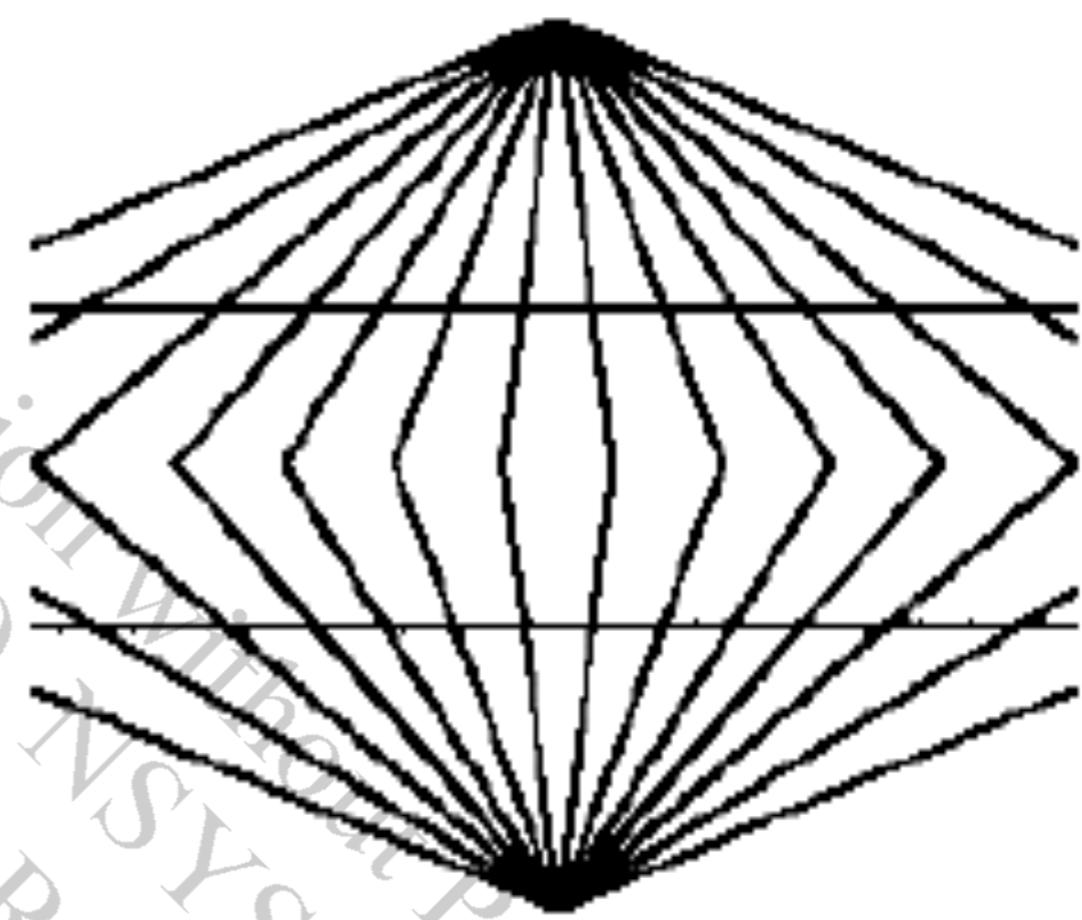


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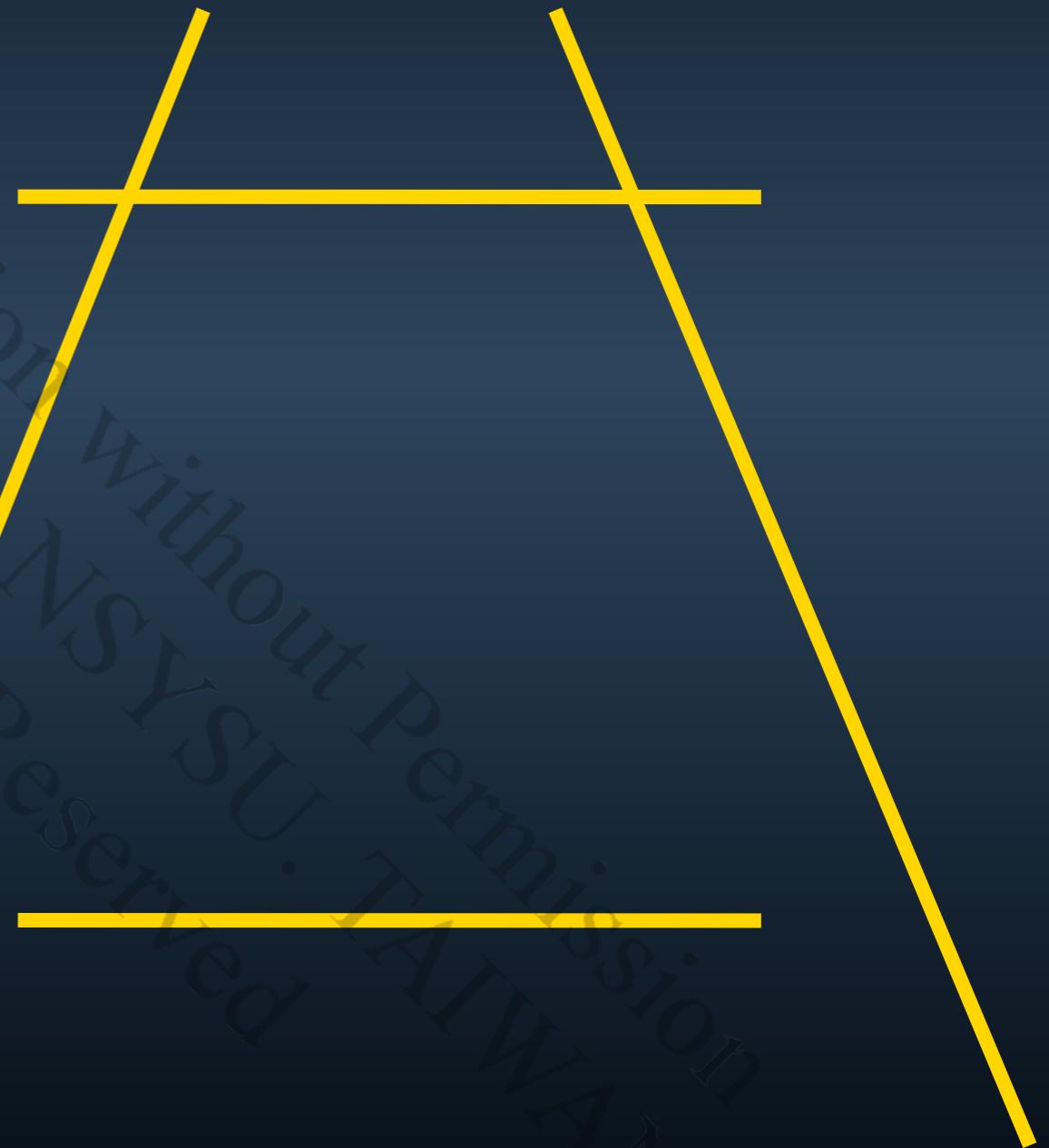
Hering Illusion (1861)

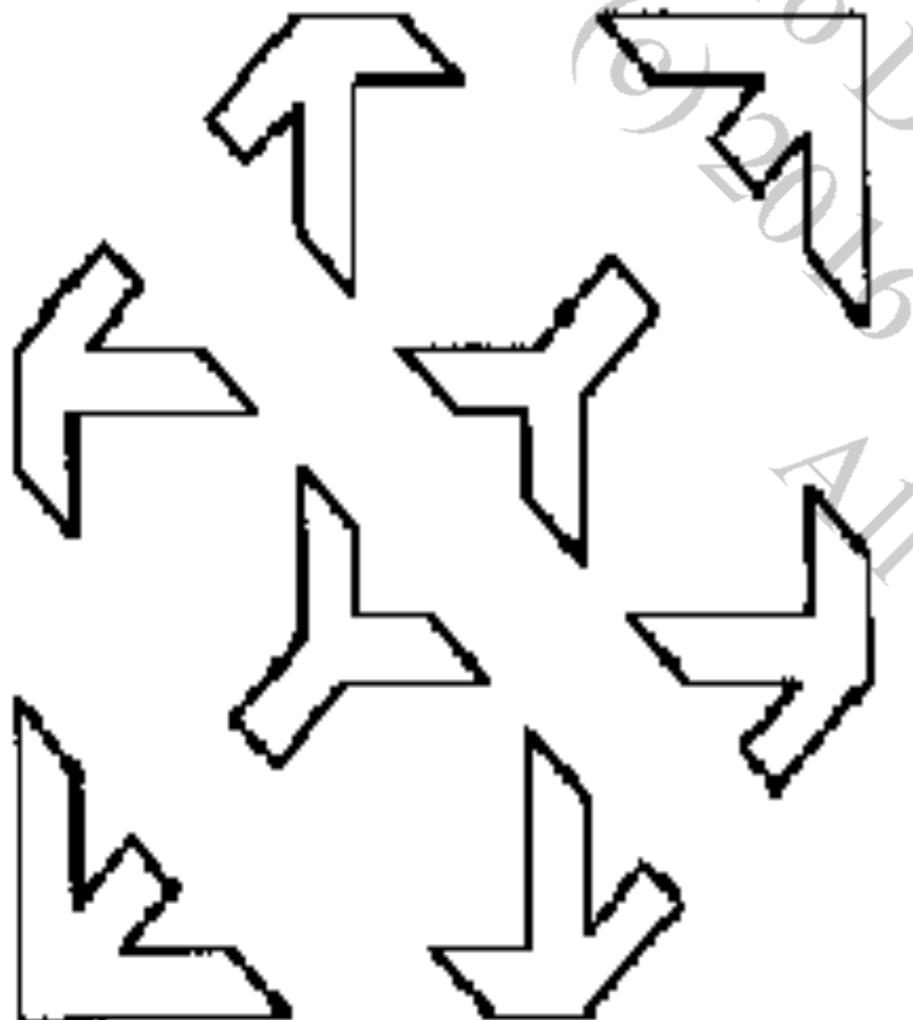


Wundt Illusion (1896)



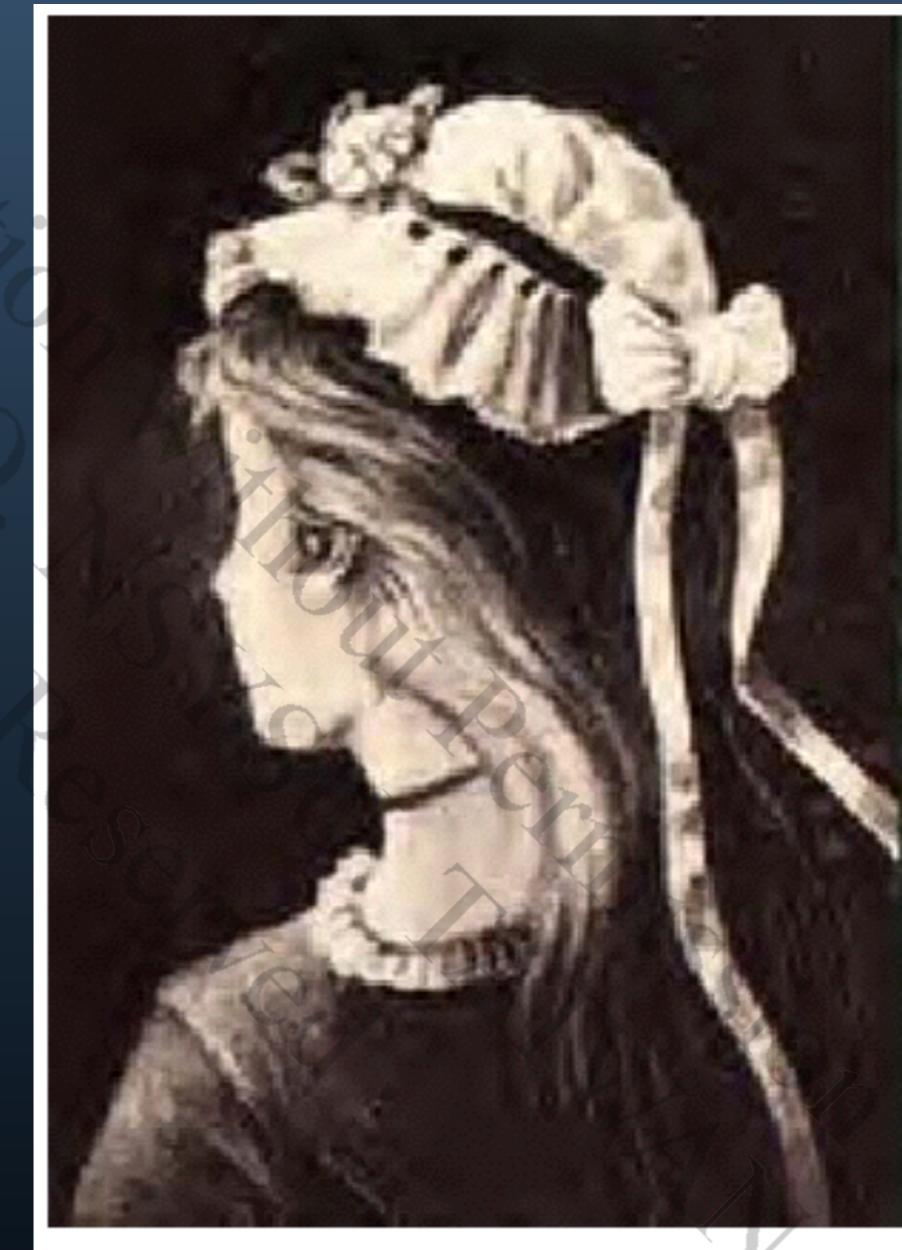
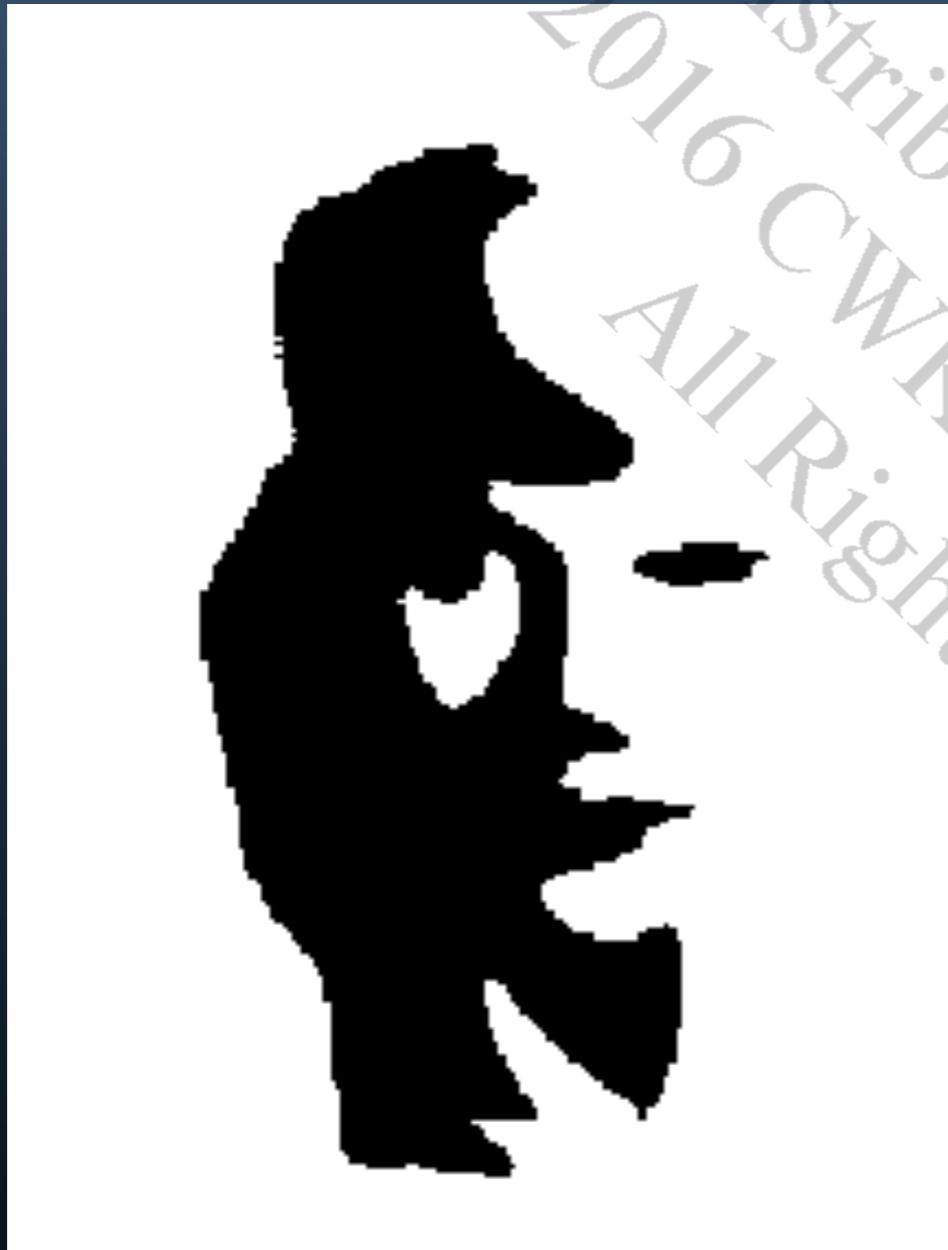
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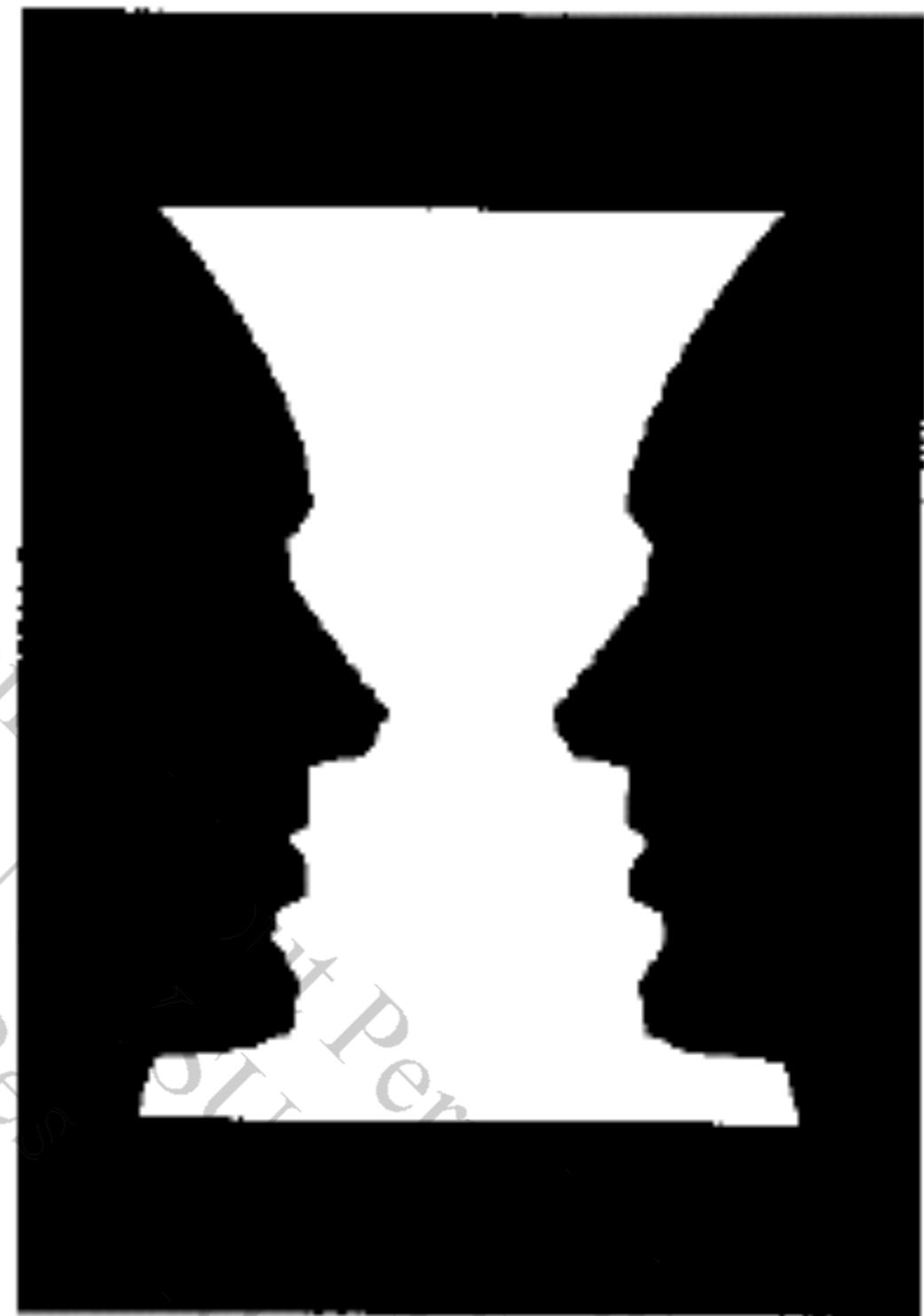
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Ambiguities





Young Girl/Old Woman



Vase/Faces

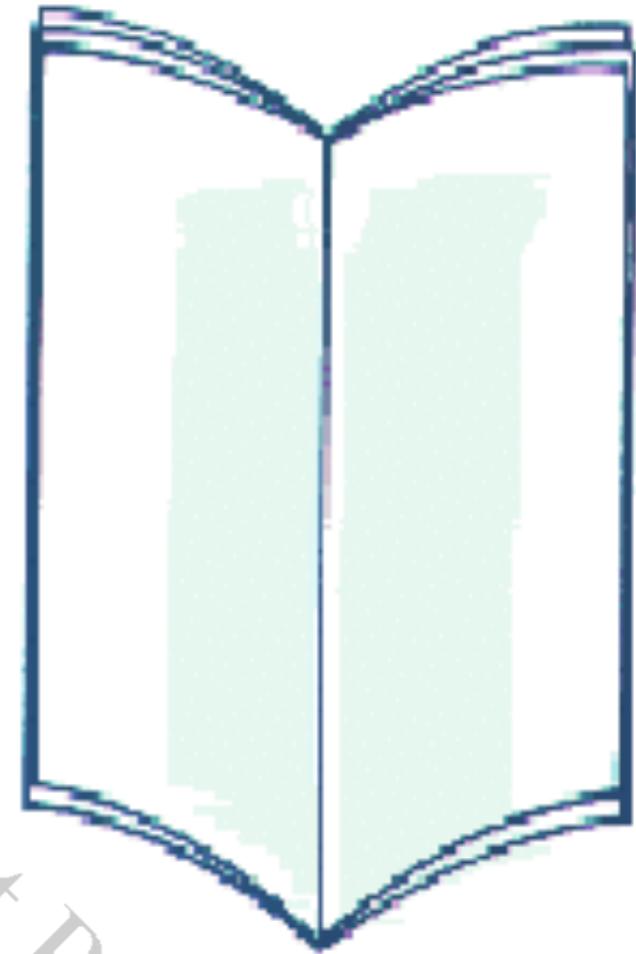


American Indian or an Eskimo? two faces or one?



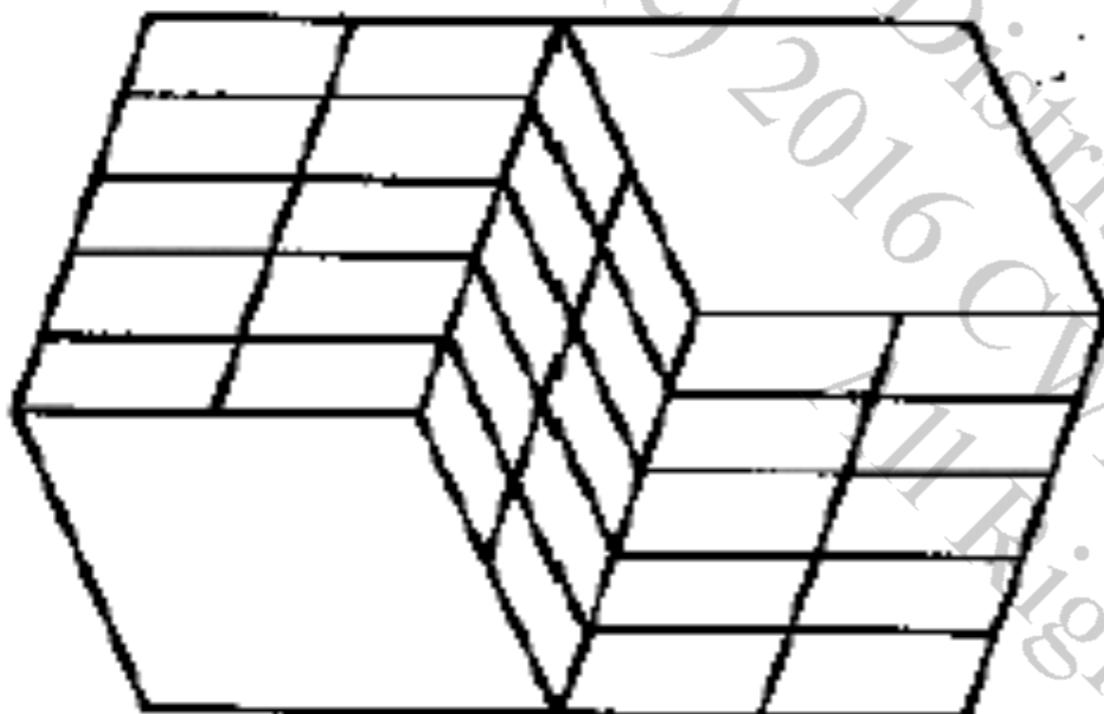
a bird or a rabbit?

a book open toward or away from you?

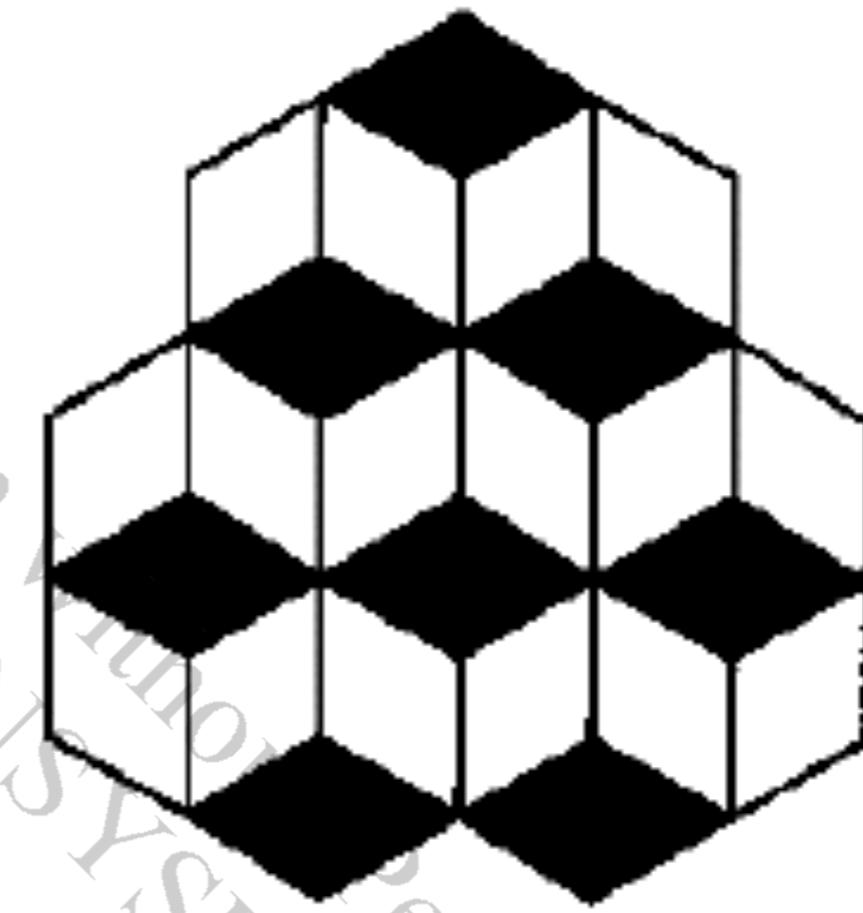




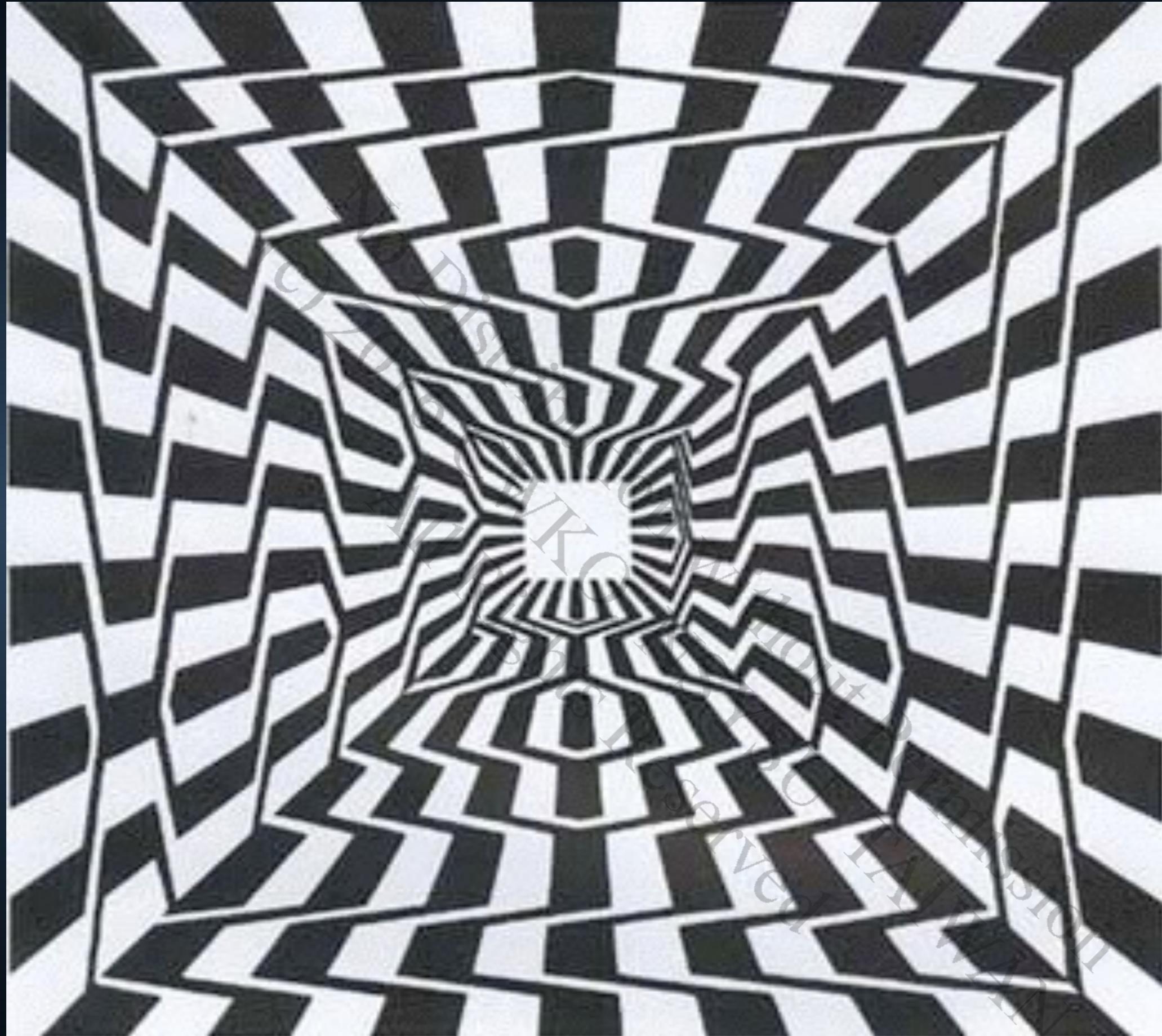
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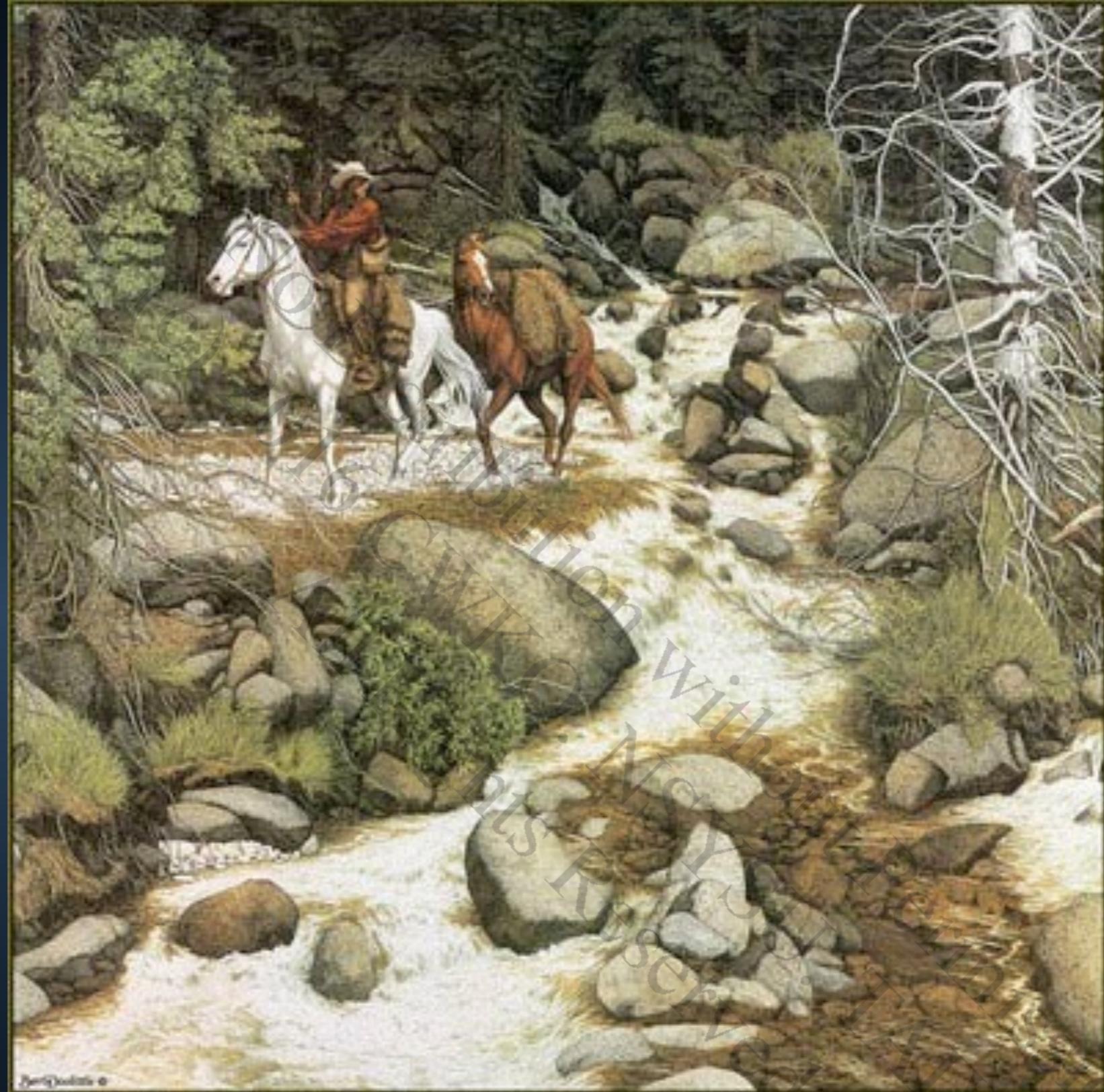


Pop In/Pop Out



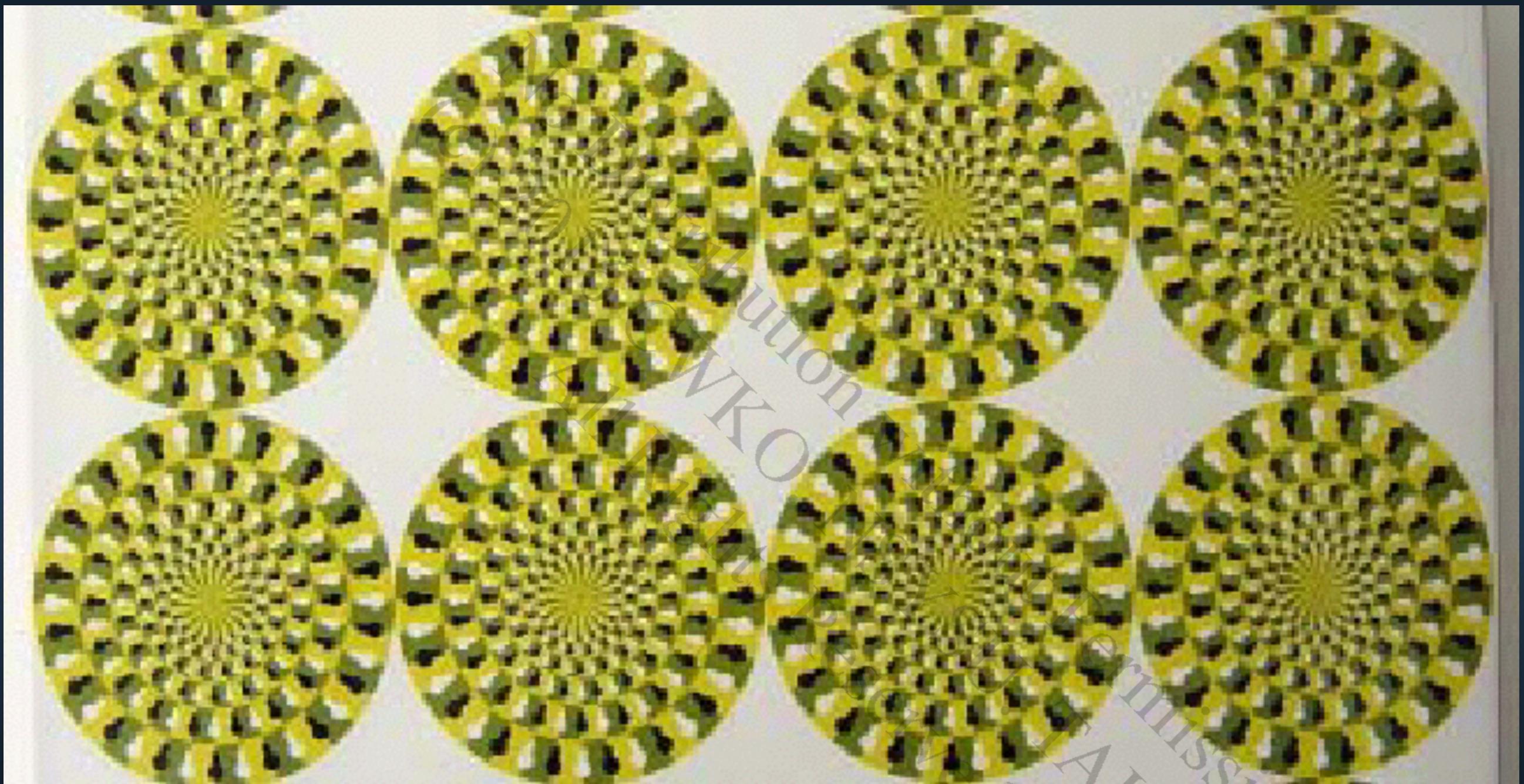
Six Cubes/Seven Cubes

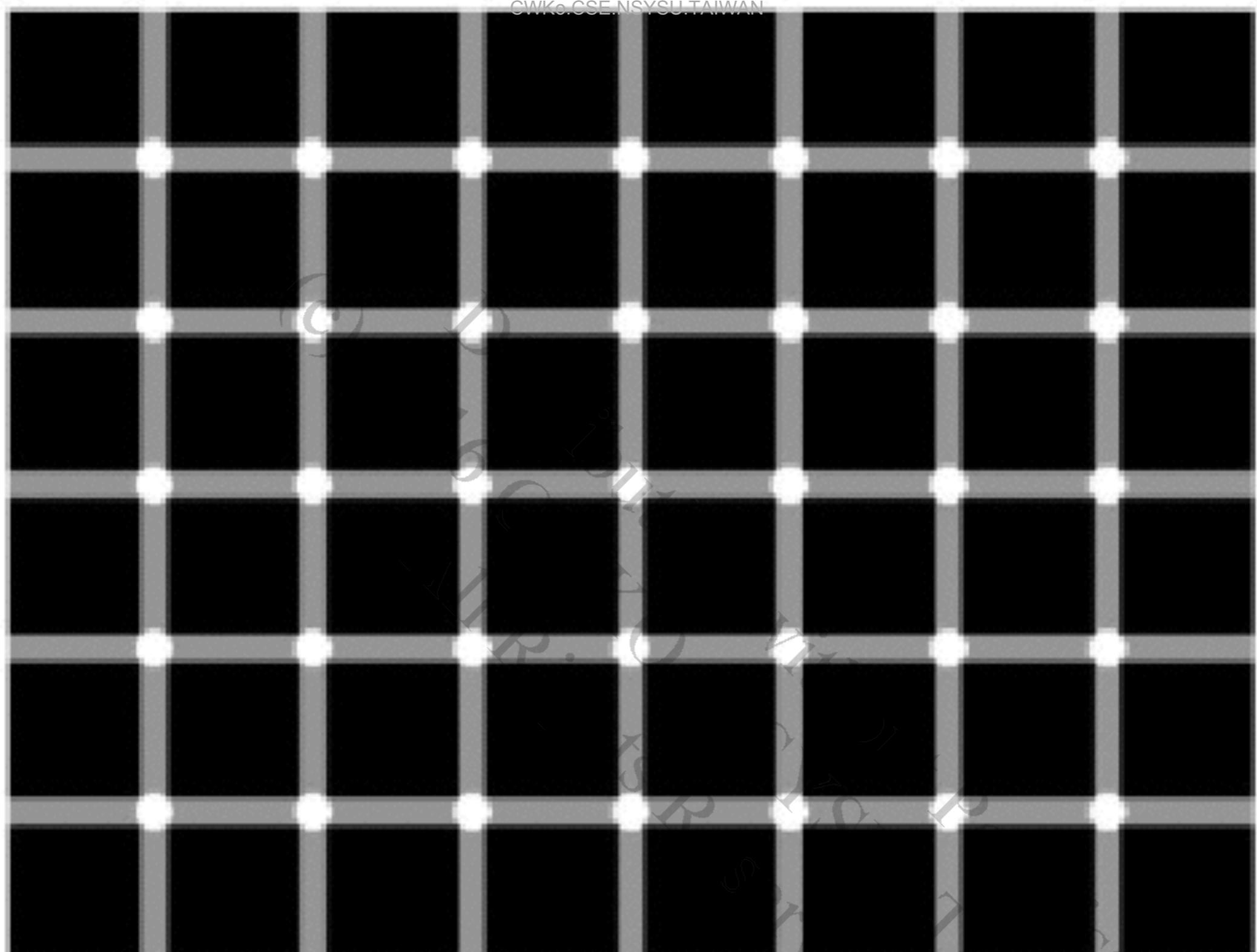




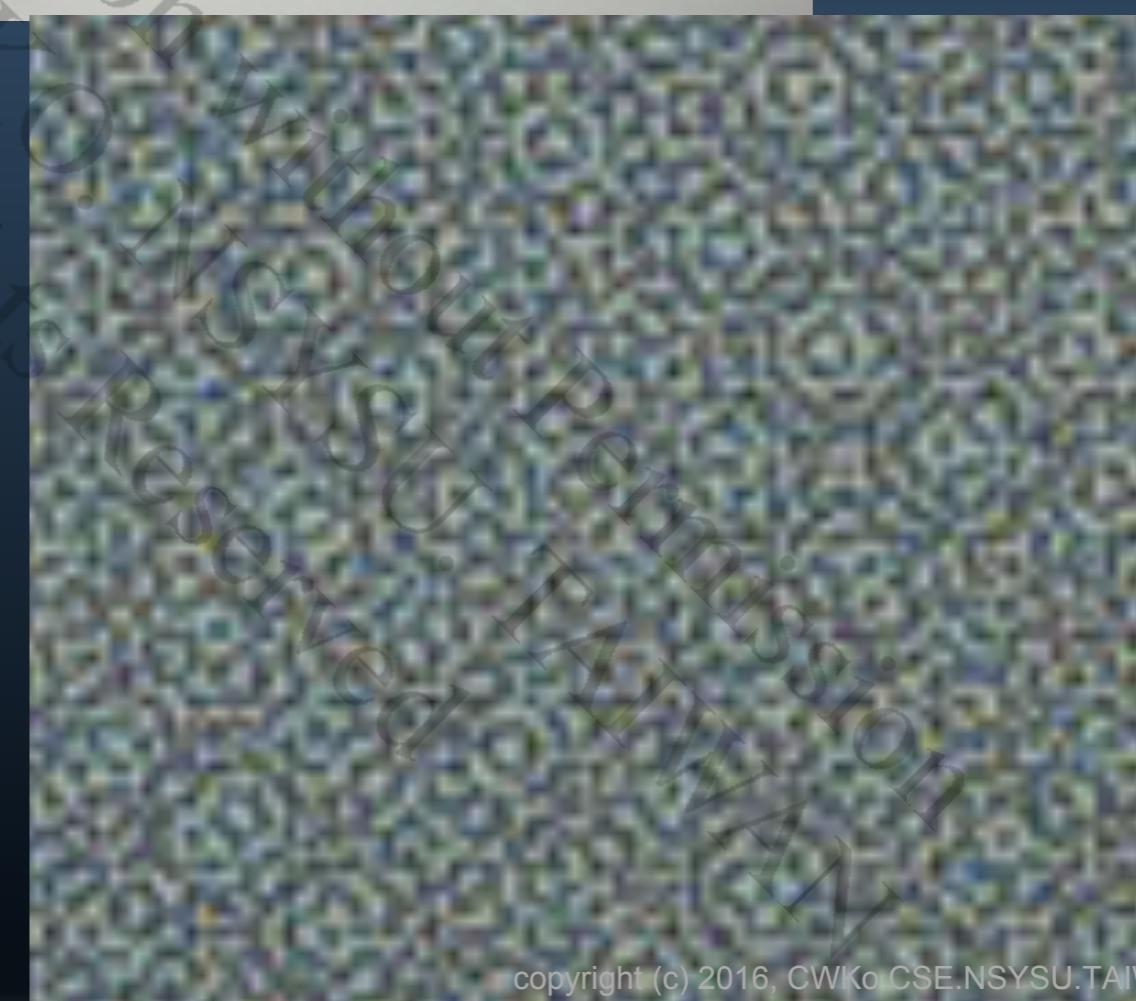
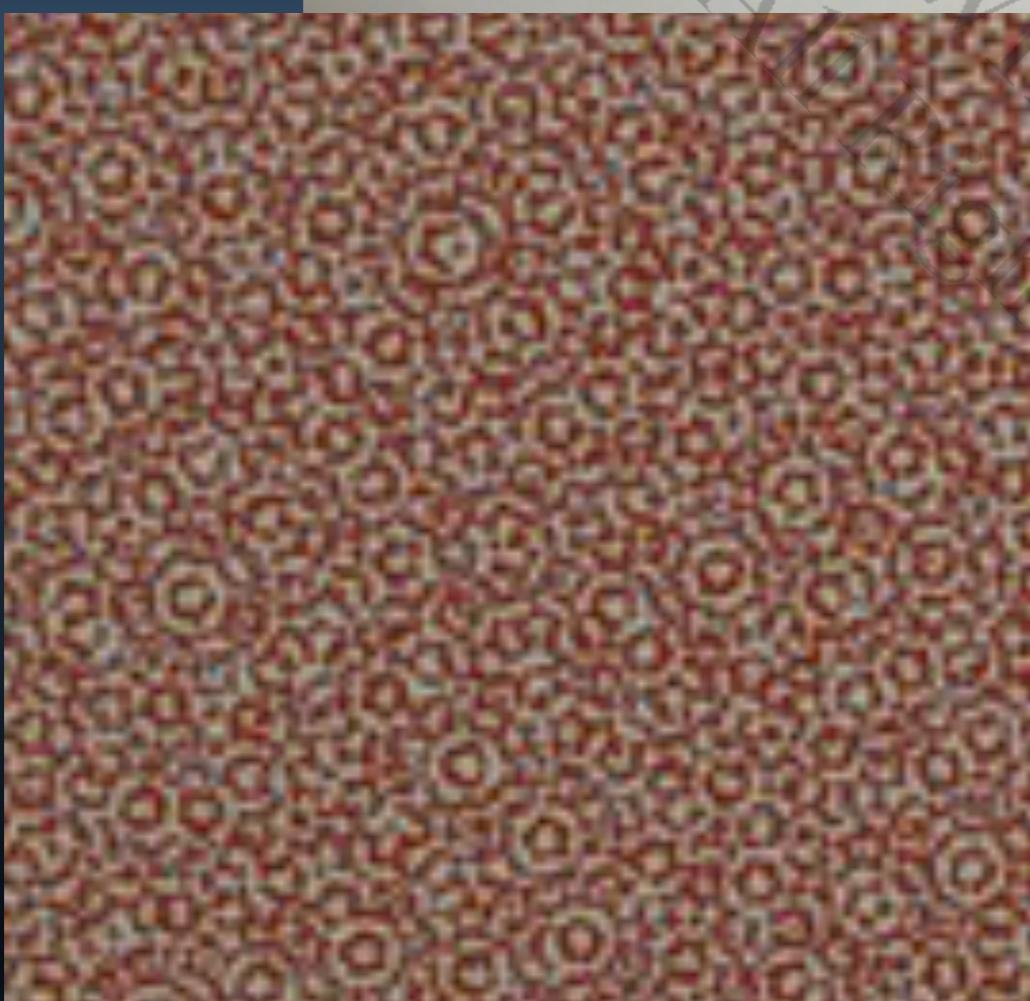
圖中棕色的馬為何要回頭看呢？原來他發現周圍有很多老人在朝他微笑。你看到了嗎？

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Count the black dots! :o)



Digitalized Image

Sampling & Quantization

Image Acquisition

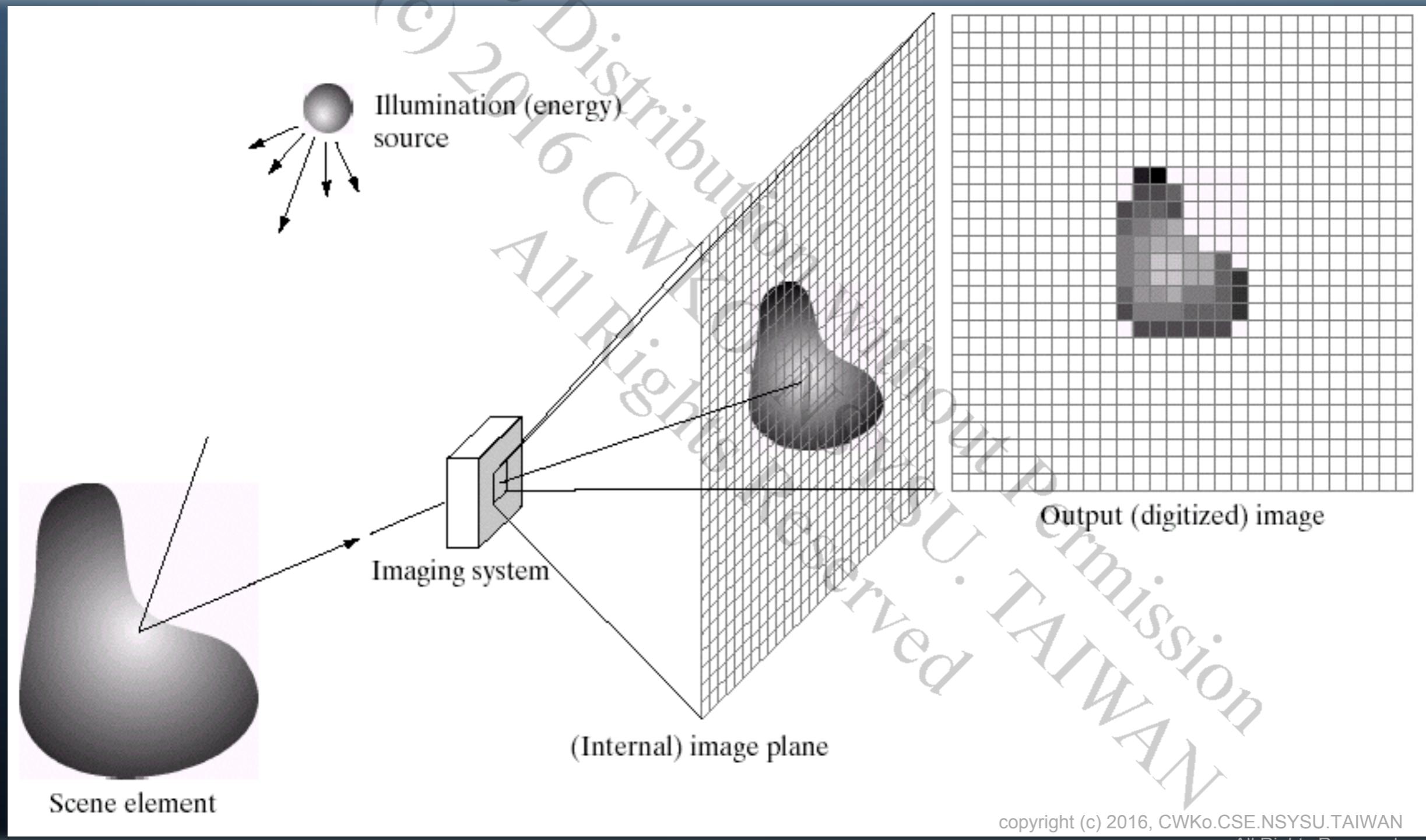
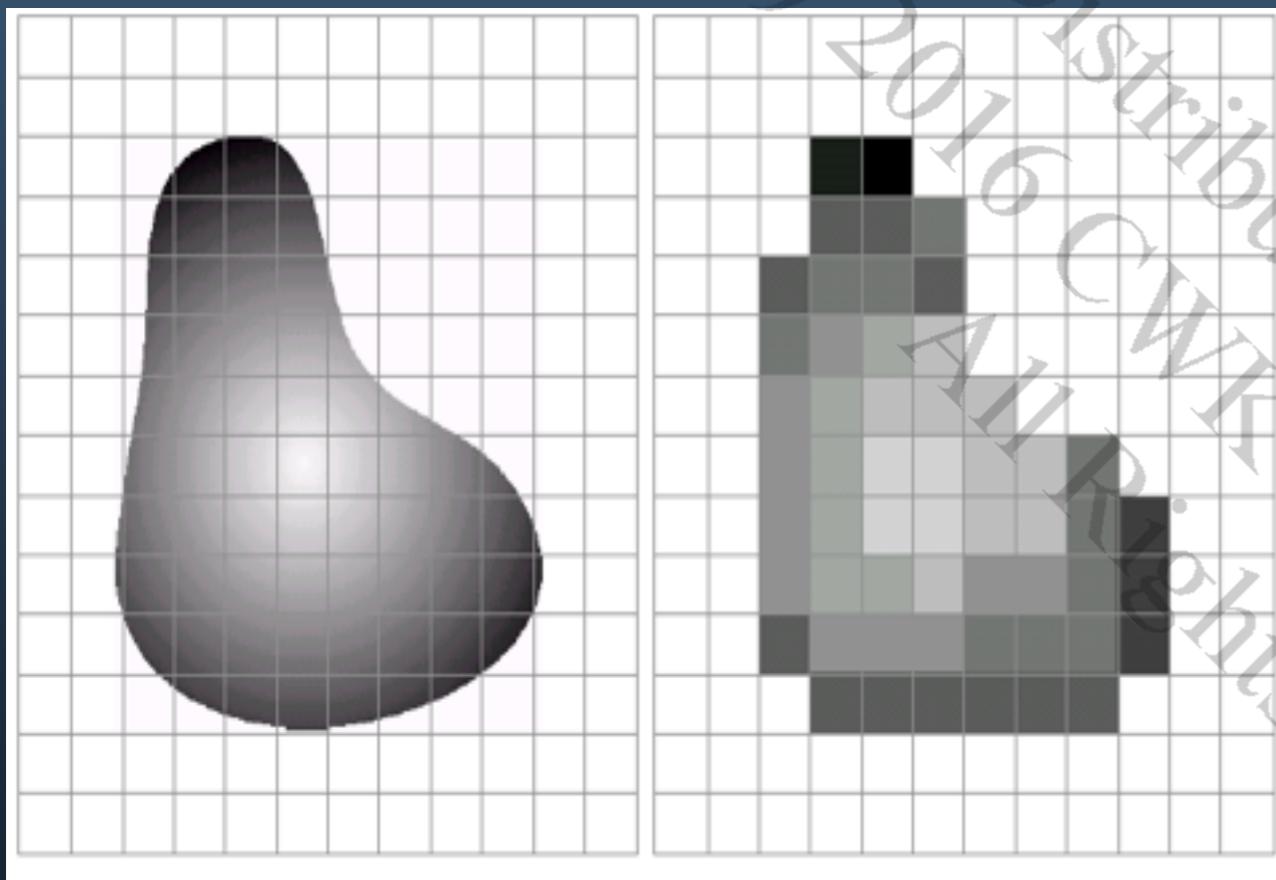


Image Sampling



$f(x,y) = ??$

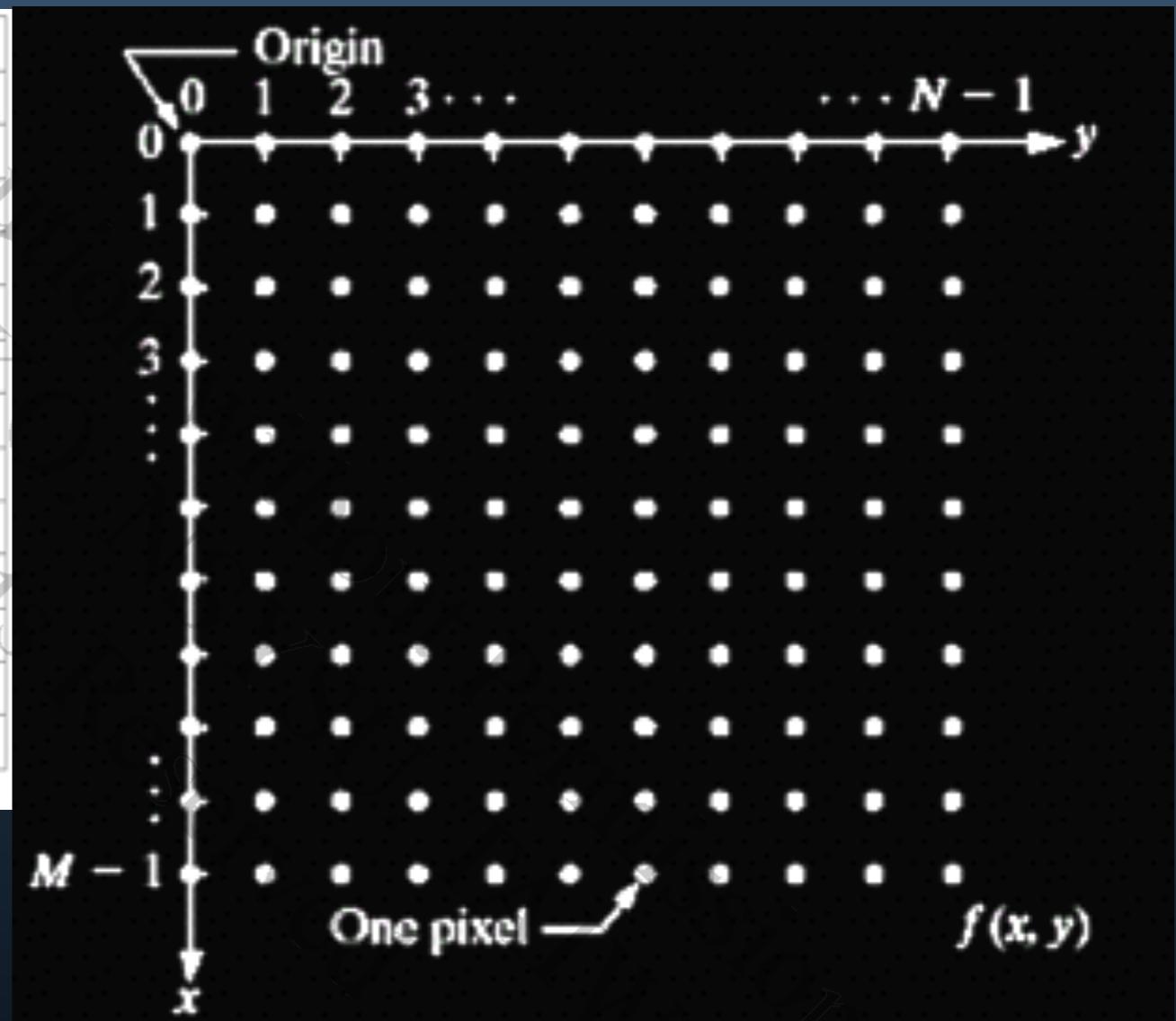


Image Sampling & Quantization

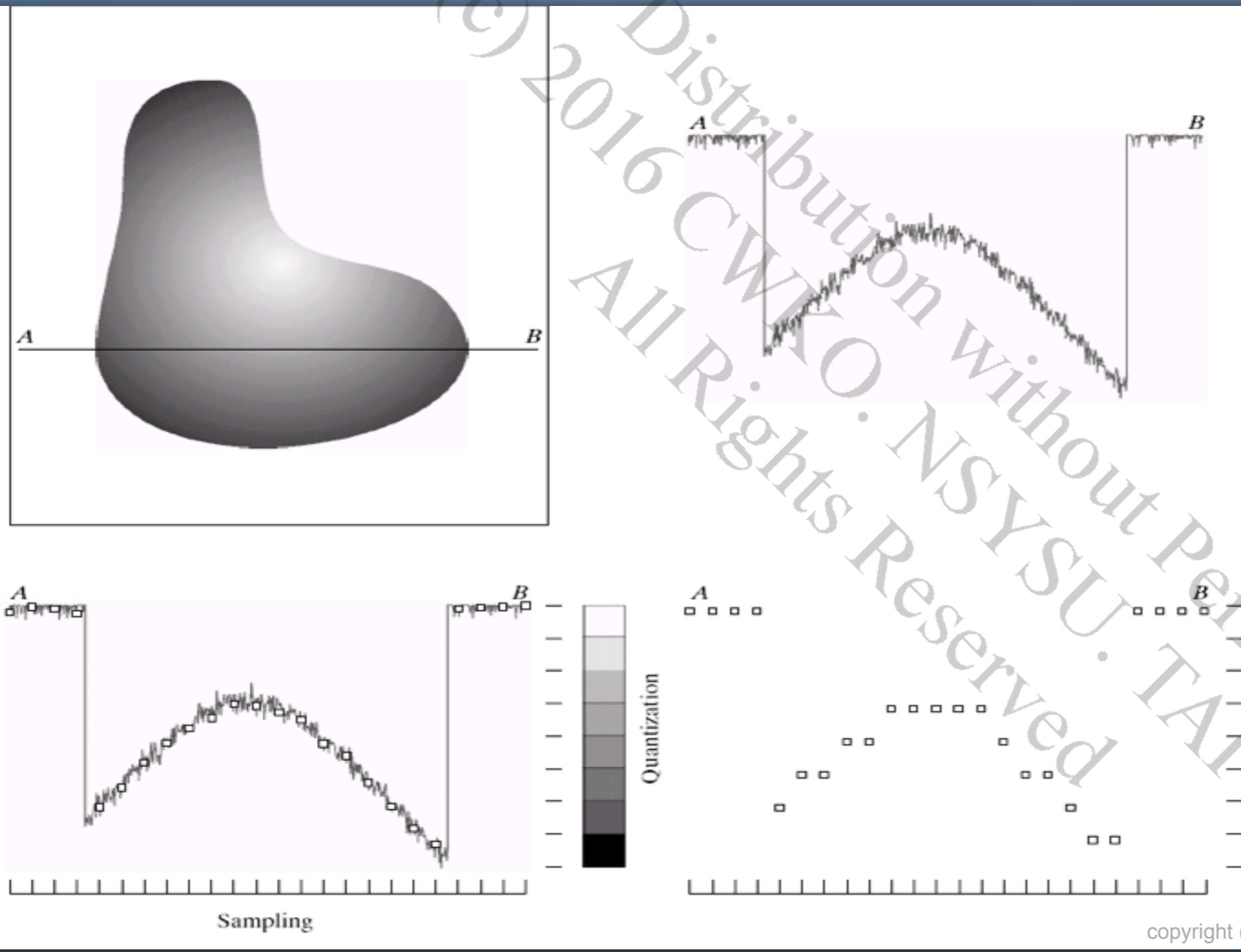
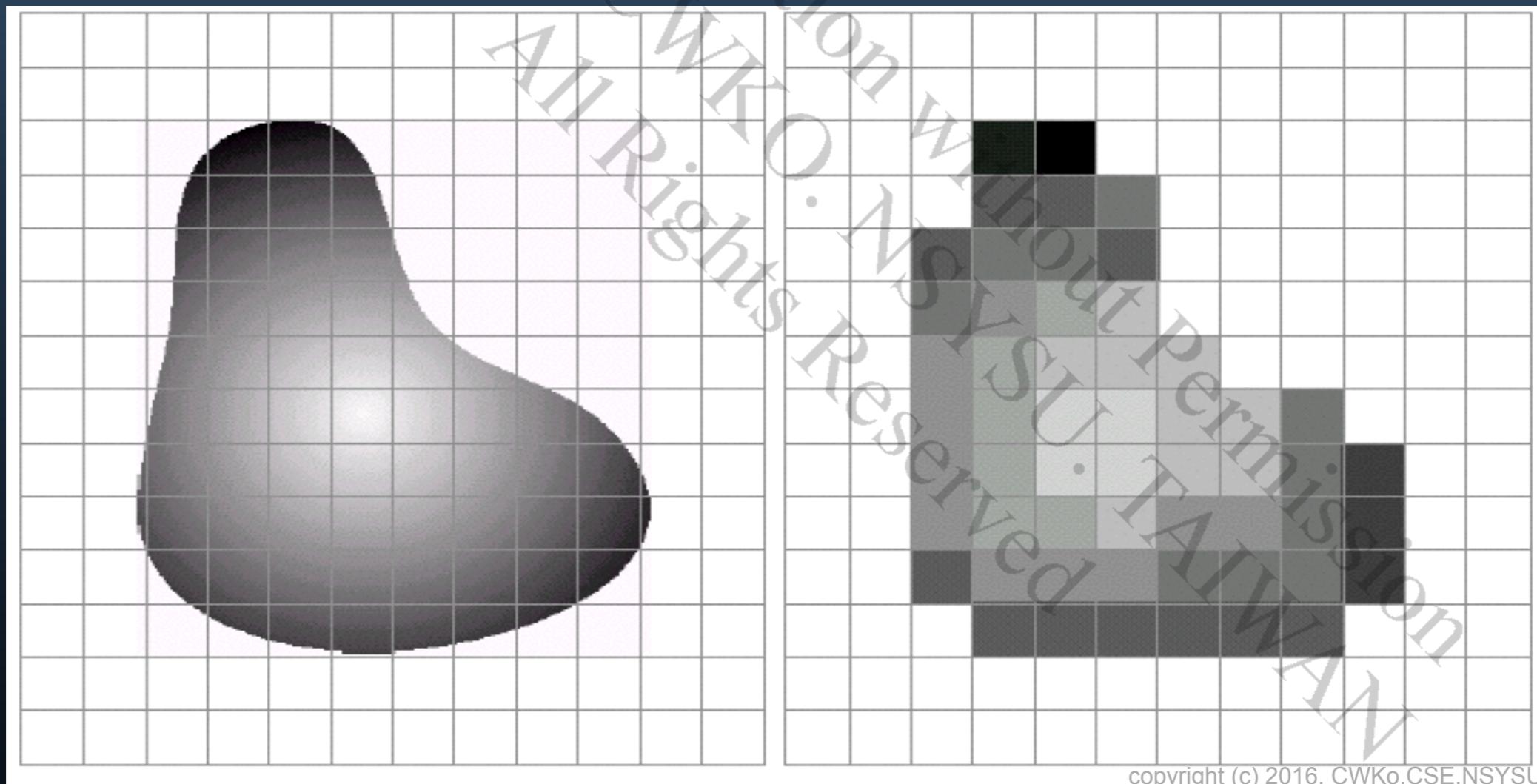
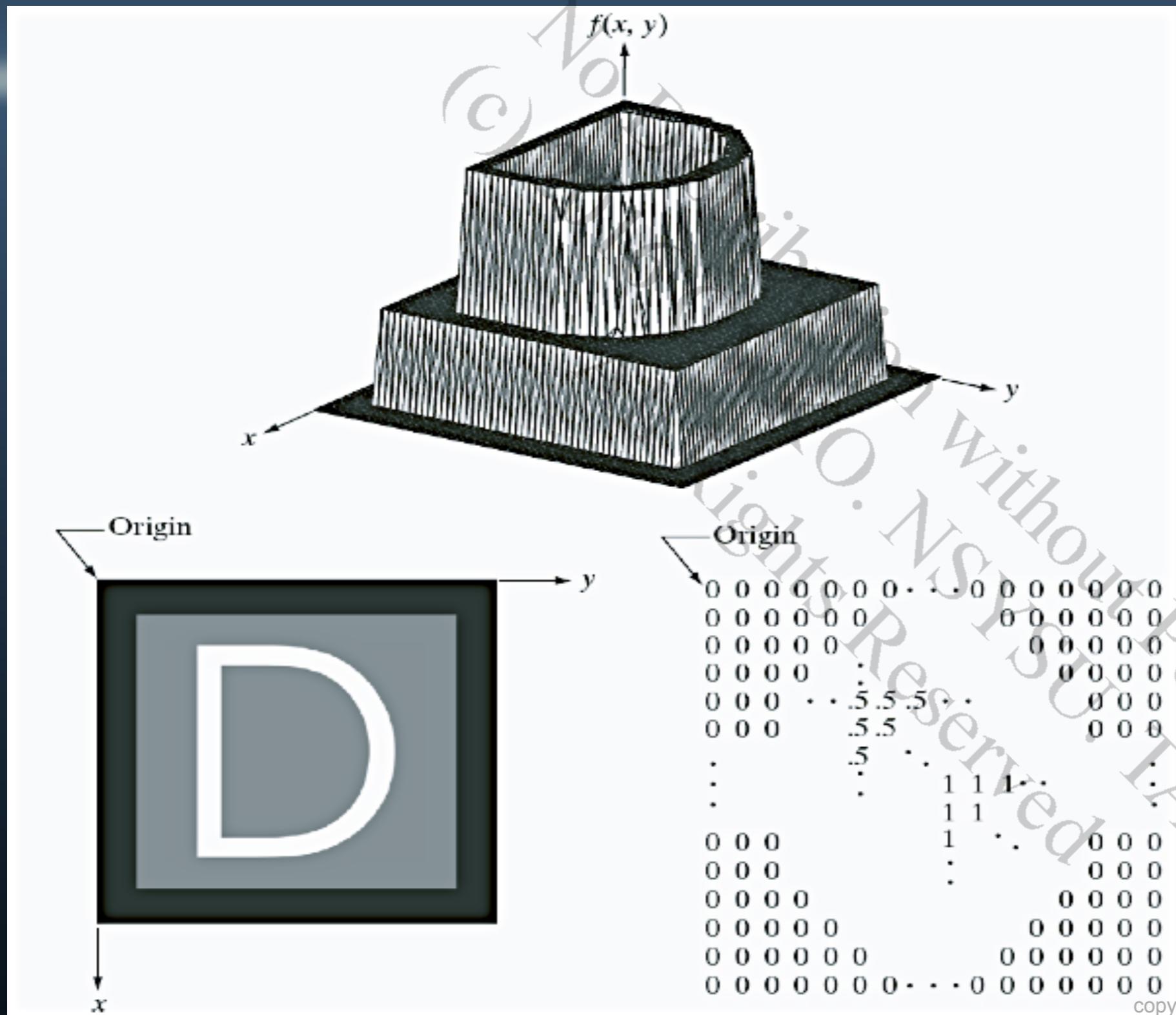


Image Sampling & Quantization

- Sampling - 座標值的數位化 - Spatial resolution
- Quantization - 灰階值的數位化 - Intensity resolution

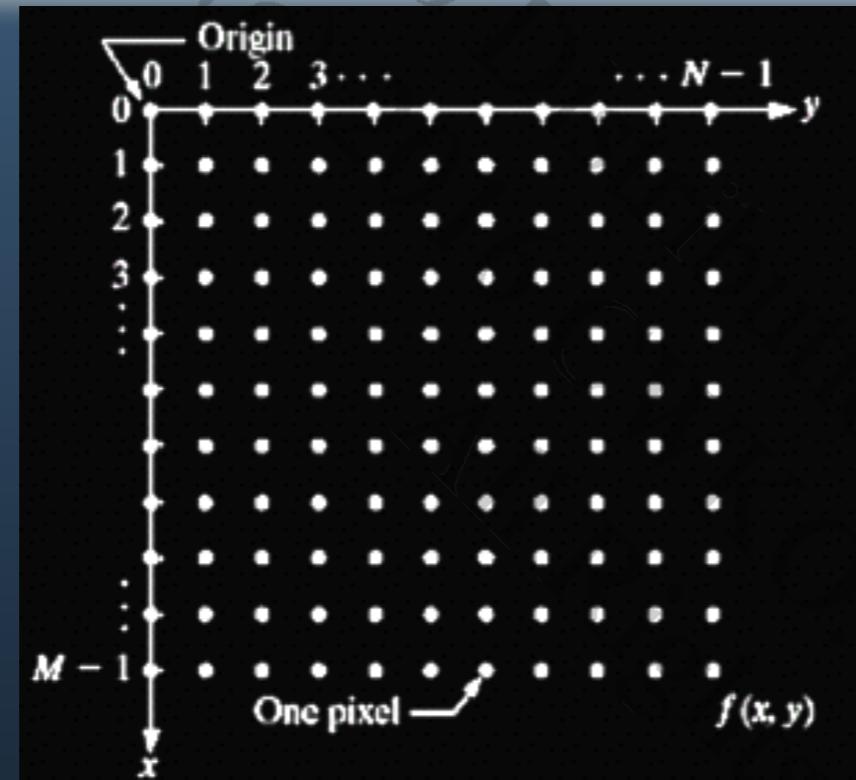


Digital Image Representation



$f(x, y)$

Digital Image Representation



Origin

0	0	0	0	0	0	0	· · ·	0	0	0	0	0	0	0
0	0	0	0	0	0			0	0	0	0	0	0	0
0	0	0	0	0				0	0	0	0	0	0	0
0	0	0	0		·			0	0	0	0	0	0	0
0	0	0	· ·	5	5	5	· ·			0	0	0	0	0
0	0	0		5	5					0	0	0	0	0
·		5		·	·					·				·
·			·	·		1	1	1	· ·					·
·					1		1							·
0	0	0			1		·	·		0	0	0	0	0
0	0	0				·				0	0	0	0	0
0	0	0	0							0	0	0	0	0
0	0	0	0	0						0	0	0	0	0
0	0	0	0	0	0	0	· · ·	0	0	0	0	0	0	0

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}$$

Digital Image Representation

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}$$



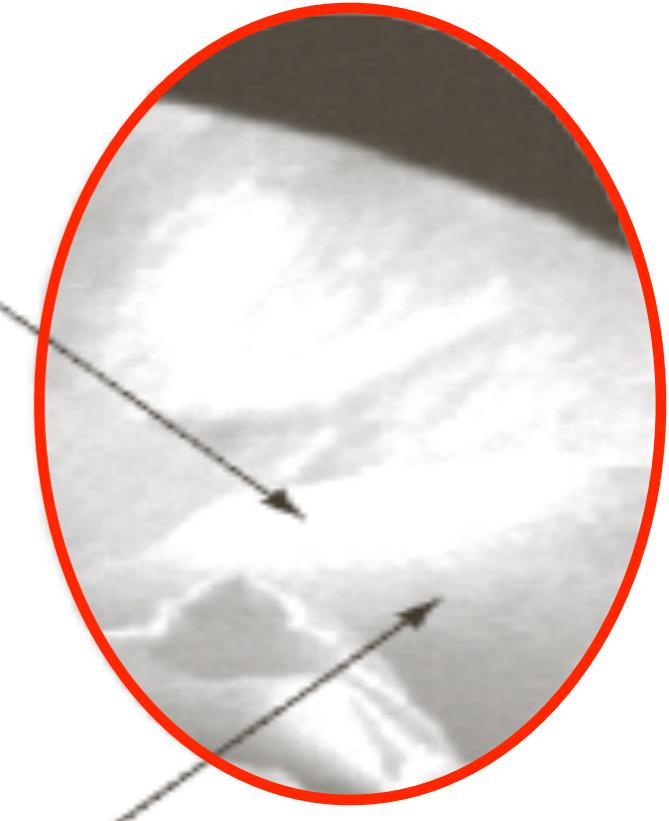
$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \dots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \dots & a_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{M-1,0} & a_{M-1,1} & \dots & a_{M-1,N-1} \end{bmatrix}$$

Digital Image Representation

- $L = 2^k$
Gray level $\rightarrow [0, L-1]$
- Bits required to store a image:
 $b = M \times N \times k$
- 256 (L) gray level \rightarrow 8 (k)-bit image
- 1024×1024 , 8-bit image
 $\rightarrow 3,145,728$ bits



Saturation



Noise

Spatial Resolution

- Sampling is the principal factor determining the spatial resolution of an image.
- Dots (pixel) per inch

Spatial Resolution

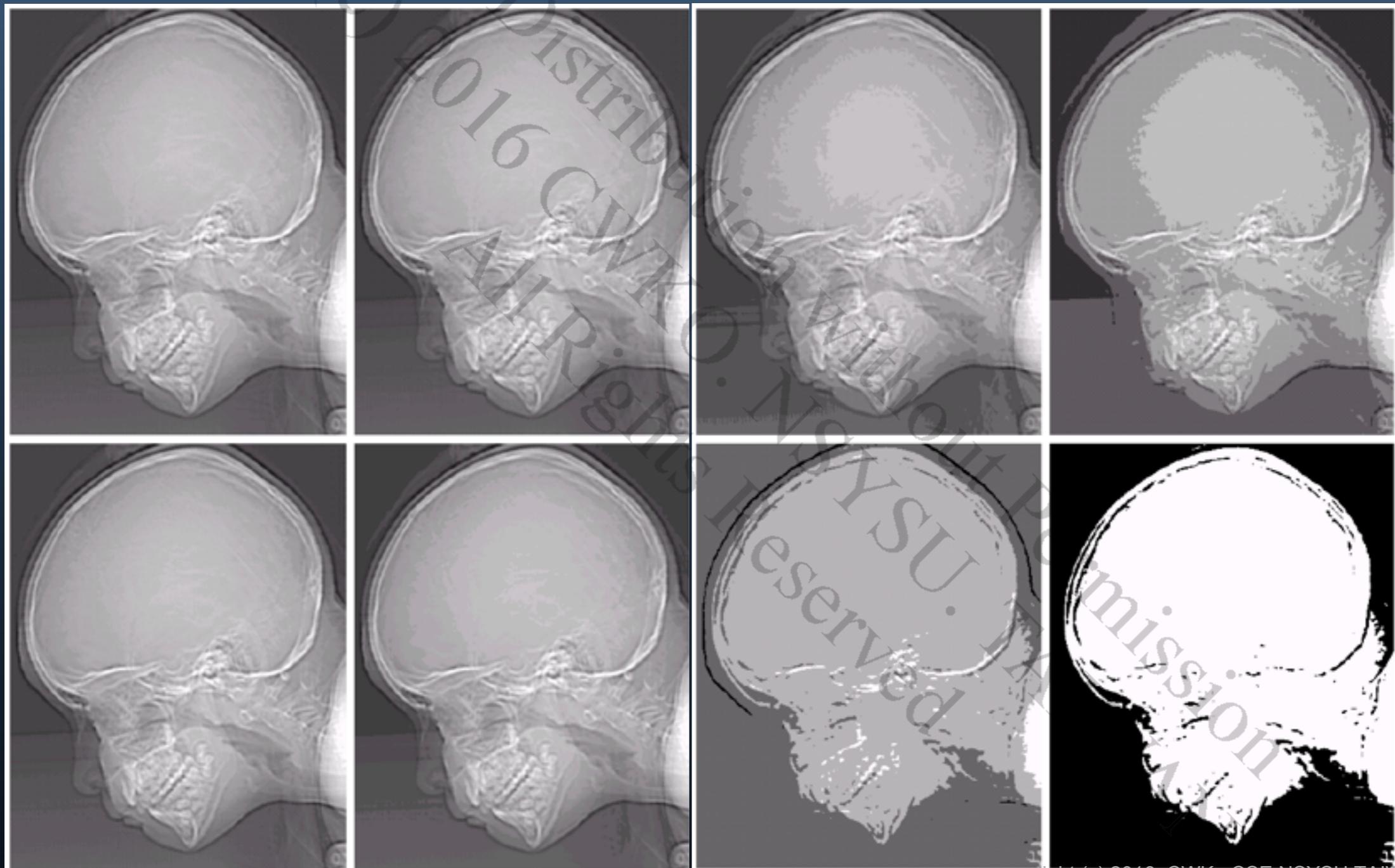
1250 dpi, 3692x2812



72 dpi, 213x162



Gray-level Resolution



Aliasing

- Band-limited function:
the highest frequency is finite and the function is of unlimited duration
- Shannon sampling theorem
If the band-limited function is sampled at a rate equal to or greater than twice its highest frequency, it is possible to recover completely the original function
- Under-sampling → aliasing

20.45

L



FL
IR
IE
TA
140
4
20.45

SL
SP
I
8.
-60.
1

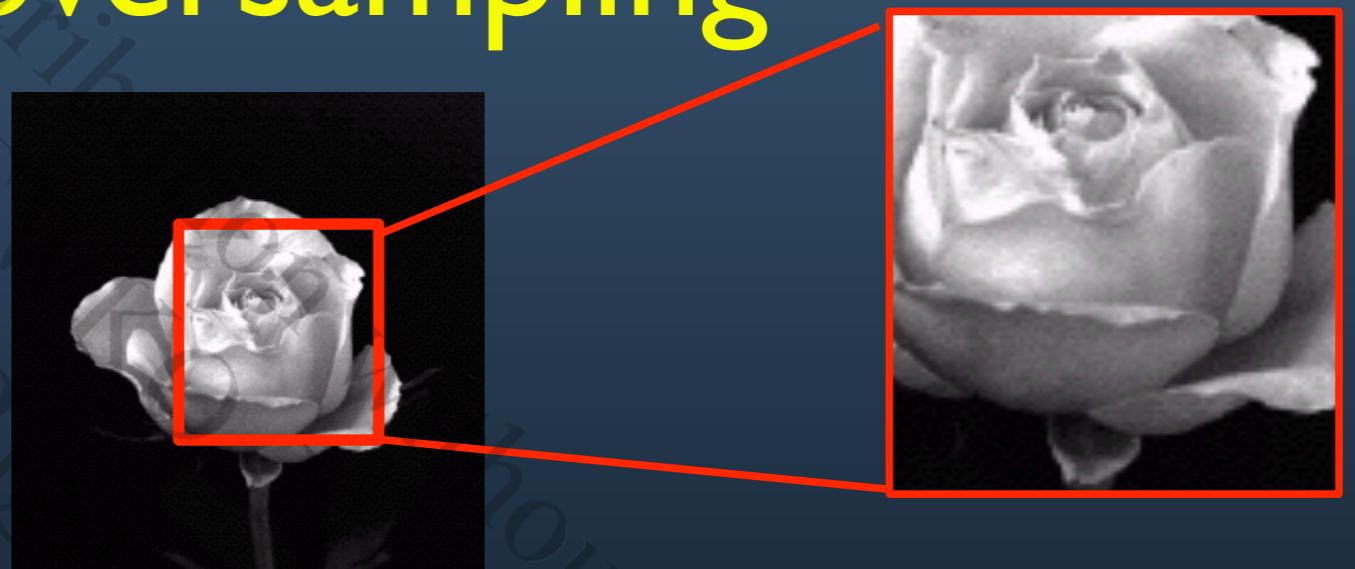
Image Resizing

Interpolation, Zooming & Shrinking

Image Zooming and Shrinking

- **Zooming → oversampling**

- Interpolation



- **Shrinking → undersampling**

- Interpolation
for non-integer factor

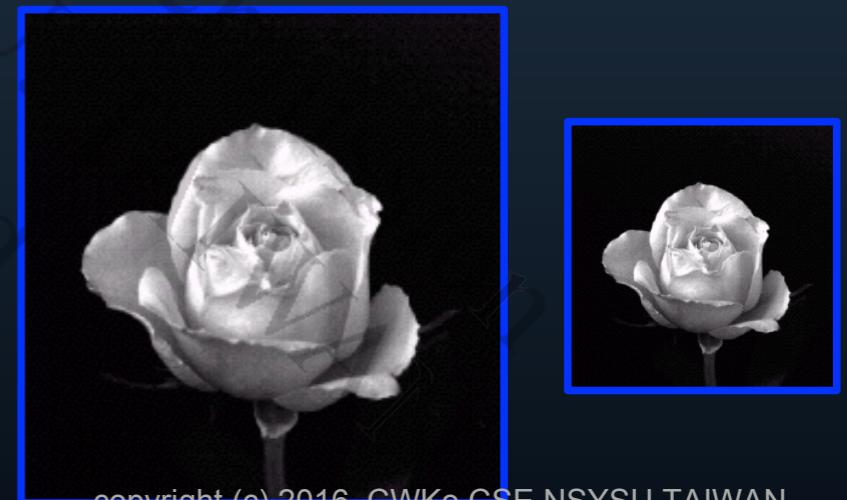
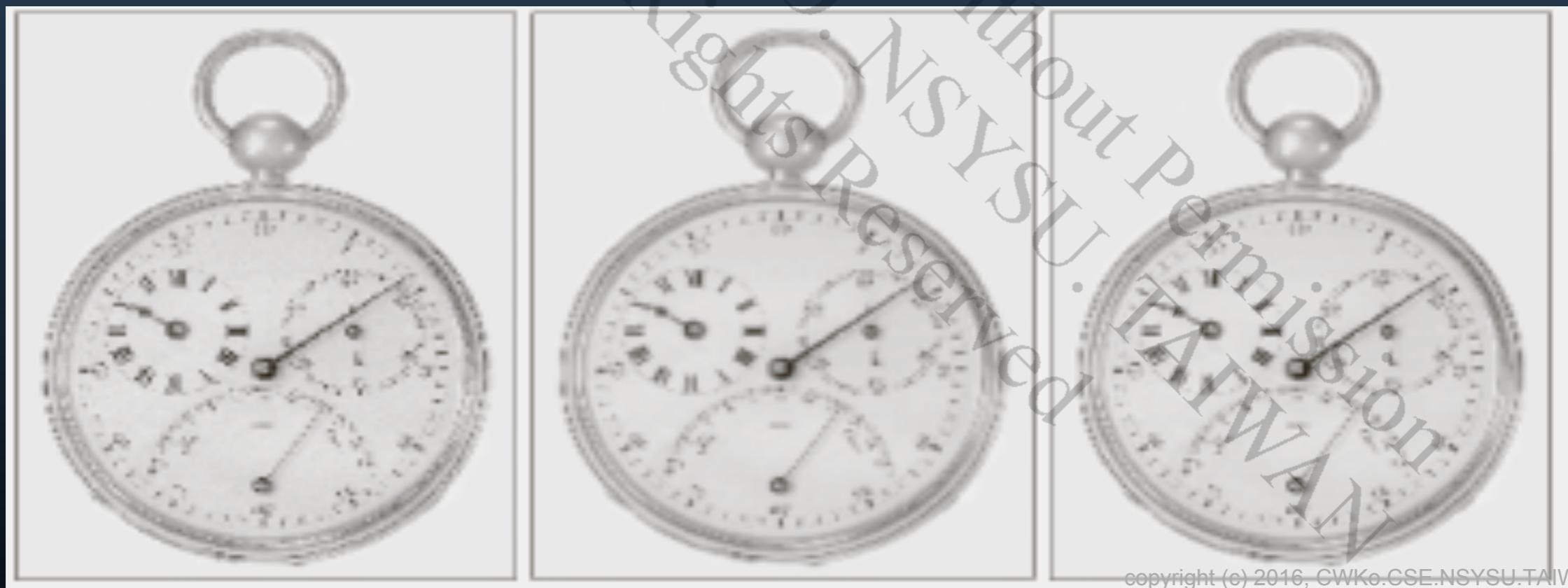
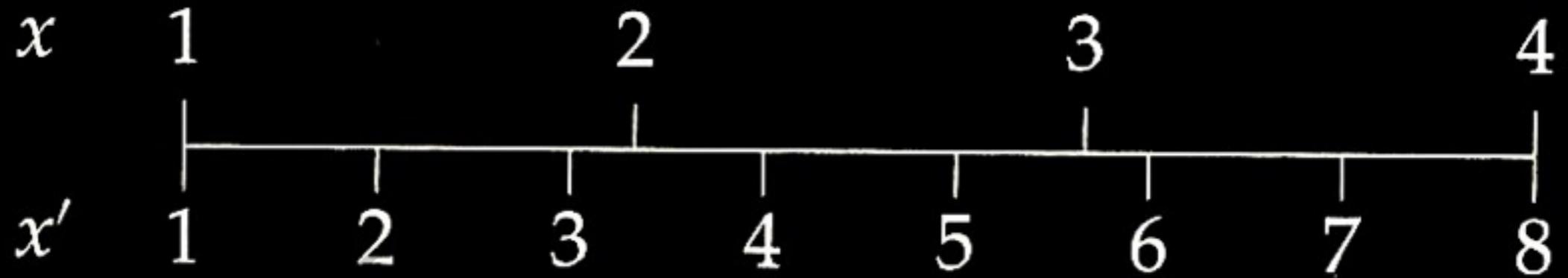


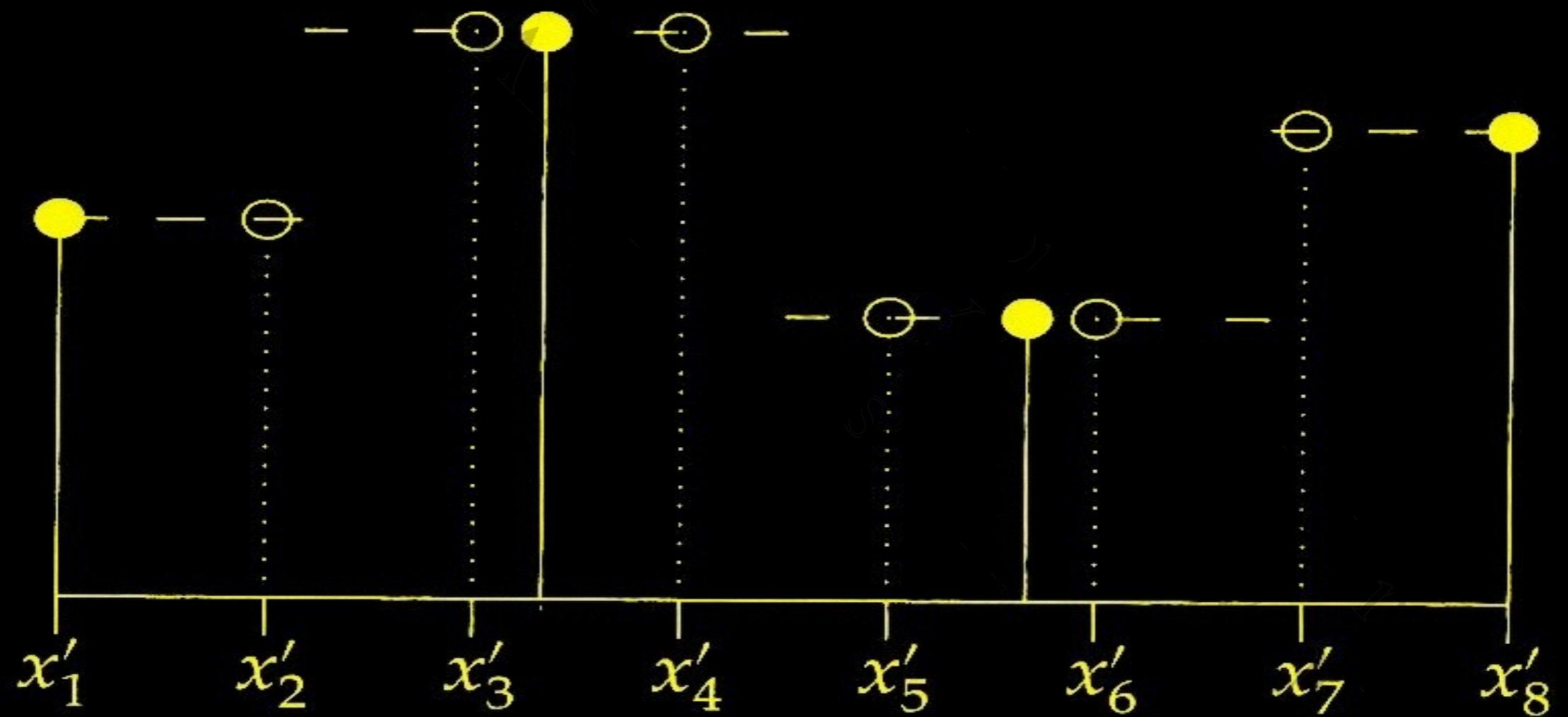
Image Interpolation

- Nearest neighbor interpolation
- Bilinear interpolation
- Bicubic interpolation

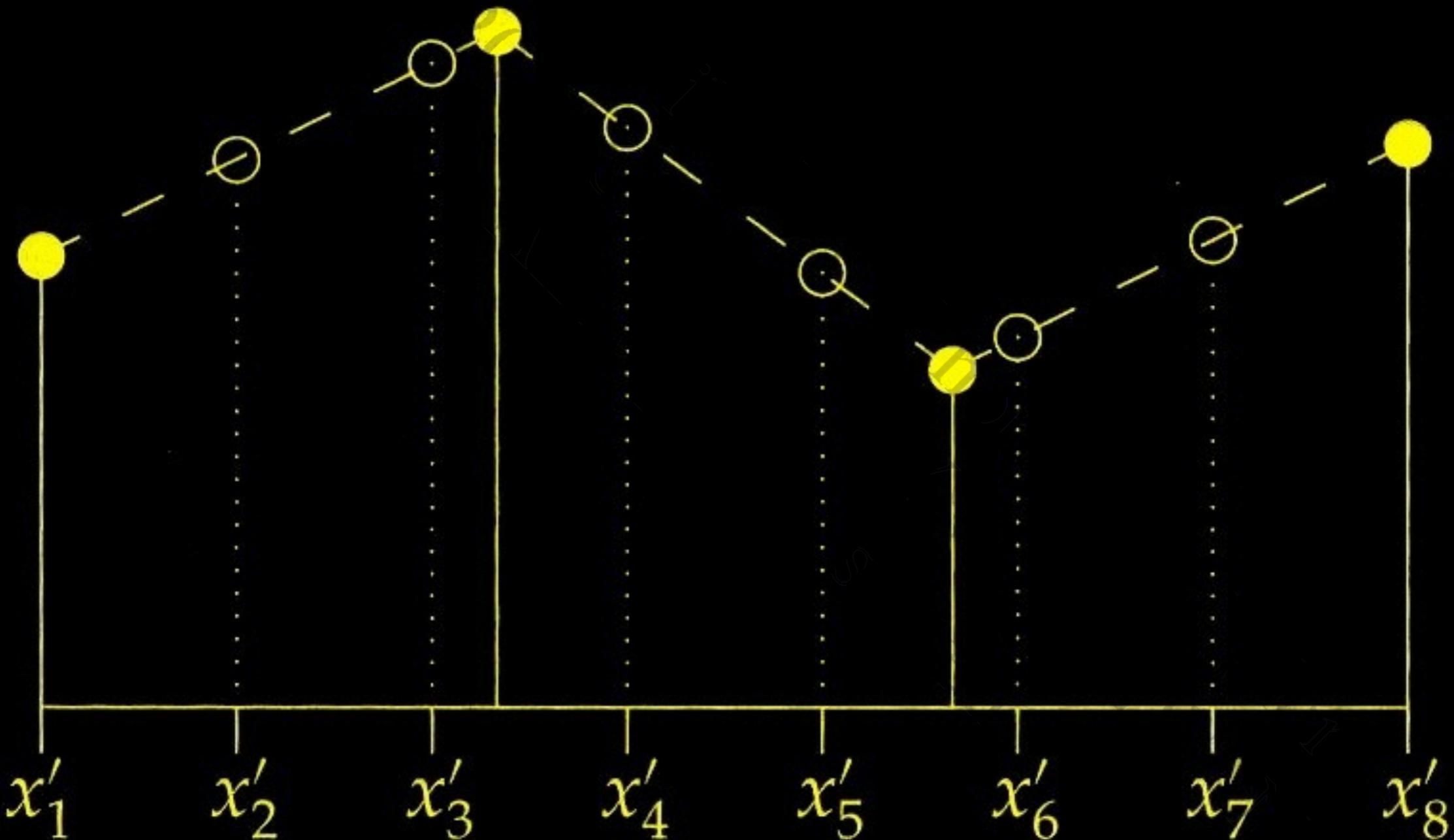




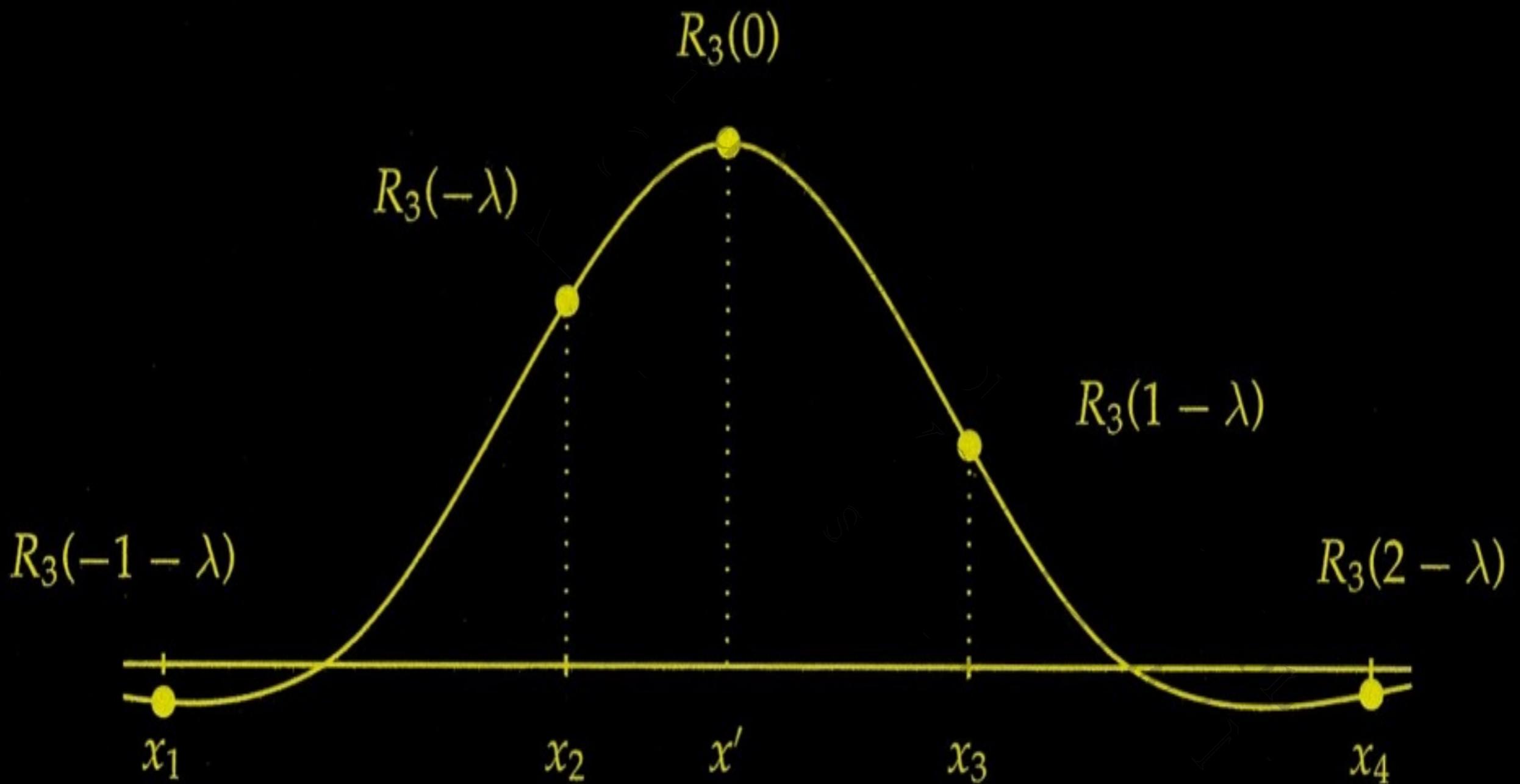
Nearest-Neighbor Interpolation



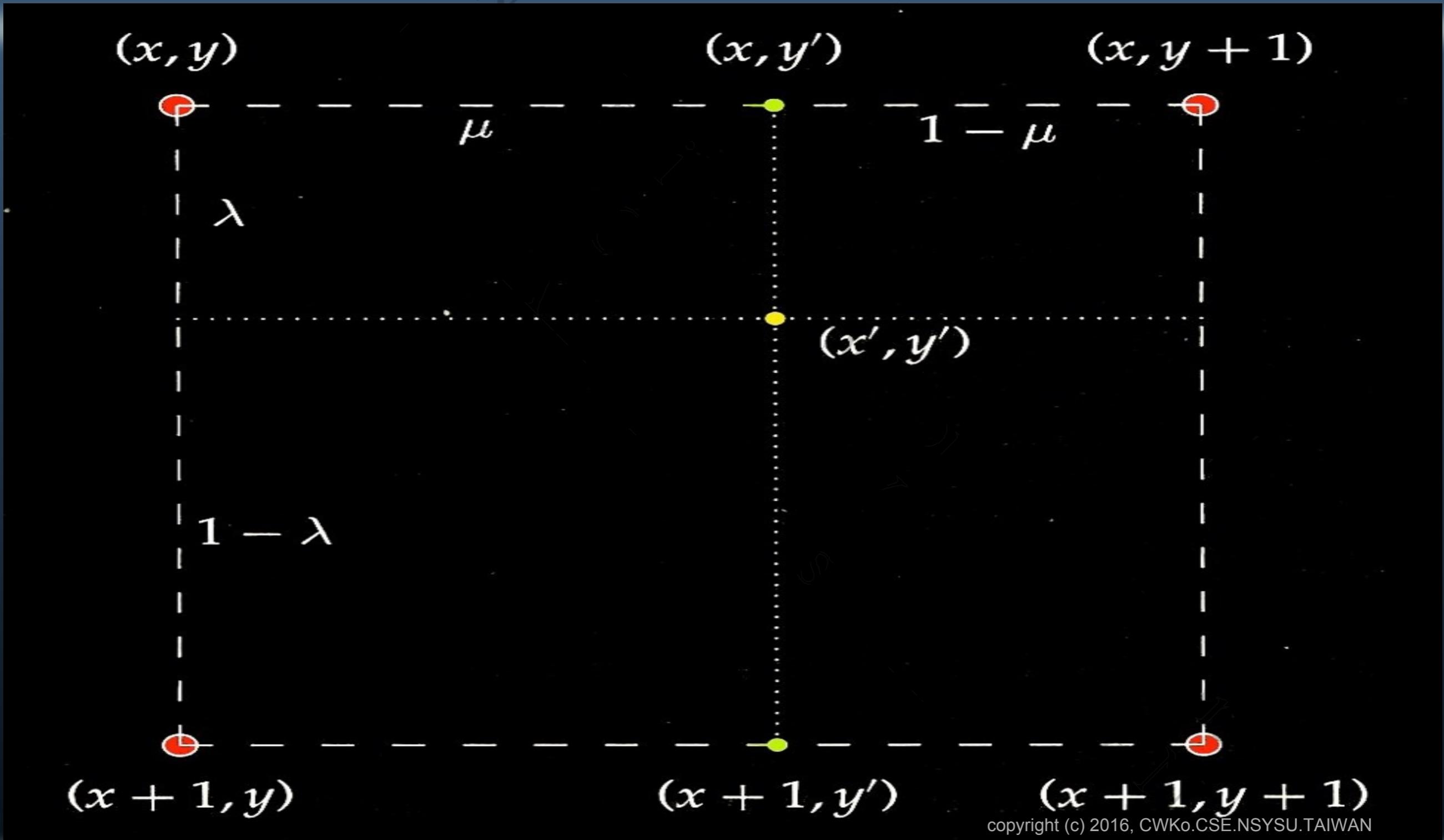
Linear interpolation



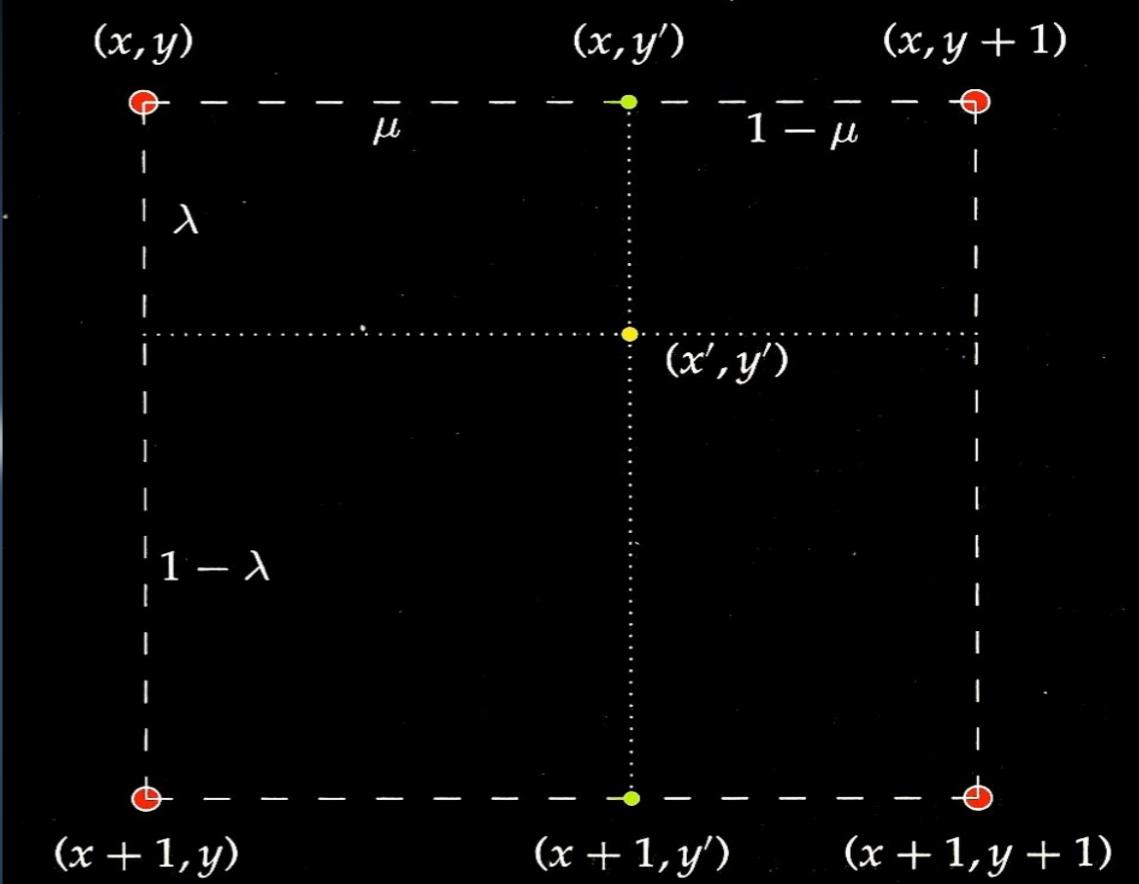
Cubic interpolation



Bilinear interpolation

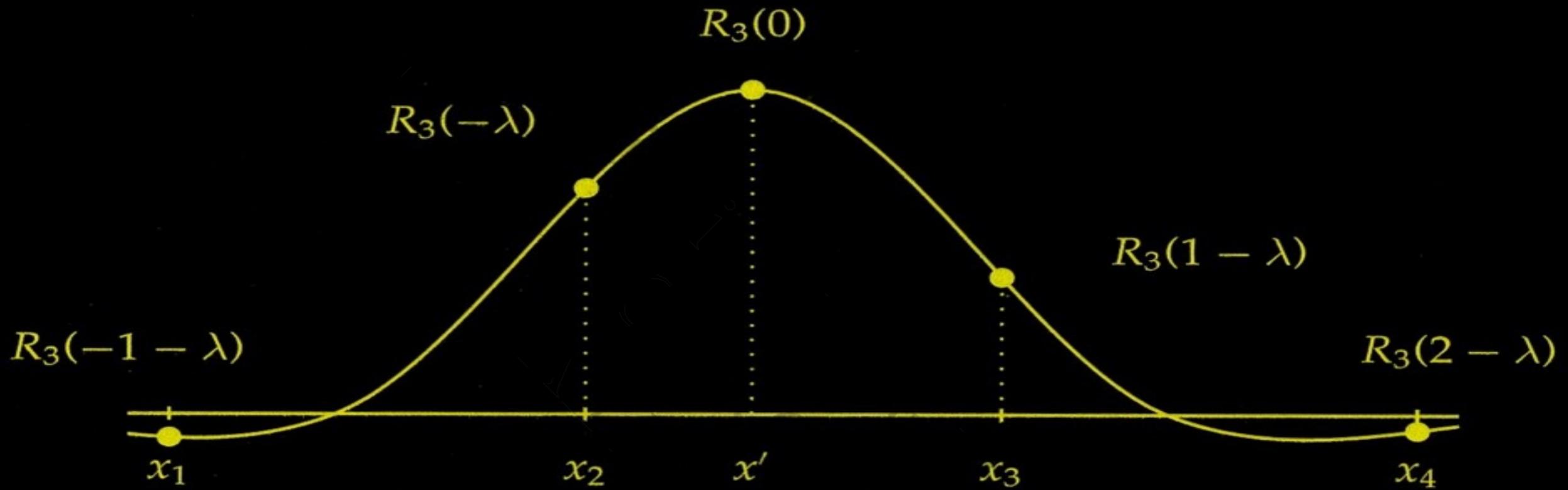


Bilinear interpolation



$$\begin{aligned}
 f(x', y') = & \lambda(\mu f(x+1, y+1) + (1-\mu)f(x+1, y)) \\
 & + (1-\lambda)(\mu f(x, y+1) + (1-\mu)f(x, y)) \\
 f(x', y') = & \lambda\mu f(x+1, y+1) + \lambda(1-\mu)f(x+1, y) + \\
 & (1-\lambda)\mu f(x, y+1) + (1-\lambda)(1-\mu)f(x, y)
 \end{aligned}$$

Cubic interpolation



$$f(x') = R_3(-1 - \lambda)f(x_1) + R_3(-\lambda)f(x_2) + R_3(1 - \lambda)f(x_3) + R_3(2 - \lambda)f(x_4)$$

$$R_3(u) = \begin{cases} 1.5|u|^3 - 2.5|u|^2 + 1 & \text{if } |u| \leq 1 \\ -0.5|u|^3 + 2.5|u|^2 - 4|u| + 2 & \text{if } 1 < |u| \leq 2 \end{cases}$$

Mathematical Tools

Pixel-by-pixel Operation

Neighbors of a Pixel

- 4-neighbors of p
 - $N_4(p)$
- 4-diagonal neighbors of p
 - $N_D(p)$
- 8-neighbors of p
 - $N_8(p)$

2	1	2
1	0	1
2	1	2

Distance Measures

- Distance function

$$D(p,q) \geq 0 \quad (D(p,q) = 0 \text{ iff } p = q)$$

$$D(p,q) = D(q,p)$$

$$D(p,z) \leq D(p,q) + D(q,z)$$

$p(x,y)$

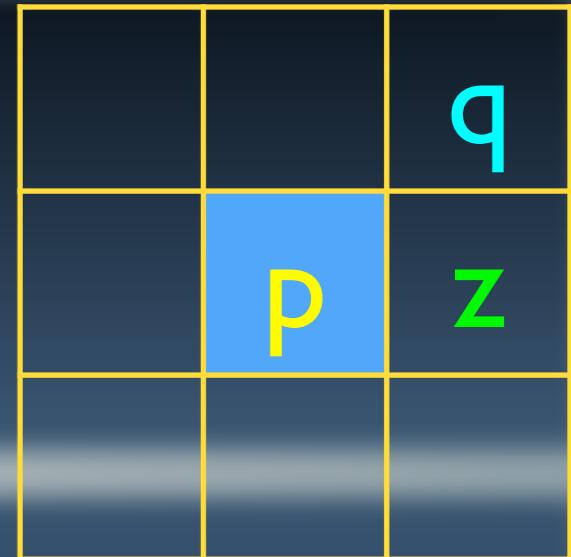
$q(s,t)$

$z(v,w)$

		q
	P	z
$p(x,y)$	$q(s,t)$	$z(v,w)$

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Distance measures



- Euclidean distance

$$D_e(p,q) = \left[(x-s)^2 + (y-t)^2 \right]^{1/2}$$

- D₄ distance (city-block distance)

$$D_4(p,q) = |x - s| + |y - t|$$

- D₈ distance (chessboard distance)

$$D_8(p,q) = \max(|x - s|, |y - t|)$$

Array vs. Matrix Operations

- **Array Product — A.*B**

$$\begin{pmatrix} \boxed{a_{11}} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \boxed{b_{11}} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} \boxed{a_{11}b_{11}} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{pmatrix}$$

- **Matrix Product — A*B**

$$\begin{pmatrix} \boxed{a_{11}} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \boxed{b_{11}} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} \boxed{a_{11}b_{11} + a_{12}b_{21}} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

Linear Operation

$$H[f(x,y)] = g(x,y)$$

- H is a linear operator if :

$$H[a_i f_i(x,y) + a_j f_j(x,y)]$$

$$= a_i H[f_i(x,y)] + a_j H[f_j(x,y)]$$

$$= a_i g_i(x,y) + a_j g_j(x,y)$$

Example - Sum

$$f_1 = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}$$

$$\sum \left\{ \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix} \right\} = \sum \left\{ \begin{bmatrix} 6 & 7 \\ 6 & 10 \end{bmatrix} \right\} = 29$$

$$\sum \left\{ \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} \right\} + \sum \left\{ \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix} \right\} = 7 + 22 = 29$$

Sum operator is linear!

Example - Max

$$f_1 = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}$$

$$\max \left\{ (1) \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + (-1) \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix} \right\} = \max \left\{ \begin{bmatrix} -6 & -3 \\ -2 & -4 \end{bmatrix} \right\} = -2$$

$$(1)\max \left\{ \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} \right\} + (-1)\max \left\{ \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix} \right\} = 3 - 7 = -4$$

Max operator is nonlinear!

Arithmetc Operations

$$s(x,y) = f(x,y) + g(x,y)$$

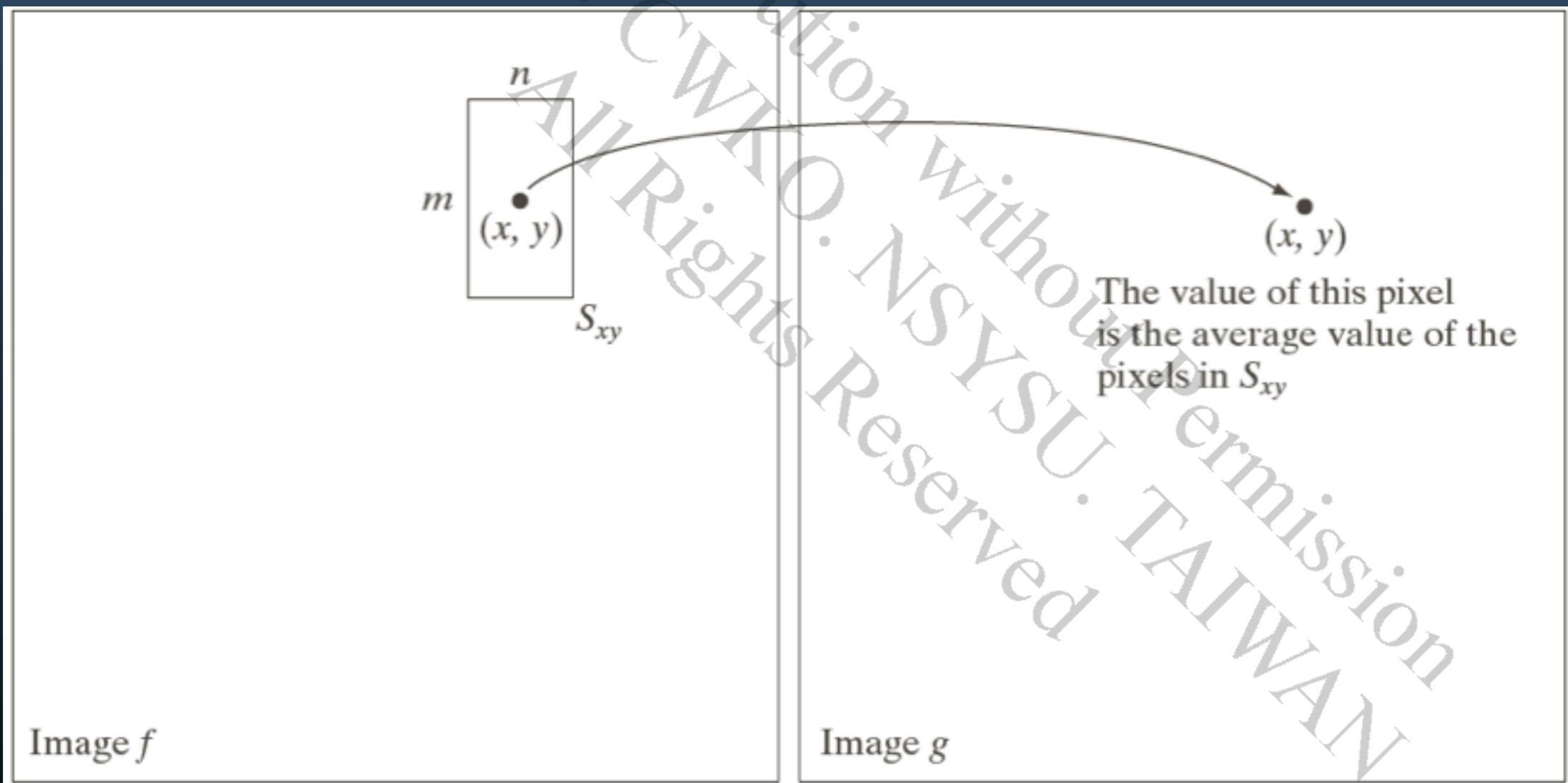
$$d(x,y) = f(x,y) - g(x,y)$$

$$p(x,y) = f(x,y) \times g(x,y)$$

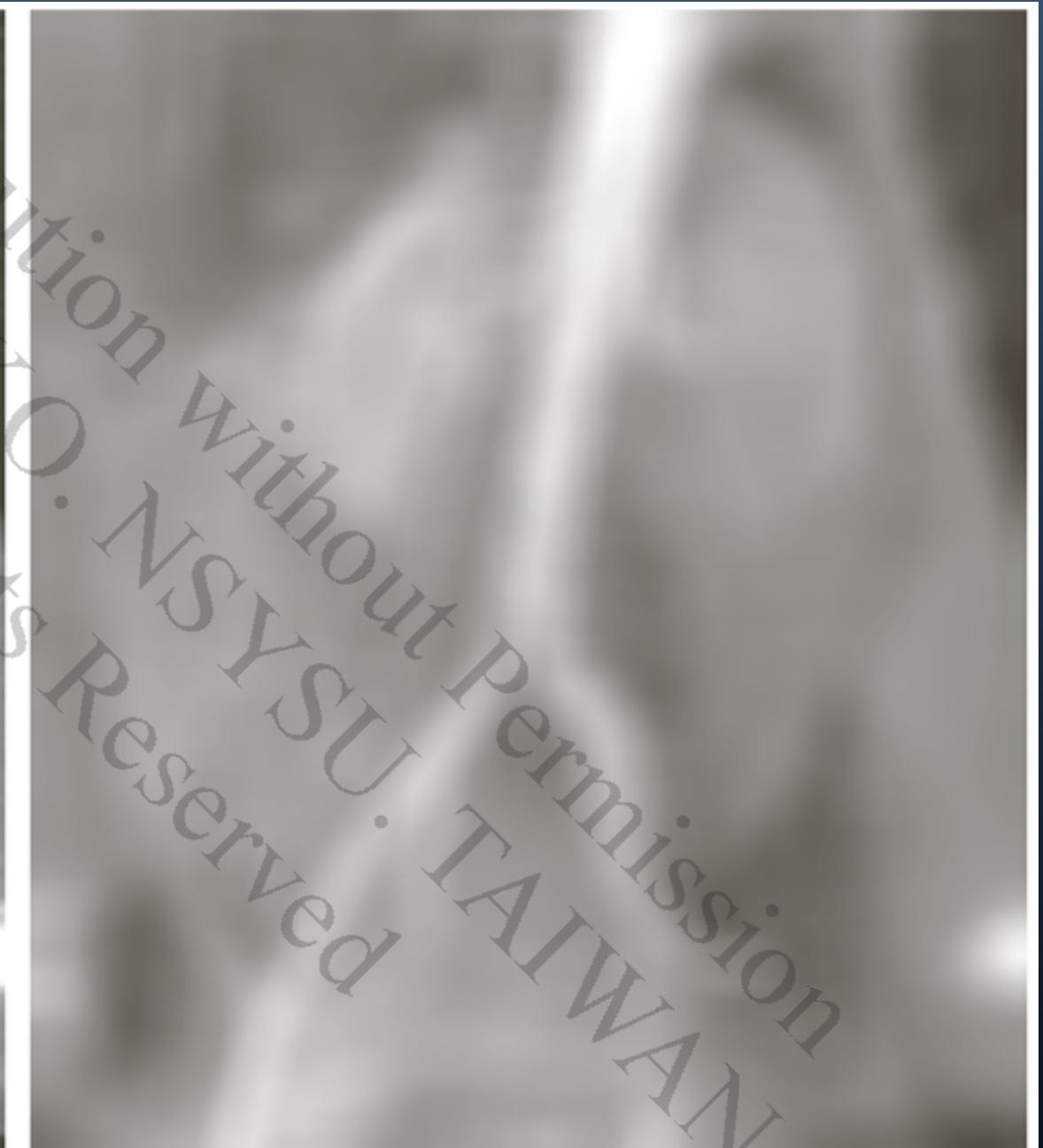
$$v(x,y) = f(x,y) \div g(x,y)$$

Local Averaging

- Neighborhood processing



Local Averaging



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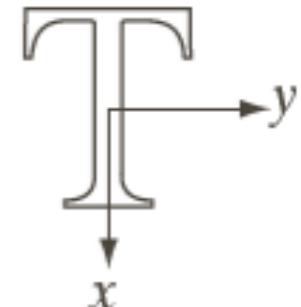
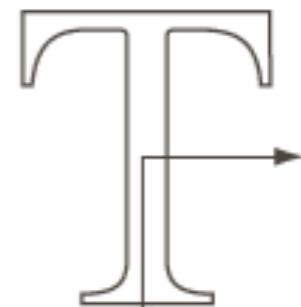
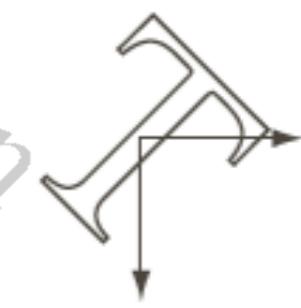
Spatial Coordinate Transformations

$$(x, y) = T \{(v, w)\}$$

$$\begin{bmatrix} x & y & 1 \end{bmatrix} = \begin{bmatrix} v & w & 1 \end{bmatrix} T$$

$$T = \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

Spatial Coordinate Transformations

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v \cos \theta - w \sin \theta$ $y = v \cos \theta + w \sin \theta$	

Next Lecture

Intensity Transformations & Spatial Filtering